An Overview of Sparsity with Applications to Compression, Restoration, and Inverse Problems

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Signal/image $f(t)$ in the time/spatial domain

Decompose $f$ as a superposition of atoms

$$f(t) = \sum_i \alpha_i \psi_i(t)$$

$\psi_i$ = basis functions
$\alpha_i$ = expansion coefficients in $\psi$-domain

Classical example: Fourier series
$\psi_i$ = complex sinusoids
$\alpha_i$ = Fourier coefficients

Modern example: wavelets
$\psi_i$ = “little waves”
$\alpha_i$ = wavelet coefficients

More exotic example: curvelets (more later)
Taking images apart and putting them back together

- Frame operators $\Psi, \tilde{\Psi}$ map images to sequences and back
- Two sequences of functions: $\{\psi_i(t)\}, \{\tilde{\psi}(t)\}$
- Analysis (inner products):

$$\alpha = \tilde{\Psi}[f], \quad \alpha_i = \langle \tilde{\psi}_i, f \rangle$$

- Synthesis (superposition):

$$f = \Psi^*[\alpha], \quad f = \sum_i \alpha_i \psi_i(t)$$

- If $\{\psi_i(t)\}$ is an orthobasis, then

$$\|\alpha\|_{\ell_2}^2 = \|f\|_{L_2}^2 \quad \text{(Parseval)}$$

$$\sum_i \alpha_i \beta_i = \int f(t)g(t) \, dt \quad \text{(where } \beta = \tilde{\Psi}[g])$$

$$\psi_i(t) = \tilde{\psi}_i(t)$$

i.e. all sizes and angles are preserved

- Overcomplete tight frames have similar properties
ACHA Mission: construct “good representations” for “signals/images” of interest

Examples of “signals/images” of interest

- Classical: signal/image is “bandlimited” or “low-pass”
- Modern: smooth between isolated singularities (e.g. 1D piecewise poly)
- Cutting-edge: 2D image is smooth between smooth edge contours

Properties of “good representations”

- sparsifies signals/images of interest
- can be computed using fast algorithms ($O(N)$ or $O(N \log N)$ — think of the FFT)
Example: The discrete cosine transform (DCT)

- For an image $f(t, s)$ on $[0, 1]^2$, we have

$$\psi_{\ell,m}(t, s) = 2\lambda_\ell\lambda_m \cdot \cos(\pi\ell t) \cos(\pi ms), \quad \lambda_\ell = \begin{cases} 1/\sqrt{2} & \ell = 0 \\ 1 & \text{otherwise} \end{cases}$$

- Closely related to 2D Fourier series/DFT, the DCT is real, and implicitly does symmetric extension
- Can be taken on the whole image, or blockwise (JPEG)
Image approximation using DCT

Take 1% of “low pass” coefficients, set the rest to zero

original

approximated

rel. error = 0.075
Image approximation using DCT

Take 1% of “low pass” coefficients, set the rest to zero

original

approximated

rel. error = 0.075
Image approximation using DCT

Take 1% of largest coefficients, set the rest to zero (adaptive)

original

approximated

rel. error = 0.057
Image approximation using DCT

Take 1% of *largest* coefficients, set the rest to zero (adaptive)

original

approximated

rel. error = 0.057
Wavelets

\[ f(t) = \sum_{j,k} \alpha_{j,k} \psi_{j,k}(t) \]

- **Multiscale**: indexed by scale \( j \) and location \( k \)
- **Local**: \( \psi_{j,k} \) analyzes/represents an interval of size \( \sim 2^{-j} \)
- **Vanishing moments**: in regions where \( f \) is polynomial, \( \alpha_{j,k} = 0 \)
2D wavelet transform

- Sparse: few large coeffs, many small coeffs
- Important wavelets cluster along edges
Multiscale approximations

Scale = 4, 16384:1

rel. error = 0.29
Multiscale approximations

Scale = 5, 4096:1

rel. error = 0.22
Multiscale approximations

Scale = 6, 1024:1

rel. error = 0.16
Multiscale approximations

Scale $= 7, \, 256:1$

rel. error $= 0.12$
Multiscale approximations

Scale = 8, 64:1

rel. error = 0.07
Multiscale approximations

Scale = 9, 16:1

rel. error = 0.04
Multiscale approximations

Scale = 10, 4:1

rel. error = 0.02
Image approximation using wavelets

Take 1% of largest coefficients, set the rest to zero (adaptive)

original

approximated

rel. error = 0.031
DCT/wavelets comparison

Take 1% of *largest* coefficients, set the rest to zero (adaptive)

DCT

rel. error = 0.057

wavelets

rel. error = 0.031
Linear approximation

- Linear \( S \)-term approximation: keep \( S \) coefficients in fixed locations

\[
f_S(t) = \sum_{m=1}^{S} \alpha_m \psi_m(t)
\]

- projection onto fixed subspace
- lowpass filtering, principle components, etc.

- Fast coefficient decay \( \Rightarrow \) good approximation

\[
|\alpha_m| \lesssim m^{-r} \quad \Rightarrow \quad \| f - f_S \|_2^2 \lesssim S^{-2r+1}
\]

- Take \( f(t) \) periodic, \( d \)-times continuously differentiable,
  \( \Psi = \) Fourier series:

\[
\| f - f_S \|_2^2 \lesssim S^{-2d}
\]

*The smoother the function, the better the approximation*

Something similar is true for wavelets ...
Nonlinear approximation

- Nonlinear $S$-term approximation: keep $S$ largest coefficients

$$f_S(t) = \sum_{\gamma \in \Gamma_S} \alpha_{\gamma} \psi_{\gamma}(t), \quad \Gamma_S = \text{locations of } S \text{ largest } |\alpha_m|$$

- Fast decay of sorted coefficients $\Rightarrow$ good approximation

$$|\alpha|_{(m)} \lesssim m^{-r} \quad \Rightarrow \quad \|f - f_S\|_2^2 \lesssim S^{-2r+1}$$

$|\alpha|_{(m)} = m\text{th largest coefficient}$
Linear v. nonlinear approximation

- For $f(t)$ *uniformly smooth* with $d$ “derivatives”

  $S$-term approx. error

<table>
<thead>
<tr>
<th>Method</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fourier, linear</td>
<td>$S^{-2d+1}$</td>
</tr>
<tr>
<td>Fourier, nonlinear</td>
<td>$S^{-2d+1}$</td>
</tr>
<tr>
<td>Wavelets, linear</td>
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<tr>
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<td>$S^{-2d+1}$</td>
</tr>
</tbody>
</table>

- For $f(t)$ *piecewise smooth*

  $S$-term approx. error

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<td>$S^{-2d+1}$</td>
</tr>
</tbody>
</table>

Nonlinear wavelet approximations *adapt* to singularities.
Wavelet adaptation

Piecewise polynomial $f(t)$

Wavelet coeffs $\alpha_{j,k}$
Approximating Pau with $S$-terms...

Approximation curves

$S \rightarrow$

wavelet nonlinear, DCT nonlinear, DCT linear
Approximation comparison

original

DCT linear (.075)

DCT nonlinear (.057)

wavelet nonlinear (.031)
The ACHA paradigm

Sparse representations yield algorithms for (among other things)

1. compression,
2. estimation in the presence of noise (“denoising”),
3. inverse problems (e.g. tomography),
4. acquisition (compressed sensing)

that are

- fast,
- relatively simple,
- and produce (nearly) optimal results
Compression
Transform-domain image coding

- Sparse representation $\Rightarrow$ good compression
  Why? Because there are fewer things to code

- Basic, “stylized” image coder
  1. Transform image into sparse basis
  2. Quantize
     Most of the xform coefficients are $\approx 0$
     $\Rightarrow$ they require very few bits to encode
  3. Decoder: simply apply inverse transform to quantized coeffs
Classical example: JPEG (1980s)
- standard implemented on every digital camera
- representation = Local Fourier
discrete cosine transform on each $8 \times 8$ block

Modern example: JPEG2000 (1990s)
- representation = wavelets
  Wavelets are much sparser for images with edges
- about a factor of 2 better than JPEG in practice
  half the space for the same quality image
JPEG vs. JPEG2000

Visual comparison at 0.25 bits per pixel (≈ 100:1 compression)

(Images from David Taubman, University of New South Wales)
Sparse transform coding is asymptotically optimal

Donoho, Cohen, Daubechies, DeVore, Vetterli, and others...

- The statement “transform coding in a sparse basis is a smart thing to do” can be made mathematically precise
- Class of images $\mathcal{C}$
- Representation $\{\psi_i\}$ (orthobasis) such that
  \[ |\alpha|_n \lesssim n^{-r} \]
  for all $f \in \mathcal{C}$ ($|\alpha|_n$ is the $n$th largest transform coefficient)
- Simple transform coding: transform, quantize (throwing most coeffs away)
- $\ell(\epsilon) =$ length of code (# bits) that guarantees the error $< \epsilon$ for all $f \in \mathcal{C}$ (worst case)
- To within log factors
  \[ \ell(\epsilon) \asymp \epsilon^{-1/\gamma}, \quad \gamma = r - 1/2 \]
- For piecewise smooth signals and $\{\psi_i\} =$ wavelets, no coder can do fundamentally better
Statistical Estimation
Statistical estimation setup

\[ y(t) = f(t) + \sigma z(t) \]

- \( y \): data
- \( f \): object we wish to recover
- \( z \): stochastic error; assume \( z_t \) i.i.d. \( N(0, 1) \)
- \( \sigma \): noise level

The quality of an estimate \( \tilde{f} \) is given by its risk (expected mean-square-error)

\[ \text{MSE}(\tilde{f}, f) = E\|\tilde{f} - f\|_2^2 \]
Transform domain model

\[ y = f + \sigma z \]

Orthobasis \( \{ \psi_i \} \):

\[
\langle y, \psi_i \rangle = \langle f, \psi_i \rangle + \langle z, \psi_i \rangle \\
\tilde{y}_i = \alpha_i + z_i
\]

- \( z_i \) Gaussian white noise sequence
- \( \sigma \) noise level
- \( \alpha_i = \langle f, \psi_i \rangle \) coordinates of \( f \)
Classical estimation example

- Classical model: signal of interest $f$ is **lowpass**

  **time domain**

  **Fourier domain**

- Observable frequencies: $0 \leq \omega \leq \Omega$
- $\hat{f}(\omega)$ is nonzero only for $\omega \leq B$
Classical estimation example

- Add noise: \( y = f + z \)

\[ \text{time domain} \quad y(t) \quad \begin{array}{c} \text{y(t)} \\ \end{array} \quad \text{Fourier domain} \quad \hat{y}(\omega) \quad \begin{array}{c} \hat{y}(\omega) \\ \end{array} \]

Observation error: \( E\|y - f\|^2_2 = E\|\hat{y} - \hat{f}\|^2_2 = \Omega \cdot \sigma^2 \)

- Noise is spread out over entire spectrum
Classical estimation example

- Optimal recovery algorithm: lowpass filter ("kill" all $\hat{y}(\omega)$ for $\omega > B$)

![Graph showing original and recovered errors](image)

\[ E\|\hat{y} - \hat{f}\|_2^2 = \Omega \cdot \sigma^2 \]
\[ E\|\tilde{f} - \hat{f}\|_2^2 = B \cdot \sigma^2 \]

- Only the lowpass noise affects the estimate, a savings of \((B/\Omega)^2\)
Modern estimation example

- Model: signal is **piecewise smooth**
- Signal is sparse in the **wavelet domain**

\[ f(t) \rightarrow j, k \]

Again, the \( \alpha_{j,k} \) are concentrated on a small set

This set is **signal dependent** (and unknown a priori)
\[ \Rightarrow \text{we don’t know where to “filter”} \]
Ideal estimation

\[ y_i = \alpha_i + \sigma z_i, \quad y \sim \text{Normal}(\alpha, \sigma^2 I) \]

- Suppose an “oracle” tells us which coefficients are above the noise level
- Form the oracle estimate

\[ \tilde{\alpha}_i^{\text{orc}} = \begin{cases} 
    y_i, & \text{if } |\alpha_i| > \sigma \\
    0, & \text{if } |\alpha_i| \leq \sigma 
\end{cases} \]

*keep the observed coefficients above the noise level, ignore the rest*

- Oracle Risk:

\[ E \| \tilde{\alpha}_i^{\text{orc}} - \alpha \|_2^2 = \sum_i \min(\alpha_i^2, \sigma^2) \]
Ideal estimation

- **Transform coefficients** $\alpha$
  - Total length $N = 64$
  - $\#$ nonzero components $= 10$
  - $\#$ components above the noise level $S = 6$

\[
E\|y - \alpha\|_2^2 = N \cdot \sigma^2
\]
\[
E\|\tilde{\alpha}_{orc} - f\|_2^2 = S \cdot \sigma^2
\]
Interpretation

\[
MSE(\tilde{\alpha}^{orc}, \alpha) = \sum_i \min(\alpha_i^2, \sigma^2)
\]

- Rearrange the coefficients in decreasing order
  \[|\alpha|_1^2 \geq |\alpha|_2^2 \geq \ldots \geq |\alpha|_N^2\]
- \(S\): number of those \(\alpha_i\)'s s.t. \(\alpha_i^2 \geq \sigma^2\)

\[
MSE(\tilde{\alpha}^{orc}, \alpha) = \sum_{i>S} |\alpha|_i^2 + S \cdot \sigma^2
\]
\[
= \|\alpha - \alpha_S\|_2^2 + S \cdot \sigma^2
\]
\[
= \text{Approx Error} + \text{Number of terms} \times \text{noise level}
\]
\[
= \text{Bias}^2 + \text{Variance}
\]

- The sparser the signal,
  - the better the approximation error (lower bias), and
  - the fewer \# terms above the noise level (lower variance)

"Can we estimate as well without the oracle?"
Denoising by thresholding

- **Hard-thresholding** (“keep or kill”)
  \[ \tilde{\alpha}_i = \begin{cases} y_i, & |y_i| \geq \lambda \\ 0, & |y_i| < \lambda \end{cases} \]

- **Soft-thresholding** (“shrinkage”)
  \[ \tilde{\alpha}_i = \begin{cases} y_i - \lambda, & y_i \geq \lambda \\ 0, & -\lambda < y_i < \lambda \\ y_i + \lambda, & y_i \leq -\lambda \end{cases} \]

- Take \( \lambda \) a little bigger than \( \sigma \)
- **Working assumption**: whatever is above \( \lambda \) is signal, whatever is below is noise
Denoising by thresholding

- Thresholding performs (almost) as well as the oracle estimator!
- Donoho and Johnstone: Form estimate $\tilde{\alpha}^t$ using threshold $\lambda = \sigma \sqrt{2 \log N}$,
  \[
  \text{MSE}(\tilde{\alpha}^t, \alpha) := E\|\tilde{\alpha}^t - \alpha\|_2^2 \leq (2 \log N + 1) \cdot (\sigma^2 + \sum_i \min(\alpha_i^2, \sigma^2))
  \]
- Thresholding comes within a log factor of the oracle performance
- The $(2 \log N + 1)$ factor is the price we pay for not knowing the locations of the important coeffs
- Thresholding is simple and effective
- Sparsity $\Rightarrow$ good estimation
Recall: Modern estimation example

- Signal is **piecewise smooth**, and sparse in the **wavelet domain**
  - time domain $f(t)$
  - wavelet domain $\alpha_{j,k}$

\[ t \rightarrow j, k \rightarrow \]

- noisy signal $y(t)$
- noisy wavelet coeffs
Thresholding wavelets

- Denoise (estimate) by soft thresholding

noisy signal

$t \rightarrow$

recovered signal

$t \rightarrow$

noisy wavelet coeffs

$j, k \rightarrow$

recovered wavelet coeffs

$j, k \rightarrow$
Denoising the Phantom

noisy

lowpass filtered

wavelet thresholding, $\lambda = 3\sigma$

Error = 25.0

Error = 42.6

Error = 11.0
Inverse Problems
Linear inverse problems

\[ y(u) = (Kf)(u) + z(u), \quad u = \text{measurement variable/index} \]

- \( f(t) \) object of interest
- \( K \) linear operator, indirect measurements

\[ (Kf)(u) = \int k(u, t) f(t) \, dt \]

Examples:
- Convolution ("blurring")
- Radon (Tomography)
- Abel

- \( z = \text{noise} \)
- **Ill-posed:** \( f = K^{-1}y \) not well defined
Solving inverse problems using the SVD

\[ K = U \Lambda V^T \]

\[ U = \text{col}(u_1, \ldots, u_n), \quad \Lambda = \text{diag}(\lambda_1, \ldots, \lambda_n), \quad V = \text{col}(v_1, \ldots, v_n) \]

- \( U \) = orthobasis for the measurement space,
- \( V \) = orthobasis for the signal space

Rewrite action of operator in terms of these bases:

\[ y(\nu) = (Kf)(\nu) \iff \langle u_\nu, y \rangle = \lambda_\nu \langle v_\nu, f \rangle \]

- The inverse operator is also natural:

\[ \langle v_\nu, f \rangle = \lambda_\nu^{-1} \langle u_\nu, y \rangle, \quad f = V \begin{pmatrix} \lambda_1^{-1} \langle u_1, y \rangle \\ \lambda_2^{-1} \langle u_2, y \rangle \\ \vdots \end{pmatrix} \]

- But in general, \( \lambda_\nu \to 0 \), making this unstable
Deconvolution

- Measure \( y = Kf + \sigma z \), where \( K \) is a convolution operator.

- Singular basis: \( U = V = \text{Fourier transform} \)

\[
\{ \langle e_{\nu}, f \rangle \} \times \{ \lambda_{\nu} \} + \text{noise} = \{ \langle h_{\nu}, y \rangle \}
\]
Regularization

- Reproducing formula
  \[ f = \sum_{\nu} \lambda_{\nu}^{-1} \langle u_{\nu}, Kf \rangle v_{\nu} \]

- Noisy observations
  \[ y = Kf + \sigma z \quad \iff \quad \langle u_{\nu}, y \rangle = \langle u_{\nu}, Kf \rangle + \sigma \hat{z}_{\nu} \]

- Multiply by damping factors \( w_{\nu} \) to reconstruct from observations \( y \)
  \[ \tilde{f} = \sum_{\nu} w_{\nu} \lambda_{\nu}^{-1} \langle u_{\nu}, y \rangle v_{\nu} \]

  want \( w_{\nu} \approx 0 \) when \( \lambda_{\nu}^{-1} \) is large (to keep the noise from exploding)

- If spectral density \( \theta_{\nu}^2 = |\langle f, v_{\nu} \rangle|^2 \) is known, the MSE optimal weights are
  \[ w_{\nu} = \frac{\theta_{\nu}^2}{\theta_{\nu}^2 + \sigma^2} = \frac{\text{signal power}}{\text{signal power} + \text{noise power}} \]

  This is the Wiener Filter
Ideal damping

- In the SVD domain:
  \[ y_\nu = \theta_\nu + \sigma_\nu z_\nu \]

  \[ y_\nu = \langle u_\nu, y \rangle, \quad \theta_\nu = \langle f, v_\nu \rangle, \quad \sigma_\nu = \sigma / \lambda_\nu, \quad z_\nu \sim \text{iid Gaussian} \]

- Again, suppose an oracle tells us which of the \( \theta_\nu \) are above the noise level

- Oracle “keep or kill” window (minimizes MSE)

  \[ w_\nu = \begin{cases} 1 & |\theta_\nu| > \sigma_\nu \\ 0 & \text{otherwise} \end{cases} \]

  Take \( \tilde{\theta}_\nu = w_\nu y_\nu \) (thresholding)

- Since \( V \) is an isometry, oracle risk is

  \[ E \| f - \tilde{f} \|_2^2 = E \| \theta - \tilde{\theta} \|_2^2 = \sum_\nu \min(\theta_\nu^2, \sigma_\nu^2) \]
Interpretation

\[ \text{MSE} = \sum_{\nu} \min(\theta_{\nu}^2, \sigma_{\nu}^2) \]

\[ = \sum_{\nu:|\theta_{\nu}|\lambda_{\nu} \leq \sigma} \theta_{\nu}^2 + \sum_{\nu:|\theta_{\nu}|\lambda_{\nu} > \sigma} \frac{\sigma^2}{\lambda^2} \]

\[ = \text{Bias}^2 + \text{Variance} \]

- Again, concentration of the \( \theta_{\nu} := \langle f, v_{\nu} \rangle \) on a small set is critical for good performance
- But the \( v_{\nu} \) are determined only by the operator \( K \) !
Typical Situation

- Convolutions, Radon inversion (tomography)
- \((v_\nu) \sim \text{sinusoids}\)
- \(f\) has discontinuities (earth, brain, ...)
- SVD basis is *not* a good representation for our signal

Fortunately, we can find a representation that is simultaneously
- almost an SVD
- A sparse decomposition for object we are interested in
Example: Power-law convolution operators

- **$K$** = convolution operator with Fourier spectrum $\sim \omega^{-1}$

- Wavelets have dyadic (in scale $j$) support in Fourier domain

- Spectrum of $K$ is almost constant (within a factor of 2) over each subband
The Wavelet-Vaguelette decomposition (WVD)

Donoho, 1995

- Wavelet basis $\{\psi_{j,k}\}$ sparsifies piecewise smooth signals
- Vaguelette dual basis $u_{j,k}$ satisfies

$$\langle f, \psi_{j,k} \rangle = 2^{j/2} \langle u_{j,k}, Kf \rangle$$

(basis for the measurement space)

- For power-law $K$, vaguelettes $\approx$ orthogonal, and $\approx$ wavelets

Wavelet - Vaguelette decomposition is almost an SVD for Fourier power-law operators
Deconvolution using the WVD

- Observe \( y = Kf + \sigma z \),
  \( K = 1/|\omega| \) power-law operator, \( z = \text{iid Gaussian noise} \)

- Expand \( y \) in vaguelette basis

\[
v_{j,k} = \langle u_{j,k}, y \rangle
\]

almost orthonormal, so noise in new basis is \( \approx \) independent

- Soft-threshold

\[
\tilde{v}_{j,k} = \begin{cases} v_{j,k} - \gamma \text{sign}(v_{j,k}) & |v_{j,k}| > \gamma_j \\ 0 & |v_{j,k}| \leq \gamma_j \end{cases}
\]

for \( \gamma_j \sim 2^{j/2} \sigma \)

- Weighted reconstruction in the wavelet basis

\[
\tilde{f}(t) = \sum_{j,k} 2^{j/2} \tilde{v}_{j,k} \psi_{j,k}(t)
\]
Deconvolution example

Measure \( y = K f + \sigma z \), where \( K = 1/|\omega| \)

- \( f(t) \) signal
- convolution kernel
- observed \( y(t) \) + noise =

WVD recovery

Wiener Filter recovery
Later this week: Acquisition (Compressed Sensing)
Wavelets and geometry

- Wavelet basis functions are isotropic
  ⇒ they cannot adapt to *geometrical structure*
- Curvelets offer a more refined scaling concept...
Curvelets

Candes and Donoho, 1999–2004

New multiscale pyramid:

- Multiscale
- Multi-orientations
- *Parabolic scaling (anisotropy)*

\[ \text{width} \approx \text{length}^2 \]
Curvelets in the spatial domain

Curvelets parameterized by scale, location, and orientation
Example curvelets
Curvelet tiling in the frequency domain

wavelet
curvelet
Piecewise-smooth approximation

- Image fragment: $C^2$ smooth regions separated by $C^2$ contours
- Fourier approximation
  \[ \|f - f_S\|_2^2 \lesssim S^{-1/2} \]
- Wavelet approximation
  \[ \|f - f_S\|_2^2 \lesssim S^{-1} \]
- Curvelet approximation
  \[ \|f - f_S\|_2^2 \lesssim S^{-2} \log^3 S \]
  (within log factor of optimal)
Application: Curvelet denoising I

Zoom-in on piece of phantom

noisy

wavelet thresholding

curvelet thresholding
Application: Curvelet denoising II

Zoom-in on piece of Lena

wavelet thresholding
curvelet thresholding
Summary

- Having a sparse representation plays a fundamental role in how well we can:
  - compress
  - denoise
  - restore
  signals and images

- The above were accomplished with relatively simple algorithms (in practice, we use similar ideas + a bag a tricks)

- Better representation (e.g. curvelets) $\rightarrow$ better results

- Wednesday and Friday:
  We will see how sparsity can play a role in *data acquisition*