

Validated Numerics – Computer Lab 1

Problem 0: Install octave and the package octave-interval. The latter is done by issuing the command `pkg install -forge interval` from the octave prompt. Starting a new session, issue `pkg load interval` from the octave prompt. Additional packages (e.g. octave-symbolic) can be installed and loaded in a similar fashion.

Problem 1: Write a program that computes the factorial $n! \stackrel{\text{def}}{=} 1 \cdot 2 \cdot \dots \cdot n$ of a given integer n (integers are declared as `int8`, `int16`, `int32`, `int64`). Use your program to compute the 20 first factorials. Do you notice anything strange? If so, try to explain what is happening. What happens when you perform the computations with floating point numbers?

Problem 2: Write a program that computes the smallest positive machine representable number η_M , and the machine epsilon ε_M . What are the values you get? Try to print them in hexadecimal form too (use `num2hex`).

Problem 3: Define the function $f(x, y) = 9x^4 - y^4 + 2y^2$. Your objective is to compute $f(40545, 70226)$. Write a program that evaluates the function using each of the formats `int64`, `single`, and `double`. What is the correct answer?

Problem 4: Write a program that switches the rounding mode on your computer. [Hint: for octave, use `__setround__(x)` from the interval package. For matlab use `system_dependent('setround', x)`, where x takes one of the values `{-inf, 0, 0.5, inf}`]. Make your program compute $1/10$ in various rounding modes. Make sure you output enough decimals, or even better – print the results in hexadecimal.

Problem 5: Try out some features of the octave interval package. Experiment with different versions of the interval constructor like `infsup('0.1')`, `infsup(0.1)`, `infsup(1)/10` etc. Redo exercise 3 in interval arithmetic.

Problem 6: Using the interval routines from the interval package, compute $f(1 + 2^{-k}[-1, 1])$ for $k = 1, \dots, 100$, where

$$f(x) = e^{\sin e^{\cos x + 2x^5}}.$$

Do you observe a shrinking and/or scaling of the enclosures?