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Reconstruction of the "new physics" Lagrangian which gives neutrino masses

Abstract :

The reconstruction of different see-saw models from possible low energy measurements is explored. We work in the limit where one neutrino mass eigenvalue is zero. The see-saw models that we consider contain two or three singlet neutrinos. We find that the measurement of flavour violating leptonic decays can probably exclude some models but is not enough for a reconstruction of the whole high energy Lagrangian. To do this we need further information about the neutrino Yukawa coupling matrix or bounds on the heavy singlet neutrino masses from Leptogenesis.

Key words :
Supersymmetry, See-saw, Flavour violating leptonic decays, Effective Lagrangian

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Contents

1 Introduction

The observed atmospheric and solar neutrino oscillations show us clearly that neutrinos have masses. However the neutrinos in the Standard Model (SM) do not have a mass. The SM does not know any right handed neutrinos with which the left handed neutrinos could couple, via Yukawa couplings, to the Higgs boson and obtain a mass. The see-saw mechanism is a elegant method to introduce a small neutrino mass. In a simplified way, the see-saw mechanism introduces right handed neutrinos, which are singlets under the SM gauge group $SU(3) \times SU(2) \times U(1)$, i.e. they do not have any gauge interactions. These right handed neutrinos can have a Majorana mass with a scale M. Furthermore via the coupling to the Higgs one gets a Dirac mass matrix at a scale m_D. Then one can show that the effective mass scale of the left handed neutrino mass matrix is given by $m_\nu = \frac{m_D^2}{M}$ [4]. If we assume that M is much larger than the electroweak scale we can explain the small neutrino masses. A more detailed presentation of the see-saw mechanism is given in section 2.1.

Although the SM of high energy physics describes very well presently known phenomena, except for neutrino masses, it is clear that the SM is not the end of high energy theories. Between the today explored energies around the electroweak scale $M_W \approx 100\,\text{GeV}$ and the Planck scale $M_P = 2.4 \times 10^{18}\,\text{GeV}$, where quantum gravitational effects become important, it seems to be almost clear that some new physics exists. Linked to this fact is the famous "hierarchy problem". By this term we mean the problem that the Higgs mass is very sensitive to almost any imaginable extension of the SM. Because of experiments at the electroweak scale we know that the Higgs mass must be of the order of $m_H = 100\,\text{GeV}$. However the Higgs mass gets quadratically divergent contributions from each Dirac fermion or scalar particle via loop diagrams. Even if the particle does not directly couple to the Higgs, via higher loop effects, it can yield quadratically divergent corrections to the Higgs mass. These corrections yield a Higgs mass which is much bigger than the actual mass that one can extract form electroweak scale observations. One possibility to get rid of these divergent terms is to introduce a symmetry between bosons and fermions. If for every boson a fermionic partner and vice versa exist, than the different signs in the loop diagrams lead to a complete cancellation of all quadratically divergent corrections to the Higgs mass. Such a symmetry between fermions and bosons is called a "supersymmetry" [14].

In our see-saw model we introduce exactly these new particles that cause the divergent radiative corrections to the Higgs mass. This is the first reason why we will consider a see-saw model in a supersymmetric framework. The second reason is, that we are going to use the supersymmetric neutrino (=sneutrino) mass matrix as one of our weak scale observables to reproduce the high energy parametrisation of the see-saw mechanism, i.e. the Majorana mass matrix and the Yukawa coupling matrix (also look at section 2.1 for more details) [7]

One interesting feature of the see-saw mechanism is, that it might be possible to explain the baryon asymmetry of the universe. As the introduced heavy right handed neutrinos are Majorana particles, they violate lepton number conservation. Furthermore one introduces new CP violating phases in the new Yukawa matrices. These two facts together can yield a lepton asymmetry, which can be transferred into a baryon asymmetry by SM processes, which can be large enough to explain the baryon asymmetry measured today. It is remarkable that a model introduced to explain small observed neutrino masses could probably also explain the baryon asymmetry in the universe, a problem that seems not at all be related to neutrino masses. [9]

The aim of this internship is to find a way to distinguish between models with a different number of heavy right-handed neutrinos based on experimental data from "low-energy" measurements. In this context "low-energy" measurements are obtained from experiments at energies that can be reached today or in the near future but much smaller than the mass of the heavy right-handed neutrinos. We will focus on models with two and three neutrino singlets.

2 General framework of this work

2.1 See-saw, a brief introduction

Neutrinos play a very special role in particle physics. They are the only up to know known particles that are not well described by the SM. Also in the $SU(2) \times U(1)$ model of the electroweak interaction they have a very special role. In this standard model of electroweak interaction the right handed field $\Psi_R = \frac{1}{2}(1+\gamma_5)\Psi$ and the left-handed field $\Psi_L = \frac{1}{2}(1-\gamma_5)\Psi$ have different $SU(2)$ properties. Left-handed fields are organised in doublets i.e. $I_W(\Psi_L) = \frac{1}{2}$, where I_W is the weak isospin. Right-handed fields are singlets under $SU(2)$, i.e. $I_W(\Psi_R) = 0$. We organize our left-handed leptons in a isospin doublet $L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$, where the charged lepton (i.e. the e_L, μ_L, τ_L) have $I_W^3 = -\frac{1}{2}$ and the neutrinos $(\nu_{eL}, \nu_{\mu L}, \nu_{\tau L})$ have $I_W^3 = \frac{1}{2}$. The electric charge of the particles is given by:

$$Q = I_W^3 + \frac{Y_W}{2} \tag{1}$$

With Y_W being the weak hypercharge of the $U(1)$ symmetry and I_W^3 being the third component of the weak isospin. From this formula we can reconstruct the hypercharge of our particles. We find

$$\begin{aligned} e_L \text{ and } \nu_L \text{ have } Y_W &= -1 \\ e_R \text{ has } Y_W &= -2 \\ \nu_R \text{ has } Y_W &= 0 \end{aligned} \tag{2}$$

From this we can see that right-handed neutrinos cannot have gauge interactions because they are singlets under the $SU(2) \times U(1)$ gauge symmetry. That is why we cannot tell (up to now) if right-handed neutrinos exist at all. However the ν_R can couple to the ν_L via a third particle as long as the coupling is invariant under $SU(2) \times U(1)$. We look e.g. at a coupling

$$\mathscr{L} \sim (\bar{\nu}_L \, \bar{e}_L) \, H \, \nu_R \tag{3}$$

In order to be invariant under our gauge group we see immediately that H must be a $SU(2)$ doublet and must have a weak hypercharge $Y_W = -1$. This particle is the Higgs boson, that couples in the SM to the up quark. It is the charge conjugate of the down type Higgs, which has hypercharge $Y_W = 1$ and couples to the electron. When the electroweak symmetry breaks down and H gets a vacuum expectation value this Yukawa coupling leads to an effective Dirac mass term for the neutrinos:

$$\mathscr{L}^{Dirac} = -\bar{\nu}_{Li}(m_D)_{ij}\nu_{Rj} + c.c. \tag{4}$$

Here i and j are a family indices. Before we will describe how we obtain this effective Dirac mass term, we will have a look at general Dirac and Majorana mass terms. [16, 18]

2.1.1 Dirac vs. Majorana mass

In general two types of mass terms exist: Dirac mass terms and Majorana mass terms. As neutrinos have no electric charge, they can have the two types of mass terms. The other charged particles in the SM can only have Dirac masses because Majorana masses would violate charge conservation. But this means that neutrinos with a Majorana mass would violate lepton number conservation. This fact could explain lepton number violating processes as the double neutrino-less beta decay, which has not been found until today but which is strongly searched for. In order to understand the origin of Majorana and Dirac masses, we will give a short introduction to Weyl spinors, which turn out to useful in describing mass terms. The action of a Lorentz transformation on a Dirac 4-spinor $\Psi(x)$ is given by:

$$\Psi(x) \to \Psi'(x) = S(\Lambda)\Psi(x) \tag{5}$$

The operator $S(\Lambda)$ reads

$$S(\Lambda) = exp\left[\frac{i}{2}\lambda_{\mu\nu}\Sigma^{\mu\nu}\right] \tag{6}$$

Here the $\lambda_{\mu\nu} = -\lambda_{\nu\mu}$ are the six parameters of a general Lorentz transformations (three boosts and three rotations). The $\Sigma^{\mu\nu}$ are the generators of the Lorentz group, which are

$$\Sigma^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu] \tag{7}$$

and satisfy the commutation relations

$$[\Sigma_{\mu\nu}, \Sigma_{\rho\sigma}] = i\left(g_{\nu\rho}\Sigma_{\mu\sigma} - g_{\mu\rho}\Sigma_{\nu\sigma} + g_{\mu\sigma}\Sigma_{\nu\rho} - g_{\nu\sigma}\Sigma_{\mu\rho}\right) \tag{8}$$

In general the Dirac matrices satisfy the following relation

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \tag{9}$$

Where our metric is

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{10}$$

It is useful to consider a particular representation for the Dirac matrices. We will use the chiral representation:

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \quad \gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{11}$$

Of course the σ^i are the famous 2×2 Pauli matrices. We set $\sigma^\mu = (\sigma^0, \sigma^i) = (1, \sigma^i)$ and analogue $\bar{\sigma}^\mu = (1, -\sigma^i)$. So we can write

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \tag{12}$$

Now we can calculate the generators of the Lorentz group from equation (8)

$$\Sigma^{\mu\nu} = i\begin{pmatrix} \sigma^{\mu\nu} & 0 \\ 0 & \bar{\sigma}^{\mu\nu} \end{pmatrix} = \frac{i}{4}\begin{pmatrix} \sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu & 0 \\ 0 & \bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu \end{pmatrix} \tag{13}$$

Now we can see why we used the chiral representation of the Dirac matrices. In this representation the transformation martix $S(\Lambda)$ takes a block diagonal form

$$S(\Lambda) = \begin{pmatrix} e^{-1/2\lambda_{\mu\nu}\sigma^{\mu\nu}} & 0 \\ 0 & e^{-1/2\lambda_{\mu\nu}\bar{\sigma}^{\mu\nu}} \end{pmatrix} = \begin{pmatrix} s & 0 \\ 0 & s^{-1\dagger} \end{pmatrix} \tag{14}$$

From this it is clear to see that the upper and the lower components of a Dirac spinor Ψ transform in a different way under Lorentz transformation. In [18] it is shown that the operator s satisfies the following conditions:

$$\sigma^2 s^{-1T}\sigma^2 = s \quad \text{and} \quad s^{-1\dagger} = \sigma^2 s^* \sigma^2 \tag{15}$$

So a given Dirac spinor, which can be written as two 2-spinors

$$\Psi = \begin{pmatrix} \chi \\ \bar{\eta} \end{pmatrix} \tag{16}$$

transforms under a Lorentz transformation like

$$\chi \to s\chi \quad \text{and} \quad \bar{\eta} \to s^{-1\dagger}\bar{\eta} \tag{17}$$

Up to now the bar over the η has no meaning. Later on we will see that it stands for the complex conjugate. Knowing equation (17) we can look at the transformation behaviour of the following term (which will turn out to be quite useful in some lines)

$$(i\sigma^2\chi) \to i\sigma^2\chi' = i\sigma^2 s\chi = s^{-1T}(i\sigma^2\chi) \tag{18}$$

So we have

$$(i\sigma^2\chi)^T = (i\sigma^2\chi)^T s^{-1} \tag{19}$$

Looking at the last line, we can construct a Lorentz invariant term easily by multiplying the last term with χ:

$$(i\sigma^2\chi)^T\chi \text{ is Lorentz invariant} \tag{20}$$

If we write the spinor

$$\chi = \chi_\alpha = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \tag{21}$$

explicitly and we set

$$\begin{pmatrix} \chi^1 \\ \chi^2 \end{pmatrix} = \chi^\alpha = (i\sigma^2\chi) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} \chi_2 \\ -\chi_1 \end{pmatrix} \tag{22}$$

we can re-express equation (20) as

$$\chi^\alpha\chi_\alpha \text{ is Lorentz invariant} \tag{23}$$

Another way to express χ^α is

$$\chi^\alpha = \epsilon^{\alpha\beta}\chi_\beta \tag{24}$$

where $\epsilon^{\alpha\beta}$ is the completely antisymmetric two dimensional tensor with $\epsilon^{12} = -\epsilon^{21} = 1$ and $\epsilon^{11} = \epsilon^{22} = 0$. Analogous we will write

$$\chi_\alpha = \epsilon_{\alpha\beta}\chi^\beta \tag{25}$$

Here $\epsilon_{\alpha\beta}$ is the inverse tensor to $\epsilon^{\alpha\beta}$. So we have $\epsilon_{12} = -1$. This notation may remind the reader of notation used in relativistic calculations. But one should be careful when one starts to manipulate these types of expressions. There is one difference to the relativistic notation:

$$\begin{aligned} \chi^\alpha\chi_\alpha =& \chi^1\chi_1 + \chi^2\chi_2 = \chi_2\chi_1 - \chi_1\chi_2 \\ =& -\chi_2\chi^2 - \chi_1\chi^1 = -\chi_\alpha\chi^\alpha \end{aligned} \tag{26}$$

In the notation of relativistic equations the minus sing does not appear. So one has to be very careful when doing calculations in this new notation. If we look at two different 2-spinors, which as they form Dirac particles obey the Grassmann algebra, we find

$$\chi\eta \equiv \chi^\alpha\eta_\alpha = -\chi_\alpha\eta^\alpha = \eta^\alpha\chi_\alpha \equiv \eta\chi \tag{27}$$

After having treated the upper component χ of a Dirac spinor sufficiently, we will now look at the under component $\bar\eta$, which transform under Lorentz transformation in a different way than χ. Therefore we use "dotted" indices for the $\bar\eta$:

$$\bar\eta = \bar\eta^{\dot\alpha} = \begin{pmatrix} \bar\eta^{\dot 1} \\ \bar\eta^{\dot 2} \end{pmatrix} \tag{28}$$

Be careful here! One should realize and keep in mind that for the upper 2-spinor we defined in equation (21) $\chi = \chi_\alpha$ that the un-indexed spinor has an under index. For the other 2-spinor however we used an upper dotted index for the un-indexed 2-spinor. In a very similar way as we did before one can find the Lorentz invariant of the $\bar\eta$ spinor:

$$(-i\sigma^2\bar\eta)^T\bar\eta \text{ is Lorentz invariant} \tag{29}$$

We find a similar notation for the Lorentz invariant

$$\bar\eta^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\bar\eta_{\dot\beta}, \quad \bar\eta_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\bar\eta^{\dot\beta} = \begin{pmatrix} -\bar\eta^{\dot 2} \\ \bar\eta^{\dot 1} \end{pmatrix} \tag{30}$$

The ϵ tensor here has numerically the same values as for the upper 2-spinor, $\epsilon^{\dot{1}\dot{2}} = 1$ and $\epsilon_{\dot{1}\dot{2}} = -1$. So finally we have four types of Weyl spinors: $\chi^\alpha, \chi_\alpha, \bar{\chi}^{\dot{\alpha}}, \bar{\chi}_{\dot{\alpha}}$ which transform under a Lorentz transformation in the following ways

$$\begin{aligned}
\chi_\alpha &\to \chi'_\alpha = s_\alpha^{\ \beta}\chi_\beta \\
\chi^\alpha &\to \chi'^\alpha = (s^{-1T})^\alpha_{\ \beta}\chi^\beta \\
\bar{\chi}^{\dot{\alpha}} &\to \bar{\chi}'^{\dot{\alpha}} = (s^{-1\dagger})^{\dot{\alpha}}_{\ \dot{\beta}}\bar{\chi}^{\dot{\beta}} \\
\bar{\chi}_{\dot{\alpha}} &\to \bar{\chi}'_{\dot{\alpha}} = (s^*)_{\dot{\alpha}}^{\ \dot{\beta}}\bar{\chi}_{\dot{\beta}}
\end{aligned} \tag{31}$$

By comparing these equations we can now justify our choice of notation for $\bar{\eta}$. Dotted spinors are the complex conjugate spinors of undotted spinors

$$\bar{\chi}^{\dot{\alpha}} = (\chi^\alpha)^*, \qquad \bar{\chi}_{\dot{\alpha}} = (\chi_\alpha)^* \tag{32}$$

To simplify life, the left-handed and right-handed Dirac spinors and their adjoints will be written down here once:

$$\begin{aligned}
\Psi_L &= \frac{1}{2}(1-\gamma_5)\Psi_D = \begin{pmatrix}\chi_\alpha \\ 0\end{pmatrix} \\
\Psi_R &= \frac{1}{2}(1+\gamma_5)\Psi_D = \begin{pmatrix}0 \\ \bar{\eta}^{\dot{\alpha}}\end{pmatrix} \\
\bar{\Psi}_L &= \bar{\Psi}_D\frac{1}{2}(1+\gamma_5) = \begin{pmatrix}0 & \bar{\chi}_{\dot{\alpha}}\end{pmatrix} \\
\bar{\Psi}_R &= \bar{\Psi}_D\frac{1}{2}(1-\gamma_5) = \begin{pmatrix}\eta^\alpha & 0\end{pmatrix}
\end{aligned} \tag{33}$$

Once again, pause one moment to realize the difference between the "bars" that we use. A bar over a Dirac spinor expresses the adjoint spinor $\bar{\Psi} = \Psi^\dagger\gamma^0$. However the bar over a Weyl 2-spinor is nothing but the complex conjugate: $\bar{\chi}_{\dot{\alpha}} = (\chi_\alpha)^*$. Although all the notation introduced here does not seem to simplify life, we will later on be glad to have introduced it.

We can use our new notation the first time when we write down a Dirac mass term. A Dirac mass term is the kind of mass term that the reader might be used to from quantum field theory courses and the standard model. He always involves a left handed spinor and a right handed spinor. We will take the mass m_D to be real.

$$\mathscr{L}^{Dirac} = -m_D\bar{\Psi}_D\Psi_D = -m_D(\bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_L) = -m_D(\bar{\chi}_{\dot{\alpha}}\bar{\eta}^{\dot{\alpha}} + \eta^\alpha\chi_\alpha) \tag{34}$$

One clearly sees that the Dirac mass term is a Lorentz invariant term made out of two **different** Weyl spinors. Of course, one can also imagine a mass term which is made out of **one single** Weyl spinor and its complex conjugate. We will call such a mass term Majorana mass term.

$$\mathscr{L}^{Majorana} = -m_M^L(\bar{\chi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} + \chi^\alpha\chi_\alpha) = -m_M^L\bar{\Psi}_M\Psi_M \tag{35}$$

Here we called the Majorana mass m_M^L (don't worry about the superscript L, it does not have any meaning up to now). In the last part of the equation we imposed that we can write the Majorana mass term analogue to the Dirac mass term as a simple multiplication of one 4-spinor with its adjoint 4-spinor. The so called Majorana spinor must have the form

$$\Psi_M = \begin{pmatrix}\chi_\alpha \\ \bar{\chi}^{\dot{\alpha}}\end{pmatrix} \text{ and } \bar{\Psi}_M = \begin{pmatrix}\chi^\alpha & \bar{\chi}_{\dot{\alpha}}\end{pmatrix} \tag{36}$$

As the Majorana spinor contains only one Weyl spinor, he can only have one independent chirality projection. We will, e.g. take these projections to be left handed projections

$$\begin{aligned}
\Psi_L &= \frac{1}{2}(1-\gamma_5)\Psi_D = \begin{pmatrix}\chi_\alpha \\ 0\end{pmatrix} \\
\overline{\Psi_L} &= \bar{\Psi}_D\frac{1}{2}(1+\gamma_5) = \begin{pmatrix}0 & \bar{\chi}_{\dot{\alpha}}\end{pmatrix}
\end{aligned} \tag{37}$$

Now we ask ourselves how we can reconstruct the right handed spinors from this left handed spinor. For a Dirac spinor this would not be possible, because the left handed and the right handed spinors are completely independent from each other. However for a Majorana spinor we should be able to find an operator how links the left handed spinor to the right handed spinor. We will see in a few lines that we can do this with the charge conjugation operator. Therefore we define the charge conjugation matrix C which must obey the following relations.

$$\begin{aligned} C^\dagger = C^{-1} =& C^T = -C \\ C^\dagger \gamma^\mu C =& -(\gamma^\mu)^T \end{aligned} \tag{38}$$

In the chiral representation of the γ matrices that we chose earlier, one finds that C can take the following form:

$$C = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} \epsilon_{\alpha\beta} & 0 \\ 0 & \epsilon^{\dot{\alpha}\dot{\beta}} \end{pmatrix} \tag{39}$$

Equipped with this matrix we construct the right handed spinor from $\overline{\Psi_L}^T$:

$$(\Psi^c)_R \equiv (\Psi_L)^c = C(\overline{\Psi_L})^T = \begin{pmatrix} 0 \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} \tag{40}$$

Here we already used the definition of the charge conjugated fields:

$$\Psi^c = C\bar{\Psi}^T \tag{41}$$

So we find for the Majorana spinor the following condition, called Majorana condition

$$(\Psi_M)^c = C(\bar{\Psi}_M)^T = \Psi_M \tag{42}$$

This is the famous condition that says that a Majorana particle is its own antiparticle. So if neutrinos were Majorana particles, they would not preserve lepton number and therefore would not have any lepton number. The Majorana behaviour of neutrinos is searched for in double neutrino-less beta decay experiments. Up to now this decay has only been seen in the Heidelberg-Moscow collaboration but not yet been confirmed by another experiment [12].

One last formula, which we will use later on. The Majorana mass term can be rewritten by using equations (41) and (37)

$$\mathscr{L}^{Majorana} = -\frac{1}{2} m_M^L \left[\overline{\Psi_L}(\Psi^c)_R + \overline{(\Psi^c)}_R \Psi_L \right] = -\frac{1}{2} m_M^L \left[\overline{\Psi_L} C (\overline{\Psi_L})^T + \Psi_L^T C \Psi_L \right] \tag{43}$$

From this equation we can see why we used the subscript L on the Majorana mass. One can write another Majorana mass term by replacing all L by R. [16]

2.1.2 The See-Saw mechanism

The most general mass term for neutrinos can be written in the one family case

$$\mathscr{L}_{mass} = -m_D[\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L] - \frac{1}{2} m_M^L [\nu_L^T C \nu_L + \overline{\nu_L} C \overline{\nu_L}^T] - \frac{1}{2} m_M^R [\nu_R^T C \nu_R + \overline{\nu_R} C \overline{\nu_R}^T] \tag{44}$$

The Dirac mass term is allowed by our $SU(2) \times U(1)$ and can come from renormalisable Yukawa couplings of neutrinos with the standard model Higgs. The electroweak symmetry breakdown then gives a Dirac mass term

$$m_D = Y^\nu < H_u > \tag{45}$$

Where Y^ν is the Yukawa coupling constant and $< H_u > \approx 180\,\text{GeV}$. As already said before, this mass term is lepton number conserving. However in the case of three families, due to off diagonal terms in the Dirac mass matrix, it will not conserve lepton family number any more. In order to get neutrino masses of the order of eV, we need incredible small Yukawa couplings $Y_\nu \sim 10^{-10}$.

The right handed Majorana mass term is also allowed by our gauge symmetry because the right handed neutrinos are singlets under this symmetry. In contrast to the Dirac mass, the m_M^R term is completely independent of any mass scale. Of course this mass term breaks lepton number conservation. The other Majorana mass term m_M^L violates the $SU(2) \times U(1)$ symmetry and can therefore not be an explicit term. But it can arise after the electroweak symmetry breaking. In the see saw model we will however not need this left handed Majorana mass term and therefore set the following hierarchy

$$m_M^R \equiv M >> m_D >> m_M^L \approx 0 \tag{46}$$

Following [16] we can rewrite the neutrino mass terms in the following matrix form

$$\mathscr{L}_{mass} = -\frac{1}{2} \begin{pmatrix} \overline{\nu_L} & \overline{(\nu^c)_L} \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} (\nu^c)_R \\ \nu_R \end{pmatrix} + h.c. \tag{47}$$

In the limit of big Majorana mass terms $M >> m_D$ we find the two eigenvalues of this matrix $\mathcal{M}$ to be

$$m_1 \simeq \frac{m_D^2}{M}; \text{ and } m_2 \simeq M \tag{48}$$

We see that in this limit we obtain one neutrino which is very heavy, $m_2 = M >> m_D$ and one very light neutrino, $m_1 = \frac{m_D^2}{M} << m_D$. The mass matrix is diagonalized by a unitary transformation

$$Diag(m_1, m_2) = U^\dagger \mathcal{M} U \tag{49}$$

where the transformation matrix is given by

$$U = \begin{pmatrix} 1 & \frac{m_D}{M} \\ -\frac{m_D}{M} & 1 \end{pmatrix} \tag{50}$$

This matrix transforms from the mass basis, in which the mass matrix is diagonal, with mass eigenstates ν_1, ν_2, into the basis of our well known left-handed and the yet unknown right-handed neutrino.

$$\begin{aligned} \begin{pmatrix} (\nu^c)_R \\ \nu_R \end{pmatrix} &= U \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \end{pmatrix} \\ \begin{pmatrix} \bar{\nu}_L & \overline{(\nu^c)}_L \end{pmatrix} &= \begin{pmatrix} \overline{(\nu_1^c)}_L & \overline{\nu_2^c)}_L \end{pmatrix} U^\dagger \end{aligned} \tag{51}$$

In the basis of the mass eigenstates the mass part of the Lagrangian reads

$$\mathscr{L}_{mass} = -\frac{1}{2} m_1 \left[\overline{(\nu_1^c)}_L \nu_{1R} + \bar{\nu}_{1R} (\nu_1^c)_L \right] - \frac{1}{2} m_2 \left[\overline{(\nu_2^c)}_L \nu_{2R} + \bar{\nu}_{2R} (\nu_2^c)_L \right] \tag{52}$$

The right and left-handed neutrinos are approximately given by

$$\begin{aligned} \nu_R &= \nu_{2R} - \frac{m_D}{M} \nu_{1R} \\ \nu_L &= (\nu_1^c)_L + \frac{m_D}{M} (\nu_2^c)_L \end{aligned} \tag{53}$$

We see that the right handed neutrinos consists mainly of the heavy neutrino ν_2, whereas the left handed-neutrinos is mostly the light neutrino ν_1. If m_2 is sufficiently heavy, one would not have seen any evidence yet for the existence of ν_2. However , the see-saw mechanism allows a light neutrino ν_1 to exist, which is essentially the same as the massless left handed neutrino of the standard model.
This whole presentation of the see-saw mechanism was done for only one family, but it can trivially be extended to a multi family case. In order to produce neutrino masses of the order of eV, under the assumption that the Yukawa couplings are of the order of one, we need to introduce the right handed neutrinos at a mass scale $M \sim 10^{13}$ GeV. So it is clear, that until now, and also in the near future we have no possibility to measure the right-handed neutrinos at that mass scale.

2.2 Introduction to supersymmetry

Before we can really write down the see-saw mechanism in the SUSY environment, we will have a very brief look at supersymmetry and introduce the basic notation that we will need in order to write down the SUSY see-saw. However this is not at all a very pedagogical introduction. Such an introduction would be far beyond the scope of this rapport. Several good introductions into SUSY exist among which I would like to recommend the one written by S. Martin [14]. This introduction into SUSY is the base of this very brief introduction to the SUSY see-saw.

One can show that the SM Lagrangian is invariant under a transformation that transforms a fermionic field into a bosonic field and vice versa. Such a transformation is called supersymmetry transformation. The operator that generates these transformations must be an anti commutating spinor with:

$$Q \mid \text{Boson}\rangle = \mid \text{Fermion}\rangle \text{ and } Q \mid \text{Fermion}\rangle = \mid \text{Boson}\rangle \tag{54}$$

The single-particle states of a supersymmetric theory are put together into irreducible representations of the supersymmetry algebra, called supermultiplets. These bosonic and fermionic fields in one supermultiplet are called superpartners of each other. One can show that the operator Q commutes with the squared mass operator $-P^2$ and therefore particles, which are in the same supermultiplet must have equal masses. Furthermore the supersymmetry operator commutes with the generator of the gauge transformations. By consequence, particles living in the same supermultiplet must have the same charge, weak isospin and colour degrees of freedom. In each supermultiplet we have the same number of fermionic and bosonic degrees of freedom. The simplest way to construct a supermultiplet is to take one single Weyl fermion (with two spin helicity states) and two real scalar fields, which will be put into one complex scalar field (also two degrees of freedom). This combination of two component Weyl spinor and complex scalar field is called *chiral*, or *matter* or *scalar* supermultiplet. Another simple possibility for a supermultiplet contains a spin-1 vector boson, which is at least before the breakdown of the gauge symmetry massless. This massless spin-1 one vector boson has two helicity degrees of freedom. So the corresponding fermionic superpartner will be a spin 1/2 Weyl fermion. These superpartners of our gauge-bosons are called *gauginos* and form together with the gauge-boson a so called *gauge* or *vector* supermultiplet.

Remember that we found that particles and their corresponding superpartners should have the same mass. However until today we have not found any supersymmetric particles. If there were superparticles with the same mass as their corresponding particles, we would have seen them already long ago. This tells us already that SUSY cannot be an exact symmetry. It must be a broken symmetry, where the superparticles have masses much higher than their corresponding particles. This SUSY breaking is technically achieved via the introduction of a "soft" mass term in the Lagrangian.

$$\mathscr{L} = \mathscr{L}_{SUSY} + \mathscr{L}_{soft} \tag{55}$$

The term $\mathscr{L}_{SUSY}$ contains all of the gauge and Yukawa interactions and preserves supersymmetry invariance. The term $\mathscr{L}_{soft}$ violates supersymmetry and contains explicit mass terms and coupling parameters for the superparticles. These soft mass terms are introduced at a mass scale well above TeV but still smaller than the Planck scale ($m_{Pl} \approx 10^{19}$ GeV). At this scale all superparticles have the same mass. Through the renormalization group equation which depend on the interactions of the different superparticles, one gets different masses for different superparticles at the scale of supersymmetric masses. The lightest SUSY particles are expected to be at several hundred GeV.

In a renormalisable supersymmetric field theory, the interactions and masses of all particles are determined by their gauge transformation properties and by the superpotential W. The superpotential is a analytic function of chiral superfields. A superfield is a single object that contains as its components all of the bosonic and fermionic fields within the corresponding supermultiplet, for example $\Phi_i \supset (\phi_i, \psi_i)$. The supermultiplet finally determines the interaction part of the Lagrangian:

$$\mathscr{L} = \mathscr{L}_{free} + \mathscr{L}_{int} \tag{56}$$

As the free part of our Lagrangian is invariant under supersymmetry, we must require that also the interaction part of the Lagrangian is invatinat under supersymmetry transformations. This restricts the possibilities for the superpotential to the following form:

$$W = \frac{1}{2} M^{ij} \Phi_i \Phi_j + \frac{1}{6} y^{ijk} \Phi_i \Phi_j \Phi_k \tag{57}$$

Where M is a mass matrix with mass dimension one and y is a Yukawa matrix with mass dimension zero. From this superpotential we can deduce the interaction Lagrangian for the interactions that involve two fermions via

$$\mathscr{L}_{int} = -\frac{1}{2} \frac{\delta^2 W}{\delta\phi_i \delta\phi_j} \psi_i \psi_j + c.c. \tag{58}$$

Now we have collected and introduced everything that we need for describing our see-saw mechanism in an supersymmetric environment.

2.3 see-saw in a SUSY environment

In the see-saw extension of the minimal supersymmetric standard model (MSSM), the leptonic part of the superpotential is given by [6]

$$W_{lep} = [\mathbf{Y^e}]_{\alpha\alpha} (L_\alpha H_d) E^c_\alpha + [\mathbf{Y}^\nu]_{\beta J} (L_\beta H_u) N^c_J + \frac{M_I}{2} N^c_I N^c_I. \tag{59}$$

This expression contains the leptonic doublet superfield L_α, the up and down type Higgs superfield $H_{u/d} = H_{2/1}$, the singlet electron superfield E_α and as a extension to the MSSM the neutrino singlet superfield N_I. For the $\{L_\alpha\}$ we use the eigenbases of $\mathbf{Y^e Y^{e\dagger}}$, for the $\{E^c_\beta\}$ we use the eigenbases of $\mathbf{Y^{e\dagger} Y^e}$ and for the $\{N_I\}$ we use the $\mathbf{M}$ eigenbases. These choices correspond to the charged lepton mass basis, referred to as the flavour basis and labelled by Greek letters, and the mass eigenstate basis of the heavy singlets, labelled by Roman capitals which run from 1 to n_N. The doublet contraction is antisymmetric $(LH_d) = E_L H^0 - N_L H^-$, the Yukawa indices are ordered left-right, and we will allow $n_N = 2$ and 3 generations of singlets N, whose masses M_I will be taken $\gg$ TeV. The resulting Lagrangian is

$$\mathcal{L} = y^e_\alpha \bar{\ell}^\alpha H^*_d e^\alpha_R + \bar{\ell}^\alpha H^*_u [\mathbf{Y}^{\nu *}]_{\alpha I} N_I + \frac{M_I}{2} \overline{N^c}_I N_I + ... + h.c. \tag{60}$$

where $y^e_\alpha \in \{y_e, y_\mu, y_\tau\}$ are the charged lepton Yukawa couplings, the singlet neutrinos are written as four-component fermions, and the ... includes sparticle interactions.

The number of degrees of freedom of the coupling constants in this Lagrangian depends strongly on n_N the number of neutrino singlets. In the case where $\mathbf{Y}^\nu$ and $\mathbf{M}$ are complex we have n_N masses $\{M_I\}$, which can be taken real by a phase choice on the $\{N_I\}$, three eigenvalues $\{y^e_\alpha\}$, which can be taken real by a relative phase choice between the $\{E^c_\alpha\}$ and $\{L_\alpha\}$, and $3 \times n_N$ complex entries in $\mathbf{Y}^\nu$, from which 3 phases can be removed by suitably choosing the phase differences between the three doublets $\{L_\alpha\}$ and the singlets $\{N_I\}$. So we expect $7 \times n_N$ real parameters. If the matrices $\boldsymbol{M}$ and $\mathbf{Y}^{\boldsymbol{\nu}}$ are restricted to be real, there would be $3 + 4 \times n_N$ real parameters. Normally the neutrino Yukawa matrix is a $3 \times n_N$ matrix. It can be diagonalized by two independent unitary transformations. One transformation $\mathbf{V}_L$ which acts on the left handed lepton doublets and is 3 and one other transformation $\mathbf{V}_R$ which acts on the neutrino singlets and is $n_N \times n_N$. So in the case of $n_N = 3$:

$$\mathbf{V}_L \mathbf{Y}^\nu \mathbf{V}_R^\dagger \quad = \quad D_\nu \equiv \text{diag } \{y_1, y_2, y_3\} \tag{61}$$

In the case of two singlets, we will embed the 3×2 Yukawa matrix into a 3×3 matrix. By consequence also $\mathbf{V}_R$ will be embedded into a 3×3 matrix in the following way

$$[\mathbf{V}_R]_{3\times3} = \begin{bmatrix} 1 & 0 \ 0 \\ 0 & [\mathbf{V}_R]_{2\times2} \\ 0 & \end{bmatrix} \tag{62}$$

We assume that the masses of the heavy singlets are much bigger than the scale of the electroweak symmetry breaking. Consequently, at low energies the right-handed neutrinos are decoupled and the corresponding effective Lagrangian contains a effective mass term for the neutrinos [7]

$$\mathbf{m}_\nu = \mathbf{m}_D \mathbf{M}^{-1} \mathbf{m}_D^T = \mathbf{Y}^\nu \mathbf{M}^{-1} \mathbf{Y}^{\nu T} v_u^2 = U \mathbf{D_m} U^T \quad , \tag{63}$$

where $v_u = \langle H_u \rangle = v \sin\beta \simeq v = 174$ GeV, $D_m = diag\{m_1, m_2, m_3\}$, and the neutrino mass differences are taken to be

$$|\Delta m_{31}^2| \simeq 2.4 \times 10^{-3} \text{ eV}^2 \qquad |\Delta m_{21}^2| \simeq 7.6 \times 10^{-5} \text{ eV}^2 \tag{64}$$

We assume that the lightest doublet neutrino is exactly massless. As only two mass differences have been measured in the neutrino sector, we still do not know anything about the hierarchy of the neutrino matrices. In our model of two right handed neutrinos we can create normal hierarchy (NH) and inverse hierarchy (IH). In the first case, where $m_1 < m_2 < m_3$, we can put the first neutrino mass to zero and set the other masses equal to the measured mass differences. In order to create inverse hierarchy, where $m_1 > m_2 > m_3$, we set the third mass to zero, the first one to $m_1 = \sqrt{|\Delta m_{31}^2|} \simeq 0.0490$ eV and finally the second mass to $m_2 = \sqrt{|\Delta m_{31}^2| - |\Delta m_{21}^2|} \simeq 0.0482$ eV.

The leptonic mixing matrix is named U and transforms between the interaction basis and the mass basis of the neutrinos $\nu_\alpha = [U]_{\alpha i} \nu_i$. It can be parametrized in the following way

$$\mathbf{U} \quad = \quad \begin{bmatrix} e^{i\alpha}\; c_{13}c_{12} & e^{i\beta}\; s_{12}c_{13} & s_{13}\; e^{-i\delta} \\ e^{i\alpha}\; (-s_{12}c_{23} - s_{23}s_{13}c_{12}\; e^{i\delta}) & e^{i\beta}\; (c_{23}c_{12} - s_{23}s_{13}s_{12}\; e^{i\delta}) & s_{23}c_{13} \\ e^{i\alpha}\; (s_{23}s_{12} - s_{13}c_{23}c_{12}\; e^{i\delta}) & e^{i\beta}\; (-s_{23}c_{12} - s_{13}s_{12}c_{23}\; e^{i\delta}) & c_{23}c_{13} \end{bmatrix} \tag{65}$$

where $c_{ij} = \cos\theta_{ij}$, etc., $\theta_{23} = 0.64$ [15], $\theta_{12} = 0.59$ [15] and $\theta_{13} = 0.18$, the central value of the recent T2K indication[1] of non-zero θ_{13} (averaged over NH and IH). Since a neutrino is massless, one of the Majorana phases α, β vanishes. In practise, we will usually neglect all phases, because we focus on CP-conserving observables.

It is nice to see that equation (63) gives us a term for the effective neutrino mass as predicted in equation (48). The only difference is, that equation (63) is a matrix equation due to different lepton families.

In our model, the neutrino mass matrix $\mathbf{m}_\nu$ is already almost determined by measurement. Only the hierarchy is not yet fixed. However we find that $\mathbf{m}_\nu$ is not enough do determine the singlet mass matrix $\mathbf{M}$ and the Yukawa couplings $\mathbf{Y}^\nu$ at the same time. Some more input is needed. We would like to parametrize the high energy theory with low energy observables. This approach is called bottom-up approach, because we start our reconstruction at low energies and calculate the Lagrangian at high energies. In the opposite approach, the top-down approach, one starts at the GUT scale and chooses some theoretically motivated textures for $\mathbf{Y}^\nu$ and $\mathbf{M}$. Requiring that the model is consistent with the observed neutrino mass matrix, one studies other low energy predictions. In our bottom-up approach, we find the matrix

$$\mathbf{P} \quad = \quad \mathbf{Y}^\nu \mathbf{Y}^{\nu\dagger} \equiv V_L^\dagger \mathbf{D}_\nu^2 V_L \tag{66}$$

is an interesting candidate that could probably be measured in the near future. It can be extracted from the slepton mass matrix. The off diagonal elements of slepton matrix might intervene in loops which induce rare leptonic decays, like $\mu \to e\gamma$. If $\mathbf{P}$ was measured completely, one could determine $\mathbf{V}_L$ and $\mathbf{D}_\nu$. Then, by inverting equation (63) with the use of equation (61) one can determine the remaining singlet mass. We will use the basis, labelled by I in which the singlet neutrino mass matrix is diagonal $\mathbf{M}^{-1} = \mathbf{D_M}^{-1} = \text{diag}\{1/M_1, ...\}$. So we find

$$V_R \mathbf{D_M}^{-1} V_R{}^T = \mathbf{D}_\nu{}^{-1} V_L U \mathbf{D}_m U^T V_L^T \mathbf{D}_\nu^{-1} v_u^2 = \mathbf{D}_\nu{}^{-1} W_L \mathbf{D}_m W_L^T \mathbf{D}_\nu^{-1} v_u^{-2} \tag{67}$$

This can also be written in the following form:

$$\mathbf{D}_m \quad = \quad W_L{}^\dagger \mathbf{D}_\nu V_R \mathbf{D_M}^{-1} V_R{}^T \mathbf{D}_\nu W_L^* v_u^2 \tag{68}$$

$\widetilde{BR}$	current bound	future
$\mu \to e\gamma$	2.4×10^{-12}[2]	$\sim 10^{-13}$, (MEG [2])
$\tau \to \mu\gamma$	2.5×10^{-7}[3]	$\sim 10^{-8}$, (super-B factories[17])
$\tau \to e\gamma$	1.9×10^{-7}[3]	$\sim 10^{-8}$, (super-B factories[17])

Table 1: Current bounds and hoped-for sensitivities to lepton flavour violating branching ratios, normalised to leptonic weak decays, as in eqn (72).

In these equations we introduced the matrix

$$W = V_L U \tag{69}$$

which transforms directly from the neutrino mass basis to the basis $\mathbf{Y}^{\nu}\mathbf{Y}^{\nu\dagger}$ diagonal. All inputs on the right hand side of equation (67) can be determined from the neutrino mass matrix and the $\mathbf{P}$ matrix. So the see saw models with $n_N \leq 3$ can be reconstructed, if both are measured.

2.4 Rare decays

Lepton flavour violating (LFV) rare decays are radiative decays that are forbidden in the SM. In a LFV rare decay a charged lepton decays into another charged lepton and a photon, but no neutrinos. The tree possibilities are

$$\mu \to e\gamma \ \ , \ \ \tau \to e\gamma \ \ , \ \ \tau \to \mu\gamma \tag{70}$$

Up to today now LFV rare decays have been measured, but a series of experiments were and are still looking for them. Table (1) summarizes measured bounds and hoped future sensitivities on leptonic rare decays. In the following paragraph we will try to explore the nature of rare decays and how they can be related to the matrix $\mathbf{P}$ that we still need to determine our see-saw models. However for a complete understanding, one has to go very deep into supersymmetry. That is why we will only provide hand-waving arguments together with approximate formula that can be found in different papers [10, 6, 5]

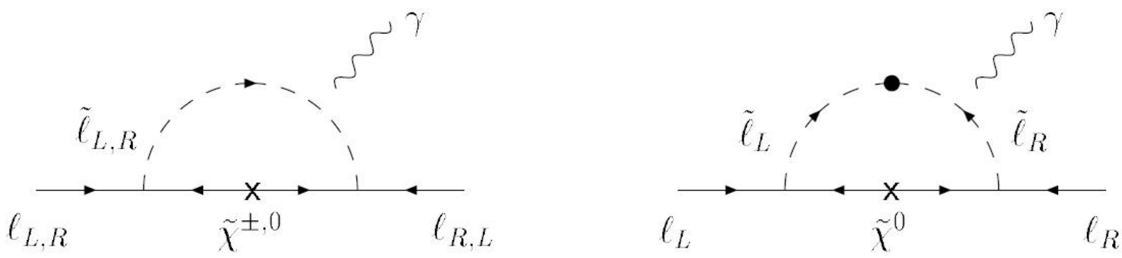

Figure 1: Chirality violating left-right contributions to rare decays. Arrows indicate the flow of fermion or scalar-partner chirality. A cross indicates a chirality violating propagator. A dot indicates a left-right mass squared mixing chiral insertion. The external photon is attached to internal charged lines in all possible ways. [10]

One diagram that leads to LFV rare decays is shown in figure (1). This diagram is not chirality-conserving. One can show, that the chirality-violating diagrams bear the dominating part of all diagrams [10]. The operators that create rare decays are dimension six dipole operators which can be written, after electroweak symmetry breaking, as

$$e\Big([\mathbf{X}_L]_{\alpha\beta}\overline{e_\alpha}\sigma^{\mu\nu}P_L e_\beta F_{\mu\nu} + [\mathbf{X}_R]_{\alpha\beta}\overline{e_\alpha}\sigma^{\mu\nu}P_R e_\beta F_{\mu\nu}\Big) \quad . \tag{71}$$

Here e without subscript is the electromagnetic coupling constant. When the coefficients of the dipole operators are known, one can calculate the branching ratios for leptonic rare decays via

$$\widetilde{BR}(l_\alpha \to l_\beta\ \gamma) = \frac{\Gamma(l_\alpha \to l_\beta\ \gamma)}{\Gamma(\ell_\alpha \to \ell_\beta \nu_\alpha \bar{\nu}_\beta)} \quad = \quad \alpha m_\alpha^3 (|X_{L\alpha\beta}|^2 + |X_{R\alpha\beta}|^2) \frac{192\pi^3}{G_F^2 m_\alpha^5} \quad . \tag{72}$$

For $\alpha = \beta = \mu$ the dipole operator (71) also contributes to the anomalous magnetic moment of the muon. The measured dipole moment of the muon differs by several standard deviations from the SM prediction. If one assumes that the whole anomaly $\delta a_\mu \simeq 29 \times 10^{-10}$ [6] comes from the same diagrams than the rare decays, one can "normalise" the rare decay ratios by the anomalous dipole moment. The coefficient in front of the dipole operator can be approximated by

$$X_{L\alpha\beta} \quad \sim \quad X_{L\alpha\alpha} \frac{\widetilde{m}^2_{\alpha\beta}}{\widetilde{m}^2} \sim \frac{\delta a_\mu}{4 m_\mu^2} m_\alpha \frac{\widetilde{m}^2_{\alpha\beta}}{\widetilde{m}^2} \tag{73}$$

This formula involves the ratio between some off-diagonal sparticle mass and the overall sparticle mass scale. This ratio receives contributions from loops involving neutrino Yukawa couplings, like in fig (2) The corrections to the flavour off-diagonal slepton masses are given by [6]

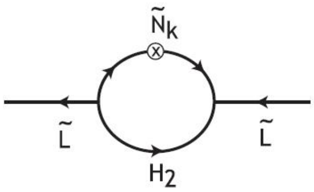

Figure 2: One example of several loop diagrams that involve Yukawa couplings and contribute to the slepton mass. The cross symbolises a soft mass insertion m_0 from the soft mass breaking Lagrangian. The two vertex are Yukawa couplings, where the slepton couples to a Higgsino and a singlet lepton.

$$\frac{\Delta \widetilde{m}^2_{L\alpha\beta}}{\widetilde{m}^2} \quad \simeq \quad -\frac{1}{4\pi^2} \left[\mathbf{Y}^\nu \mathbf{Y}^{\nu\dagger} \right]_{\alpha\beta} \tag{74}$$

If we put all the approximate formulas together that were given in this section we can calculate the branching ratios of rare decays as a function of known constants and the matrix $\mathbf{P} = [\mathbf{Y}^\nu \mathbf{Y}^{\nu\dagger}]$

$$\widetilde{BR}(l_\alpha \to l_\beta\ \gamma) \quad = \quad \frac{3\alpha |\delta a_\mu|^2}{4\pi m_\mu^4 G_F^2} \left| [\mathbf{Y}^\nu \mathbf{Y}^{\nu\dagger}]_{\alpha\beta} \right|^2 \simeq 10^{-6} \left| [\mathbf{Y}^\nu \mathbf{Y}^{\nu\dagger}]_{\alpha\beta} \right|^2 \quad . \tag{75}$$

This gives us for example a upper limit on $[\mathbf{Y}^\nu \mathbf{Y}^{\nu\dagger}]_{\mu e} \lesssim 2 \times 10^{-3}$ However, as you have noticed, this equation contains a lot of approximations. If one looks at the ratios of different branching ratios, the prefactor in front of $[\mathbf{Y}^\nu \mathbf{Y}^{\nu\dagger}]$ cancels out

$$\frac{\widetilde{BR}(\mu \to e\gamma)}{\widetilde{BR}(\tau \to \mu\gamma)} = \frac{|[\mathbf{Y}^\nu \mathbf{Y}^{\nu\dagger}]_{\mu e}|^2}{|[\mathbf{Y}^\nu \mathbf{Y}^{\nu\dagger}]_{\tau\mu}|^2} \quad . \tag{76}$$

Of course for other decays the formula is completely similar. This way we can drop a lot of our approximations, but we also loose some degree of prediction.

One last approximation that we will use widely in this report: the eigenvalues y_i of the neutrino Yukawa matrix are considered to be sufficiently hierarchical that in that in $\ell_\alpha \to \ell_\beta \gamma$ processes, only the terms involving the largest eigenvalue y_3 need to be considered:

$$|[\mathbf{Y}^\nu \mathbf{Y}^{\nu\dagger}]_{\alpha\beta}|^2 \longrightarrow y_3^4 |V_{L3\alpha} V^*_{L3\beta}|^2 \quad . \tag{77}$$

In order to make some estimations of branching ratios we set $y_3 = 1$ for all of the following part. As the matrix V_L is unitary, the elements $V_{L3\alpha}$ full fill the normalisation equation

$$\sum_{\alpha} |V_{L3\alpha}|^2 = 1 \tag{78}$$

If we work with equation (75) this implies relations between different rare decays, so that the measurement of two decay rates determines the third one. In the framework of equation (76) the normalisation equation gives the values of all three $|V_{L3\alpha}|^2$ if all three branching ratios are measured (or if two different ratios of branching ratios are measured) and therefore the constant of proportionality in equation (75) is fixed.

3 The different models (= research part)

Now we should have all the necessary tools to explore different see-saw models and look at their possibility of reconstruction. However before doing so, we will collect all the used approximations and simplifications that we use.

We look at a see-saw model in a SUSY environment with two or three singlet neutrinos. Almost all the time the reconstruction will be considered to be real. In section (3.1.2) we will justify this approximation. The neutrino mass matrix is almost determined by the neutrino oscillation data. We consider the case where the lightest neutrino has zero mass. This leaves only the hierarchy undetermined (look around equation (64)). We will consider normal and inverse hierarchy for all see-saw models. The eigenvalues of the neutrino Yukawa matrix are also assumed to be hierarchical. The biggest eigenvalues y_3 is set to 1. For the determination of the elements of $\mathbf{P}$ which are relevant for rare decays, we will only care about the biggest eigenvalue y_3, like in equation (77). A very optimistic link between branching ratios and the the $\mathbf{P}$ matrix is given by equation (75). A rather conservative, but less defective equation for rapports of branching ratios is given in equation (76).

3.1 Two singlets

The first model contains two right handed singlet neutrinos with finite masses. The motivation for this model is that it is the simplest see-saw model that can already explain all observed neutrino oscillations. It is clear that in this model we can only have two non-zero light neutrinos and one neutrino with vanishing mass.

In general there are two possibilities to write down this model. The first possibility is to use a neutrino Yukawa coupling matrix $[\mathbf{Y}^\nu]_{\alpha\mathbf{I}}$ which is not a diagonal matrix but a 3×2 matrix, so that it can connect the 3×3 light neutrino matrix to the 2×2 right handed neutrino matrix. The second possibility is to write also the right handed sector as 3×3 matrices and fill up the unused components with zeros, so that all matrices that are used can stay diagonal. We will use the latter one.

3.1.1 real case

In the first step we will consider the case in which all possible phases are zero and consequently the whole problem becomes real. In this case we can write equation (68) for normal hierarchy

$$\begin{aligned}\begin{pmatrix}0 & 0 & 0\\ 0 & m_2 & 0\\ 0 & 0 & m_3\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\ 0 & c_{WL} & s_{WL}\\ 0 & -s_{WL} & c_{WL}\end{pmatrix}\begin{pmatrix}0 & 0 & 0\\ 0 & y_2 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & c_R & -s_R\\ 0 & s_R & c_R\end{pmatrix}\begin{pmatrix}0 & 0 & 0\\ 0 & 1/M_2 & 0\\ 0 & 0 & 1/M_3\end{pmatrix} \times \\ \times \begin{pmatrix}1 & 0 & 0\\ 0 & c_R & s_R\\ 0 & -s_R & c_R\end{pmatrix}\begin{pmatrix}0 & 0 & 0\\ 0 & y_2 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & c_{WL} & s_{WL}\\ 0 & -s_{WL} & c_{WL}\end{pmatrix} v_u^2\end{aligned} \tag{79}$$

Where $c_{WL,R}$ stands for $cos(\theta_{WL,R})$ and analogue for $s_{WL,R}$. We see that our problem simplifies to the diagonalisation of a 2×2 matrix. This is due to the fact that we do not use the basis of the charged leptons in this equation. The matrix W transforms directly form the neutrino mass basis into the neutrino Yukawa interaction basis. We can completely determine W by measuring only one rare decay. As shown before the rare decay $\ell_a \to \ell_\beta\gamma$ is proportional to the matrix element

$$\mathbf{P}_{\alpha\beta} = [\mathbf{Y}^\nu \mathbf{Y}^{\nu\dagger}]_{\alpha\beta} = \mathbf{U}\mathbf{W}_L^\dagger \mathbf{D}_Y^2 \mathbf{W}_L \mathbf{U}^\dagger \tag{80}$$

In the real case U is completely given and W_L depends only on one single parameter θ_{WL}. Consequently one measured branching ratio can already determine the whole W_L matrix. But this also means that the measurement of one single rare decay will fix the other two rare decays. So one very appealing feature of this model is, that it introduces relations between the different rare decays, which can be tested if rare decays are measured. By measuring two rare decays we could already exclude the model if it does not describe reality.

In principle this model can be completely reconstructed. Even an analytic reconstruction is possible. The right handed neutrino mass eigenvalues are given by:

$$\begin{aligned}\frac{1}{M_2} &= \frac{1}{y_2^2}(m_2 c_{WL}^2 + m_3 s_{WL}^2) + \frac{1}{y_2} c_{WL} s_{WL} \frac{c_R}{s_R}(m_2 - m_3)\\ \frac{1}{M_3} &= \frac{1}{y_2^2}(m_2 c_{WL}^2 + m_3 s_{WL}^2) - \frac{1}{y_2} c_{WL} s_{WL} \frac{s_R}{c_R}(m_2 - m_3)\end{aligned} \tag{81}$$

The equation that determines θ_R is

$$\frac{1}{y_2}\frac{c_{WL}s_{WL}}{c_R s_R}(s_R^4 - c_R^4)(m_2 - m_3) = m_2\left(s_L^2 - \frac{1}{y_2^2}c_{WL}^2\right) + m_3\left(c_{WL}^2 - \frac{1}{y_2^2}s_{WL}^2\right) \tag{82}$$

If we assume that the upper border on the branching ration of $\mu \to e\gamma$ is the actual value we are left with only one free parameter, namely y_2. This assumption can be justified by the fact that the value of θ_L does almost not change weather the value of $|\mathbf{P}_{\mu e}|$ is approximately $2 \cdot 10^{-3}$ or zero. Of course this fine-tuning of the angle changes the the branching ratios, but on our reconstruction of the right handed neutrino mass matrix it has almost no influence.

In this approximation the decay rates for τ rare decays are already fixed. However when we impose the value of $\mathbf{P}_{\mu e}$ we have to solve an equation in sin and cos of θ_{WL}.

$$\mathbf{P}_{\mu e} = |V_{L3\mu}V_{L3e}| = |(s_{WL}U_{ej} + c_{WL}U_{ej+1})(s_{WL}U_{\mu j} + c_{WL}U_{\mu j+1})| \tag{83}$$

(where $j = 1$ for the inverse hierarchy, $j = 2$ for a normal hierarchy,and $s_W = \sin\theta_{WL}$, etc.). We can implement a small value for $\mathbf{P}_{\mu e}$ either by $|V_{L3e}| < 10^{-2}$ or by $|V_{L3\mu}| < 10^{-2}$. This gives us two solutions for the mixing angle θ_{WL} in W.

In the case of normal hierarchy be get for the first solution $\theta_L = 2.37$, where $|V_{L3\mu}|$ is small. This tells us already that $\tau \to \mu\gamma$ is also small and therefore beyond the reach of future experiments. The branching ratios can be estimated via equation (75) $\widetilde{\mathrm{BR}}(\tau \to e\gamma) = 5.5 \cdot 10^{-8}$ and $\widetilde{\mathrm{BR}}(\tau \to \mu\gamma) = 3,3 \cdot 10^{-11}$. The second solution $\theta_L = 2.83$, where $|V_{L3e}|$ is small yields the inverse picture. We find that $\tau \to e\gamma$ cannot be measured in the near future. The branching ratios are given by $\widetilde{\mathrm{BR}}(\tau \to e\gamma) = 1,4 \cdot 10^{-11}$ and $\widetilde{\mathrm{BR}}(\tau \to \mu\gamma) = 1,1 \cdot 10^{-7}$. In both cases we have one branching ratio at the same order as the μ decay and one very big branching ratio at the order of $10^{-8} - 10^{-7}$ which will be measurable in the next generation experiments. If we use the more carefully equation (76), we obtain the same qualitative results. There we assume that $V_{L3\mu} \to 0$ or $V_{L3e} \to 0$. In these limits we can calculate the different ratios of rare decays and find the same results for the ratios as above. We can just not make any prediction for the actual quantitative values of rare decays.

In the inverse hierarchy model a similar procedure yields a first solution for the mixing angle $\theta_L = 0.845$ with branching ratios $\widetilde{\mathrm{BR}}(\tau \to e\gamma) = 4.2 \cdot 10^{-8}$ and $\widetilde{\mathrm{BR}}(\tau \to \mu\gamma) = 1,2 \cdot 10^{-13}$. For this solution $|V_{L3\mu}|$ is small and therefore also $\tau \to \mu\gamma$ is smaller than we can measure in the near future. If we take a closer look at this case we find that the branching ratio of $\tau \to e\gamma$ is calculated by the formula $\widetilde{BR}(\tau \to e\gamma) \sim 10^{-6}s_{13}^2$. This might be measurable for cooperative supersymmetric values and s_{13} near the current T2K central value 0.18. But for a smaller mixing angle and uncooperative SUSY masses this might become to small to measure in the next generation experiments. The second solution for the angle is $\theta_L = 2.56$ where $|V_{L3e}|$ is small. Consequently we are not surprised when we calculate the branching ratios $\widetilde{\mathrm{BR}}(\tau \to e\gamma) = 1,3 \cdot 10^{-12}$ and $\widetilde{\mathrm{BR}}(\tau \to \mu\gamma) = 2,1 \cdot 10^{-7}$ and find that $\tau \to e\gamma$ will not be measurable.

In a more general point of view, our model of two right handed neutrinos and hierarchical neutrino Yukawa eigenvalues almost always yields one τ rare decay at the order of the of the μ rare decay or even smaller and the other τ branching ratio at the order of $10^{-8} - 10^{-7}$. Consequently, this model can be tested in the very near future. Only in the inverse hierarchy we can have a case where no branching ratio can be measured in the near future.

We find that only one mass of the right handed neutrinos depends on the yet unfixed second neutrino Yukawa eigenvalue. For reasons of leptogenesis we want to have the lightest right handed neutrino mass to be of the order of 10^{10} GeV. This restricts the space for our free parameter for the normal hierarchy to approximately

$$0,002 \leq y_2 \leq 0,007 \tag{84}$$

For inverse hierarchy we obtain a restriction to

$$0,003 \leq y_2 \leq 0,009 \tag{85}$$

In these limits the second right handed neutrino mass is found to be $M_3 \approx 2 \cdot 10^{15}$ GeV in the hierarchical case and $M_3 \approx 6 \cdot 10^{14}$ GeV in the inverse hierarchy. It is interesting to compare the disintegration ratio of the lightest right handed neutrino to the expansion rate of the universe at a temperature equal to the mass of this neutrino. The singlet neutrino disintegration ratio is given by [9]

$$\Gamma_i = \frac{[\mathbf{Y}^{\nu\dagger}\mathbf{Y}^{\nu}]_{ii}}{8\pi} M_i \tag{86}$$

The expansion rate of the universe can be calculated via

$$H = \frac{1,66 \cdot \sqrt{g_*} T^2}{m_P l} \tag{87}$$

Where $m_{Pl} = 1,2 \cdot 10^{19}$ GeV is the Planck mass and g_* is the number of degrees of freedom at a certain temperature. At our temperature of about $T = M_2 \approx 10^{10}$ GeV it is given by $g_* = 107$ [13]. For both hierarchies we obtain a ratio of

$$\frac{\Gamma_1}{H} \sim 45 \tag{88}$$

So we are in the region of strong washout.

3.1.2 complex case

We will now have a look at the reconstruction in the case where the phases are unequal to zero. As only two of our three neutrinos have masses, we need only one Majorana phase in the PMNS Matrix U. We can set $\alpha = 0$ without loss of generality. The W matrix can be written with phases

$$W = e^{i\phi_g} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{WL}e^{i\gamma/2} & -s_{WL}e^{-i\sigma/2} \\ 0 & s_{WL}e^{i\sigma/2} & c_{WL}e^{-i\gamma/2} \end{pmatrix} \tag{89}$$

We also want to determine the mixing angle θ_L from the $\mu \to e\gamma$ decay. We will look at the case of normal hierarchy.

$$\mathbf{P}_{\mu e} = \left| \left(e^{\frac{1}{2}i(\gamma+\sigma+2\beta-2\delta)} c_{WL} s_{13} + c_{13} s_{12} s_{WL} \right) \cdot \left(c_{WL} c_{13} s_{23} + e^{\frac{1}{2}i(\gamma+\sigma+2\beta)} s_{WL} (c_{12} c23 - e^{-i\delta} s_{12} s_{13} s_{23}) \right) \right| \tag{90}$$

In general this is of the order of $\frac{1}{2}$. If we want to have an value of the order of 10^{-3} or even smaller, one of the two brackets must be small. This is only possible if at the same time the real part and the imaginary part are small. This gives us one equation for the mixing angle θ_L and one equation for the phases involved. For the first term we find that

$$\begin{aligned} \frac{1}{2}(\gamma + \sigma + 2\beta - 2\delta) &\approx 0, \pm\pi, \pm 2\pi, ... \\ \theta_L \approx \arctan\left(\frac{s_{13}}{c_{13}s_{12}}\right) &= 0.316 \end{aligned} \tag{91}$$

If in both equations the approximately is replaced by equal, the first term vanishes completely. If the second term should vanish, we have to impose

$$\begin{aligned} \frac{1}{2}(\gamma + \sigma + 2\beta) &\approx \operatorname{arccot}\left(\frac{\cos\delta - \frac{c_{12}c_{23}}{s_{12}s_{13}s_{23}}}{\sin\delta}\right) = \operatorname{arccot}\left(\frac{\cos\delta - 11.2}{\sin\delta}\right) \\ \theta_L &\approx -0.77 \end{aligned} \tag{92}$$

Here also, if both equations are exactly fulfilled we obtain zero for the second term. In general we see, that the condition of small $\mu \to e\gamma$ branching ratio gives us certain restrictions on the possible phases.

A very important question to answer is, if the predictions of the other two branching ratios depend strongly on the choice of the phases. If we have a look at the other branching ratios, we find as a first result that also these two decay rates do only depend on the sum of $\gamma + \sigma$. If the phases vary in such a way that they still satisfy equation (91) or (92) and if $\mathbf{P}_{\mu e}$ stays constant we find that the other two branching ratios vary. But their overall variations over the whole parameter space are not bigger than a factor two. This factor two might be crucial for an actual determination of the branching ratios, but in the scope of our approximations this factor two will not influence our conclusions.

The phases also have an influence on the reconstructed Majorana masses. The first observation that we make is, that the Majorana mass eigenvalues become complex. However by one phase rotation we can absorb one phase in the $\mathbf{W}$ matrix. So at the end we only have one complex mass eigenvalue. Finally we look at the absolute values of our Majorana masses and compare them to their value in the real case. We find that the variation does not exceed a factor four. This will also not have any influence on our conclusions.

In the following we will neglect phases in our reconstruction procedure. As we are not interested in CP violation and the variations of the physical observables for different phases are rather small, we will place all our following analysis in a real case.

3.2 Three singlets

This section studies the prospects for reconstruction of a seesaw model with three heavy singlets N_I. In this model, all the light neutrinos can be massive. We study the limit where the smallest light neutrino mass $m_{\nu,min} \to 0$, to represent the experimental situation where $m_{\nu,min} \ll \sqrt{\Delta m^2_{sol}}$ is unknown. This limit could arise [11] when one of the singlets is very heavy, $1/M_3 \to 0$, so does not contribute to the observed light masses, or when one of the $\mathbf{Y}^\nu$ eigenvalues does not contribute to the light masses, $y_1 \to 0$. We wish to know if such three singlet models can be distinguished from the two-singlet model of the previous section. All phases are neglected (set to zero) in this section.

3.2.1 One infinite mass eigenvalue

This model is quite similar to the model of two singlet because the third right handed neutrino introduced here, is completely decoupled and has no physical effect any more. However there is one big difference between the two models. In the model of three singlets the Yukawa matrix has three non zero eigenvalues. Consequently the matrices W or V_L and V_R are full 3×3 matrices without generic zero components. They are both described by three mixing angles and can be written in the same form as the PMNS matrix, with other angles of course. If we set $V_L^\dagger$ in the form of the PMNS matrix we find that the components that determine the branching ratios for rare decays depend only on two of the three angles. So we need to measure all three branching ratios to determine two mixing angles.

$$\begin{aligned}\frac{\widetilde{BR}(\mu \to e\gamma)}{\widetilde{BR}(\tau \to e\gamma)} &= \frac{|V_{L3\mu}|^2}{|V_{L3\tau}|^2} = \frac{s_{L23}^2}{c_{L23}^2} \\ \frac{\widetilde{BR}(\mu \to e\gamma)}{\widetilde{BR}(\tau \to \mu\gamma)} &= \frac{|V_{L3e}|^2}{|V_{L3\tau}|^2} = \frac{s_{L13}^2}{c_{L23}^2 c_{L13}^2}\end{aligned} \tag{93}$$

Consequently this model does not predict the third branching ratio when we measured the two branching ratios like the model with two singlets does. (This is different from the two singlet model of the previous section). However, if equation (75) is imposed, then measuring two ratios determines the other. For instance, in the limiting case where $V_{L3e} = s_{VL13} \simeq 0$, then the rates for $\mu \to e\gamma$ and $\tau \to e\gamma$ vanish (at leading order), and $\tau \to \mu\gamma$ can have any value. More interesting would be the parameter space where $|V_{L3\mu} V_{L3e}^*|$ is small but non-zero, corresponding to an observed rate for $\mu \to e\gamma$. Then measuring either of $\tau \to \mu\gamma$ or $\tau \to e\gamma$ would determine the other — this is illustrated in figure 3. This correlation is not a consequence of the supersymmetric seesaw model; rather, it follows from assuming that:

1. only the biggest Yukawa eigenvalue is important for determining the branching ratios of rare decays, and,
2. that the small branching ratio of $\mu \to e\gamma$ imposes that $|V_{L3\mu}|$ or $|V_{L3e}|$ must be small (=almost zero)

So if the prediction of figure 3 was verified, it would be consistent with the three-singlet seesaw, as well as any other MFV-like model satisfying the above two assumptions. And if some other ratios of branching ratios were measured, it could likely be accomodated in a supersymmetric three-singlet seesaw model by relaxing the above assumptions.

Furthermore the third mixing anlge cannot be determined from rare decays. In order to determine the third angle and the eigenvalues of Yukawa matrix we have to measure the sneutrino mass matrix. So in principle, if we know the whole $\mathbf{P}$ matrix and the neutrino mass matrix $\mathbf{m}_\nu$, we can reconstruct the Yukawa couplings and the Majorana mass matrix.

In order to calculate the mass eigenvalues of the Majorana mass matrix we can use equation(67). We already know that the matrix has one eigenvalue equal to zero. So we can use the formula given by [8]

$$M_1, M_2 = \frac{1}{2}\left[\mathrm{Tr}[\mathbf{M}] \pm \sqrt{(\mathrm{Tr}[\mathbf{M}])^2 - 4(M_{11}M_{22} + M_{11}M_{33} + M_{22}M_{33} - M_{23}^2 - M_{12}^2 - M_{13}^2)}\right] \tag{94}$$

Finally this gives us the dependence of the right handed neutrino masses on our still not determined parameters, as the Yukawa eigenvalues and the mixing angles in $\mathbf{W}$. The explicit formula is very long and does not bring any great insight. By putting actual numbers into the yet unknown parameters we find that the first eigenvalue of the Majorana mass does almost not depend on y_2 but depends quadratically on y_1. The same statement can be made for the second mass eigenvalue with y_1 and y_2 interchanged. But we find it crucial to know the values of the mixing angles in W. By looking all over the parameter space, we could change the value of M_1 with fixed y_1 and y_2 by four orders of magnitude. Even if we are in one of the maximal or minimal points, we can still adjust the Yukawa eigenvalues in such a way that we obtain $M_1 \sim 10^{10}$ GeV, as needed in Leptogenesis. This tells us, that even if we can measure all rare decays we cannot yet determine whether the lightest singlet mass has the value needed for Leptogenesis or not.

We find that for generic choices of the mixing angles in V_L, the singlet masses M_1 and M_2 have the form $\sim v^2 y_i^2/\overline{m}$, as plotted in figure 4, where $\overline{m} = \sqrt{\Delta m_{sol}^2} + \sqrt{\Delta m_{atm}^2}$. This is unsurprising, since the light masses corresponding to eqn (64) are much more degenerate than the squared eigenvalues of $\mathbf{Y}^\nu$. So a given M_I is usually controlled by a single y_i^2, and the singlet mixing angles in V_R are generically

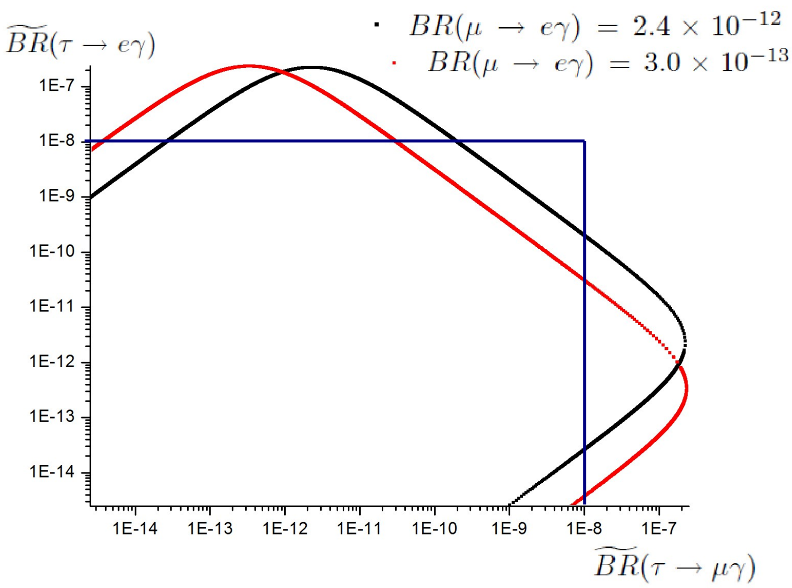

Figure 3: Branching ratios for $\tau \to \mu\gamma$ and $\tau \to e\gamma$ (normalised to leptonic decays as in eqn (72)) in the hypothetical cases that $\mu \to e\gamma$ was observed, with $BR(\mu \to e\gamma) = 2.4 \times 10^{-12}$ (upper rising-towards-the-left black line), or $BR(\mu \to e\gamma) = 3.0 \times 10^{-13}$ (lower rising-towards-the-left red line). The $\widetilde{BR}$s are estimated with eqn (75). The estimated sensitivity of superB factories is outside the (blue) rectangle. This plot can also be interpreted as an upper bound on the $\widetilde{BR}$s which are consistent with a $\mu \to e\gamma$ bound: the allowed region is to the left and below the diagonal line sloping-up-to-the-left. The horizontal and vertical cutoffs are uncertain because they are determined by our estimate of C_ν^X.

small. That is, V_R usually can be written as a rotation matrix with small mixing angles multiplied by either the identity matrix or a permutation.

It might seem logical if we want reproduce the limit of two singlets in this model. This can easily be achieved by putting the θ_{WL12} and θ_{WL13} in W to zero. Then the left handed neutrinos do not couple any more to y_1. Consequently y_1 becomes completely irrelevant for the reconstruction. The angle θ_{WL23} plays the role of θ_{WL} in the two singlet model. In this limit the eigenvalue M_1 does not depend any more on y_1 but now it depends quadratic on y_2.

One interesting question might be if we can distinguish between the system of two and three singlets by measuring rare decay rates? Not necessarily. If the measured branching ratios accord with the relations predicted by the two singlet see-saw, we cannot find a difference because the three singlet model can also explain every constellation of branching ratios that are possible in the two singlet model. But if the branching ratios do not accord with the predicted relations, we can already exclude the two singlet see-saw model.

3.2.2 One zero Yukawa eigenvalue

In this section, we briefly discuss the case where the lightest neutrino mass vanishes, $m_{\nu min} \to 0$, because one Yukawa eigenvalue, which we take to be y_1, goes to zero.

Equation (68) can be rewritten:

$$W_L \mathbf{D}_m W_L^T = \mathbf{D}_\nu V_R \mathbf{D_M}^{-1} V_R^T \mathbf{D}_\nu v_u^2 \quad (95)$$

where $\mathbf{D}_\nu$ has only two non zero entries on the diagonal and therefore the right hand side of this equation has only a 2×2 sub matrix with non-zero entries. Furthermore, we know that the doublet neutrino mass eigenstates in $\mathbf{D}_m$ do not couple to the vanishing first Yukawa eigenvalue y_1. Therefore

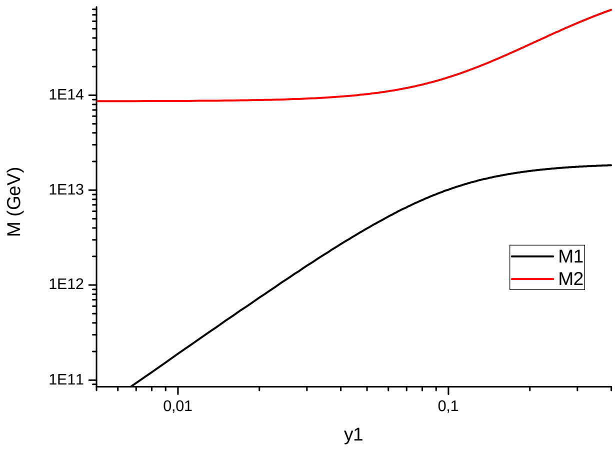

Figure 4: General dependence, in the three-singlet model where $1/M_3 \to 0$, of the singlet masses on the smallest Yukawa eigenvalue y_1. In this plot, $y_2 = .01$ and $y_3 = 1$. The angles in W_L were taken similar to those of U: $\theta_{WL13} = .18$, $\theta_{WL12} = .6$, and $\theta_{WL23} = .6$. Different values of the mixing angles can shift the normalisation of the axes.

the two angles θ_{W12} and θ_{W13} are identically zero, and a 2×2 singlet inverse mass matrix can be reconstructed using eqn (67). However, $\mathbf{D}_M^{-1}$ on the right hand side of eqn (95) still contains the three singlet masses. Three equations do not determine three mass eigenvalues and three mixing angles, so the whole high energy Lagrangian cannot be reconstructed.

This limit reduces to the case of two right handed neutrinos. The right hand side of equation (95) can be rewritten in terms of two "effective" mass eigenvalues and one "effective" mixing angle. Of course the effective values are functions of the three initial mass eigenvalues and the three mixing angles, but we can only determine the effective values. So we cannot observe any difference between the two models unless we can measure an operator $[\mathbf{d}]$ of dimension 6. With this operator we can distinguish between the two models, but we cannot perform a complete reconstruction [5].

4 Summary and conclusion

We looked at different simplified models of see-saws with two and three heavy singlet neutrinos. We always took one neutrino mass to be zero. By this the current measured values from neutrino oscillations almost fix the neutrino mass matrix. Only the hierarchy is not yet fixed. We considered both normal and inverse hierarchy. We were interested in the reconstruction of these models and how they might be tested. Therefore we paid a lot of attention to rare decays and how they can be used to reconstruct our models or how they can test predictions of different models. For the reconstruction we used the neutrino mass matrix $\mathbf{m}_\nu$ (equation (63)) and $\mathbf{P}$ (equation (66)). However, in a realistic world the operator $\mathbf{P}$ will not be completely measurable. That is why we made some further assumptions about it, e.g. hierarchical eigenvalues. Almost the whole analysis was done in the case where all matrices were real.

In our limit of one zero neutrino mass eigenvalue, we can already reproduce all measured oscillation by a model with two singlets. In this model, branching ratios are determined by only one mixing angle in the matrix W. By consequence the measurement of one rare decay determines the other two decay rates if we work with equation (75). If we restrict ourselves to equation (76) we make less approximations but we are only able to determine all ratios of branching ratios if we have measured already one rapport of branching ratios. If we assume that the actual value of $\mu \to e\gamma$ is of the order of the current upper bound, we find, by using equation (75) that one τ decay rate is of the order of $\mu \to e\gamma$ and the other one is $\widetilde{BR}(\tau \to \ell\gamma) \gtrsim 10^{-8}$ and therefore testable in the next generation experiments. This is completely true for normal hierarchy. For inverse hierarchy it is possible to have all branching ratios under the limit of next generation experiments.

The only free parameter in this model is the Yukawa eigenvalue y_2. By forcing one heavy singlet to have a mass at the order of $\sim 10^{10}$ GeV we can restrict the parameter space for $y_2 \sim 10^{-3} - 10^{-2}$. The model can be reconstructed completely if we can measure $\mathbf{m}_\nu$ and $\mathbf{P}$.

The other model we considered is the see-saw model with three singlet neutrinos. There we made the difference between one version of this model where one heavy singlet is so heavy, that he is decoupled and one version where one Yukawa eigenvalue is zero. In the first version we found that the measurement of one rare decay rate gives a correlation between the remaining two rates. All decay rates are determined by two mixing angles. In this model he have, after measuring rare decays, still three free parameters: two of the three Yukawa eigenvalues $y_1,\ y_2$ and one mixing angle in W. However in principle we can reconstruct the model.

The second version of the three singlet model with one zero Yukawa eigenvalue cannot be reconstructed even if we measure $\mathbf{P}$ completely. The model contains three singlets, but seems to be effectively like a model with two singlets. We can only reconstruct effective singlet masses and mixing angles. Without further input this model cannot be distinguished from the $2N$ model.

One problem encountered several times in this report is that equation (75) is not very well determined. As long as we have not measured supersymmetry, we cannot give very exact values for branching ratios for lepton flavour violating decays. We have to make a lot of approximations to be able to give some numbers. If we consider the safer equation (76) we loose some predictive power of our models. Measurements of SUSY at the LHC would solve this problem. In general we find that the measurement of rare decays can help to exclude certain see-saw models, but cannot reconstruct the models completely. Therefore we need further input. One possible input would be the measurement of sparticle masses and their decays at colliders.

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