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From Spin Ice to Kagomé Planes: The Kasteleyn Transition in kagomé spin ice

1. The Kasteleyn Transition
2. Neutron scattering, simulation and experiment
3. Finite size scaling and worm commensurability

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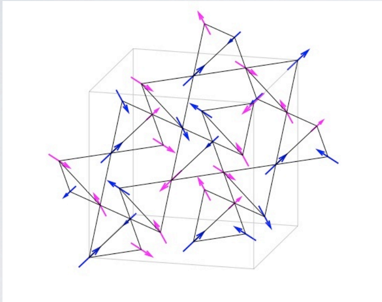
From Spin Ice to Kagomé Planes: The Kasteleyn Transition in kagomé spin ice

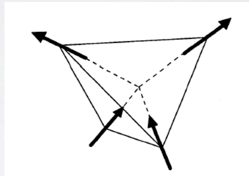
Adam Harman-Clarke, Tom Fennell,
Jean-Yves Fortin, Steven Bramwell

Ice rules in spin ice

Spin ice materials $Ho_2Ti_2O_7$, $Dy_2Ti_20_7$

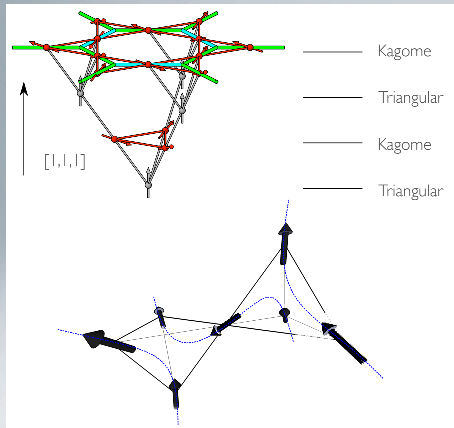


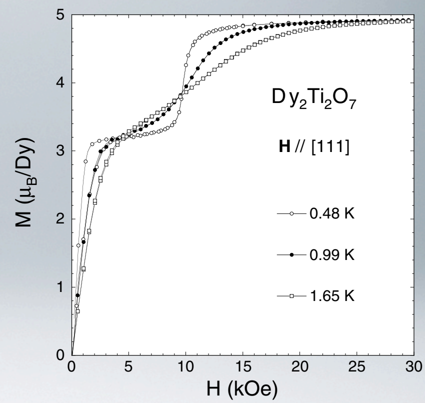
Magnetic ice rules
two-in two-out



Harris et al, Phys. Rev. Lett.**79**, 2554-2557 (1997)

KAGOME ICE



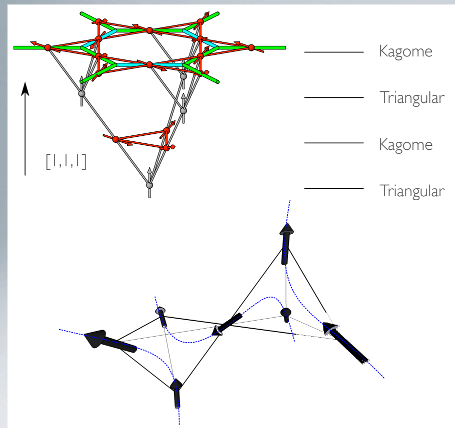


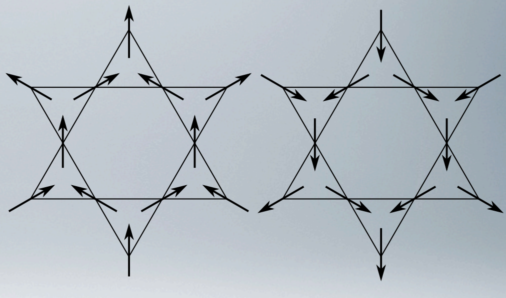
$$\nabla \cdot \mathbf{M}(\mathbf{r}) = 0$$

$$\boxed{S_0 = \frac{3}{4}\frac{S_{\Delta antiferro}}{4} \approx 0.06R}$$

Matsuhira *et al.* J o P: Cond. Mat., 2002
Udagawa *et al.* JPSJ, 2002
Wills *et al.* PRB, 2002
Isakov *et al.* PRB, 2004
Moessner and Sondhi, PRB, 2003

KAGOME ICE





Broken Z_2 symmetry

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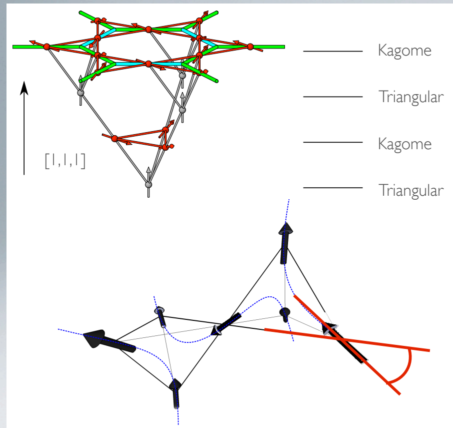
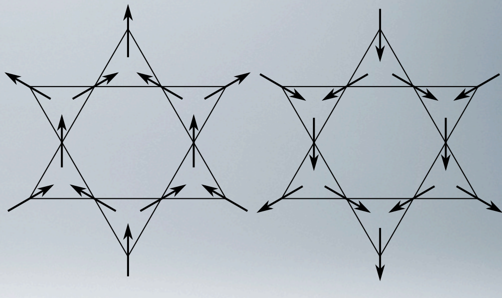
KAGOME ICE





Broken Z_2 symmetry

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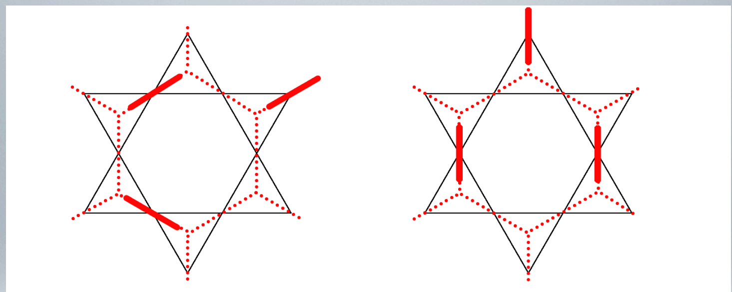
Moessner and Sondhi, PRB, 2003

THE KASTELEYN TRANSITION

Spin to dimer mapping

Kasteleyn, J. of Math. Phys. 1963
Moessner and Sondhi, PRB, 2003

THE KASTELEYN TRANSITION



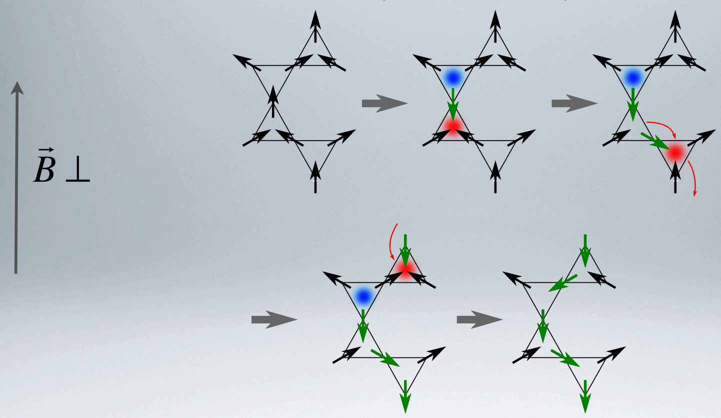
Disordered
Finite entropy

Ordered
Zero entropy

Kasteleyn, J. of Math. Phys. 1963
Moessner and Sondhi, PRB, 2003

WORM EXCITATIONS

(CLOSED LOOPS)



At transition

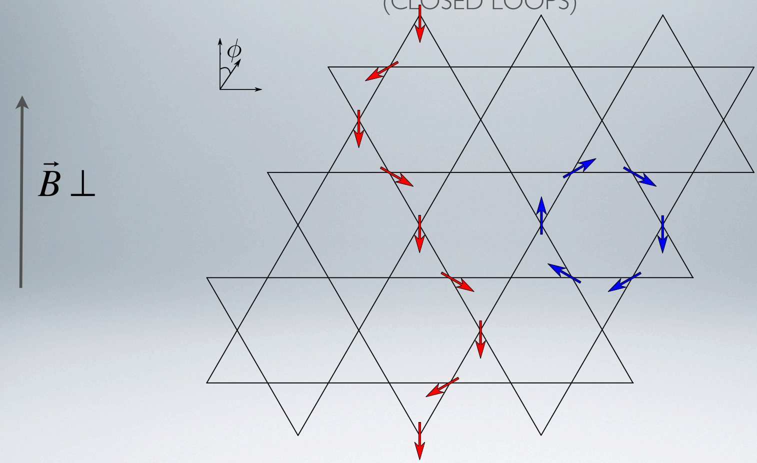
$\delta G = \delta\varepsilon - T_K \delta S = 0$

energy loss
=entropy gain

Only long loops
change M

WORM EXCITATIONS

(CLOSED LOOPS)



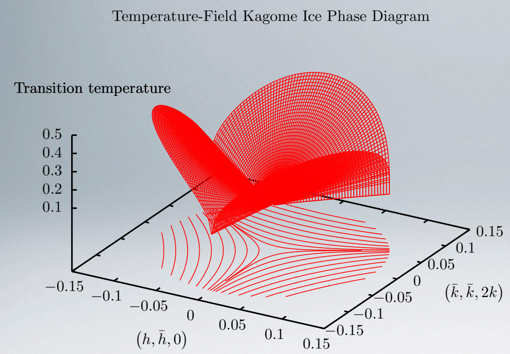
At transition

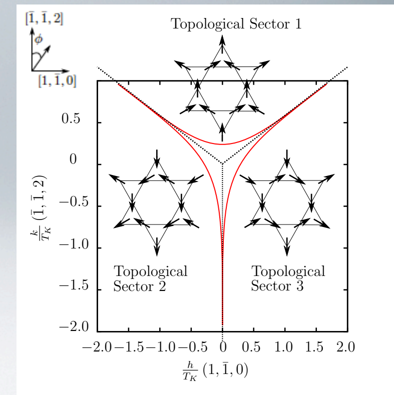
$$\delta G = \delta\varepsilon - T_K \delta S = 0$$

energy loss
=entropy gain

Only long loops
change M

PHASE DIAGRAM





Thermodynamics of the K transition

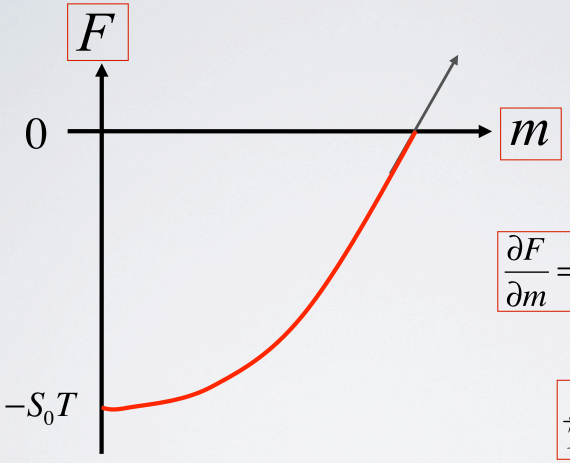
internal energy U is constant for all (Pauling) states

Magnetic Helmholtz Free energy is pure Entropy!

$$\boxed{F=-TS(m,N)}\qquad \boxed{dF=-SdT+Nhdm}$$

$$\boxed{h=\frac{1}{N}\frac{\partial F}{\partial m}=-\frac{T}{N}\frac{\partial S}{\partial m}}\qquad \boxed{\chi=\frac{\partial m}{\partial h}=-\frac{1}{T}\frac{N}{\partial^2 S/\partial m^2}}$$

Kasteleyn transition:
S=>0 for finite h/T



$$\frac{\partial F}{\partial m} = -T\frac{\partial S}{\partial m} = Nh$$

$$\frac{h}{T} \; finite$$

KASTELEYN transition in collective paramagnet

LANDAU FREE ENERGY

$$x = m_{sat} - m \qquad \frac{G_L}{NT} = \frac{F - Nhm}{NT} = \frac{1}{T}(h - h_K)x + \frac{\alpha_2}{2}x^2 + \frac{\alpha_3}{3}x^3 \ldots$$

Landau critical exponents:

$$\beta = 1, \gamma = \alpha = 0$$

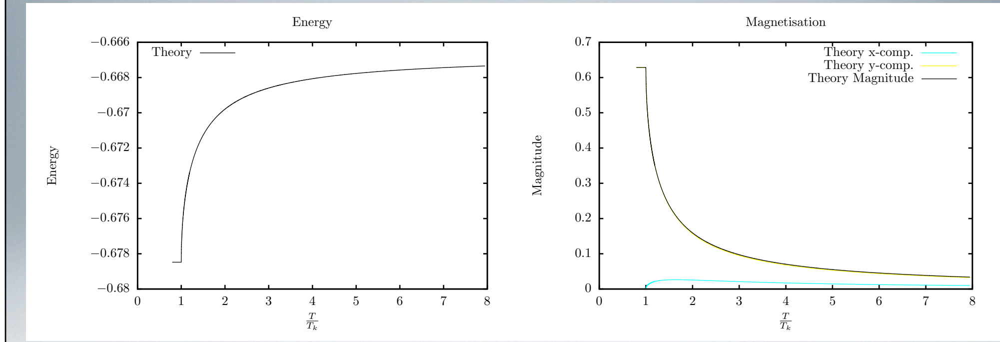
Upper critical dimension is 3
therefore mean field exponents will
be subject to corrections in 2D.
Fluctuation modified values:

$$\beta = 1/2, \gamma = \alpha = 1/2$$

$\alpha = 1/2$ for $d = 2$ in agreement with the exact solution[3] and $\alpha = (3 - d)/2$ more generally for $1 < d < 3$;

Jaubert *et al.* J. Phys.: Conf. Ser. 2009
Bhattacharjee *et al.* J. Stat. Phys. 1983

THERMODYNAMICS



Asymmetric, 1st/2nd order character, biaxial magnetisation

Neutron scattering - “in plane”, polarized, unpolarized

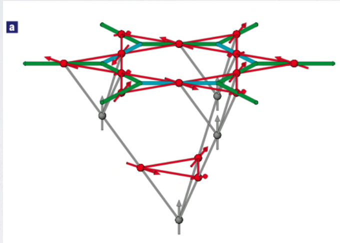
$$S(Q)=\left\langle \sum_R \vec{S}_\perp(R).\vec{S}_\perp(0)\exp(i\vec{Q}.\vec{R})\right\rangle$$

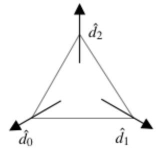
$\vec{S}_\perp(R)$ is component perp to Q and parallel to neutron polarization

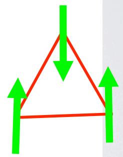
$$\begin{aligned}\hat{d}_0 &= [-\sqrt{3}/2, -1/2]\\ \hat{d}_1 &= [\sqrt{3}/2, -1/2]\\ \hat{d}_2 &= [0, 1]\end{aligned}$$

$$\boxed{\sigma_i = \vec{S}_i.\vec{d}_i}$$

$\boxed{\vec{B}}$





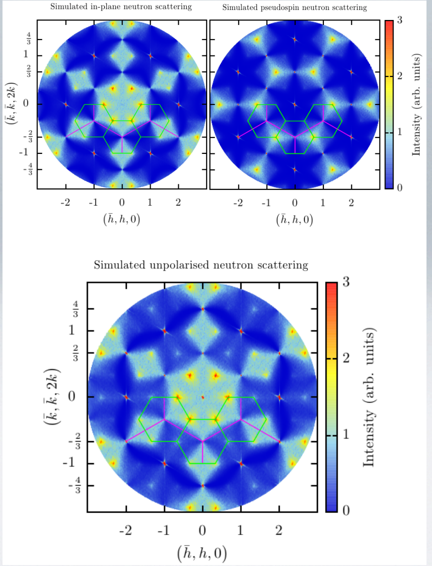


Neutrons polarized along <111> could measure pseudo-spins

In plane <111>

Unpolarized is sum of both

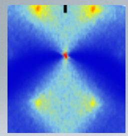
ZERO FIELD NEUTRON SCATTERING

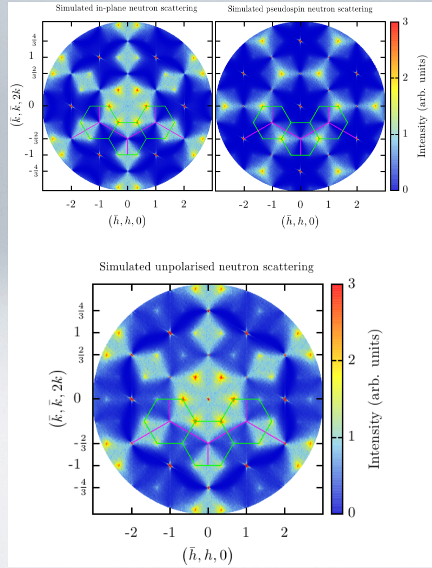


10800 spins,
50 different spin configurations.

ZERO FIELD NEUTRON SCATTERING

Pinch points





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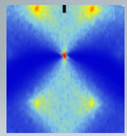
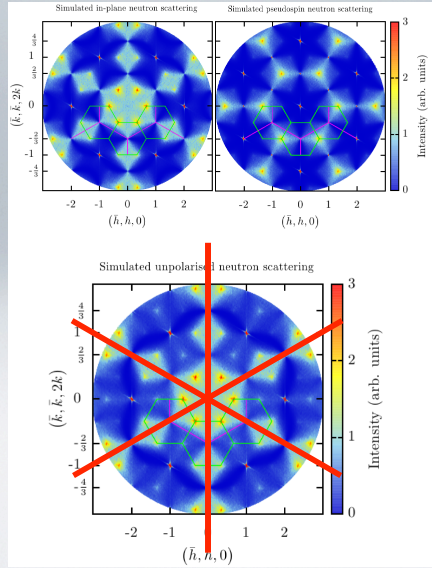


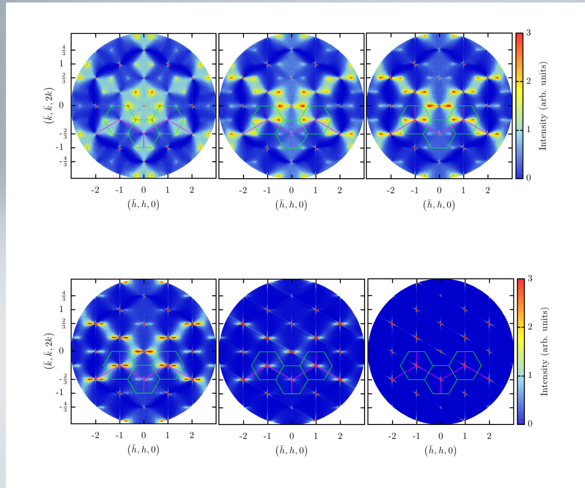


10800 spins,
50 different spin configurations.

High symmetry

NEUTRON SCATTERING FINITE FIELD

ϕ=0



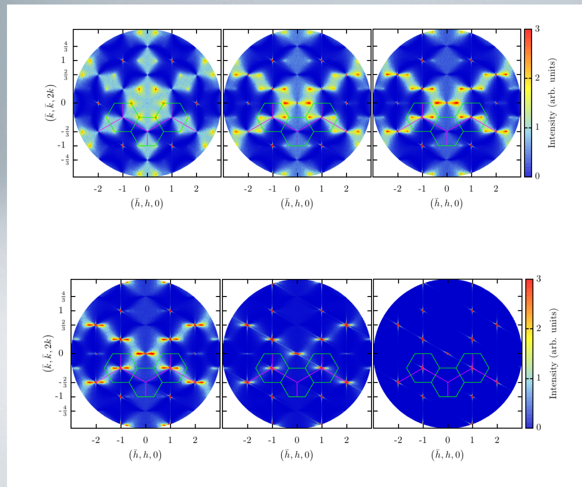
M.S. predictions

- Drifting peaks
- Reduced symmetry
- Different perpendicular correlation lengths
- Asymmetry of transition

Moessner and Sondhi, PRB, 2003

NEUTRON SCATTERING FINITE FIELD

$\phi=0$



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NEUTRON SCATTERING
FINITE FIELD

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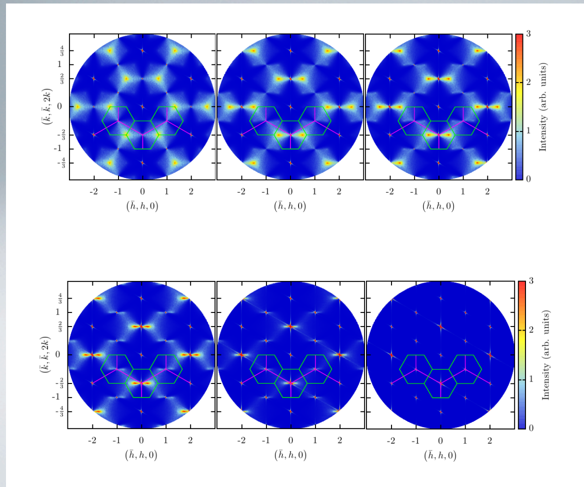

M.S. predictions

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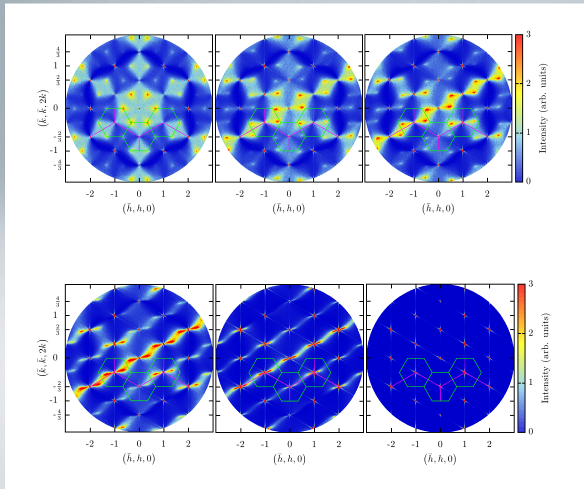
Moessner and Sondhi, PRB, 2003

NEUTRON SCATTERING FINITE FIELD

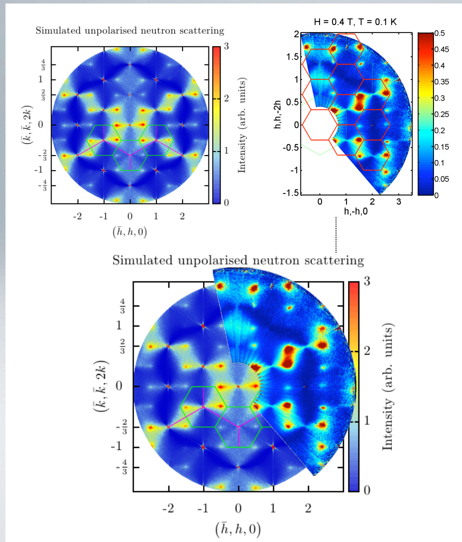
ϕ≠0



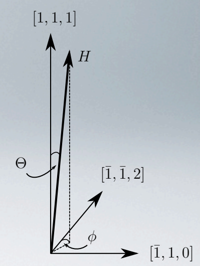
Note:

- Symmetry
- Peak shape
- Correlation lengths

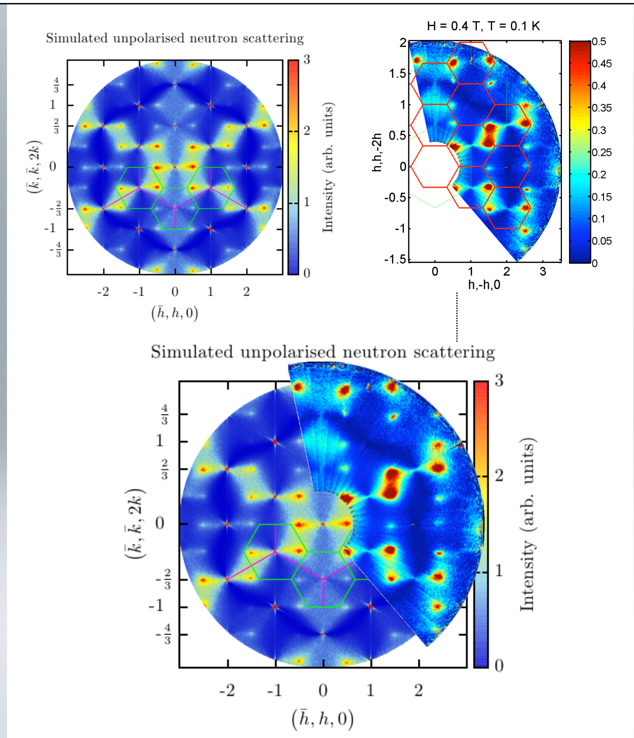
COMPARISON WITH EXPERIMENT



$Ho_2Ti_2O_7$
0.1K, 0.4T
ILL



Fennell *et al.* Unpub.
Fennell *et al.* Nat. Phys., 2007
Morris *et al.* Science, 2009

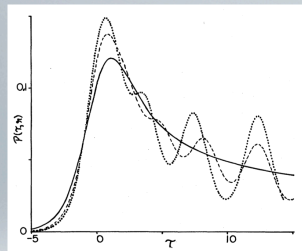


Agreement is qualitative

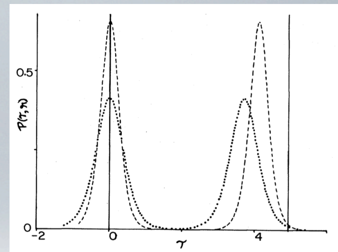
$$\frac{h}{T_{\exp}} = \frac{10\mu_B H_{\exp} \sin(3^0)}{k_B T_{\exp}} \approx 1.4$$

$$\frac{h}{T_{th}} \approx 0.12$$

FINITE SIZE SCALING



Short, wide, R→∞



Tall, thin, R→0

$$R = \frac{N^2}{M}$$

Specific heat of brick lattice

Bhattacharjee and Nagle, PRA, 1981

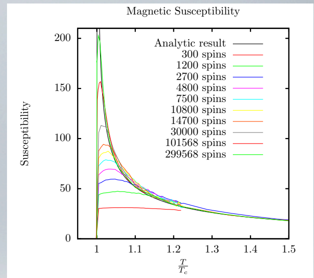
COMMENSURATE FIELD BEHAVIOUR

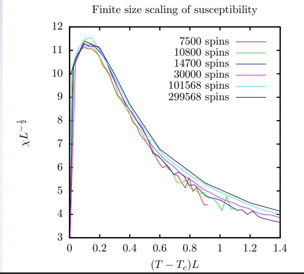
$$\chi = t^{-\gamma}\mathcal{P}\left(\frac{\xi_\perp}{L_\perp}, \frac{\xi_\parallel}{L_\parallel}\right)$$

$$\begin{aligned}\chi &= t^{-\gamma}\mathcal{P}\left(\frac{1}{|t|^{\nu_\perp}L}, \frac{1}{|t|^{\nu_\parallel}L}\right)\\ &= t^{-\gamma}\times\left(|t|^{\nu_\parallel}L\right)^{\frac{\gamma}{\nu_\parallel}}\mathcal{Q}\left(\frac{1}{|t|^{\nu_\perp}L}, \frac{1}{|t|^{\nu_\parallel}L}\right)\\ &= L^{\frac{\gamma}{\nu_\parallel}}\mathcal{Q}\left(\frac{1}{|t|^{\nu_\perp}L}, \frac{1}{|t|^{\nu_\parallel}L}\right)\end{aligned}$$

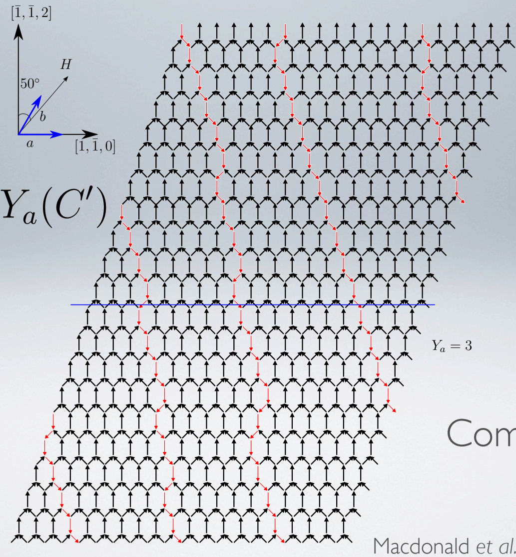
$$\lim_{|t|\to 0}\chi = L^{\frac{\gamma}{\nu_\parallel}}\mathcal{Q}\left(\frac{1}{|t|^{\nu_\parallel}L}\right)$$

$$\gamma = \tfrac{1}{2} \quad \nu_\parallel = 1 \quad \nu_\perp = \frac{1}{2}$$





WINDING NUMBER



$$W = Y_a(C) - Y_a(C')$$

$$\Gamma = \frac{1}{W} \propto R$$

Commensurability

Macdonald *et al.* J. Phys.: Cond. Mat. 2011

INCOMMENSURATE FIELD BEHAVIOUR

$$\lim_{|t|\to 0} \chi = L^{\frac{\gamma}{\nu_\parallel}} \mathcal{Q}\left(\frac{1}{|t|^{\nu_\parallel} L}\right)$$

$$\gamma = \tfrac{1}{2} \quad \nu_\parallel = 1 \quad \nu_\perp = \frac{1}{2}$$

INCOMMENSURATE FIELD BEHAVIOUR

$$\lim_{|t|\to 0} \chi = L^{\frac{\gamma}{\nu_\perp}} \mathcal{Q}\left(\frac{1}{|t|^{\nu_\perp} L}\right)$$

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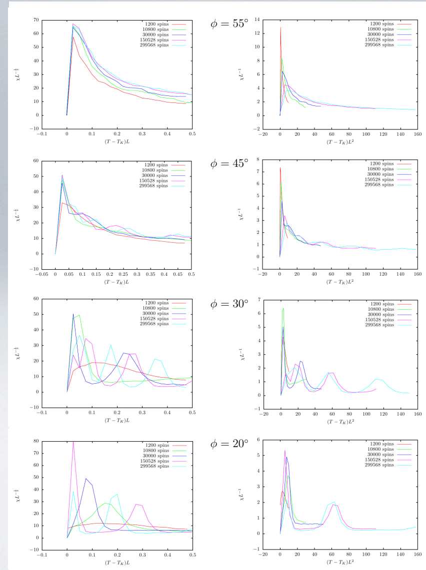
INCOMMENSURATE FIELD BEHAVIOUR

$\nu_{\parallel}$ scaling

$\nu_{\perp}$ scaling



$$\lim_{|t|\to 0} \chi = L^{\frac{\gamma}{\nu_\parallel}\nu_\perp} \mathcal{Q}\left(\frac{1}{|t|^{\nu_\perp} L}\right)$$

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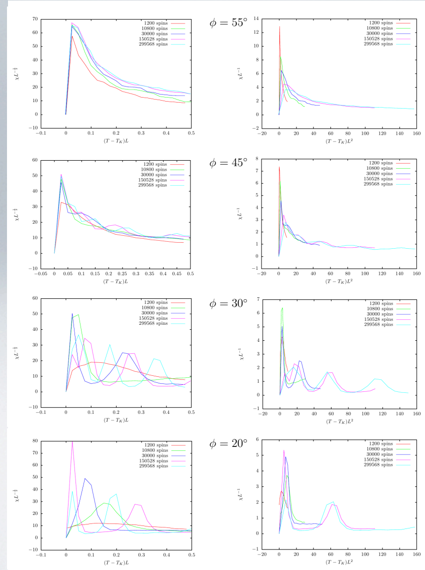
INCOMMENSURATE FIELD BEHAVIOUR

$\nu_{\parallel}$ scaling



$\nu_{\perp}$ scaling

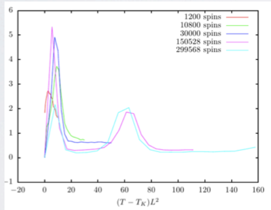
Quantised behaviour!

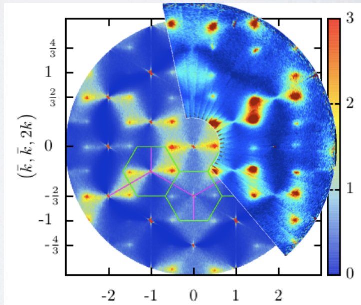
$$\lim_{|t|\to 0} \chi = L^{\frac{\gamma}{\nu_\parallel}\nu_\perp} \mathcal{Q}\left(\frac{1}{|t|^{\nu_\perp} L}\right)$$

$$\gamma = \frac{1}{2} \quad \nu_{\parallel} = 1 \quad \nu_{\perp} = \frac{1}{2}$$

Conclusions

1. The Kasteleyn Transition appears in model <111> spin ice
2. A vestige of this transition can be observed in experiment
3. Finite size scaling exposes the topological nature of the K-transition





From Spin Ice to Kagomé Planes:
Natal, December 2011