Seismic imaging simulation

(a) Estimated (16× faster, SNR=9.6 dB).

(b) Estimation error (Figure 2b minus 5(a)).

(c) Cross-correlation estimate.

- Result produced with 16× “compression” in the computations
- Can even take this example down to 32×
Randomly modulated integration

- Uses a standard “slow” ADC preceded by a “fast” binary mixing
- Mixing circuit much easier to build than a “fast” ADC
- In each sampling interval, the signal is summarized with a random sum
- Sample rate $\sim$ total active bandwidth
Random modulated integration in time and frequency

Input signal $x(t)$

\[
\times
\]

Pseudorandom sequence $p_c(t)$

\[
\equiv
\]

Modulated input

Input signal $X(\omega)$

\[
\ast
\]

Pseudorandom sequence spectrum $P_c(\omega)$

\[
\equiv
\]

Modulated input and integrator (low-pass filter)
This architecture is being implemented as part of DARPA's Analog-to-Information program.
Analog-to-digital converter state-of-the-art

The pipelined structure and unknown structure have the best overall performance, so that they are best suited for applications with high performance requirements, such as wireless transceiver applications and military use [3]. SAR ADCs have widely ranging sampling rates, though they are not the fastest devices. Still, these devices are popular for their range of speeds and resolutions as well as low cost and power dissipation. It can be seen that there is a borderline of sampling rate at around 30 Ms/s separating the sigma-delta and flash ADCs. Sigma-delta ADCs have the highest resolution with relatively low sampling rates from kilosamples per second to megasamples per second, while flash ADCs have the highest sampling rates up to Gsps due to their parallel structure but with a resolution limited to no more than 8 b due to nonlinearity.

We are also interested in the envelope of the sample distributions in this plot since such an envelope indicates the performance limitations. It is reasonable to extract the envelope information based on the ADCs with the highest performance to postulate the design challenges and technology trends.

In Figure 1, if Walden’s claim that $P$ is relatively constant is true, according to (1), the envelope line should show that a 3 dBs/s increment in $f_s$ corresponds to a 1-b reduction in resolution. However, Figure 1 shows that the real tradeoff is 1 b/2.3 dBs/s. Compared to the 1 b/3 dBs/s slope hypothesis, there is an improvement in $P$ at low sampling rates and degradation at high sampling rates. This trend indicates that the ADC performance boundary is varying with sampling rate, as illustrated by Figure 2 where ENOB is plotted versus the sampling rate.

As stated previously, noise and distortion cause most of the performance degradation in practical ADCs. The internal sample-hold-quantize signal operations are nonlinear, and those effects are represented as equivalent noise effects so that they can be unified into noise-based equations to simplify the performance analysis. Therefore, besides thermal noise, we have two additional noise sources, quantization noise [2] and aperture-jitter noise [1].

**THERMAL NOISE**

Thermal noise by itself [1] has a 1 b/6 dBs/s relationship to sampling frequency assuming Nyquist sampling [2]. However, it is usually overwhelmed by the capacitance noise since the S/H stage, as the input stage of an ADC, shows strong capacitive characteristics. Therefore, the capacitance noise (modeled as $kT/C$ noise [4], where $k$ is Boltzmann’s constant, $T$ is the temperature, and $C$ is the capacitance) is usually assumed as the input noise floor.

**QUANTIZATION NOISE**

The signal distortion in quantization is modeled as quantization noise with a signal-to-quantization-noise ratio (SQNR) definition of

\[
\text{SQNR} = 10 \log_{10} \left( \frac{S}{N} \right)
\]

where $S$ is the signal power and $N$ is the noise power.

**The bad news starts at 1 GHz**

(Le et al ’05)
Analog-to-digital converter state-of-the-art

From 2008...

(Lots of RF signals have components in the 10s of gigahertz...)
Spectrally sparse RF signals
Randomly modulated integration receiver

- Random demodulator being built at part of DARPA A2I program (Emami, Hoyos, Massoud)
- Multiple (8) channels, operating with different mixing sequences
- Effective BW/chan = 2.5 GHz
  Sample rate/chan = 50 MHz
- Applications: radar pulse detection, communications surveillance, geolocation
Goal: acquire an *ensemble* of $M$ signals

- Bandlimited to $W/2$
- “Correlated” $\rightarrow$ $M$ signals are $\approx$ linear combinations of $R$ signals
Sampling correlated signals

Goal: acquire an ensemble of $M$ signals

Bandlimited to $W/2$

“Correlated” $\rightarrow$ $M$ signals are $\approx$ linear combinations of $R$ signals
Sensor arrays
**Low-rank matrix recovery**

- Given *P* linear samples of a matrix,
  \[ y = A(X_0), \quad y \in \mathbb{R}^P, \quad X_0 \in \mathbb{R}^{M \times W} \]

  we solve
  \[ \min_X \|X\|_* \text{ subject to } A(X) = y \]

  where \( \|X\|_* \) is the **nuclear norm**: the sum of the singular values of \( X \).
Low-rank matrix recovery

- Given \( P \) linear samples of a matrix,
  \[
  y = \mathcal{A}(X_0), \quad y \in \mathbb{R}^P, \quad X_0 \in \mathbb{R}^{M \times W}
  \]
  we solve
  \[
  \min_X \|X\|_* \quad \text{subject to} \quad \mathcal{A}(X) = y
  \]
  where \( \|X\|_* \) is the nuclear norm: the sum of the singular values of \( X \).

- If \( X_0 \) is rank-\( R \) and \( \mathcal{A} \) obeys the mRIP:
  \[
  (1 - \delta)\|X\|_F^2 \leq \|\mathcal{A}(X)\|_2^2 \leq (1 + \delta)\|X\|_F^2 \quad \forall \ \text{rank-2}R \ X,
  \]
  then we can stably recover \( X_0 \) from \( y \). \hspace{1cm} (Recht et. al ’07)
Low-rank matrix recovery

- Given $P$ linear samples of a matrix,
  \[ y = A(X_0), \quad y \in \mathbb{R}^P, \quad X_0 \in \mathbb{R}^{M \times W} \]
  we solve
  \[
  \min_{X} \|X\|_* \quad \text{subject to} \quad A(X) = y
  \]
  where $\|X\|_*$ is the nuclear norm: the sum of the singular values of $X$.

- If $X_0$ is rank-$R$ and $A$ obeys the mRIP:
  \[
  (1 - \delta)\|X\|_F^2 \leq \|A(X)\|_2^2 \leq (1 + \delta)\|X\|_F^2 \quad \forall \text{rank-}2R \ X,
  \]
  then we can stably recover $X_0$ from $y$. \hspace{1cm} \text{(Recht et. al '07)}

- An 'generic' (iid random) sampler $A$ (stably) recovers $X_0$ from $y$ when
  \[
  \#\text{samples} \gtrsim R \cdot \max(M, W) \\
  \gtrsim RW \quad \text{(in our case)}
  \]
If the signals are spread out uniformly in time, then the ADC and modulators can run at rate

\[ \varphi \gtrsim RW \log^{3/2}(MW) \]

Requires signals to be (mildly) spread out in time
Summary

- Main message of CS:
  - We can recover an $S$-sparse signal in $\mathbb{R}^N$ from
    $\sim S \cdot \log N$ measurements
  - We can recover a rank-$R$ matrix in $\mathbb{R}^{M \times W}$ from
    $\sim R \cdot \max(M, W)$ measurements

- Random matrices (iid entries)
  - easy to analyze, optimal bounds
  - universal
  - hard to implement and compute with

- Structured random matrices (random sampling, random convolution)
  - structured, and so computationally efficient
  - physical
  - much harder to analyze, bound with extra log-factors
Compressive sensing tells us ...

**Sensing...**

- ... we can sample *smarter* not faster
- ... we can replace front-end acquisition complexity with back-end computing
- ... injecting randomness allows us to *super-resolve* high-frequency signals (or high-resolution images) from low-frequency (low-resolution) measurements
- ... the acquisition process can be *independent* of the types of signals we are interested in
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**Mathematics...**
- ... there are unique *sparse* solutions to underdetermined systems of equations
- ... random projections keep sparse signals separated
- ... a seemingly impossible optimization program (subset selection) can be solved using a tractable amount of computation