Geometrical pumping with a Bose-Einstein condensate

Abstract:
We realized a quantum “charge” pump for a Bose-Einstein condensate (BEC) in the ground band of a novel bipartite magnetic lattice. In this topological lattice, the bands are characterized by non-trivial topological invariants: the Zak phases. For each band, the Zak phase is determined by that band’s integrated Berry curvature, a geometric quantity defined at each crystal momentum. We probed this Berry curvature in an adiabatic charge pump experiment. Unlike topological charge pumps in filled bands, that yield quantized pumping; our BEC occupied just a single momentum state allowing us access to the local geometry. Still, like topological charge pumps, for each pump cycle we observed an overall displacement (here, not quantized) and a modulation of the atomic wavepacket in each unit cell. Our magnetic lattice enabled us to observe this modulation by measuring the BEC’s magnetization. While our periodic modulation shifted the lattice by one unit cell per cycle, counterintuitively the displacement of the BEC, solely determined by the underlying Berry curvature, was less than this classically anticipated displacement.

Key words: Cold atom experiment, spin pump mechanism, Berry-phase, Zak-phase, topological insulators Bose-Einstein condensate

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Acknowledgement

I would like to express my sincere gratitude to my supervisor Ian Spielman for the opportunity to join the RbK-team and for his excellent supervision. I would like to thank the whole RbK-team for the warm welcome and for their help. Furthermore, I would like to thank William Phillips for his discussion and help with my presentation. Most of all, I would like to thank Hsin-I Lu. For her patience and time she spend explaining and discussing the experimental and theoretical background of the experiment.

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1 Introduction

One branch of cold atomic research studies model Hamiltonian relevant to solid state physics phenomena. The advantages of studying these problems in the context of cold atom experiment lie in the unprecedented degree of control on the experimental parameters. These experiments can be seen as “quantum simulations” as proposed 1982 by R. Feynman. During this internship we study the problem of quantum transport by means of adiabatic charge pumps.

Motion in lattices is conventionally understood in terms of Bloch band structure: metals are materials with partially filled bands, while insulators have completely filled bands. In this context, a topological charge pump is a counterintuitive system, where charge motion accompanies the adiabatic and periodic modulation of an insulating lattice. This phenomena can be pictured as an Archimedes’ screw, which pumps via a rotating spiral. Thouless showed that this conduction is quantized for band insulators and is completely govern by the band topology [1, 2]. Thouless charge pumps have not been realized in solid state devices, although charge transport has been demonstrated via periodically varying potentials in devices such as quantum dots, quantum wires, and quasi one-dimensional (1D) channel [3, 4, 5, 6]. Very recently topological charge pumps have been observed in cold atom insulators [7, 8]. Here we break from this established paradigm and create a quantum charge pump for a BEC occupying a single crystal momentum state [9, 10, 11], labeled $q$ and show that this charge pump results in non-quantized motion sensitive to the local geometry rather than the global band topology. The theoretical description of this phenomena is presented in the first part of this report sec. 2.1.

We experimentally investigated the quantum transport in a new kind of 1D topological, bipartite magnetic lattice. This new lattice is presented in the sec. 2.4. We periodically move the potential energy minima of the lattice by one lattice site per cycle. However the displacement of the wavepacket is solely determined by aggregate local geometry, revealing the quantum nature of the charge pump process. The quantum charge pump mechanism was simulated and the experimental results are compared to theoretical and numerical results.

Quantum charge pumps resulting from the underlying Berry curvature for systems in single $q$ have not been demonstrated experimentally before this work. Despite its importance, it has received rare theoretical studies. In this internship, I simulated the charge pumping in real space and predicted parameters for observing the motion in the experiment. In parallel, I formulated the theory to explain the experimental observations, as well as the real space simulations. I contributed to the process of preparing a manuscript about this experiment, which will be submitted and parts of this report are directly linked to this manuscript.

2 Theory of adiabatic transport

2.1 The Berry phase

Since the discovery of the Berry phase in 1984 [12], it has become an important unifying concept in quantum mechanics. It applies to the situation where a Hamiltonian $H(R(t))$ depends on a time-dependent parameter set $R(t)$ with components $R^\mu(t)$, that are varied in an adiabatic manner. The Hamiltonian $H(R(t))$ has an instantaneous eigenbasis $\{|n(R(t))>\}$ with instantaneous eigenvalues $\epsilon_n(R(t))$. Initially the system is in one of the eigenstate $|n(R(0))>$. During the time evolution the
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initial state acquires two phases of different origin:

\[ |\Psi_n(t)\rangle = e^{i(\phi_{\text{Berry}} + \phi_{\text{dyn}})} |n(R(t))\rangle. \] (1)

A dynamical phase \( \phi_{\text{dyn}} = -\frac{1}{\hbar} \int_0^t \epsilon_n(R(t')) dt' \) and a geometric phase \( \phi_{\text{Berry}} = \int_C A_n(R) \cdot dR \) expressed in terms of a line integral with the Berry connection \( A_n(R) = i < n(R) | \partial_n(R) | n(R) > \). The Berry connection is a gauge dependent quantity and therefore not observable. For closed paths the Berry phase can be expressed in terms of the Berry curvature \( \Omega_{\mu\nu} \), as the surface integral over the surface \( S \) surrounded by the path \( C \)

\[ \phi_{\text{Berry}} = \int_S dR^\mu \wedge dR^\nu \Omega_{\mu\nu}, \] (2)

with the Berry curvature

\[ \Omega_{\mu\nu} = \frac{\partial A_\nu}{\partial R^\mu} - \frac{\partial A_\mu}{\partial R^\nu} = i \left[ < \frac{\partial n(R)}{\partial R^\mu} | \frac{\partial n(R)}{\partial R^\nu} > - < \frac{\partial n(R)}{\partial R^\nu} | \frac{\partial n(R)}{\partial R^\mu} > \right], \] (3)

which is gauge independent quantity and also an observable quantity. The Berry curvature \( \Omega_{\mu\nu} \) is a geometrical object describing the curvature of the underlying parameter space. In physical sense it acts as a fictious magnetic field in the parameter space, just as a real magnetic field gives rise to the Aharonov-Bohm phase for charged particles. The Berry phase is a gauge-invariant physical quantity. For closed surfaces the Berry phase is quantized in units of \( 2\pi \) (Chern numbers) due to the Chern–Gauss–Bonnet theorem, connecting geometry to topology. [13]

2.2 Topological charge pumping

In 1983 Thouless predicted [14] a charge pump in topological band insulators by a slow variation of the Hamiltonian parameters in a closed loop. The scenario applies to electrons in filled bands as found in condensed matter physics. However the concept can be extended to the case of bosons in a lattice. The rubidum atoms in our experiment form a Bose-Einstein condensate, in which all the atoms approximately occupy the same state and can be described by the same wave function. In the considered situation the Hamiltonian \( H(t) \) is periodic in time

\[ H(t + T) = H(t). \] (4)

The Hamiltonian is varied slowly such that the adiabatic theorem can be applied. In a one-dimensional lattice and in a single particle picture, the wave function \( |\Psi_n^q(x)\rangle \) is described by the quasi-momentum \( q \) in the \( n \)th band and in cell periodic Bloch functions \( |n^q_n(x)\rangle = e^{-iqx} |\Psi_n(x)\rangle \). The quantity which characterizes the charge pump\(^1\) for neutral atoms is the center of mass \( < x > \), which corresponds to the polarization \( P = e < x > \) for charged particles with charge \( e \). We note that the quantity

\[ < x > = \int_{-\infty}^{\infty} x |\Psi(x)|^2 dx \] (5)

\(^1\)Although the atoms in the considered experiments are neutral, we still use the term “charge” pump to describe the motion of the neutral atoms. Most literature describes the situation in condensed matter physics, where the considered particles are electrons. In this spirit we keep the word "charge".
is usually ill-defined in a perfect crystal for either extended wave functions or periodic boundary conditions [15]. However in our experiment the definition in eq. (5) is a well defined quantity, since the trap limits the extend of the wave function $\Psi(x)$ and $\Psi(x)$ decays exponentially.

The mean center of mass position is related to the velocity operator $\hat{v}$ by $<x>(t) = \int_0^t \hat{v} \, dt'$. In momentum space\(^2\) the velocity operator is given by

$$\hat{v} = \frac{\partial H(q,t)}{\partial \hbar q}. \quad (6)$$

In first order perturbation theory the eigenstate evolves as

$$|u_n(q,t)\rangle = |u_n(q,t)\rangle - i\hbar \sum_{n'\neq n} \frac{<u_n^\prime(t)|\hat{v}|u_n(t)>}{\epsilon_n - \epsilon_{n'}}. \quad (7)$$

The expectation value for the velocity operator $\hat{v}$ is then given by

$$<\hat{v}(q)> = \frac{\partial \epsilon_n(q)}{\partial \hbar q} - \Omega_{q\ell}. \quad (8)$$

The first term in eq. (8) vanishes if the state $|u_n(q,t)\rangle$ stays at the minimum of the energy band\(^3\). For a single $q$ vector the charge pump is then given by\(^4\)

$$<x(t)> = -\frac{1}{2\pi} \int_0^t \Omega_{q\ell} \, dt'. \quad (9)$$

\[^{13}\] The charge pump mechanism is therefore probing the time averaged Berry curvature $\Omega_{q\ell}$ at quasi-momentum $q$, where $\frac{\partial \epsilon_n}{\partial q} = 0$.

2.3 Complementary view of the charge pump mechanism

For full cycles expression eq. (9) for $<x(T)>$ can be derived directly from the expression

$$<x> = <\Psi|\hat{x}|\Psi>. \quad (10)$$

We can express $|\Psi(x,T)\rangle$ in terms of the initial state $|\Psi(x,0)\rangle$, the dynamical phase $\phi_{dyn}$ and the geometrical phase $\phi_{Berry}$

$$|\Psi(x,T)\rangle = e^{i(\phi_{dyn} + \phi_{Berry})}|\Psi(x,0)\rangle. \quad (11)$$

In momentum space the expectation value of $x$ can be evaluated

$$<x(T)> = \frac{i}{2\pi} <\Psi(q,T)|\frac{\partial}{\partial q}|\Psi(q,T)> = -\left[ \frac{d\phi_{Berry}}{dq} + \frac{d\phi_{dyn}}{d\hbar q} + <\Psi(q,0)|\frac{\partial}{\partial q}|\Psi(q,0)> \right], \quad (12)$$

\(^2\)The velocity operator in momentum space is given by $\hat{v}(q) = e^{-iqx}\hat{v}e^{iqx} = e^{-iqx}\frac{\partial}{\partial q}[H,x]e^{iqx}$.

\(^3\)This situation applies in our experiment, see more sec. 4.2.

\(^4\)The factor $\frac{1}{2\pi}$ was added in order to obtain the correct normalization.
with the initial position value \( i < \Psi(q,0) \frac{\partial}{\partial q} |\Psi(x,0)\rangle \) and the dynamical contribution, which vanishes in the considered experimental situation. Leading to the expression

\[
<x(T)> = -\frac{d\phi_{\text{Berry}}}{dq} + <x(0)>
\]  

(13)

This expression is equivalent to eq. (9) (see appendix 8).

### 2.4 Artificial gauge fields for neutral atoms

Optical Raman lattices can be used to create lattice Hamiltonians, which mimic Hamiltonians found in condensed matter physics. The Hamiltonian engineered in our system resembles the Rice-Mele model (see sec. 2.4.1). The following paragraphs describe how light-atom interactions can give rise to an effective magnetic field coupling with the magnetic moment of the atoms. This magnetic interaction can be chosen such that it creates a lattice potential which depends on the spin state.

The Hamiltonian of an alkali atom in its ground state manifold without coupling to laser fields is given by

\[
H_0 = \frac{p^2}{2m} \mathbf{1} + H_{hf} + \frac{\mu_B}{\hbar} B \cdot (g_J \hat{J} + g_I \hat{I}) + H_Q,
\]

(14)

where \( \frac{p^2}{2m} \mathbf{1} \) is the kinetic energy; \( H_{hf} \) is the hyperfine splitting, which is a constant for the considered atoms\(^5\); the nuclear angular moment \( \hat{I} \) and the sum of the orbital \( \hat{L} \) and electronic spin \( \hat{S} \) angular moment \( \hat{J} = \hat{L} + \hat{S} \) with corresponding Landé g-factors. Finally, the quadratic Zeeman shift is given by

\[
H_Q = -\frac{\epsilon}{\hbar} (\hbar^2 \mathbf{1} - \hat{F}_z^2),
\]

(15)

with \( \epsilon/\hbar = \epsilon_B B^2 \), where \( \epsilon_B = 71.77 \text{ Hz}/\text{e}^2 \) and the z-component of the operator \( \hat{F} = \hat{J} + \hat{I} \). With the field strengths used in the experiment, the quadratic Zeeman shift can be treated as a small perturbation.

For alkali atoms the \( g_I \) factor is small compared to \( g_J \) and can be neglected. In a semi-classical treatment the atom-light interaction in the dipole approximation is given by

\[
H_L = \sum_{i,j} E_i \chi_{ij} E_j,
\]

(16)

with \( E_i \) the spatial components of the optical electric field and \( \chi_{ij} \) the atomic polarizability, which in general depends on the electronic angular momentum \( \hat{J} \). The polarizability \( \chi_{ij} \) can written in the form as the sum \( \chi_{ij} = u_s \delta_{ij} + i \frac{2\mu_B}{\hbar} J \epsilon_{ijk} \) of a irreducible scalar \( u_s \) with the Kronecker delta \( \delta_{ij} \), a vector \( u_v \) with the Levi-Civita tensor \( \epsilon_{ijk} \) and second rank tensor contribution, which can be neglected in the especially\(^5\) for the ground state of the considered rubidium atoms is in the total angular momentum \( F = 1 \), for which the sum of spin and the orbital momentum is \( J = 1/2 \) and the nuclear angular momentum \( I = 3/2 \).
case of rubidium. This leads to

\[ H_L = u_s E^* \cdot E + i \frac{u_v (E^* \times E)}{\hbar} \cdot \mathbf{j}, \]  

(17)

where the second term \( B_{\text{eff}} = i \frac{u_v (E^* \times E)}{\hbar} \) can be interpreted as an effective magnetic field coupling to the electronic angular momentum, which adds up with the real magnetic field in eq. (14). The first term describes a scalar potential \( V(x) = u_s E^* \cdot E \). With the chosen laser wavelength in the charge pump experiment, the scalar potential vanishes \( (u_s \approx 0) \) [16]. [17, 18]

Raman and radio-frequency Hamiltonian. The effective magnetic field \( B_{\text{eff}} \) can be engineered to generate a state dependent lattice Hamiltonian; depending on its \( m_f \) quantum number the atoms experience different potential energies. A pair of counterpropagating laser beams A and B with two different frequencies \( \omega \) and \( \omega + \delta \omega \) shine on the atom cloud. The difference in frequency \( \delta \omega \) is close to the Zeeman splitting \( \omega_Z \approx \delta \omega \), which is created by an external magnetic field \( B \). This provides a coupling between the different hyperfine states (see fig. 2). The detuning of the Raman coupling to the Zeeman splitting is denoted as \( \delta = \omega_Z - \delta \omega \). The Raman transition is a two-photon process. An atom can absorb a photon from beam A and re-emit a photon to beam B, obtaining a momentum kick of \( 2k_R \) with \( k_R \approx \frac{\omega_A/B}{c} \) with \( c \) the speed of light. A single Raman lattice has already been created [19], using one pair of counterpropagating Raman beams and one RF coupling field. A second pair of Raman beams “A/B” imparts an opposite momentum of \( -2k_R \) relative to the first pair and creates a topological non-trivial magnetic lattice (see sec. 6.2). As shown in fig. 3, polarizations of Raman beams in each pair are linearly polarized. The frequency of the RF source corresponds to the frequency difference of the Raman beams \( \omega_{\text{RF}} = \Delta \omega \). The effective magnetic field \( B_{\text{eff}} \) is created by the electric field \( E = E_1 + E_2 \) of the

Figure 2: The external bias field \( B = B_0 e_z \) creates a Zeeman splitting in the F=1 \(^{87}\)Rb state. [16]

Figure 3: Schematic set-up of the two Raman coupling A and B, the RF couplings and the external magnetic field \( B_z \).

\(^6\)The concept can be extended to non counterpropagating lasers.

\(^7\)Values in cold atom experiments are usually given in units of the single photon recoil momentum \( k_R = \frac{2\pi}{\lambda} \) and the recoil energy \( E_R = \frac{\hbar^2 k_R^2}{2m} \) with the wavelength \( \lambda \) of the Raman laser.
Raman beams (see eq. (17))

\[
E_1 = E_1 \left[ e^{i(k_R z - \omega t)} e_y + e^{-i(k_R x + (\omega + \delta \omega)t)} e_z \right],
\]

\[
E_2 = E_2 \left[ e^{-i(k_R x + \omega t)} e_x + e^{i(k_R x - (\omega + \delta \omega)t)} e_y \right],
\]

Both pairs of Raman beams are created by the same laser and the fiber lengths are kept at the same lengths, such that the relative phase between the two pairs of Raman beams vanishes. The magnetic field giving the RF coupling changes the splitting of the energy levels. We assumed a constant magnetic field in space, which is a valid approximation given the wavelength of the RF source \( \lambda_{RF} \approx 400 \) m. In the interaction picture together with the rotating frame approximation the Hamiltonian \( H = H_0 + H_L \) reads as

\[
H = \frac{\hbar^2 p^2}{2m} \mathbb{I} + \frac{1}{\sqrt{2}} \left\{ \Omega \cos(2k_R x) + \Omega_{RF} \cos \phi \right\} \hat{F}_x - \frac{1}{\sqrt{2}} \left\{ \delta \Omega \sin(2k_R x) + \Omega_{RF} \sin \phi \right\} \hat{F}_y + \delta \hat{F}_z + H_Q,
\]

with \( \tilde{\Omega} = \frac{\eta}{\kappa} (|E_1| + |E_2|) \) and \( \delta \Omega = \frac{\eta}{\kappa} (|E_1| - |E_2|) \). The RF coupling strength is denoted \( \Omega_{RF} \) and the phase between the RF- and Raman coupling \( \phi \). The Hamiltonian can be rewritten in the compact form \( H = \frac{\hbar^2 p^2}{2m} + \Omega \cdot \hat{F} + H_Q \). The Raman and RF lattice can be understood intuitively as follows: A two-photon Raman coupling changes the atomic momentum by \( \pm 2k_R \hat{e}_x \) and the internal state from \( |m_i > \rightarrow |m_{i+1} > \). The additional RF coupling affects only the spin degree of freedom \( |m_i > \rightarrow |m_{i+1} > \).

2.4.1 The Rice-Mele model

Our model Hamiltonian can be understood in the context of the Mele-Rice (without energy difference on site A and B it is also called the Su-Schrieffer-Heeger (SSH) model [20]).

**The Rice-Mele model** The Rice-Mele model was first studied in the context of Polyacetylene \( (C_2H_2)_n \) and describes a chain of dimers A-B (see fig. 4) [21]. The Hamiltonian of a chain of length \( M \) and hopping parameters \( t \) and \( t' \) is given by

\[
\mathcal{H} = \sum_{n=1}^{M} \left\{ t a_n^\dagger b_n + t' a_n^\dagger b_{n-1} + \text{h.c.} \right\} + \Delta \sum_{n=1}^{M} \left\{ a_n^\dagger a_n - b_n^\dagger b_n \right\},
\]

where \( a_n^\dagger \) creates a particle on-site A and \( b_n^\dagger \) respectively on site B and \( \Delta \) is the energy offset between the A and B side. The creation and annihilation operators fulfill the standard commutation relation:

\[
[a_i, a_j^\dagger] = \delta_{i,j}, [b_i, b_j^\dagger] = \delta_{i,j}, \text{and all other combinations } [a_i^{(1)} / b_i^{(1)}, a_j^{(1)} / b_j^{(1)}] = 0
\]

**Mapping to the Rice-Mele model** The double Raman and RF Hamiltonian can be mapped to the Rice-Mele model in the limit of \( \tilde{\Omega} >> \delta \Omega, \tilde{\Omega} >> \Omega_{RF} \), \( \delta \approx 0 \) and in the \( x \) basis for the spins, denoted as \( |m_x > \). The two distinct site population A and B correspond to spin up and down in the experiment, while \( m_x = 0 \) is neglected. The \( \Omega_x \) term creates a lattice potential for the spin up and down spin state, while the \( |m_x = 0 > \) state is unaffected by the lattice. The spin nature of the lowest
Figure 5: The magnetic lattice in the Rice-Mele model with spin up and down sites, two distinct hopping parameters $t$ and $t'$ and energy difference between the spin up and down sites $\Delta$.

state depends on $\Omega_{RF} \cos(\phi)$ and is either a spin up or a spin down state, which leads to the on-site energy difference

$$\Delta = \Omega_{RF} \cos \phi.$$  \hfill (23)

By introducing Wannier functions for the up $|\phi_\uparrow(x)\rangle$ and down $|\phi_\downarrow(x)\rangle$ states, one obtains the hopping parameters

$$t \approx \frac{<\phi_\uparrow(x)|\Omega_y \hat{F}_y|m_x = 0><m_x = 0|\Omega_y \hat{F}_y|\phi_\downarrow(x)>}{\Delta}$$ \hfill (24)

$$t' \approx \frac{<\phi_\downarrow(x)|\Omega_y \hat{F}_y|m_x = 0><m_x = 0|\Omega_y \hat{F}_y|\phi_\uparrow(x)>}{\Delta},$$ \hfill (25)

with the translation invariant $|m_x = 0\rangle$ state.

In sec. 6.2 we calculate the Zak phase, which is the same as for the SSH model, proving the topological equivalence.

3 Experimental set-up

3.1 Laser cooling

Laser cooling for alkali atoms is performed in three stages. The Rb beam source is slowed down by a Zeeman-slower. Furthermore the atoms are trapped and cooled down in a magneto-optical trap (MOT). In the last step the atoms are cooled down further by evaporative cooling and a phase transition to a Bose-Einstein condensate (BEC) occurs at the order of nanokelvins.

3.2 Traps

During evaporative cooling the atoms are trapped by a crossed dipole trap, which is essential to the charge pump experiment. Dipole trapping is caused by a spatially dependent laser field. The interaction between atoms and the laser field can be described by a two level system and in the dipole approximation. In second order perturbation theory and within the RWA approximation, one obtains a potential seen by the atoms

$$U(r) = \frac{3\pi c^2}{2\omega_c^3 \Delta} I(r),$$ \hfill (26)
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with the speed of light $c$, the dipole matrix element $\Gamma$, the detuning $\Delta^8$ and the spatial intensity profile of the laser $I(r)$. And a scattering rate $\Gamma_{SC}$

$$\Gamma_{SC} = \frac{3\pi c^2}{2\hbar \omega_c^2} \left( \frac{\Gamma}{\Delta} \right)^2 I(r)$$  \hspace{1cm} (27)

[22]. In order to prevent heating, the scattering needs to be minimized, which can be obtained by a far detuned laser field. For a Gaussian beam profile and a red detuned laser ($\Delta < 0$), as used in the experiment, the potential can be approximated by a harmonic trap. A trap with variable trapping potential in the $x - y$ plane can be created by a crossed-beam dipole trap created by two parallel polarized lasers. [16]

In the experiment a laser beam of $\lambda = 1064\,\text{nm}$ is used to create a harmonic trap along the lattice direction

$$V(x) = \frac{1}{2} m\omega_{trap}^2 x^2.$$  \hspace{1cm} (28)

The oscillation of the condensate in the direction of the lattice was measured and it corresponds to a trapping frequency $\omega_{trap} = 90\,\text{s}^{-1}$.

3.3 Imaging techniques

Real space images  The imaging is based on resonant absorption imaging. Light with intensity is shining on the atom cloud and a camera registers the intensity behind the atomic cloud $I_{\text{out}}$. In the case of low intensity imaging, $I_{\text{out}}$ is related to the atomic density via Beer’s law:

$$I_{\text{out}}(x,y) = I_0 e^{-\sigma \int |\Psi(x,y,z)|^2 dz},$$  \hspace{1cm} (29)

with the resonant absorption cross-section $\sigma$. In order to obtain an image which does not destroy the atom cloud in the $F = 1$ state, imaging is performed in the $F = 2$ state. A microwave pulse is used to transform approximately 5% of the atoms to the $F = 2$ state, where the imagine process is applied. A absorption laser is resonant to the $F = 2 \rightarrow F' = 3$ transition ($^5S_{1/2} \rightarrow ^5P_{3/2}$). In this sense, it can be seen as a non-destructive measurement, since the 95% of the atoms are unaffected. The final image is obtained by subtracting a background image, containing the contribution from room light. Probe light passing through the atomic cloud is collected by a set of lenses with a magnification of 6 and imaged by a CCD camera with a pixel size of 5.6 $\mu\text{m}$ (see fig. 6). This gives 0.93 $\mu\text{m}$ in the atom plane, which corresponds to a resolution of approximately two lattice sites. The resolution sets a limit for the sensitivity of the displacement in the charge pump measurement. In previous experiments it has been shown that a sensitivity of approximately 0.3 $\mu\text{m}$ root mean square (rms) can be achieved [23]. This experiment was conducted in the low interacting regime. The ground state for a non-interacting harmonic oscillator is a Gaussian profile. The image taken represents the density distribution integrated

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In this section $\Delta$ is assigned to the detuning, and should not be confused with the on-site energy difference in the Mele-Rice model. With this choice we follow the standard notation.
over the direction in which the light is shined (z-direction). In the case of an assumed 3D Gaussian distribution, the integrated image is still a 2D Gaussian distribution centered around \( x_0, y_0 \) and the width \( \sigma_x, \sigma_y \). The atomic density \( \rho(x, y) \) is then given by

\[
\rho(x, y) = |\Psi(x, y)|^2 \approx \frac{1}{\sqrt{2\pi\sigma_x^2\sigma_y^2}} e^{-\frac{(x-x_0)^2}{2\sigma_x^2} - \frac{(y-y_0)^2}{2\sigma_y^2}}. \tag{30}
\]

In order to obtain the center of the cloud, we fit a Gaussian 2D distribution.

**Momentum space and spin population measurements**

In order to measure the spin population and the momentum distribution, the time-of-flight (TOF) technique is used, where the dipole trap and the lattice is switched off and the atom cloud expands during 19 ms. The quasi-momentum \( q \) of the lattice is projected onto the momentum \( k \) of a free particle. For sufficiently long times \( t \), after which the initial size of the cloud can be neglected, an absorption image can be taken, which together with the time \( t \) provides the information about the quasi-momentum \( q \). The image technique is implemented as a destructive measurement. It is also taken in the \( F = 2 \rightarrow F' = 3 \) transition and for this purpose a repump laser is employed to pump atoms out of the F=1 dark state. A different CCD-camera is used, such that the information in real and momentum space can be obtained simultaneously.

In the charge pump experiment, the momentum space measurement are used to verify the adiabaticity of the process, by monitoring the higher lattice orders.

In the charge pump experiment the momentum space measurement is combined with a Stern-Gerlach field. Due to an external magnetic field the three spin components are separated spatially. The magnetic field is aligned such that the two-dimensional TOF image contains the information about the spin projection in one direction and about the 1D momentum in the perpendicular direction (see fig. 7).

4 **Numerical calculations**

4.1 **Raman and RF Hamiltonian in momentum space**

The Hamiltonian eq. (20) describes a lattice Hamiltonian and the notion of Bloch waves can be introduced. For sufficiently large \( \bar{\Omega} \) the Hamiltonian (20) can be seen as a one dimensional (1D) magnetic lattice Hamiltonian. In a one-dimensional lattice the wave function \( \Psi^q_n \) is described by the quasi-momentum \( q \) in the \( n \)th band in cell periodic Bloch functions \( u^q_n \). The Bloch functions \( u^q_n \) share the same periodicity as the Raman and RF Hamiltonian eq. (20) and can be therefore expanded in a Fourier series \( u^q_n = \sum_i a_ie^{2ikRx} \). Inserting the Fourier series into the stationary 1D Schrödinger equation

\[
H\Psi^q_n = E_n\Psi^q_n
\]

Figure 7: Momentum space image with spin degree of freedom and some population in the higher orders at \( \pm 2kR \).
one obtains an expression which must hold without loss of generality for all \( l \) and the Hamiltonian can be rewritten in the matrix representation in the basis \( \{|l, m_f >\} \). For low energies the Hamiltonian can be expanded around \( l = 0 \). For \( l \in \{-1, 0, +1\} \) the Hamiltonian is written in appendix sec. 8. By diagonalizing the Hamiltonian the time evolution is given by

\[
|\Psi_{m_f,l}(t)> = \sum_n e^{-i\frac{\hbar}{\epsilon}E_n}|n><n|\Psi_{m_f,l}(0)>,
\]

(32)

with the spin degree of freedom \( m_f = \{+1, 0, -1\} \), for which we use the quantization axis set by the bias field \( B = B_0 e_z \), the eigenenergies \( E_n \) and eigenstates \( |n> \). Population evolution predicted from eq. (32) can be used to compare with the experimental data for calibrating the parameters (see sec. 5.1).

4.2 Band structure of the Hamiltonian

We calculated the eigenvalues of the momentum space Hamiltonian eq. (47) \( E_n(q) \) (see fig. 8). An important quantity for the charge pump mechanism is the relative phase \( \phi \) between the Raman and RF coupling (see sec. 4.3). Fig. 8 shows that independently of the phase \( \phi \), the derivative of the lowest band at \( q = 0 \) is zero, giving no dynamic phase contribution to the charge pump. It depends entirely on the Berry curvature, the geometry of the underlying parameter space (see eq. (8)).

![Figure 8: Band structure for different values of \( \phi \) during the ramping scheme.](image)

4.3 Numerical calculation of the Berry curvature

For numerical purposes an equivalent definition of the Berry phase is useful. The difference in Berry phase \( \Delta \phi_{Berry} \) between two neighboring points \( R_i \) and \( R_{i+1} \) is given by

\[
e^{i\Delta \phi_{Berry}} = \frac{\Psi(R_i)|\Psi(R_{i+1})>}{|\Psi(R_i)|\Psi(R_{i+1})>},
\]

(33)

\[
\Rightarrow \Delta \phi_{Berry} = -\text{Im} \left[ \ln \left( \frac{\Psi(R_i)|\Psi(R_{i+1})>}{\Psi(R_i)|\Psi(R_{i+1})>} \right) \right].
\]

(34)

[15] This concept can be generalized to the Berry-curvature

\[
\Omega_{q_1t_2} = \frac{\partial A_{t_2}}{\partial q_1} - \frac{\partial A_{q_1}}{\partial t_2} = -\frac{1}{\Delta q \Delta t} \text{Im} \left[ \ln \left( \frac{<u_{q_1}(t_2)|u_{q_1}(t_2+1)>}{<u_{q_1}(t_1)|u_{q_1+1}(t_1+1)>} \right) - \ln \left( \frac{<u_{q_1+1}(t_1)|u_{q_1+1}(t_1+1)>}{<u_{q_1}(t_1)|u_{q_1}(t_1+1)>} \right) \right],
\]

(35)
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Figure 9: a) Normalized berry curvature $\Omega_{q=0,\phi}$ for $\Omega_{rf} = 2.2 E_R$ and fixed $\bar{\Omega} = 6 E_R$. Arrows correspond to the experimentally realized trajectories: $\delta \Omega/\bar{\Omega} = \pm 0.71$, $\delta \Omega/\bar{\Omega} = \pm 0.31$ and $\delta \Omega/\bar{\Omega} = \pm 0$. b) Integrated Berry curvature during per cycle (displacement) $\Delta X$ during one cycle as a function of $\delta \Omega/\bar{\Omega}$ at fixed $\Omega_{RF} = 2.2 E_R$.

with the discrete point in time $t_j$ and point in momentum space $q_i$ and resulting grid size in momentum space $\Delta q$ and the time step $\Delta t$.

The Hamiltonian $H$ is $2\pi$-periodic in $\phi$, which allows for continuous charge pump mechanism by changing $\phi(t)$ linearly in time. The integrated Berry curvature $\int \Omega_{q,\phi} dt$ can be rewritten in terms of $\phi$

$$-2\pi \Delta X = \int_0^\phi \Omega_{q,\phi'} d\phi'.$$

We calculated the the Berry curvature for a condensate at $q = 0$ with eq. (35). The eigenstates of the lowest band were found by diagonalizing the Hamiltonian in momentum space eq. (47) (see section 4.1) including 10 momentum states for each of the 3 spin states. The Berry curvature corresponding to the parameters used in the experiment is shown in fig. 9 a). In the largest imbalance case $\delta \Omega/\bar{\Omega} = \pm 1$ the Berry curvature is a constant and corresponds to the single Raman case with a sliding lattice. For $|\delta \Omega/\bar{\Omega}| < 1$, the lattice corresponds to a double-well lattice with changing minima similar to the Rice-Mele model. For the case of balanced Raman coupling $\delta \Omega/\bar{\Omega} = 0$, the Berry curvature and the displacement per cycle is zero and no motion is expected, even though the local minima in real space change position as a function of the phase (see appendix 8). These calculation were used to predict the appropriate choice of parameters for the experiment (see fig. 9 b) and appendix fig. 20).

4.4 Quantum transport in a classical picture of a sliding lattice

The quantum transport by means of the presented charge pump mechanism can be understood in a more intuitive classical picture of a sliding lattice. However the classical picture of a sliding lattice cannot provide the full picture to understand the quantum transport. In the limit of a single Raman and RF coupling ($\bar{\Omega} = \delta \Omega$) the Hamiltonian eq. (20) leads to a sliding of the lattice, where the spin nature of the minima change during one cycle and the atom would be transported by one lattice
site per cycle (see appendix sec. 8). The lattice with double Raman and RF coupling shows a more complicated behavior with two minima within each lattice site (see Rice-Mele model sec. 21) and a sliding of the lattice minima. Due to the Hamiltonian’s periodicity, the lattice is equivalent to the initial lattice displaced by one lattice site after one cycle. In a classical picture one might then expect a motion which is an integer of the lattice sites. This does not correspond to the quantum mechanical picture, where the displacement per cycle can be less than 1 lattice site (see fig. 9 b).

4.5 Simulation of the time evolution

In order to take into account possible non-adiabatic effects and the influence from the trap, we simulated the behavior of the atoms under the cyclic change of Hamiltonian. We simulated the evolution of the condensate in real space in a single particle picture without taking temperature effects into account. This is a workable approximation in our situation for weakly interacting Bose-Einstein condensate (BEC). Our simulations take into account the spin degree of freedom of the condensate in the \( f=1 \) manifold, consisting of three states \( m_f = -1, 0, +1 \). For this purpose the code from the M1 internship [24] was extended to the charge pump Hamiltonian with spin degree of freedom. We propagated the wave function with spin and in 1D in the position representation \( |\Psi_{m_f}(x,t)\rangle = \hat{U}(t)|\Psi_{m_f}(x,0)\rangle \) by the time evolution operator

\[
\hat{U}(t) = \mathcal{T} \exp \left( -\frac{i}{\hbar} \int_0^t H(\tau) d\tau \right)
\]

(37)

with the time-ordering operator \( \mathcal{T} \) and the Hamiltonian \( H \) given in eq. (20). For an efficient calculation of the operator exponential in eq. (37) the split operator method [25] was used

\[
e^{(A+B)\Delta t} \approx e^{\frac{\Delta t}{2} A} e^{\Delta B} e^{\frac{\Delta t}{2} A},
\]

(38)

for two operators \( A, B \) and a small time step \( \Delta t \). The split operator method was used to break the exponential \( e^{-\frac{i\Delta t}{\hbar} H(t)} \) into two parts; part A with the light-atom interaction and the trapping potential \( H - \frac{p^2}{2m} + V(x) \) and part B with the kinetic energy \( \frac{p^2}{2m} \). The exponential of the kinetic energy was calculated in momentum space, where it is diagonal and part B in real space. Part B is non-diagonal in the spin degree of freedom and a diagonalization of the spin part at each time step and point in space was used. The real space was discretized to 20 grid points within each unit cell, which was sufficient to reconstruct the spin and space dependent Wannier functions (see fig. 10).

Imaginary time evolution We started the simulations from the ground state of the Hamiltonian, which was found by imaginary time evolution. Imaginary time evolution is a numerical tool to find the ground state of a Hamiltonian. It starts from an arbitrary state with non-zero overlap with the actual ground state\(^9\). In the time evolution operator (eq (37)) the time is replaced by the imaginary time \( \tau = it \), which is equivalent to a Wick rotation. This evolution is non-unitary and does not conserve the probability. In order to conserve the probability the wave function was normalized after each imaginary time step. The imaginary time evolution damps each state exponentially with the weight of its energy.

\(^9\)In practical, this was assured by starting from a state with a random real and imaginary part in each spin degree of freedom and at each point in space. The possibility that such a state has non-zero overlap is practically zero.
Figure 10: The wavefunction $|\Psi_{m_x}(x)\rangle$ in real space representation and in the $|m_x\rangle$ basis, forming Wannier functions in the double Raman and RF lattice. The trapping potential confines the wavefunctions ($\bar{\Omega} = 6E_R$, $\delta\Omega = 4.5E_r$, $\Omega_{RF} = 2.2E_R$ and $\phi = 0$).

Therefore the evolution converges to the ground state with the lowest energy

$$|\Psi(x, \tau)\rangle = \sum_{n=0} e^{-E_n \tau/\hbar} |n\rangle < n|\Psi(x, 0)\rangle \xrightarrow{\tau \to \infty} |0\rangle,$$

with $\{|n\rangle\}$ the eigenbasis and $E_n$ the eigenvalues of the Hamiltonian $H$.

**Simulation of the charge pump process**  We simulated the charge pump mechanism as previously described by the split operator method. As shown in fig. 11 a), we found that adiabatic pumping cycles, which are fast compared to the trap period, show a good agreement with the theoretical prediction from the integrated Berry curvature theory (eq. (9) and fig. 9). We found that for $\delta\Omega/\bar{\Omega} = 0.7$ and $\Omega_{RF} = 2.2^{10}$ the fastest ramping cycle can be implemented within $\approx 2$ ms and adiabaticity is still maintained. The displacement per cycle become less as the condensate moves up the trap and starts to oscillate in the trap. The simulations show that the condensate can move up to 12 lattice site (see fig. 11). For lower ratios of $\delta\Omega/\bar{\Omega}$ the band gap at $q = 0$ drops and the pumping cycle needs to be slower in order to maintain adiabaticity (see appendix table 1).

**5 Experimental results**

As discussed in sec. 2.4 we implemented the system with the double Raman and RF Hamiltonian (eq. 20). Experimentally we control the Raman and RF couplings $\bar{\Omega}$, $\delta\Omega$, $\Omega_{RF}$ by the intensity of the lasers and of the RF source. The time dependence of the relative phase $\phi(t)$ between the RF and the Raman coupling is introduced by adding a small shift in frequency $\delta\omega$ to the RF field

$$\omega_{RF} = \Delta\omega + \delta\omega_{RF},$$

$^{10}$These values are comparable with the range set by the experiment.
Figure 11: a) Comparison between the theoretical prediction of the charge pump $\Delta X = -\frac{1}{2\pi} \int_0^\Omega \delta \Omega \phi d\phi$ and simulations (points) for one cycle and different trajectories in the Hamiltonian parameter space (the trajectories do not only correspond to straight lines in fig. 9, they also include circles in the $\delta \Omega/\bar{\Omega} - \phi$ plane and different values of $\Omega_{RF}$).

b) Center of mass motion ($< x >$) for adiabatic cycle of $\phi$ within 2 ms and $\delta \Omega/\bar{\Omega} = 0.7$, $\Omega_{RF} = 2.2 E_R$ and $\omega_{trap} = 90 \text{s}^{-1}$. For longer time periods the influence of the trap limits the displacement induced by the charge pump and the condensate starts to oscillates.

which effectively creates a time dependent phase $\phi(t) = \delta \omega_{RF} \cdot t$ and a cyclic change in the Hamiltonian parameters with period $T = \frac{2\pi}{\delta \omega_{RF}}$.

Spin basis As described in sec. 21 the magnetic lattice Hamiltonian eq. 20 corresponds to a lattice in the $m_x$ basis. The bias field $B = B_0 e_z$ sets the quantization axis of the laboratory basis $|m_z>$ (see fig. 3). In order to obtain images in the $|m_x>$ basis, we apply a $\pi/2$-pulse. We observed that applying a RF pulse with phase of $\pi/2$ and with the same strength as the lattice RF coupling effectively rotated the three-level system into the $|m_x>$ basis. At the same time as applying the RF pulse, the Raman coupling was strongly reduced.

5.1 Calibration of the parameters

The coupling constants The first experimental step was the calibration of the Hamiltonian’s parameters $\bar{\Omega}$, $\delta \Omega$, $\Omega_{RF}$, $\delta$, $\epsilon$ and $\phi$. In the preparation phase the condensate was loaded at $q = 0$ to the $|m_z = 0>$ state and at $t = 0$ the coupling fields were instantaneously turned on. After a certain pulse time the coupling fields were abruptly switched off and the Stern-Gerlach imaging was used in order to measure the population in the different $|m_z>$ states. We observed population up to the order of $\pm 4k_R$. The coupling parameters were calibrated individually, by only pulsing on either the Raman or the RF coupling, which shows regular Rabi oscillations (see fig 12 a)). Pulsing on both Raman and RF coupling creates a complicated evolution in the spin degree of freedom. By fitting the Hamiltonian evolution of eq. (32) to the experimental data the values for the parameters in the Hamiltonian (eq. (20)) were obtained.

Calibration of the phase The phase $\phi$ between the RF and the Raman coupling is crucial for the charge pump experiment. $\phi(t)$ is controlled dynamically during the experiment by eq. (40). The value
Figure 12: Pulsing data for Raman and RF coupling. The Hamiltonian evolution eq. (32) has been fitted to the data. a) Only Raman coupling is switched on; showing regular Rabi oscillations. b) Both Raman and RF coupling is switched on, which leads to a complicated evolution; here given for $\phi = 0$.

Figure 13: a) Control of the absolute phase between the Raman and RF coupling with $\bar{\Omega} = 4.4 E_R$, $\delta \bar{\Omega} = 0 E_R$ and $\Omega_{RF} = 2.2 E_R$. The small figures show the corresponding lattice structure for $\phi$ in real space.

b) Hamiltonian Parameters during the charge pump experiment. The first 2 ms correspond to the loading of the lattice.
of the initial value of \( \phi \) is controlled by an RF-mixer. However the absolute value compared to the Raman beams is a priori not known. The spin nature of the ground state of the lattice depends directly on the staggered field \( \Omega_{RF} \cos(\phi) \) (see sec. 21). By adibatically loading the lattice at different values of \( \phi \) the absolute value of \( \phi \) was calibrated (see fig. 13 a)).

5.2 Charge pump experiment

**Loading of the lattice** After the evaporative cooling stage, the atoms need to be adiabatically loaded to the Raman and RF lattice. During the cooling process the quantization axis is set by the bias field in z-direction. The bias field can be controlled by the current in the coils, which effectively controls the detuning \( \delta \). The lattice is loaded adiabatically by ramping down the detuning \( \delta \) linearly to zero and simultaneously ramping up the linearly the coupling strengths to its final value (see first 2 ms in fig. 13 b)). During this phase of the lattice loading the band structure does not have a minimum at \( q = 0 \) (see appendix fig. 19), which induces some motion during the state preparation. However this motion is in the order of 1-2 lattice sites \( a \). By starting from a condensate in the \( m_f = 0 \) state this motion could be avoided, however the lattice Hamiltonian needs to be prepared more slowly in order to maintain an adiabatic loading scheme.

**Charge pump** One cycle of the charge pump consists of ramping the phase \( \phi(t) = \delta \omega_{RF} \cdot t \) linearly from 0 to \( 2\pi \). During the pumping the condensate changes from \( |m_x^c| = -1 \) to \( |m_x^c| = +1 \) and back (see fig. 15). The maximum speed in order to maintain adiabaticity was obtained from simulations (see sec. 4.5) and depends on \( \Omega_1 \), \( \Omega_2 \) and \( \Omega_{RF} \). The charge pump motion was observed with the real space images (see sec. 3.3) and the center of mass was determined from a Gaussian fit and further averaged over several runs. The error of the center of mass was deduced from fitting errors and the errors are plotted with 2 standard deviations in positive and negative direction. The position of the condensate before loading the lattice was detected in the same way by a non-destructive real space measurement as described in sec. 3.3.

As shown in fig. 14, we show the displacement in real space versus pumping duration. The direction of the motion depends on the sign of \( \frac{d\phi}{dt} \) and the ratio of \( \Omega_1 \) \( \Omega_2 \). For \( \frac{\Omega_1}{\Omega_2} > 0 \) and \( \frac{\Omega_1}{\Omega_2} < 0 \) the charge pump changes direction. Due to the confinement of the trap, the displacement per cycle saturates above \( \approx 10 \) cycles, which is consistent with the simulations, overlapped with the data in fig. 16 a). The Simulations do not start at \( x = 0 \) due to the dynamical motion introduced during the loading of the lattice (see sec. 5.2). In order to improve the signal to noise ratio, we averaged the displacements in 15 realizations. The data sets in fig. 16 contain different lattice loading scheme. In one third of the data the detuning \( \delta \) was ramped from initial \( 5E_R \) to \( 0 E_R \) during the loading of the lattice. In two thirds of the data the detuning was ramped from initial \( 2.5E_R \) to \( 0 E_R \). This results

![Figure 14: Charge pump experiment for \( \delta \omega_{RF} = \pm 2\pi \) 500 s\(^{-1}\) and \( \delta \omega_{RF} = 0 \) s\(^{-1}\) with linear fit. Even for a constant Hamiltonian a drift due to experimental imperfection is visible.](image-url)
Figure 15: Population during the charge pump mechanism at the beginning and the end of the pumping mechanism. The pumping mechanism stayed adiabatic for at least 110 ms. The Data was taken with the Stern-Gerlach splitter.

Figure 16: a) Experimental charge pump for different ratios of $\Omega_1 / \Omega_2$ and different $d\phi / dt$ compared with results from simulations (continuous line). The different starting points are due to the lattice loading which induces a dynamical motion in the order of 1-2 lattice sites. Negative phases correspond to negative $\delta \omega_{RF}$.

b) Experimental displacements obtained from fits in the linear regime compared to the theoretical displacement. The numbers corresponds to the different trajectories shown in fig. 9 a).
in a small difference in the starting point of the charge pump measurement. The results shown for the
simulation in fig. 16 a) consist of a weighted average of these two different loading schemes.

As a general problem, it was observed that the condensate was not entirely stable during the
charge pump experiment due to experimental imperfections. Even for $\frac{d\phi}{dt} = 0$ (constant Hamiltonian)
the condensate moved in the trap (see fig. 14). This motion has not been observed previously and it is
expected to be related to experimental imperfections. We expect that this instability is also one of the
reason why we were not able to achieve the previously achieved sensitivity of 0.3 $\mu$m rms. Hereinafter,
we subtract the motion for a constant Hamiltonian as a reference point.

**Displacement per cycle** The displacement of the charge pump can be changed by the ratio of
$\delta\Omega/\bar{\Omega}$\textsuperscript{11}. It can be varied approximately between $-1$ and $+1$ (see fig. 16 b). From the simulations of the
time evolution, we deduced that for smaller ratios $\delta\Omega/\bar{\Omega}$, $\phi(t)$ needs to be varied more slowly in order
to maintain adiabaticity (see appendix table 1). We choose additional trajectories at $\delta\Omega/\bar{\Omega} = \pm0.31$ and
$\delta\Omega/\bar{\Omega} = 0$.

**Comparison with the Berry curvature** For small displacements and short time periods the results
are consistent with the integrated Berry curvature. By fitting the measured displacement in the linear
regime, we extracted the displacement $\Delta X$ per cycle. Fig. 16 b) shows the experimental displacements
per cycle compared with the theoretical displacement calculated as in sec. 4.3. The experimental results
show good agreement with theoretical predictions, confirming the geometrical origin of the observed
charge pump mechanism.

## 6 Outlook

### 6.1 Resolving the steps of the charge pump mechanism

Recently performed experiments [8] were able to reveal the quantized motion. Fig. 17 a) shows the
motion in the $|m_x = -1 >$ state, deduced from simulations (see sec. 4.5). The size of the plateau is
within the reach of the previous experimental resolution of 0.3 $\mu$m [19], however the current resolution
due to experimental imperfections, does not allow the resolution of the plateaus.

### 6.2 Filled bands and the Zak-phase

The spin pump experiment for single $q$ also enables the possibility of spin pump experiments with filled
bands. For filled bands the motion is predicted to be topological protected number, the Zak phase and
the displacement per Hamiltonian cycle is equal to one lattice site [13]. For a filled band the Berry
phase across the Brillouin zone can be defined, which is the Zak phase $\phi_{Zak}^n$\textsuperscript{12} in a band

$$\phi_{Zak}^n = i \int_{BZ} <u_q^n|\nabla_q|u_q^n> dq,$$

\textsuperscript{11}We found that the dependency on $\Omega_{RF}$ within the experimental range is small.
\textsuperscript{12}Note that the value of the Zak phase depends on the choice of the unit cell.
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Figure 17: a) Steps of the spin pump mechanism. The parameters correspond to the parameters in path 1 (see fig. 9)

b) The Zak phase in dependency of the angle $\phi$ and the radial axes $\Omega_{rf}$.

and characterizes the charge pump

$$\Delta X = -\frac{1}{2\pi} \int_0^T dt \int_{\text{BZ}} dq \Omega_{qt} = \frac{1}{2\pi} \int_0^T dt \int_{\text{BZ}} dq \{ \partial_q A_t - \partial_t A_q \} = -\frac{1}{2\pi} \{ \phi_{Zak}(T) - \phi_{Zak}(0) \}. \quad (42)$$

The first term $\partial_q A_t$ gives zero under the periodic gauge of the Berry connection $A_t$, by the integration over whole Brillouin zone [13].

We calculated the Zak phase for the lowest band of the magnetic lattice Hamiltonian (eq. (20)) and the Zak phase changes by $2\pi$ for the same trajectory as used for single $q$ (see fig. 17 b). Topological spin pumps for filled bands occur when encircling a singularity in the Zak-phase. This singularity can be seen in fig. 17 b) for $\Omega_{RF} = 0$. Experimentally filled bands have been already achieved for different lattices by displacing the trap for some milliseconds.

7 Conclusion

We experimentally established for the first time a geometrical charge pump. In contrast to topological charge pumps, it leads to a variable displacement. It was implemented in a new type of bipartite magnetic lattice by dressing Rb atoms with two Raman and one RF coupling fields. For small displacements the results are in good agreement with prediction based on the integrated Berry curvature (see fig. 16 b). For longer pumping times the effect of the trap limits the displacement and the results are in good agreement with dynamical simulations (see fig. 16). Our results are comparable with similar experimental results in superlattices [7, 8], which have been obtained almost simultaneously. In the future, the presented geometrical charge pump could be turned into a topological charge pump by working with filled bands. The condensed matter analog of the presented geometrical charge pump would be a 1D conductor with partially filled bands, allowing a precise control of the electron displacement.
8 Appendix

Complementary view of the charge pump mechanism

The derivative of the Berry phase is equivalent to the integral over the Berry connection

\[- \frac{d\phi_{\text{Berry}}}{dq} = -i \frac{d}{dq} \int_0^T dt \langle u_q(t) | \frac{\partial}{\partial t} | u_q(t) \rangle \quad (43)\]

with an integration by parts, one obtains

\[- \frac{d\phi_{\text{Berry}}}{dq} = -i \int_0^T \left[ \langle \frac{\partial u_q(t)}{\partial q} | \frac{\partial u_q(t)}{\partial t} \rangle > - \langle \frac{\partial u_q(t)}{\partial t} | \frac{\partial u_q(t)}{\partial q} \rangle > \right] dt - \frac{i}{2\pi} \langle u_q | \frac{\partial}{\partial q} | u_q > \mid_0^T (44)\]

we obtain the same expression as in eq. (9)

\[- \frac{d\phi_{\text{Berry}}}{dq} = - \frac{1}{2\pi} \int_0^T \Omega_q dt, \quad (45)\]

where the factor \(\frac{1}{2\pi}\) was added in order to obtain the correct normalization and the additional term \(- \frac{i}{2\pi} \langle u_q | \frac{\partial}{\partial q} | u_q > \mid_0^T\) is zero for full cycles in time. This can be seen as follows, by rewriting the derivative as

\[- i \langle u_q | \frac{\partial}{\partial q} | u_q > \mid_0^T = \lim_{\epsilon \to 0} \frac{\langle u_q | u_{q+\epsilon} > \mid_0^T - 1}{\epsilon} (46)\]

At \(H(t)\) and \(H(t + T)\) the overlap between between two neighboring states \(\langle u_q | u_{q+\epsilon} >\) in the case of non-degenerate bands needs to be the same at \(t\) and \(t + T\) due to the Hamiltonian periodicity.
Lattice potential

![Figure 18](image-url)

Figure 18: Lattice potential in real space for a) a single Raman lattice $\frac{\delta \Omega}{\Omega} = 1$, b) standard setting used during the charge pump experiment $\frac{\delta \Omega}{\Omega} = 0.75$ c) balanced case $\frac{\delta \Omega}{\Omega} = 0$. 
Double Raman and RF lattice Hamiltonian in momentum space

Up to first order \( l \in \{-1, 0, 1\} \) the Hamiltonian can be written as Hamiltonian \( H = \)

\[
\begin{pmatrix}
\frac{\hbar^2}{2m} (q + 2kR)^2 + \delta & \frac{\Omega_{BR}}{2} e^{i\phi} & 0 & 0 & \frac{\Omega_r}{2} & 0 & 0 & 0 & 0 \\
\frac{\Omega_{BR}}{2} e^{-i\phi} & \frac{\hbar^2}{2m} (q + 2kR)^2 - \epsilon & \frac{\Omega_{BR}}{2} e^{i\phi} & 0 & \frac{\Omega_r}{2} & 0 & 0 & 0 & 0 \\
0 & \frac{\Omega_{BR}}{2} e^{-i\phi} & \frac{\hbar^2}{2m} (q + 2kR)^2 - \delta & 0 & \frac{\Omega_r}{2} & 0 & 0 & 0 & 0 \\
0 & \frac{\Omega_r}{2} & 0 & \frac{\hbar^2}{2m} q^2 + \delta & \frac{\Omega_{BR}}{2} e^{i\phi} & 0 & 0 & \frac{\Omega_r}{2} & 0 \\
0 & 0 & \frac{\Omega_r}{2} & \frac{\hbar^2}{2m} q^2 - \epsilon & \frac{\Omega_{BR}}{2} e^{-i\phi} & \frac{\Omega_r}{2} & 0 & 0 & \frac{\Omega_r}{2} \\
0 & 0 & 0 & \frac{\hbar^2}{2m} (q + 2kR)^2 + \delta & \frac{\Omega_{BR}}{2} e^{i\phi} & 0 & 0 & \frac{\hbar^2}{2m} (q + 2kR)^2 - \delta & \frac{\Omega_{BR}}{2} e^{-i\phi} \\
0 & 0 & 0 & 0 & \frac{\hbar^2}{2m} q^2 - \delta & \frac{\hbar^2}{2m} (q + 2kR)^2 - \delta & \frac{\hbar^2}{2m} (q + 2kR)^2 - \delta & \frac{\hbar^2}{2m} (q + 2kR)^2 - \delta & \frac{\hbar^2}{2m} (q + 2kR)^2 - \delta \\
0 & 0 & 0 & 0 & \frac{\hbar^2}{2m} q^2 + \delta & \frac{\hbar^2}{2m} (q + 2kR)^2 + \delta & 0 & \frac{\hbar^2}{2m} (q + 2kR)^2 + \delta & \frac{\hbar^2}{2m} (q + 2kR)^2 + \delta \\
0 & 0 & 0 & 0 & \frac{\hbar^2}{2m} q^2 - \epsilon & \frac{\hbar^2}{2m} (q + 2kR)^2 - \epsilon & \frac{\hbar^2}{2m} (q + 2kR)^2 - \epsilon & \frac{\hbar^2}{2m} (q + 2kR)^2 - \epsilon & \frac{\hbar^2}{2m} (q + 2kR)^2 - \epsilon
\end{pmatrix}
\]

written in the basis \( \{|l = +1, m_f = +1>, |l = +1, m_f = 0>, |l = +1, m_f = -1>, |l = 0, m_f = +1>, \ldots\} \). The Hamiltonian eq. (47) can be generalized for higher orders.
Band structure during lattice loading

Figure 19: Band structure during the lattice loading with a minimum of the first band at $q \neq 0$. Hamiltonian parameters correspond to the parameters at the midpoint during the lattice loading (compare fig. 13 b) at $t = 1 \text{ms}$).

Displacement per cycle depending

The Displacement per cycle, depends mainly on the coupling terms $\bar{\Omega}$, $\delta \Omega$ and $\Omega_{RF}$. Here we show the displacement per cycle calculated as described in sec. as a function of $\Omega_{RF}$. From these calculations we deduced the appropriate choice of parameters in order to obtain a detectable displacement after several cycles.

Figure 20: Displacement per cycle as a function of $\Omega_{RF}$ and for $\delta \Omega = 4.26 E_R$ and $\bar{\Omega} = 6 E_R$. 

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Approximate minimal time for charge pump cycle to maintain adibaticity

<table>
<thead>
<tr>
<th>$\frac{\delta \Omega}{\Omega}$</th>
<th>Approximate minimal $T$</th>
</tr>
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<tbody>
<tr>
<td>±0.71</td>
<td>1 ms</td>
</tr>
<tr>
<td>±0.31</td>
<td>3 ms</td>
</tr>
<tr>
<td>0</td>
<td>5 ms</td>
</tr>
</tbody>
</table>

Table 1: Approximate minimal time $T$ for one cycle in the charge pump experiment, which maintains adiabaticity. Results found by monitoring the spin population in momentum space in the simulations.
References


