Problem 1: Write an interval bisection routine. Your program should take as parameters a function handle, the search domain \( x \), and a tolerance \( TOL \). What are the results you get for the function \( f(x) = \sin x - \cos x \) with \( x = [-10, +10] \) and \( TOL = 0.0001 \)?

Problem 2: Write an interval Newton routine. Your program should take as parameters a function handle and a search domain \( x \). Make sure the search stops when two successive enclosures are identical. What are the results you get for the function \( f(x) = \sin(\cos(x - 3)) \) with the search domains \( x_1 = [1, 2] \), \( x_2 = [1.5, 2.5] \), and \( x_3 = [2, 3] \)?

Problem 3: Combine the ideas from the two previous problems and write a hybrid bisection/interval Newton routine. It should bisect the search domain into subintervals \( x_k \) until either \( \text{diam}(x_k) \leq TOL \), or \( 0 \notin f'(x_k) \). In the latter case the subinterval should undergo an interval Newton search. Your program should prompt the user for the search domain \( x \), and a tolerance \( TOL \). What are the results you get for the function \( f(x) = \sin(\cos(x - 3)) \) with the search domain \( x = [-10, 10] \), and \( TOL = 0.001 \)?

Problem 4: Write an interval optimizer. You may choose to use any or all of the midpoint, monotonicity, or convexity checks. Your program should take as parameters a function handle, a search domain \( x \), and a tolerance \( TOL \). What are the results you get for the function

\[
 f(x) = x^2 - \frac{1}{2} e^{-a(x - \frac{1}{2})^2}
\]

with \( a = 10000 \), \( x = [-10, +10] \) and \( TOL = 10^{-10} \)?

Problem 5: Write a uniform-step, interval integrator. Your program should take as parameters a function handle, the domain of integration \( x \), and a the number of subdomains \( N \). What results do you get for the function \( f(x) = e^x \sin x \) over the domain \( x = [-2, +2] \). and using \( N = 10, 100, 1000 \)?

Problem 6: (Only if you have time!) Make the integrator above adaptive.