EFFECTS OF DISORDER IN A QUANTUM SPIN LIQUID

Adding dirt to the Kitaev honeycomb model

John Chalker

Physics Department, Oxford University

Adam Willans, Oxford & Roderich Moessner, Dresden

Phys Rev Lett 104, 237203 (2010) and Phys Rev B 84, 115146 (2011)

Outline

Disorder in quantum magnets

- as a distraction
- as a probe
- as a source of new physics

Kitaev's honeycomb lattice model

- from spins to fluxes and free fermions

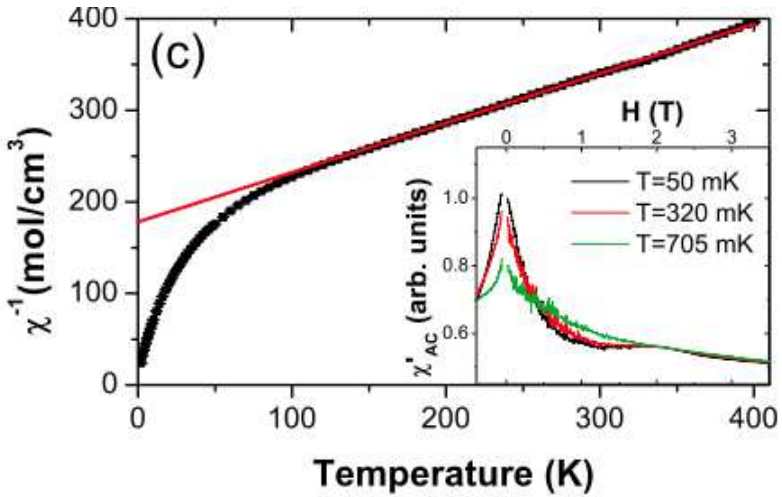
Disorder in the Kitaev model

- vacancies and moment formation
- weak exchange randomness and soft excitations

Disorder as a distraction

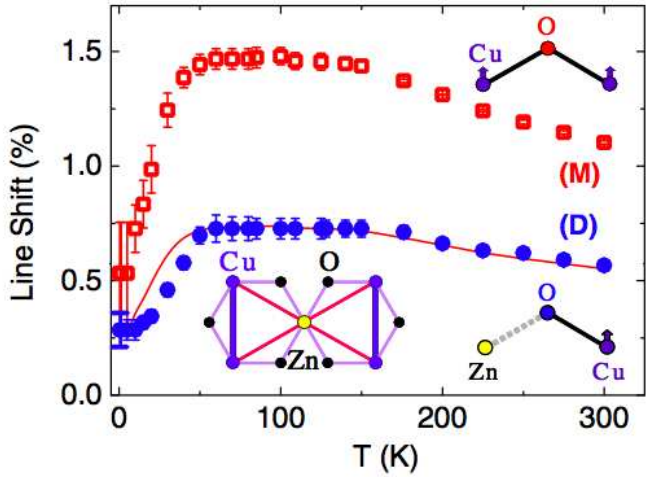
Example: $S = 1/2$ **kagome system** **$ZnCu_3(OH)_6Cl_2$**

Curie-like bulk susceptibility at low T



Helton *et al*, **PRL 98**

Finite local susceptibility



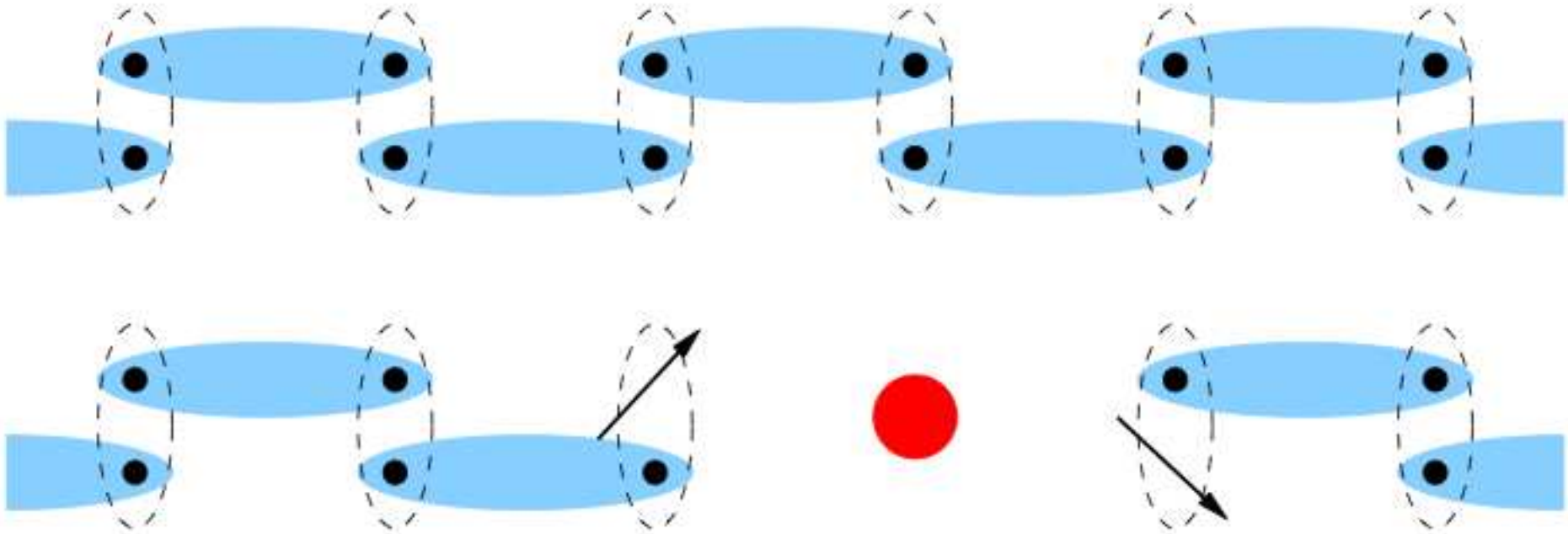
Mendels *et al*, **PRL 100**

Cu $\leftrightarrow$ Zn substitution ?

Disorder as a probe

Example: formation of spin-half moments in Haldane state

if spin-one Heisenberg chain is cut by vacancy



New physics from disorder

Example:

Random singlet phases from weak exchange randomness

in spin-half antiferromagnetic Heisenberg chain

Curie-like susceptibility at low T

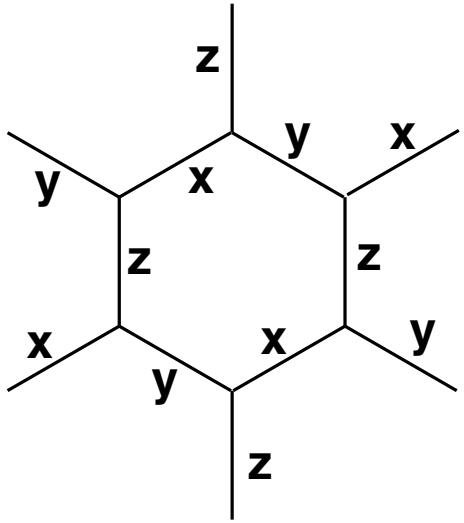
$$\chi \sim 1/[T \ln^2 T]$$

Dasgupta & Ma (1980), Bhatt & Lee (1982), D. Fisher (1994)

Kitaev's honeycomb model

Spin $S = 1/2$ quantum magnet

with strong 'spin-orbit' anisotropy



$$\mathcal{H} = -J_x \sum_{\text{x}-\text{bonds}} \sigma_i^x \sigma_j^x - J_y \sum_{\text{y}-\text{bonds}} \sigma_i^y \sigma_j^y - J_z \sum_{\text{z}-\text{bonds}} \sigma_i^z \sigma_j^z$$

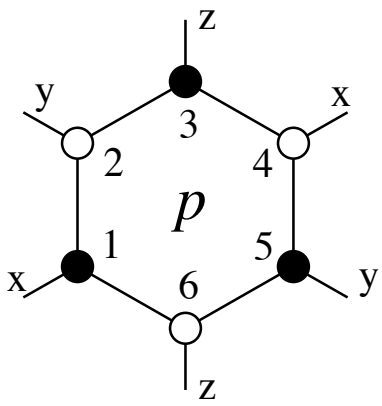
A. Kitaev, Ann. Phys. 321, 2 (2006)

Suggested realisation:

G. Jackeli and G. Khaliullin, PRL 102, 017205 (2009) **A_2IrO_3 A=Na or Li**

Emergent degrees of freedom

Static fluxes



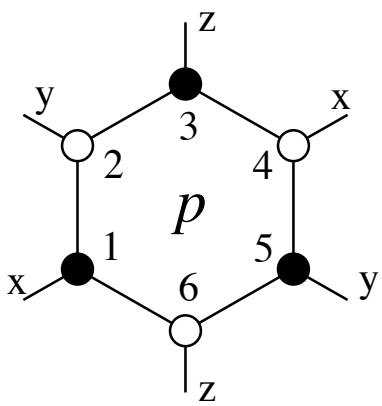
$$W_p = \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z$$

$$[W_p, H] = 0$$

$$[W_p, W_q] = 0$$

Emergent degrees of freedom

Static fluxes . . . **and** . . . **free fermions**



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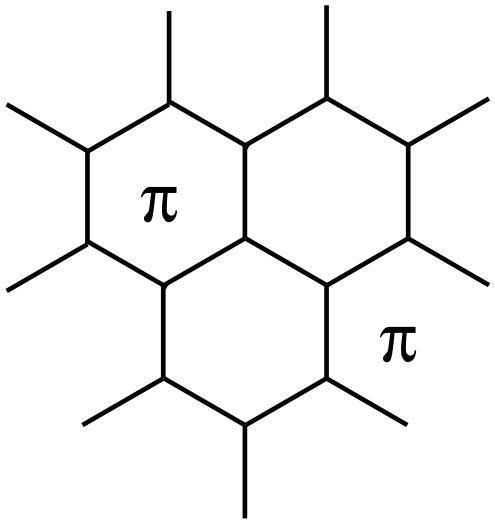


Tight binding model

hopping magnitudes J_x, J_y **&** J_z

signs set by Z_2 **fluxes**

Spin correlations ultra-short-range: $\langle \sigma_j^\alpha \sigma_k^\alpha \rangle = 0$ **for** $|\mathbf{r}_j - \mathbf{r}_k| > 1$

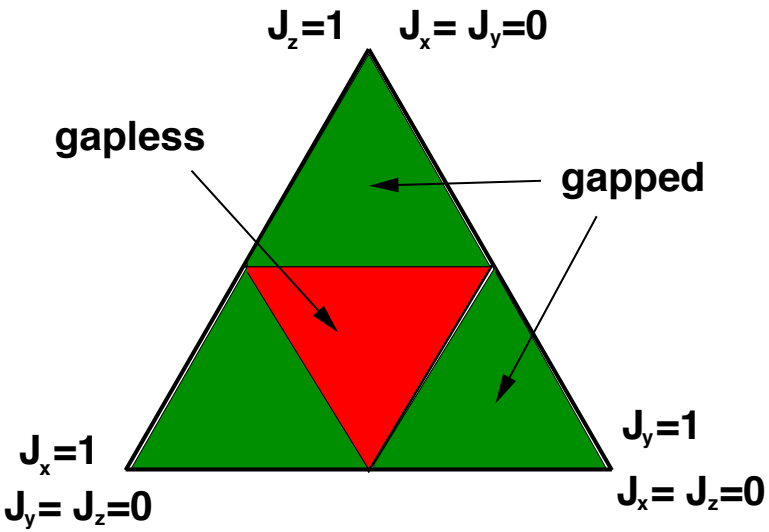
Ground state phase diagram

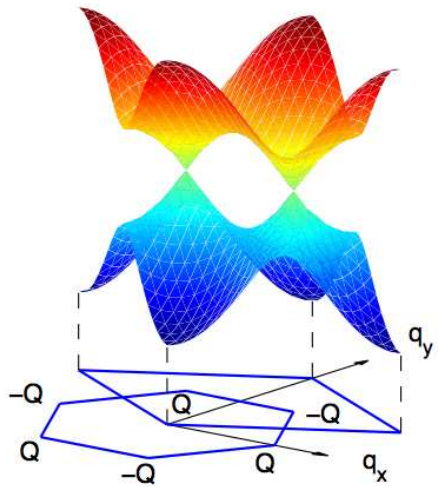
- **Gapped liquid phases for $J_z > J_x + J_y$ and permutations**

  Weakly coupled dimers – both sectors gapped

- **Gapless liquid phase around $J_x = J_y = J_z \equiv J$**

  Dirac cones in fermion spectrum – flux sector gapped





Introducing disorder

Exchange randomness

- **Weak disorder** $\delta J_\alpha \ll J_\alpha$

  generated by random strains

Gives random hopping

for fermion excitations

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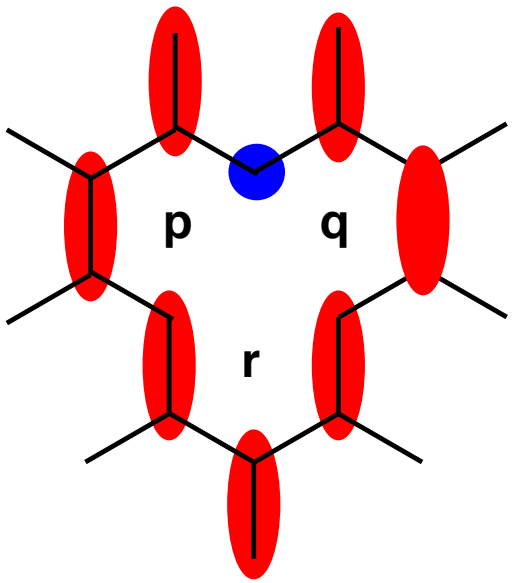
Vacancies



- **Break dimers for** $J_z \gg J_x, J_y$

  – consequences?

- **Fuse fluxes**

$$W_{\rm tot} = W_p \cdot W_q \cdot W_r$$

  – consequences?

Response to a Zeeman field

Clean system: **Zeeman energy** $\mathcal{H}_Z = -\sum_k \vec{h} \cdot \vec{\sigma}_k$

makes fluxes dynamical: $[\mathcal{H}_Z, W_p] \neq 0$

$\langle E_n | \mathcal{H}_Z | E_m \rangle \neq 0$ **only between distinct flux sectors**

Gap between flux sectors $\Rightarrow$ **finite susceptibility**

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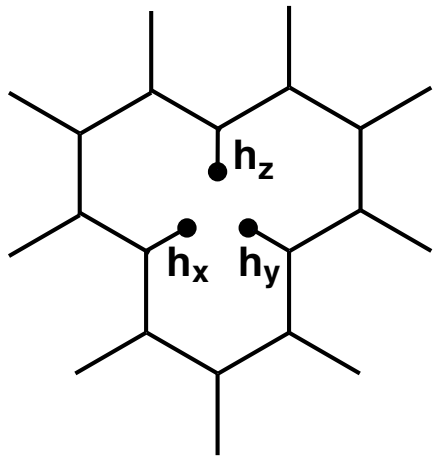
System with vacancy: **solvable with leading part of** $\mathcal{H}_Z$

$$\mathcal{H}_Z = \mathcal{H}_Z^{(0)} + \mathcal{H}_Z^{(1)}$$

$$\left[\mathcal{H}^{(0)}, W_p\right] = 0 \quad \text{for all } p.$$

$$\mathcal{H}_Z^{(0)} = -h_x \sigma_1^x - h_y \sigma_y^2 - h_z \sigma_3^z$$



missing interactions & fusion of flux sectors

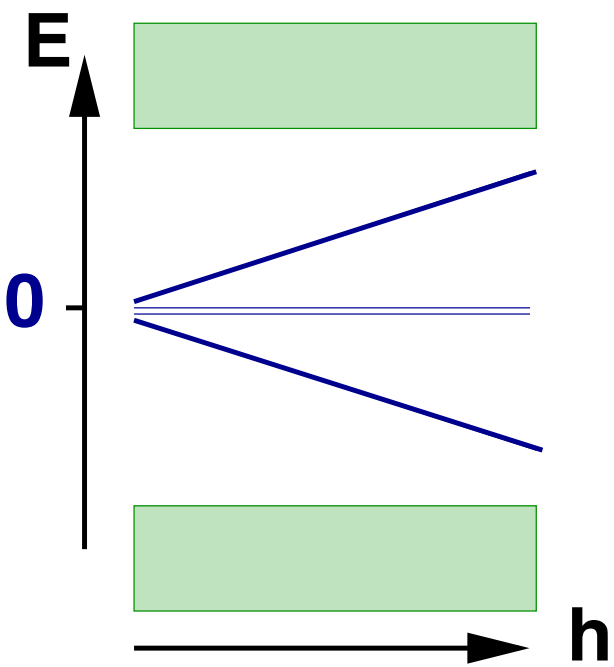
$\Rightarrow$ enhanced susceptibility & static fluxes at leading order

Isolated vacancy in gapped phase

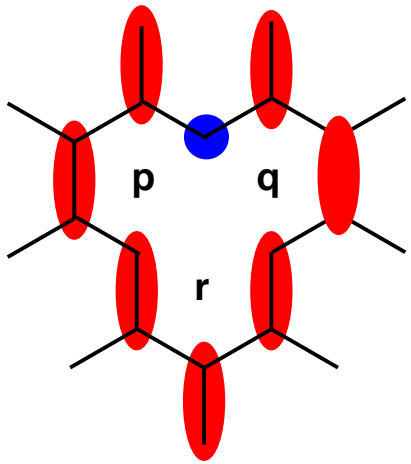
Free moment formed

Zero T: $\langle \sigma^z \rangle = g_{\rm eff}\, {\rm sgn}(h_z)$

Finite T: $\chi \propto 1/T$



- **Moment localised on single site**



- **Magnitude varies with J_α**

$g_{\rm eff} \to 1$ **for** $J_z \gg J_x, J_y$

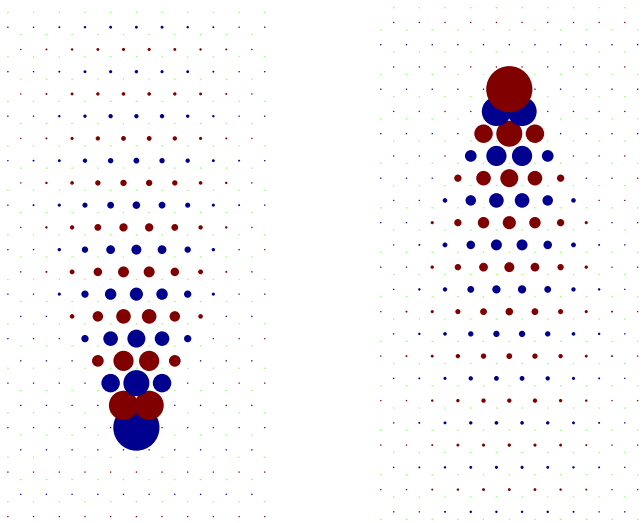
$g_{\rm eff} \to 0$ **at boundary**

with gapless phase

Interaction between vacancy moments

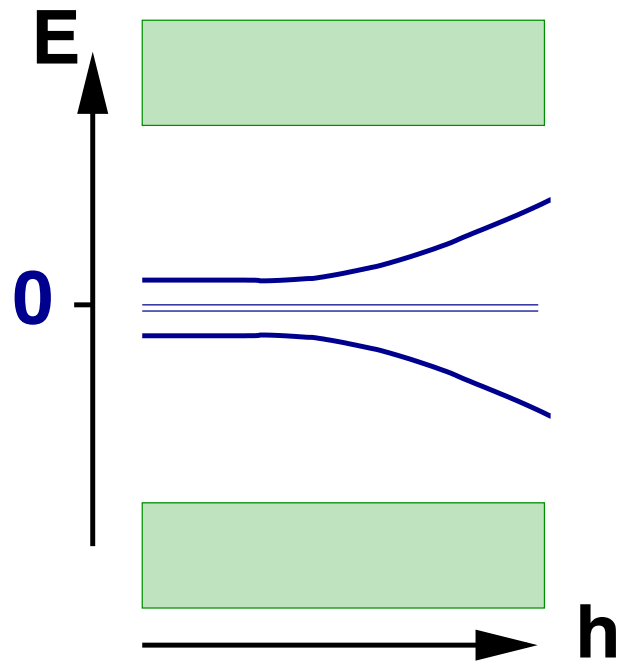
Gapped phase

Fermion zero modes localised in wedges



A-sublattice B-sublattice

Two vacancies on opposite sublattices



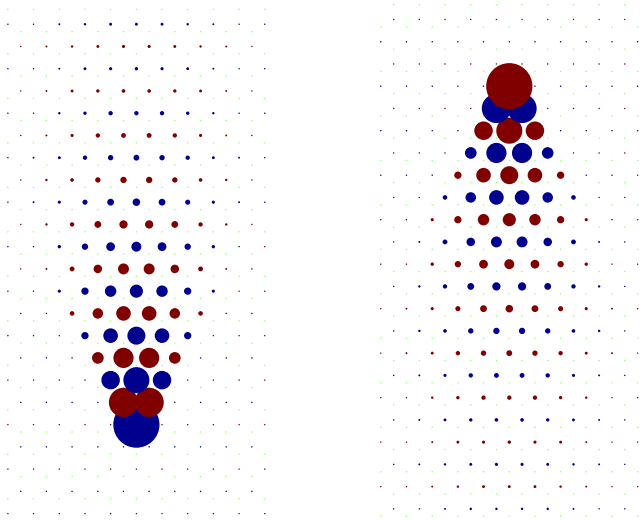
Overlapping modes $\Rightarrow$ weakly coupled moments

Interaction between vacancy moments

Gapped phase

Fermion zero modes

localised in wedges



A-sublattice B-sublattice

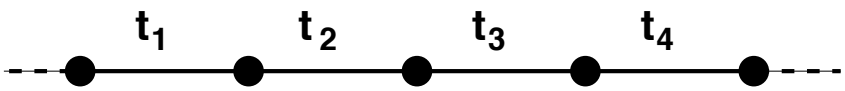
Overlapping modes

$\Rightarrow$ weakly coupled moments

Finite vacancy density $\Rightarrow$

Random bipartite hopping problem

1D version well-studied



Dyson singularity

in density of states

$$\rho(E) \sim 1/[E(\ln E)^3]$$

What happens

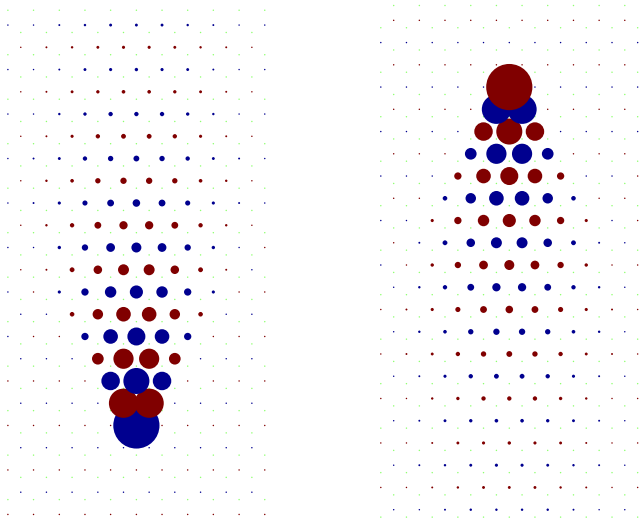
for Kitaev problem?

Interaction between vacancy moments

Gapped phase

Fermion zero modes

localised in wedges



A-sublattice **B-sublattice**

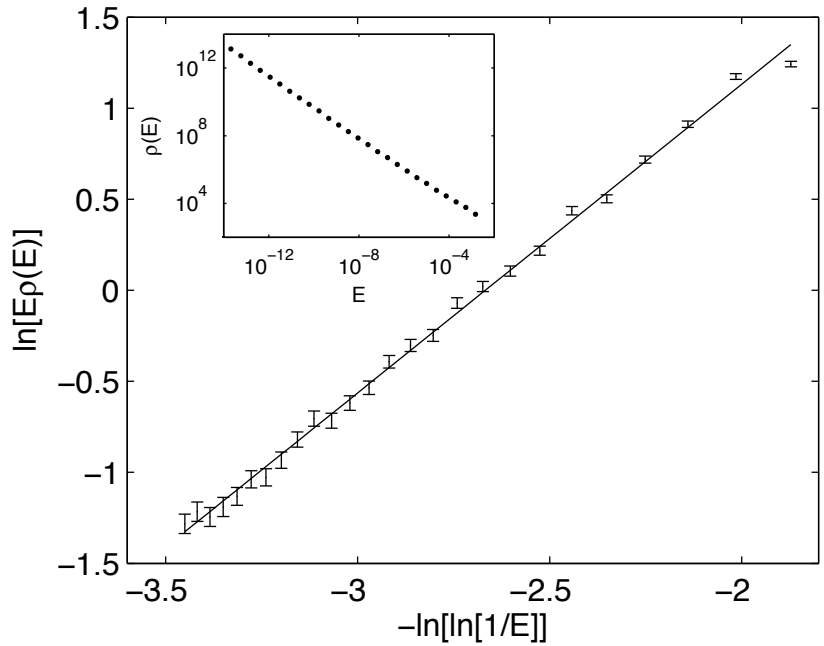
Overlapping modes

$\Rightarrow$ **weakly coupled moments**

Finite vacancy density

Many low-energy excitations

$$\rho(E) \sim 1/[E(\ln E)^x]$$



$$\chi(T) \sim 1/[T|\ln T|^x]$$

with $x \approx 1.7$

Vacancy susceptibility in the gapless phase

No free moments but spins next to vacancy easily polarised

- **Ground state without vacancies is in zero flux sector**
- **Vacancies bind fluxes — Consequences?**

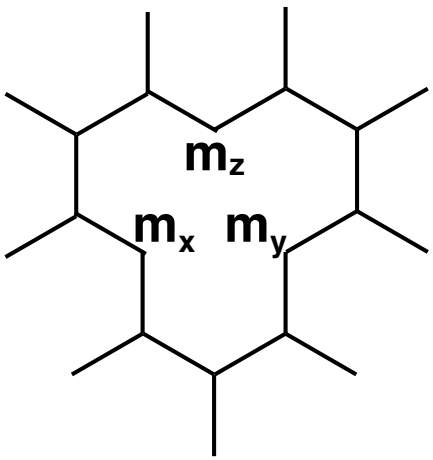
Vacancy susceptibility in the gapless phase

No free moments but spins next to vacancy easily polarised

- **Ground state without vacancies is in zero flux sector**
- **Vacancies bind fluxes — Consequences?**

Single vacancy with flux

$$\chi \propto \ln(1/T)$$



Zero-flux sector

$$\chi \propto [T \ln(1/T)]^{-1}$$

Each component of $\vec{m}$

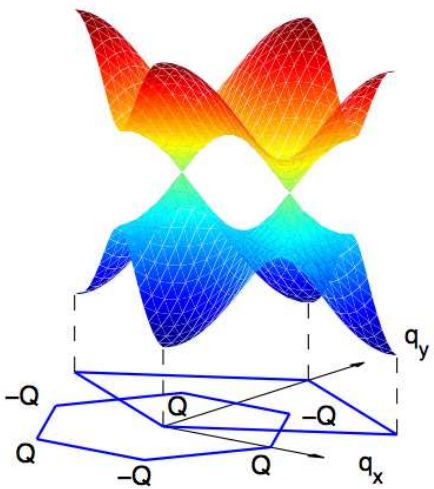
located on different site

Weak exchange randomness in gapless phase

Fermionic excitations are massless Dirac particles

Exchange disorder translates into

random vector potential in Dirac eqn



Weak exchange randomness in gapless phase

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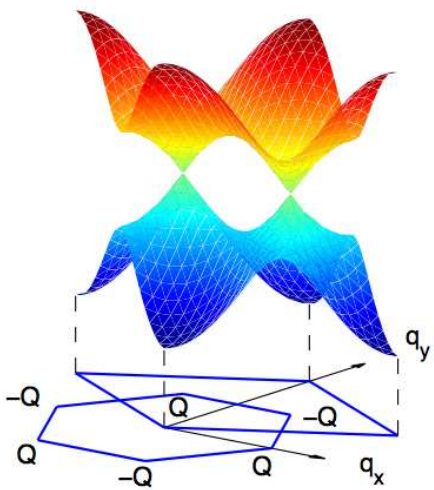


Exchange disorder translates into

random vector potential in Dirac eqn

Power-law behaviour

exponents vary with disorder strength

Ludwig et al 1994

With $\Delta \propto \langle (\delta J)^2 \rangle$ **find**

- **Density of states** $\rho(E) \propto |E|^{(1-\Delta)/(1+\Delta)}$
- **Hence heat capacity** $C \propto T^{2/(1+\Delta)}$

Summary

- **Can use Kitaev honeycomb model as lab for theorists**
  - model remains tractable with disorder
- **Disorder does not alter nature of degrees of freedom**
  - fluxes and free fermions
- **But ground states, excitation spectra and response functions all changed**
  - flux bound to vacancies
  - moment formation and singular susceptibility
  - soft excitations from vacancies or exchange disorder

From spins to fermions

— sketch of Kitaev's solution

Represent each spin using 4 Majorana fermions $(bc = -cb,\ c^\dagger = c)$

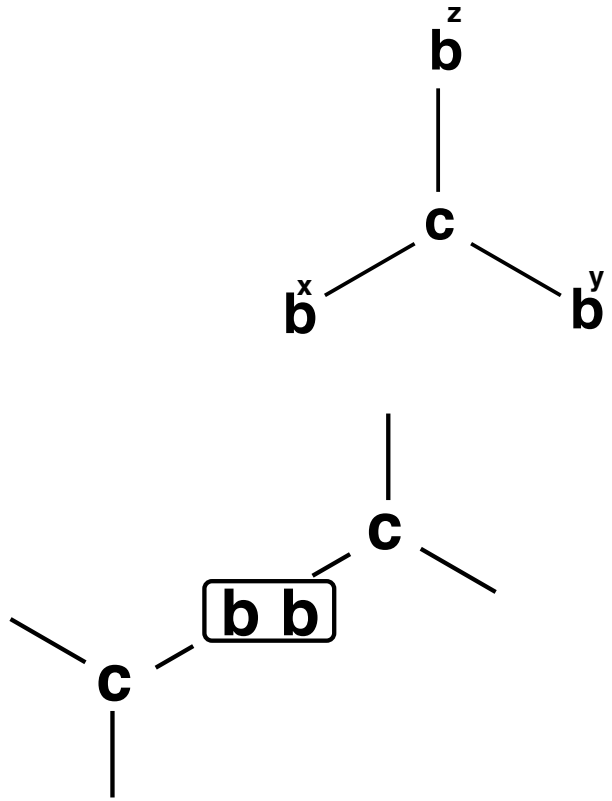
$\vec{\sigma} \to \{c, b^x, b^y, b^z\}$ **with** $\sigma_k^\alpha = \mathrm{i}\, b_k^\alpha c_k$ **so** $\sigma_j^\alpha \sigma_k^\alpha = b_j^\alpha b_k^\alpha c_j c_k$

- **Resulting** $\mathcal{H}$ **is quadratic in** c_k's
- $[\mathcal{H}, \hat{u}_{jk}] = 0$ **with** $\hat{u}_{jk} = \mathrm{i} b_j^{\alpha_{jk}} b_k^{\alpha_{jk}}$

$$\mathcal{H} = \frac{i}{4} \sum_{jk} \hat{A}_{jk} c_j c_k$$

$$\hat{A}_{jk} = \begin{cases} 2J_{\alpha_{jk}} \hat{u}_{jk} & j, k \text{ neighbours} \\ 0 & \text{otherwise} \end{cases}$$

– honeycomb tight binding model



Project to get physical states