Finite Automata in Number Theory

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Abstract. Among infinite sequences with values in a finite set, or among set of natural numbers, some are very regular such as periodic sequences or arithmetic progressions, while others, such as random sequences or random sets, cannot be described in a simple way.

Finite automata form a class of very basic Turing machines. In arithmetic, they can be used to define in a natural way sequences and sets which are said to be 'automatic'. One of the main interest of these automatic structures is that they enjoy some strong regularity without being trivial at all. They can be thus though of as lying somewhere between order and chaos, though in many aspects they appear as essentially regular. This special feature of automatic structures leads to various applications of Automata Theory to Number Theory.

This course has two main goals. On the one hand, we will present the classical part of the theory of automatic sequences. To this aim, the book of Allouche and Shallit [1] provides an accessible and complete reference. On the other hand, we will study some more advanced number theoretical questions (transcendence, Diophantine approximation, Diophantine equations...) which turn out to be intimately related to the theory of finite automata. Several recent results [2–7] shed new light on these interactions between arithmetic and automata. Some of them will be discussed in detail.

Some references

[1] J.-P. Allouche and J. Shallit, *Automatic Sequences*, Cambridge, 2002.

[2] K. Kedlaya, Finite automata and algebraic extensions of function fields, *J. Théor. Nombres Bordeaux* 18 (2006), 379–420.

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[5] Ch. Mauduit, J. Rivat, Sur un problème de Gelfond : la somme des chiffres des nombres premiers, *Annals of Math.* 171 (2010), 1591–1646.

[6] B. Adamczewski and J. Bell, On vanishing coefficients of algebraic power series over fields of positive characteristic, *Invent. Math.* 187 (2012), 343–393.

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