Wien effect in lattice Coulomb gas

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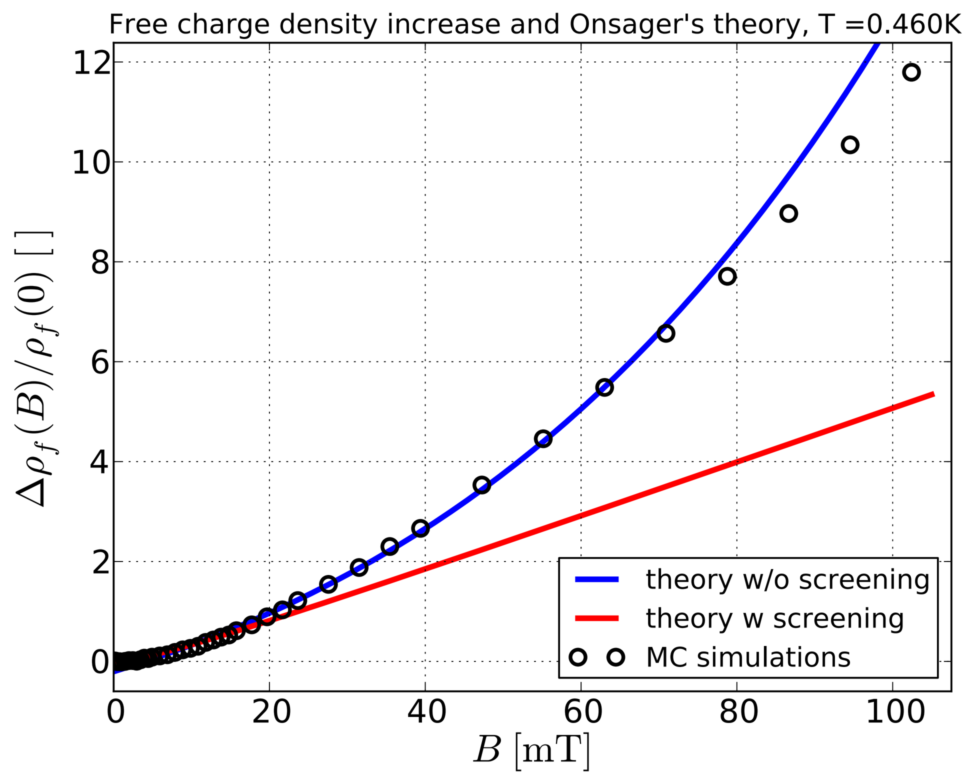
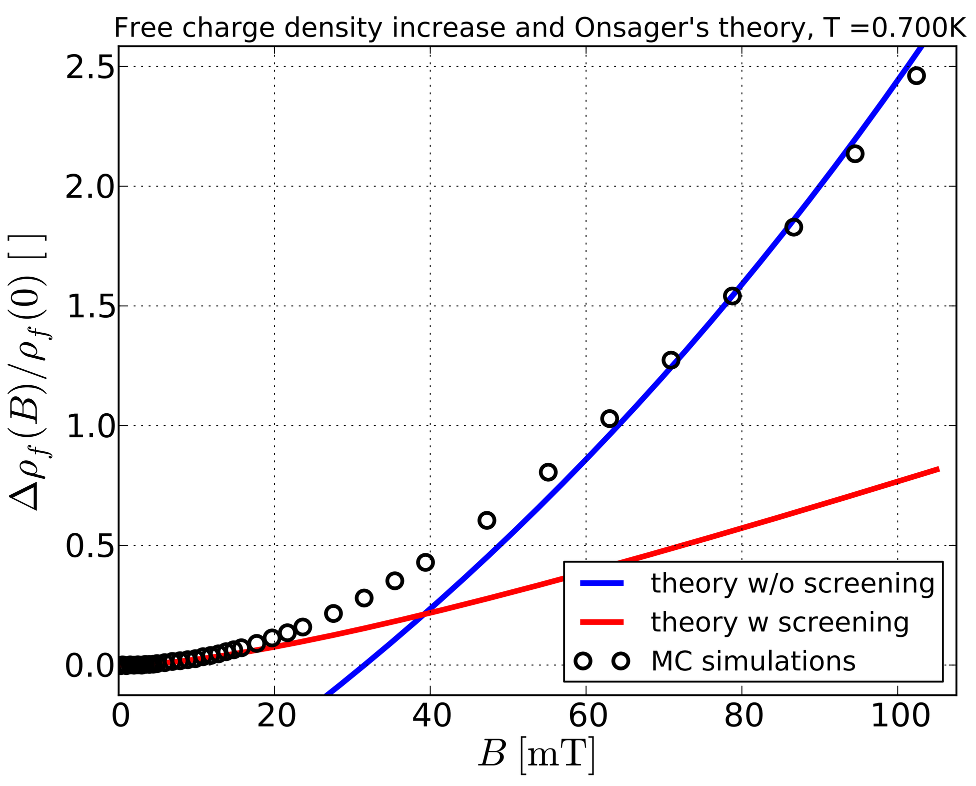
Lattice Coulomb gas

Magnetic monopoles are excitations from the 2-in-2-out spin ice configuration [1]. They carry a magnetic charge and interact Coulombically. Inspired by the correspondence between thermodynamics of spin ice and Coulomb gas with kinetic constraints [1, 2], we study effects of an external field on the diamond-lattice Coulomb gas in grand-canonical ensemble with chemical potential μ_0 per charge. In the presented results, we neglect the kinetic constraints and the fact that each charge configuration is dressed in multiple Dirac string configurations.

Charge density





Relative increase of free charge density follows closely Onsager (blue) and Liu (red) theory. The offset of the unscreened (blue) curve with B-axis corresponds to $l_B = l_D$. There are no fitting parameters.

Zero-field charge density increases with T, making screening stronger. Total charge density can be found from the Debye-Hückel theory [7], while the free charge density is given by $\exp(+\beta\mu_0)$. We assume that the increase in the free charge density is equal to the total density increase.

Outlook

Preliminary results from simulations of Coulomb gas with Dirac strings show a similar increase in the density. However, permanent currents do not occur due to the kinetic constraints. A interminent quasi-steady-state regime might appear. Our further work will explore this direction.

Wien effect

Creation of magnetic monopoles can be described by the following chemical equilibrium:

$$\text{quasi-particle vacuum} \rightleftharpoons \text{bound pairs} \rightleftharpoons \text{free charges}$$
$$\mathrm{AB} \rightleftharpoons \mathrm{A^-B^+} \rightleftharpoons \mathrm{A^-} + \mathrm{B^+}\,,$$

where pairs are described as bound if they are separated by less than the Bjerrum length l_T. Free charges are deconfined.

If external magnetic field is applied, pairs orient and the energy threshold for pair dissociation is decreased, while association is unaffected by the field, giving rise to the second Wien effect as proposed in recent experiments [4, 5]. The reaction of the system to the field can be described using the following lengths:

Bjerrum length	$l_B = \mu_0 Q^2/8\pi k_b T$
field length	$l_B = k_B T/QB$
Debye length	$l_D = \sqrt{k_B T/2\mu_0 Q^2 \rho_V}$

The resulting increase in free charges has been shown by Onsager [3] to be

$$\frac{\rho_f(B)}{\rho_f(0)} = \sqrt{\frac{K(B)}{K(0)}} = \sqrt{\frac{I_1(2\sqrt{2l_T/l_B})}{\sqrt{2l_T/l_B}}} = 1 + \frac{l_T}{2l_B} + \mathcal{O}\left(\left(\frac{l_T}{l_B}\right)^2\right)\,.$$

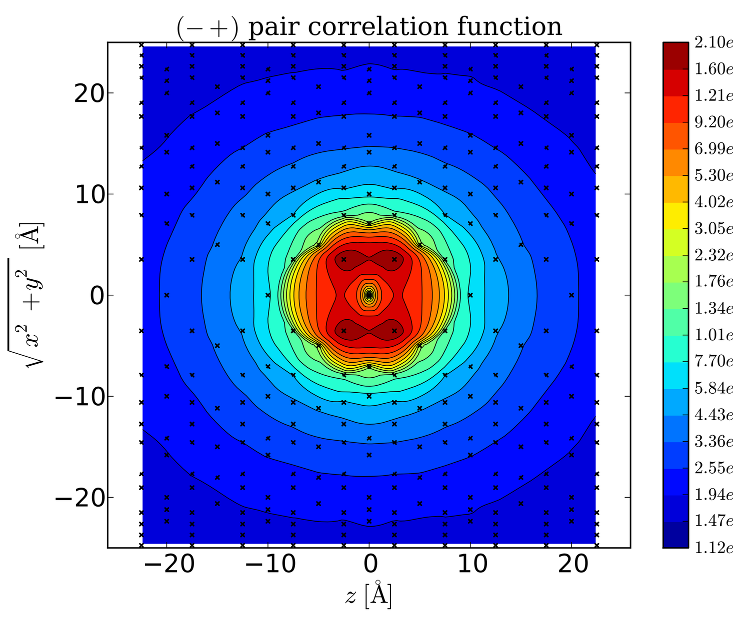
If Debye screening is significant ($l_D \sim l_B$), an extended result by Liu applies:

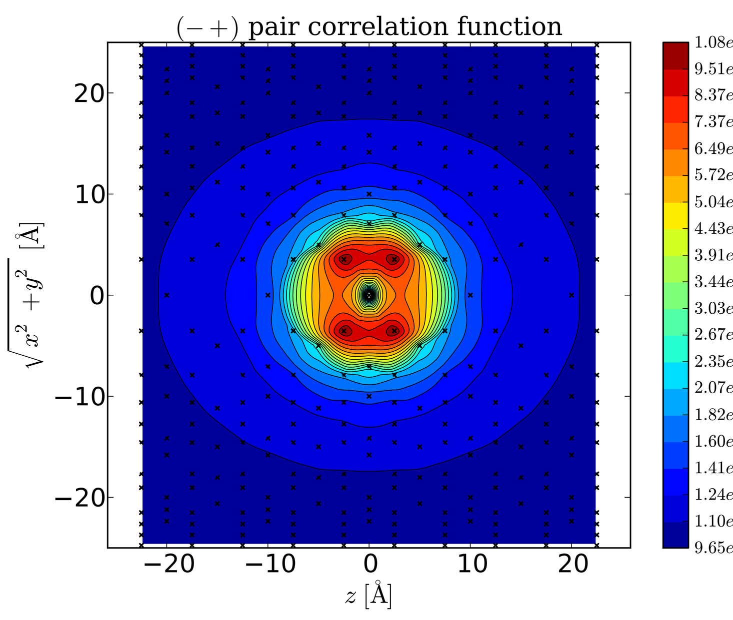
$$\frac{\rho_f(B)}{\rho_f(0)} = 1 + \frac{l_T/l_H + (2l_T/l_D)(f(l_D/l_H) - 1)}{1 + 2l_T/l_D} + \ldots\,,$$

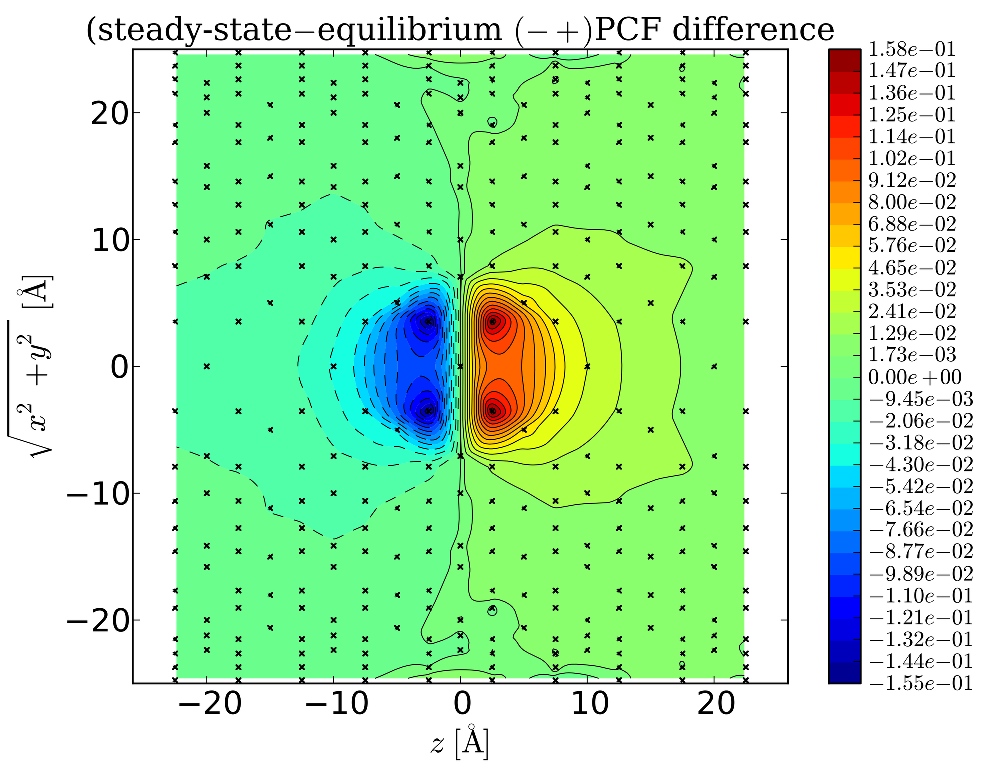
where f is defined in Ref. [6].

Pair correlations

Pair correlation functions (PCF) allow observing the structure of the Coulomb gas and its changes due to the external field. Following PCFs show the decreased extent of the ionic atmosphere due to the screening (0.46 K and 0.7 K).







The second Wien effect is a non-equilibrium steady-state process, where pairs constantly reorient and separate in the field direction, while new ones are created isotropically. Pair orientation can be observed in the PCF change with applied fields (example at 0.6 K and 20 mT).

Projections of PCF to the $\sqrt{x^2+y^2}$-z-plane. The up-down mirror symmetry serves for clarity only.

References

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