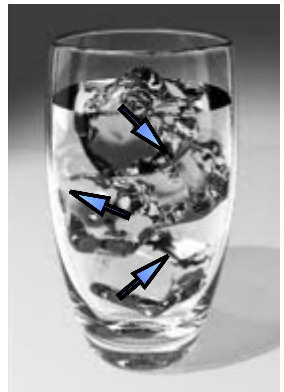


Quantum Spin Liquids: answers to questions

Leon Balents, KITP

Natal, Dec. 2011

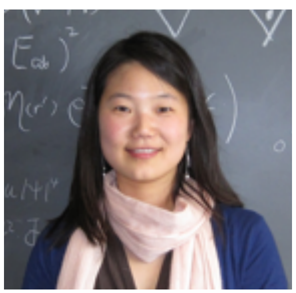
Collaborators

Lucile Savary

Kate Ross

Bruce Gaulin

Shigeki Onoda



SungBin Lee

Hong-Chen Jiang

Outline

- What are QSLs?
- What is non-local entanglement?
- How many QSLs are there?
- Are QSLs topological?
- Are QSLs stable?
- What are the differences between quantum and classical SLs?
- How could we observe QSLs?
- Where do we look?

What is a QSL?

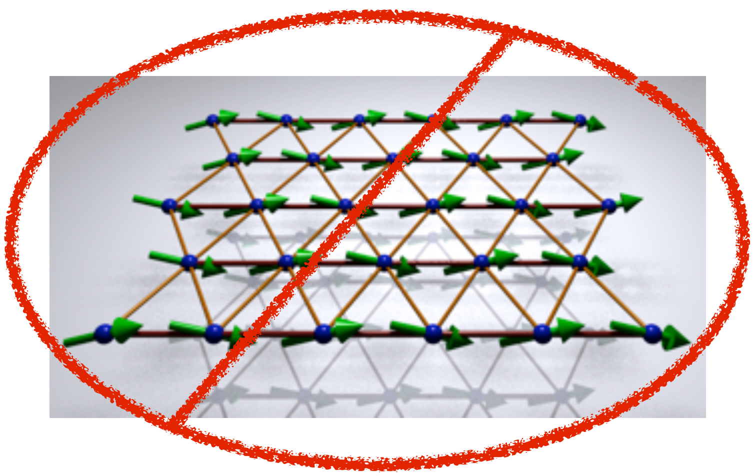
- The “layman’s” definition: a system of spins which is correlated but does not order at T=0

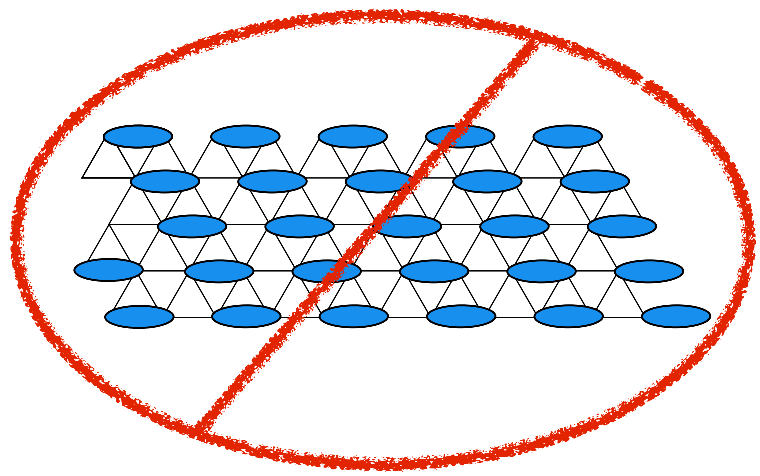
What is a QSL?

- The “layman’s” definition: a system of spins which is correlated but does not order at T=0
- Why we should ask for more:
  - This defines what it isn’t!
  - This in itself is not interesting!
  - It misses the important physics

What is a QSL?

- Let's call a QSL a ground state of a spin system with *long range entanglement*
- This means a state which cannot be regarded or even approximated as a product state over any finite blocks





What is a QSL?

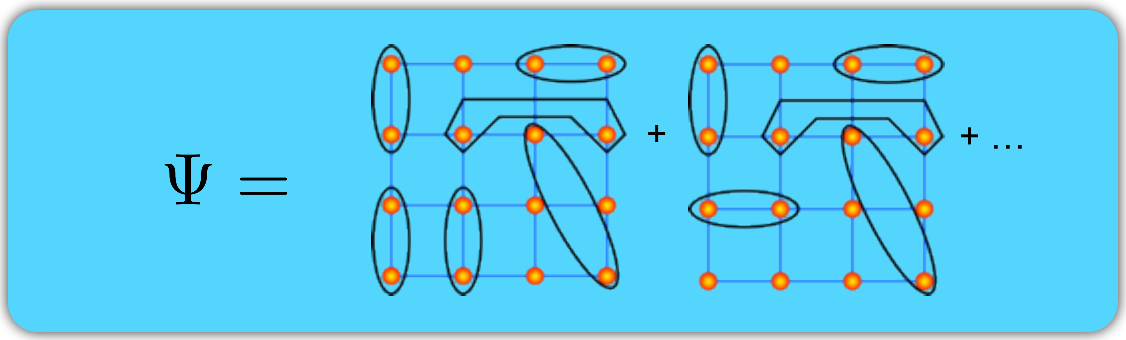
- Let’s call a QSL a ground state of a spin system with *long range entanglement*
- This means a state which cannot be regarded or even approximated as a product state over any finite blocks
- Presence or absence of spin-rotational symmetry, and indeed *any* symmetry, has nothing to do with it

What is a QSL?

- Let's call a QSL a ground state of a spin system with *long range entanglement*
- This means a state which cannot be regarded or even approximated as a product state over any finite blocks
- Indeed, you may even have a QSL *with* magnetic order (c.f. Lucile's talk tomorrow)

RVB States

- Anderson (73): ground states of quantum magnets might be approximated by superpositions of singlet "valence bonds"

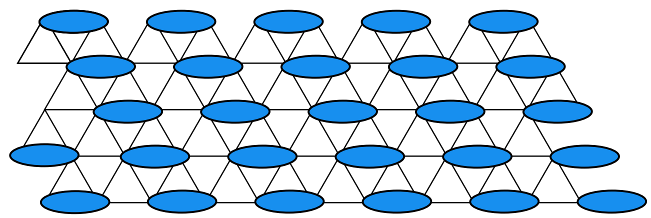


- Valence bond = singlet

$$|VB\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

VB states

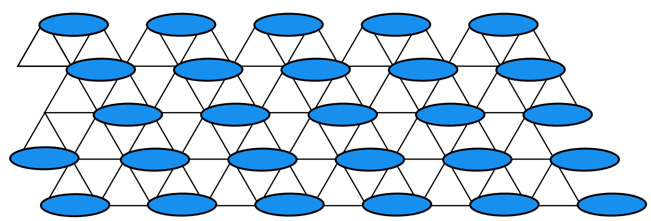
VBS



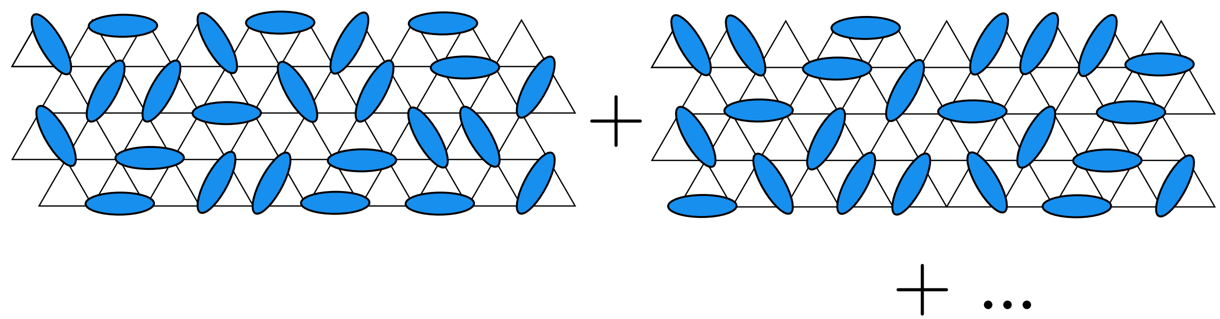
not a spin liquid

VB states

VBS



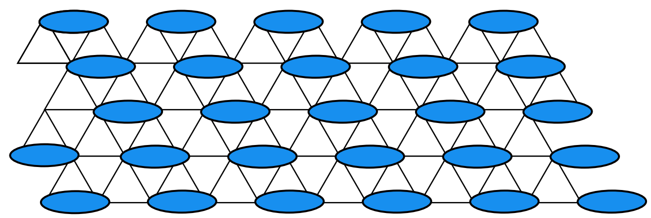
Short-range RVB



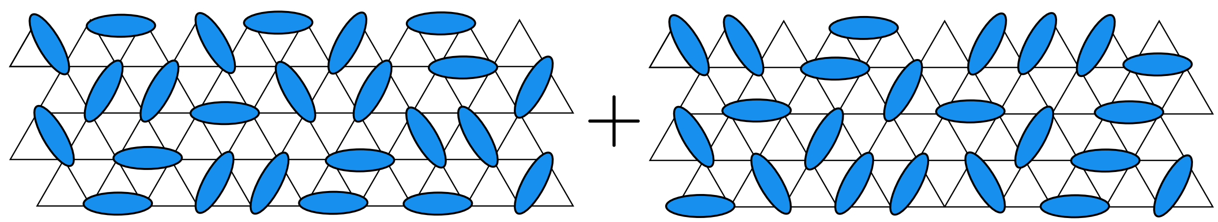
a QSL with an energy gap to break a singlet

VB states

VBS

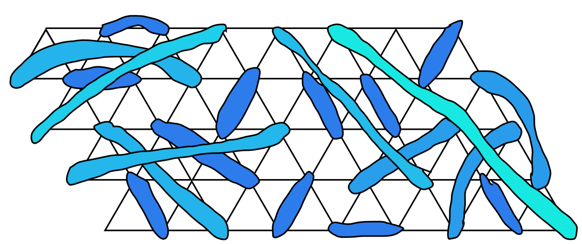


Short-range RVB



+ ...

Long-range RVB



+...

gapless spin excitations

What is non-local entanglement?

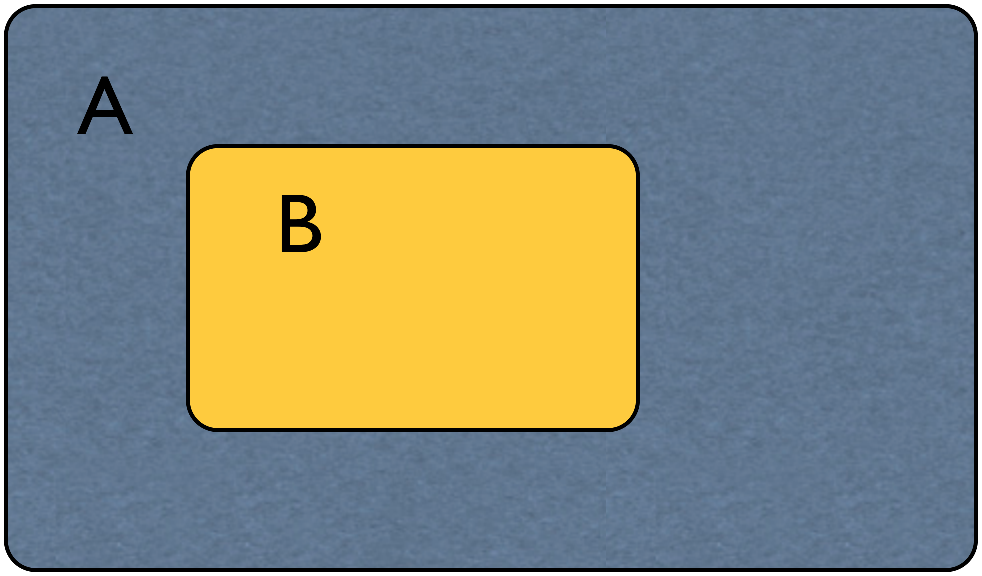
- We can say that a state has non-local entanglement if it violates the usual scaling of entanglement entropy for product or mean-field-like states

Entanglement Entropy

- Von Neumann

$$\rho_B = \mathrm{Tr}_A\left[\rho_{A\otimes B}\right]$$

$$S = -\mathrm{Tr}_B\left[\rho_B \log \rho_B\right]$$

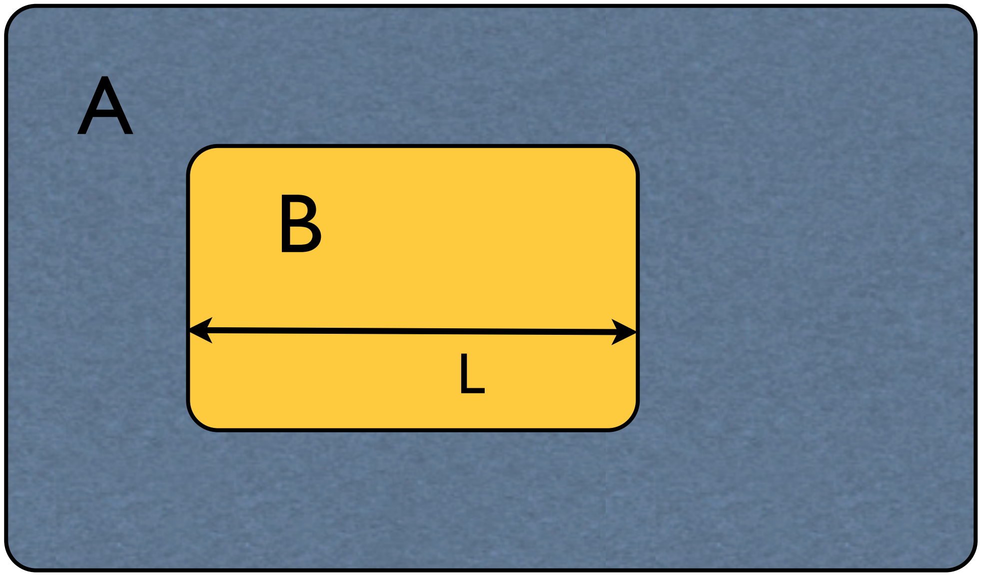


Entanglement Entropy

- Von Neumann

$$\rho_B = \mathrm{Tr}_A\left[\rho_{A\otimes B}\right]$$

$$S = -\mathrm{Tr}_B\left[\rho_B \log \rho_B\right]$$

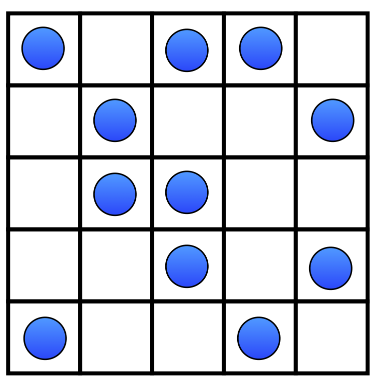
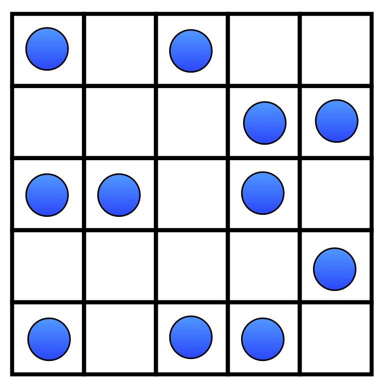
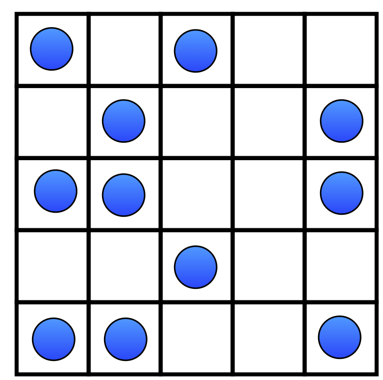


- Area law: $S \sim \sigma L^{d-1}$
  - for any product state (and any gapped GS)

Free Fermions

- A Fermi gas is a familiar example of a long-range entangled state: a product in momentum space rather than real space

$$\Psi = \prod_{k<k_F} c_k^\dagger |0\rangle$$

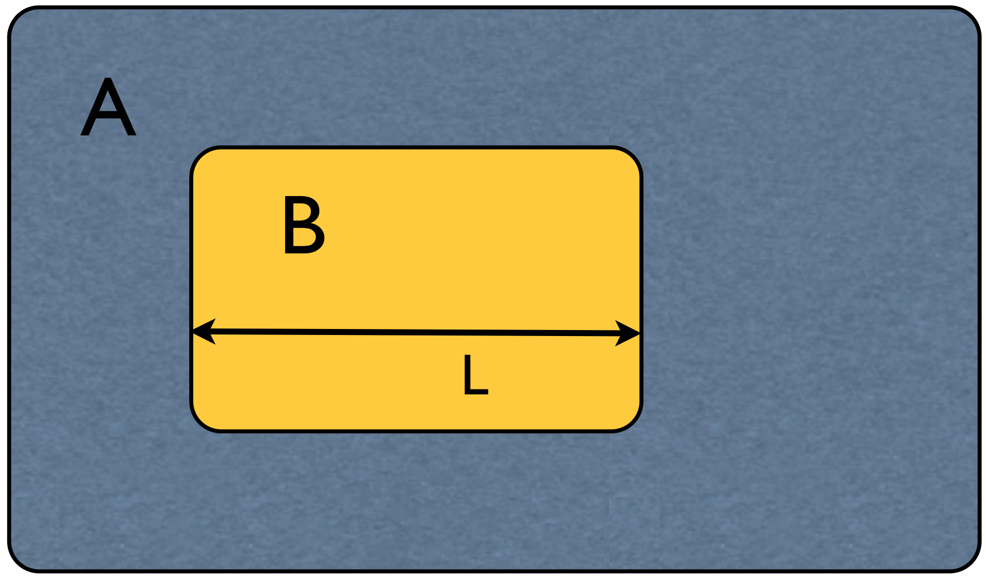
$= \; c_1$  $+c_2$  $+c_3$  $+\cdots$

Entanglement Entropy

- Von Neumann

$$\rho_B = \mathrm{Tr}_A \left[\rho_{A\otimes B}\right]$$

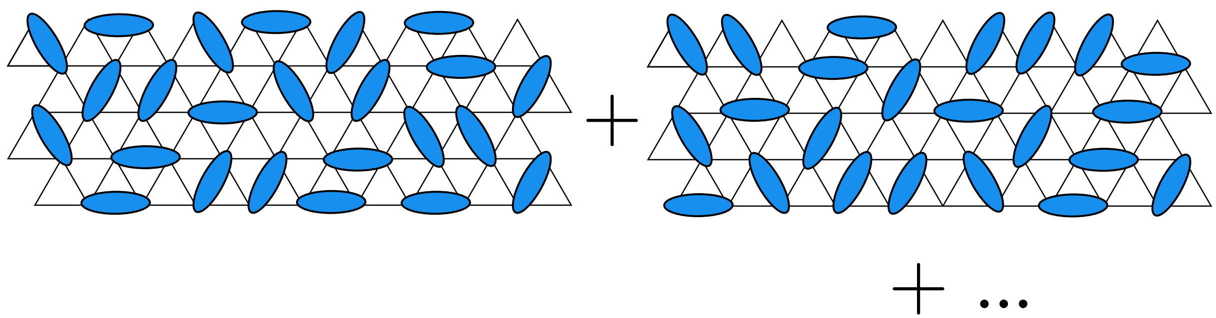
$$S = -\mathrm{Tr}_B \left[\rho_B \log \rho_B\right]$$



D. Gioev+I. Klich, 2006
M.M. Wolf, 2006

- Free fermions $S \sim \sigma L^{d-1} \log L$
  - About the largest known entanglement!

Topological phases



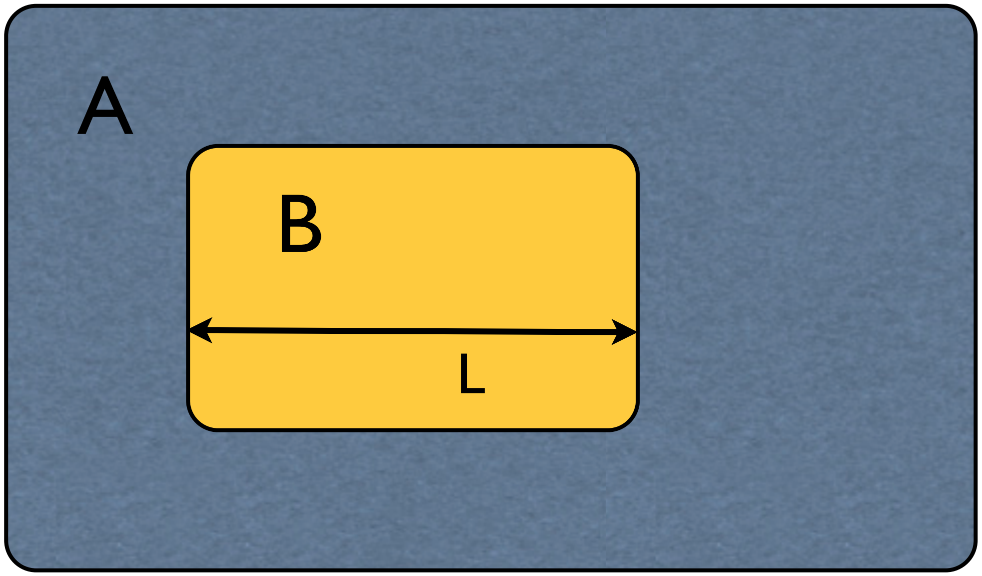
- A class of states with a gap to all excitations which violate the usual scaling by having entanglement entropy *smaller* than usual

Entanglement Entropy

- Von Neumann

$$\rho_B = \mathrm{Tr}_A \left[\rho_{A\otimes B}\right]$$

$$S = -\mathrm{Tr}_B \left[\rho_B \log \rho_B\right]$$

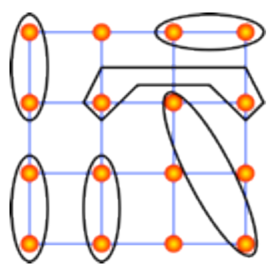
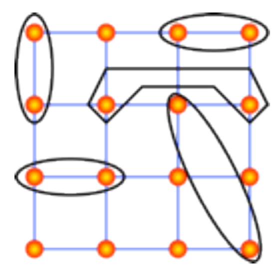


$$S_{2d} \sim \sigma L - S_{\mathrm{topo}}$$

- Topological entanglement entropy
    - e.g. $S_{topo} = \ln(2)$ for Z_2 QSL

How many QSLs are there?

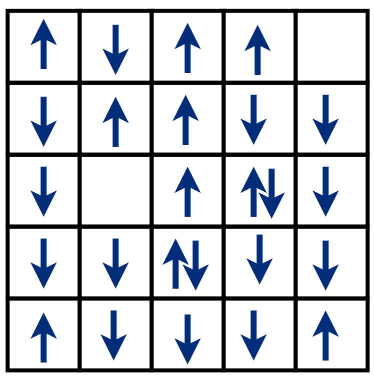
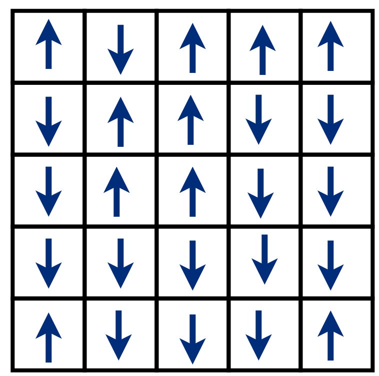
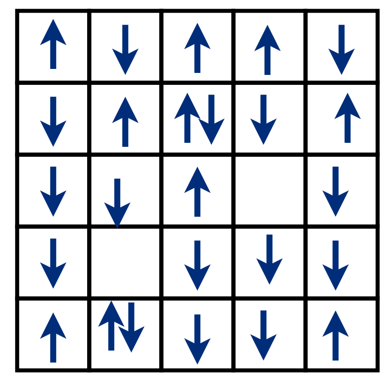
- A long-range entangled wavefunction is a complicated thing!

$\Psi = \ c_1$  $+ \ c_2$  $+ \dots$

- Very hard to work directly with all these coefficients - is there another way?

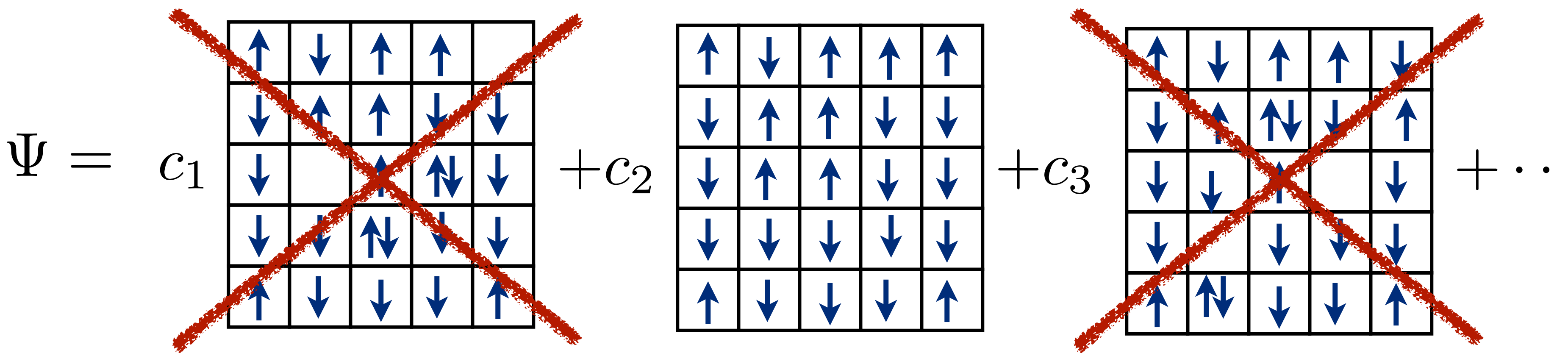
Gutzwiller Construction

- Construct QSL state from free fermi gas with spin, with 1 fermion per site (S=0)

$\Psi_0 = c_1$  $+c_2$  $+c_3$  $+\cdots$

Gutzwiller Construction

- Construct QSL state from free fermi gas with spin, with 1 fermion per site



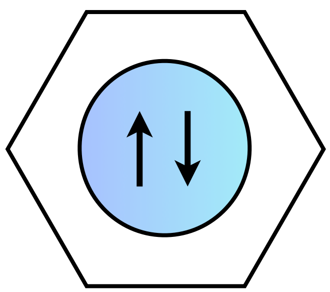
- Projection removes empty and doubly occupied sites

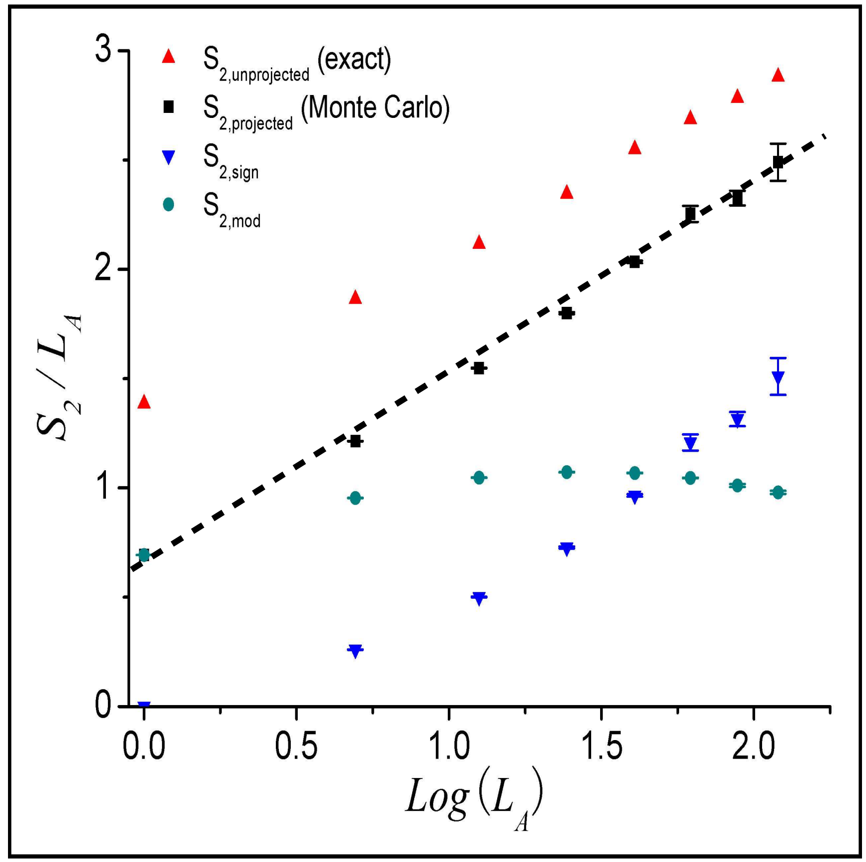
$$\Psi = P_G \Psi_0$$

Projected Fermi Sea

- Entanglement entropy of projected Fermi sea seems to show the same behavior!

$$\Psi = P_G$$





Yi Zhang *et al*, 2011

Slave particles

- Gutzwiller-type variational wavefunction uses a reference Hamiltonian

$$H_{ref} = \sum_{ij} \left[t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + \mathrm{h.c.} + \Delta_{ij} c^\dagger_{i\uparrow} c^\dagger_{j\downarrow} + \mathrm{h.c.} \right]$$

- Project

$$|\Psi_{var}\rangle = \prod_i \hat{P}_{n_i=1} |\Psi_{ref}\rangle$$

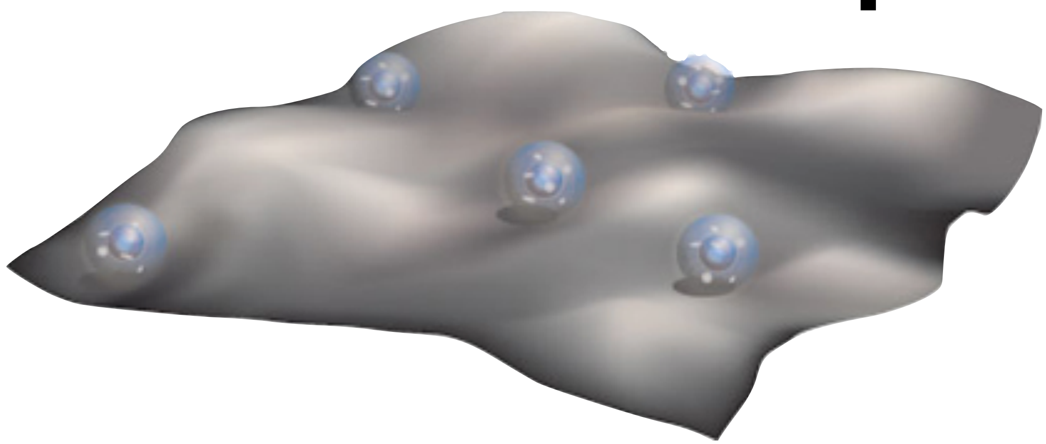
- The fermions are "slave" particles

$$\vec{S}_i = c^\dagger_{i\alpha} \frac{\vec{\sigma}_{\alpha\beta}}{2} c_{i\beta}$$

Gauge theory

- Such variational wavefunctions are approximate representations of ground states of an associated field theory
- This field theory consists of *spinons* coupled to fluctuating *gauge fields*
- Different QSLs are characterized at the most basic level by different *gauge groups*: Z_2, U(1), ...

The “landscape”



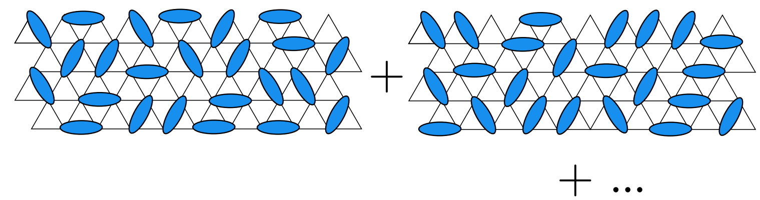
- The number of distinct QSL *phases* is huge
  - e.g. X.G. Wen has classified *hundreds* of different QSL states all with the same symmetry on the square lattice (and this is *not* a complete list!)

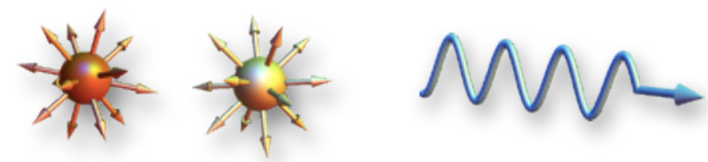
Are QSLs topological?

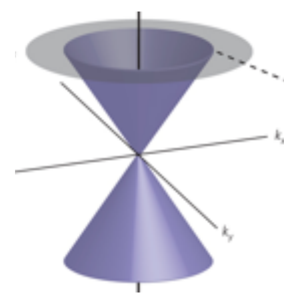
- Depends what you mean by topological!
- The *simplest* QSLs are "topological phases"
  - A gap to excitations, having non-trivial statistics and fractional charges
  - Ground state degeneracy depending on sample topology
- Others have various gapless excitations

Classes of QSLs

- Topological QSLs
  - full gap
- U(1) QSL
  - gapless emergent "photon"
- Algebraic QSLs
  - Relativistic CFT (power-laws)
- Spinon Fermi surface QSL







Are QSLs stable?

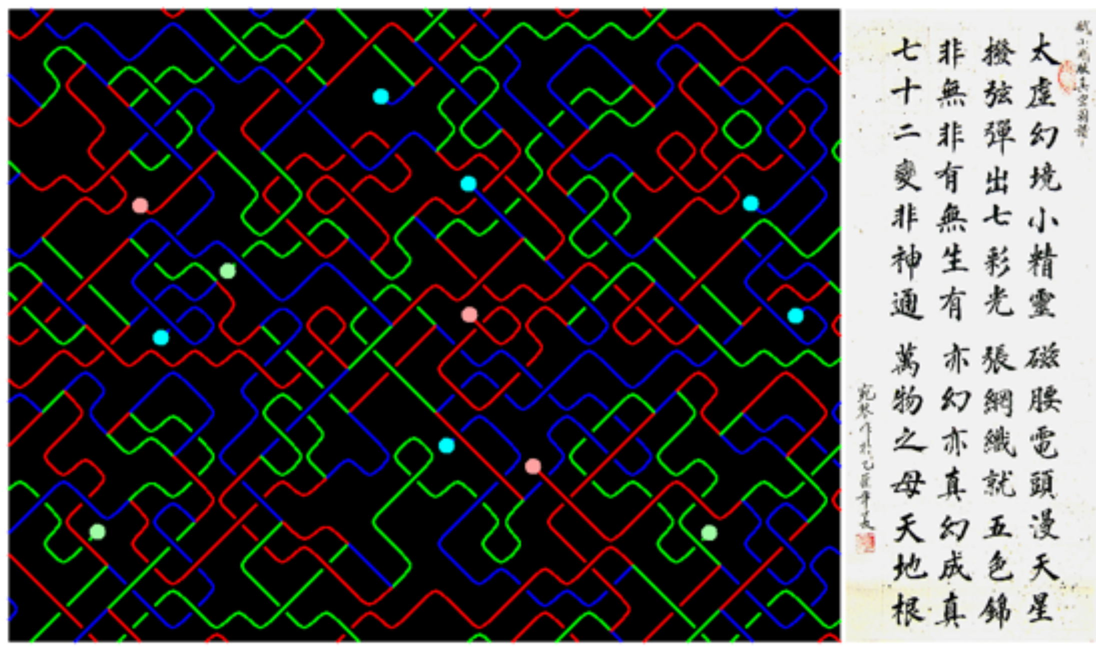
- Many QSLs are known to be stable to *all* perturbations, even those which break all possible symmetries
  - This includes all topological phases in 2 and 3 dimensions and some gapless QSLs including U(1) states in 3 dimensions
- For some others stability is less clear and may require some symmetry

Are QSLs stable?

- Some QSLs are also stable to *three dimensional coupling* (topological QSLs, etc.) or are *intrinsically 3D* (U(1) QSL) and stable
- Others (especially 2d gapless QSLs) are unstable to 3D ordering

Loop picture

- Crudely, these phases are stable because “ends” of “strings” are costly and do not modify long-distance physics if “cut” strings are short



A picture of our vacuum

A string-net theory of light and electrons

What are the differences between classical and quantum SLs?

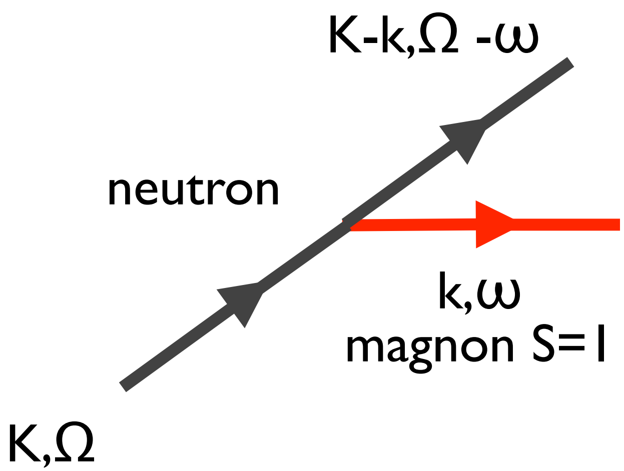
- A QSL is a ground state: one wavefunction, while a CSL is a thermal mixture of many states
- A QSL is a distinct stable phase, while a CSL requires fine tuned degeneracy of infinitely many states
- Quantum spin liquids have non-local entanglement!

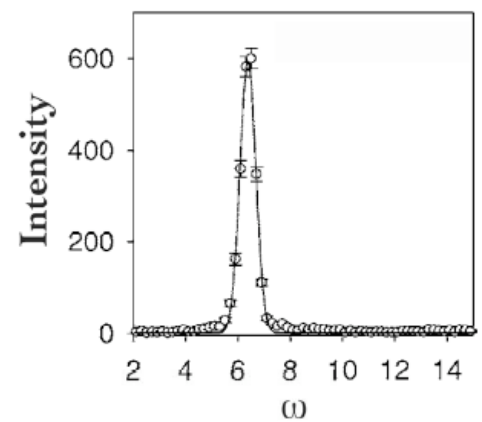
How could we observe QSLs?

- Some signatures apply to all QSLs
  - long-range entanglement (hard to measure!)
  - fractional S=1/2 "spinons" (not S=1 "triplons" like in VBS states)
- Most depend upon the particular QSL
  - e.g. measurements of spinon Fermi surface or low T thermal transport

Seeing Spinons

- A proof of principle: 1d spinons have been observed in several materials by neutron scattering
- Basic idea



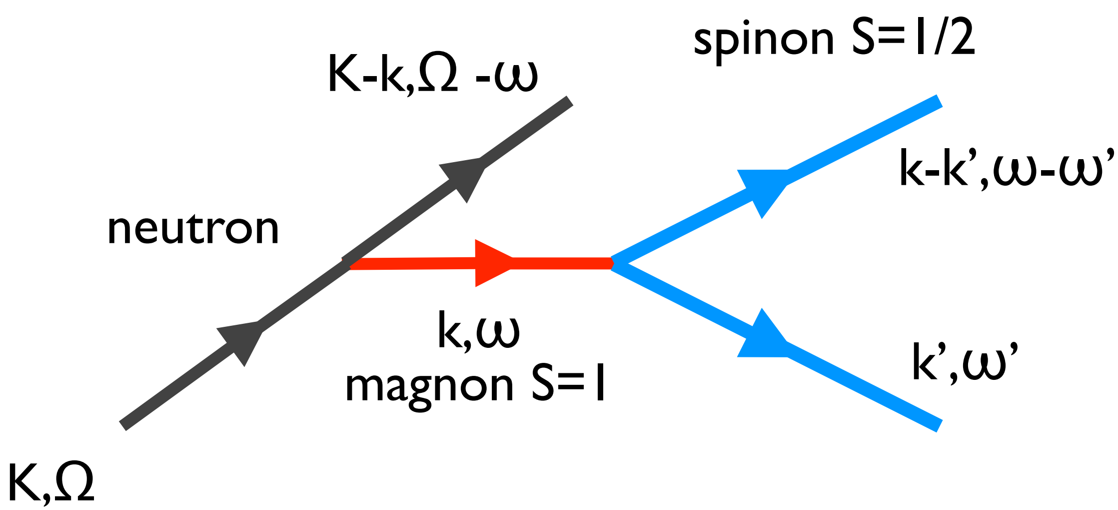


Line shape in Rb_2MnF_4

leads to a sharp peak at $\omega=\varepsilon(k)$

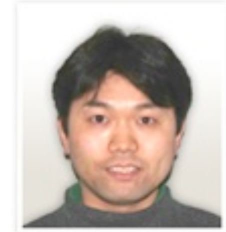
Seeing Spinons

- A proof of principle: 1d spinons have been observed in several materials by neutron scattering
- Basic idea



broad peak with $\omega=\varepsilon(k')+\varepsilon(k-k')$

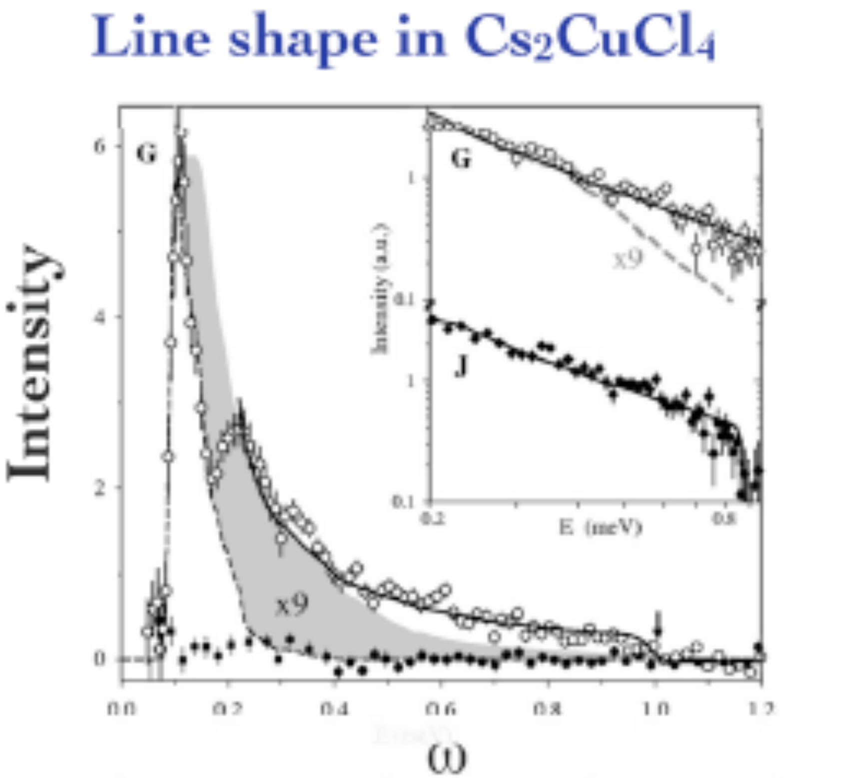
Cs_2CuCl_4

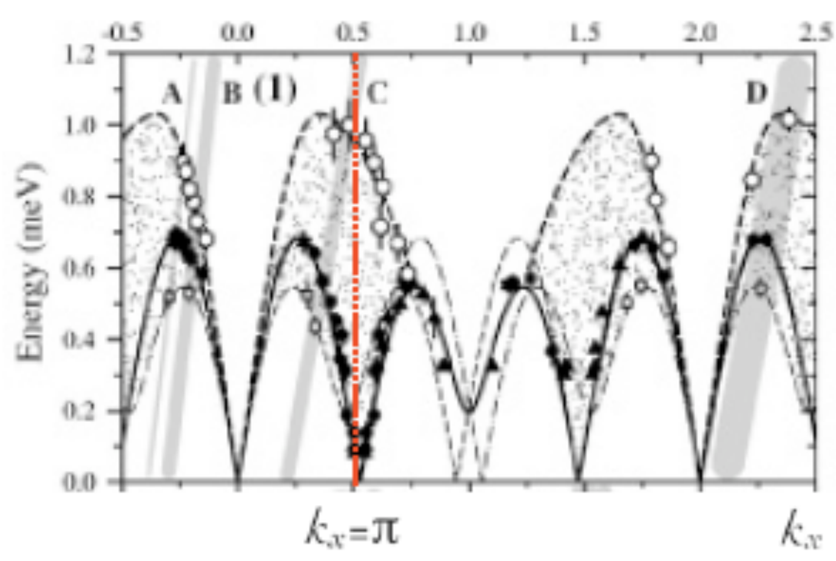


Oleg Starykh

Masanori Kohno

- "Power law" fits well to free spinon result
  - Fit determines normalization





R. Coldea

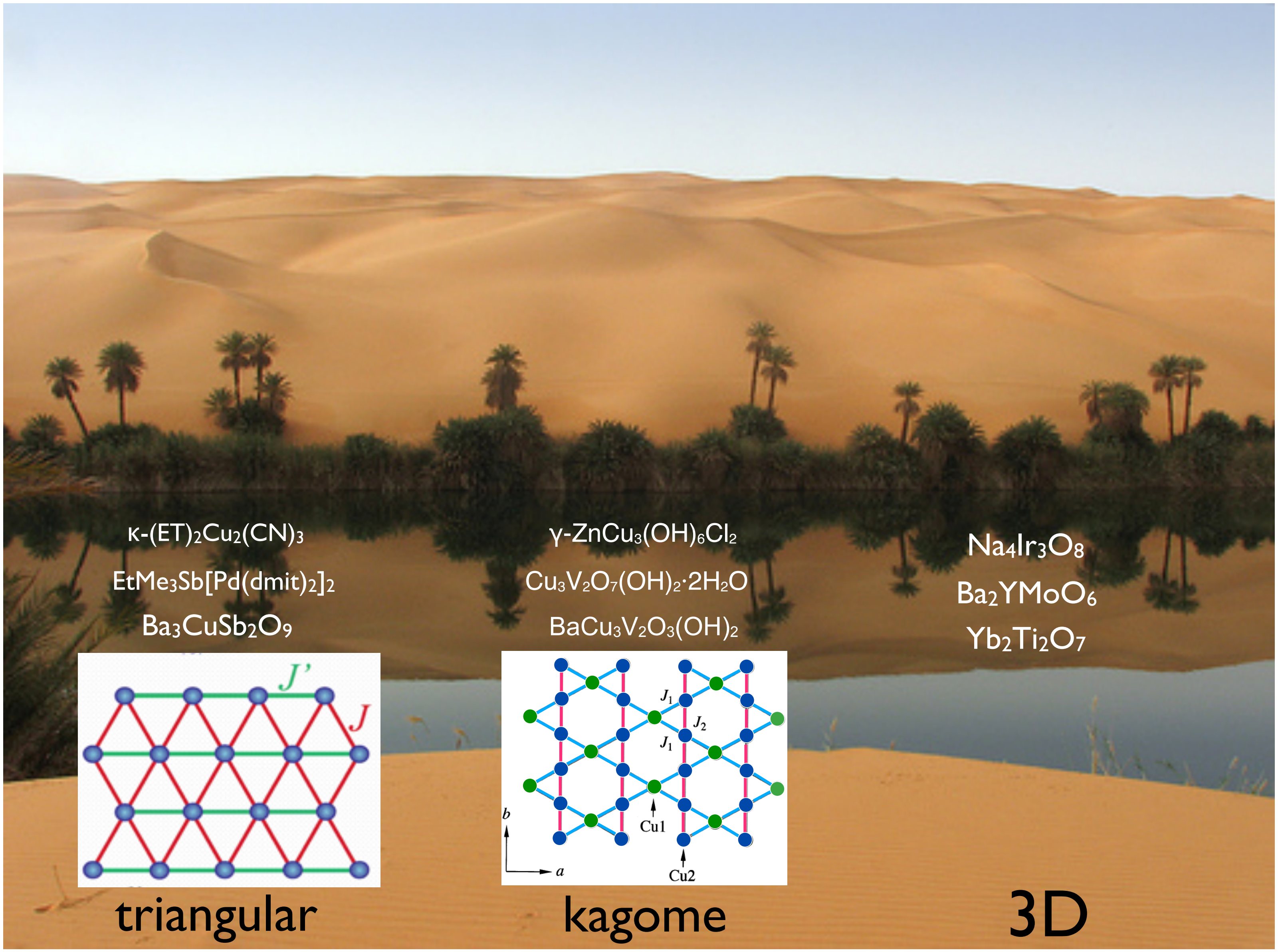
Where do we look?

- This might be the most important question - but mostly the subject of another talk!
- Recent years have seen a lot of progress *experimentally* for signs of QSLs
  - mostly this has come from searching "natural" candidates with highly geometrically frustrated lattices

Is this the landscape ?

Is this the landscape?

An Oasis?


κ-$(ET)_2Cu_2(CN)_3$
$EtMe_3Sb[Pd(dmit)_2]_2$
$Ba_3CuSb_2O_9$
J'
J
triangular
γ-$ZnCu_3(OH)_6Cl_2$
$Cu_3V_2O_7(OH)_2 \cdot 2H_2O$
$BaCu_3V_2O_3(OH)_2$
J_1
J_2
J_1
Cu1
Cu2
b
a
kagome
$Na_4Ir_3O_8$
Ba_2YMoO_6
$Yb_2Ti_2O_7$
3D

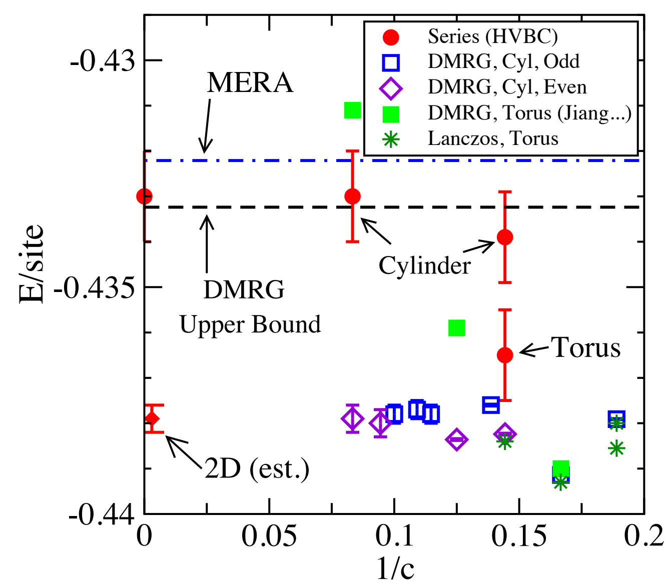
Theoretical searches

- Theory has been, until recently, mostly limited to uncontrolled approaches or very small systems
- The most influential method has been variational Gutzwiller approach, which can at least differentiate distinct QSLs
- Relatively few material-specific predictions of QSLs (but see pyrochlore talks tomorrow)

2d DMRG

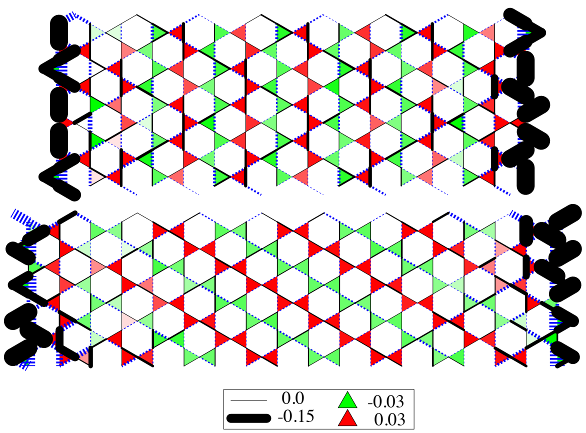
- Recent progress in DMRG promises controlled, unbiased, study of fairly realistic potential 2d QSL materials

S. Yan *et al*, 2010
Z_2 QSL on kagome lattice

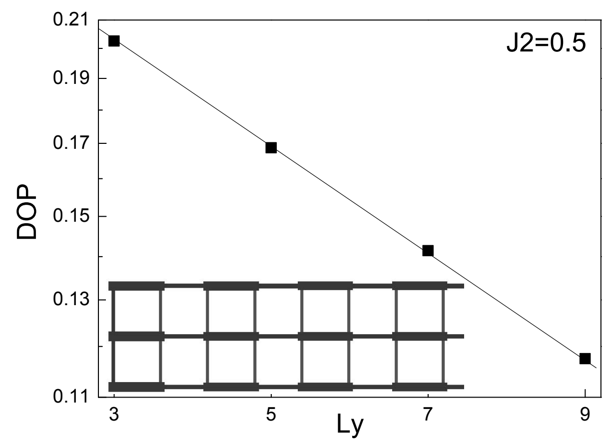


Parity effect

- On long cylinders, Z_2 QSL becomes weakly dimerized 1d state *when the circumference is odd*, with $D \sim \exp(-L_y/\xi)$

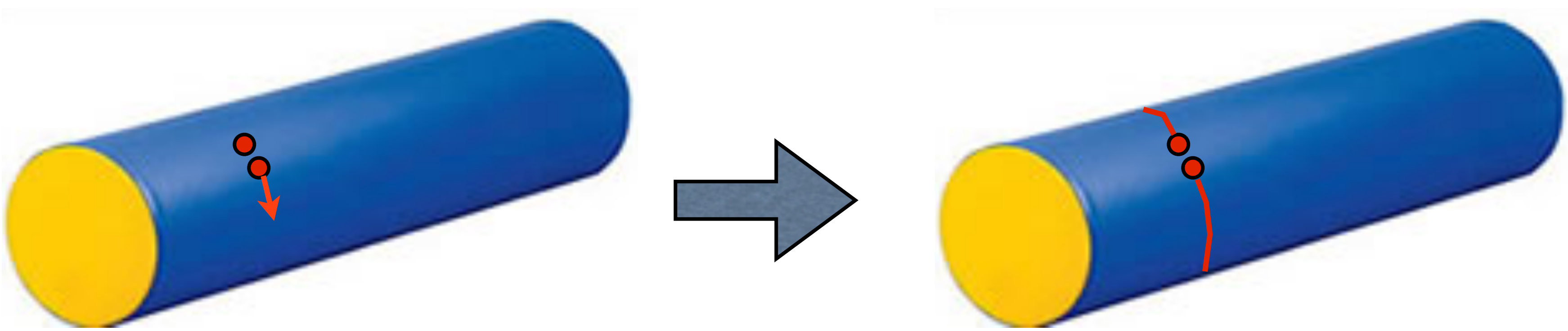


S. Yan *et al* (2010)
kagome Heisenberg



H.-C. Jiang *et al* (2011)
J_1-J_2 model

Origin of parity effect



Virtual vison pairs encircling cylinder lead to dimerization

$$\varphi_a(y + L_y) = (-1)^{aL_y}\varphi_a(y)$$

$$\psi_{\mathrm{VBS}} = (\varphi_2 + i\varphi_1)^2$$

Conclusions

- Quantum spin liquids are entirely new states of matter with remarkable long-range quantum structure
- New materials, more detailed theory, and hard work seem finally to be exposing them to experimental study