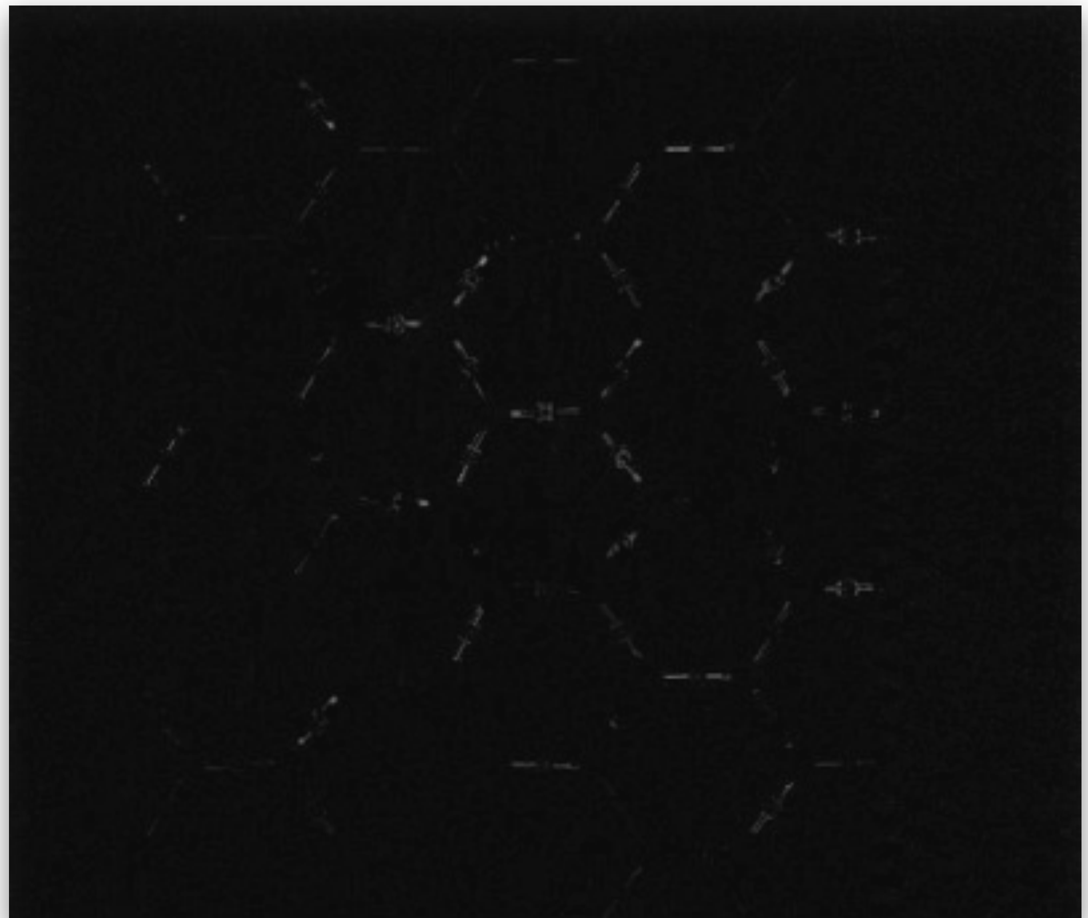
Spin Ice correlations in a Macroscopic system



Paula Mellado

in collaboration with Andres Concha and L. Mahadevan

School of Engineering and Applied sciences, Harvard University

Natal, December, 2011

Overview

We have a new experimental realization of a macroscopic version of spin ice where interactions give rise to well defined "selection rules" for the low energy patterns which vary depending on geometry and interaction strength.

This system endow us with an ideal tool to examine the dynamics and the role of intrinsic and external perturbations in a frustrated system.

Using a combination of experiments, simulations and analytical arguments we unveil the relevant parameter space determining the dynamics of the system.

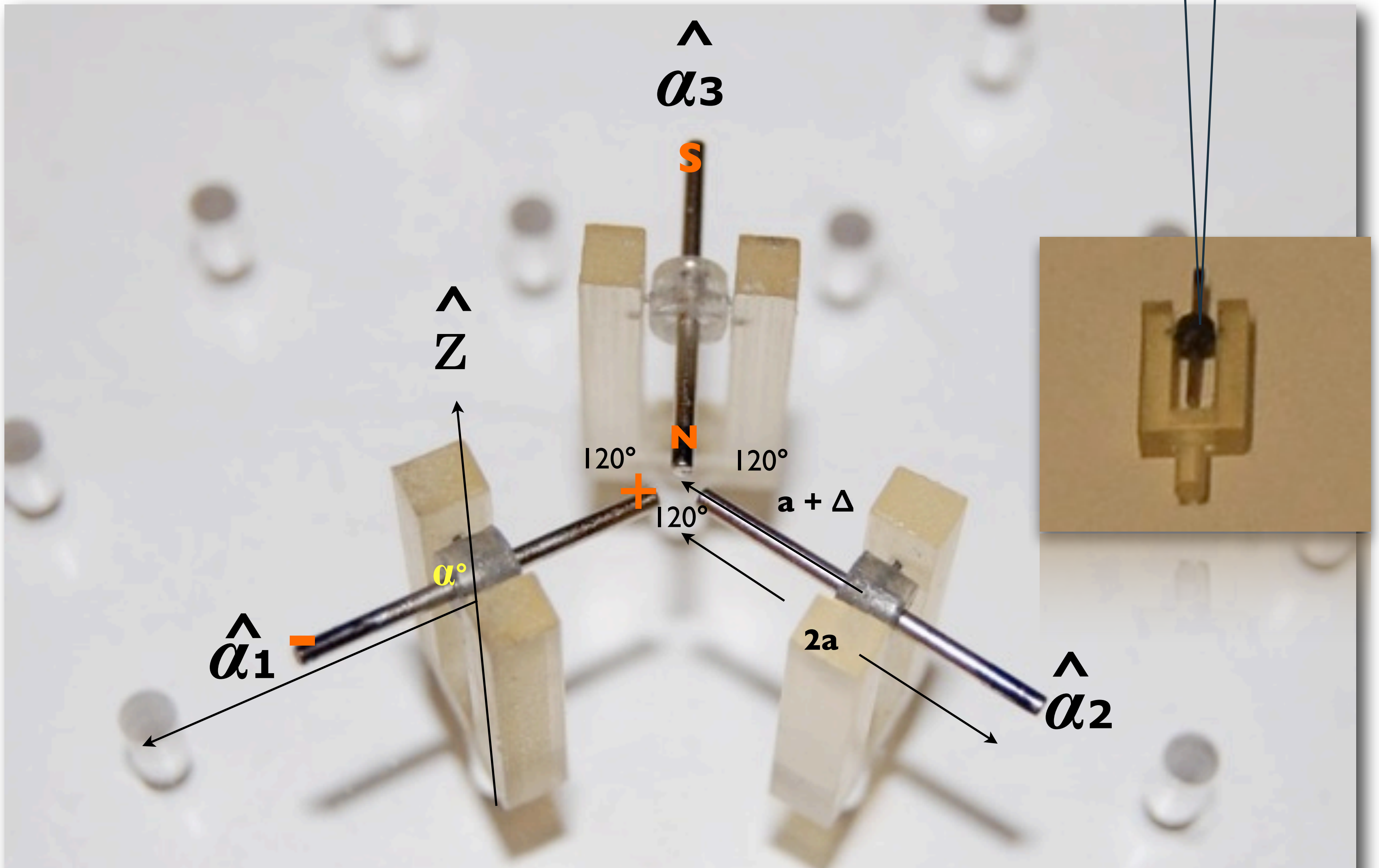
Three Magnetic Rods at θ=120° star configuration

Ferromagnetic rods which rotate continuously along the azimuthal direction, hinged at the plane that is equidistant respect to the N and S magnetic poles so that their axis of rotation cross their center of mass.

Hinges axis were made out of graphite. By design the only allowed motion is around the local axis

commercial Neodymium magnets $Nd_2Fe_{14}B$

2a = 1.9 10^-2 [m]
$2r_{rod}$ = 1.59 10^-3 [m]
mass = 0.29 10^-3 [Kg]
Δ = 4.6 10^-3 [m]
Ms = 10^6 [A/m]

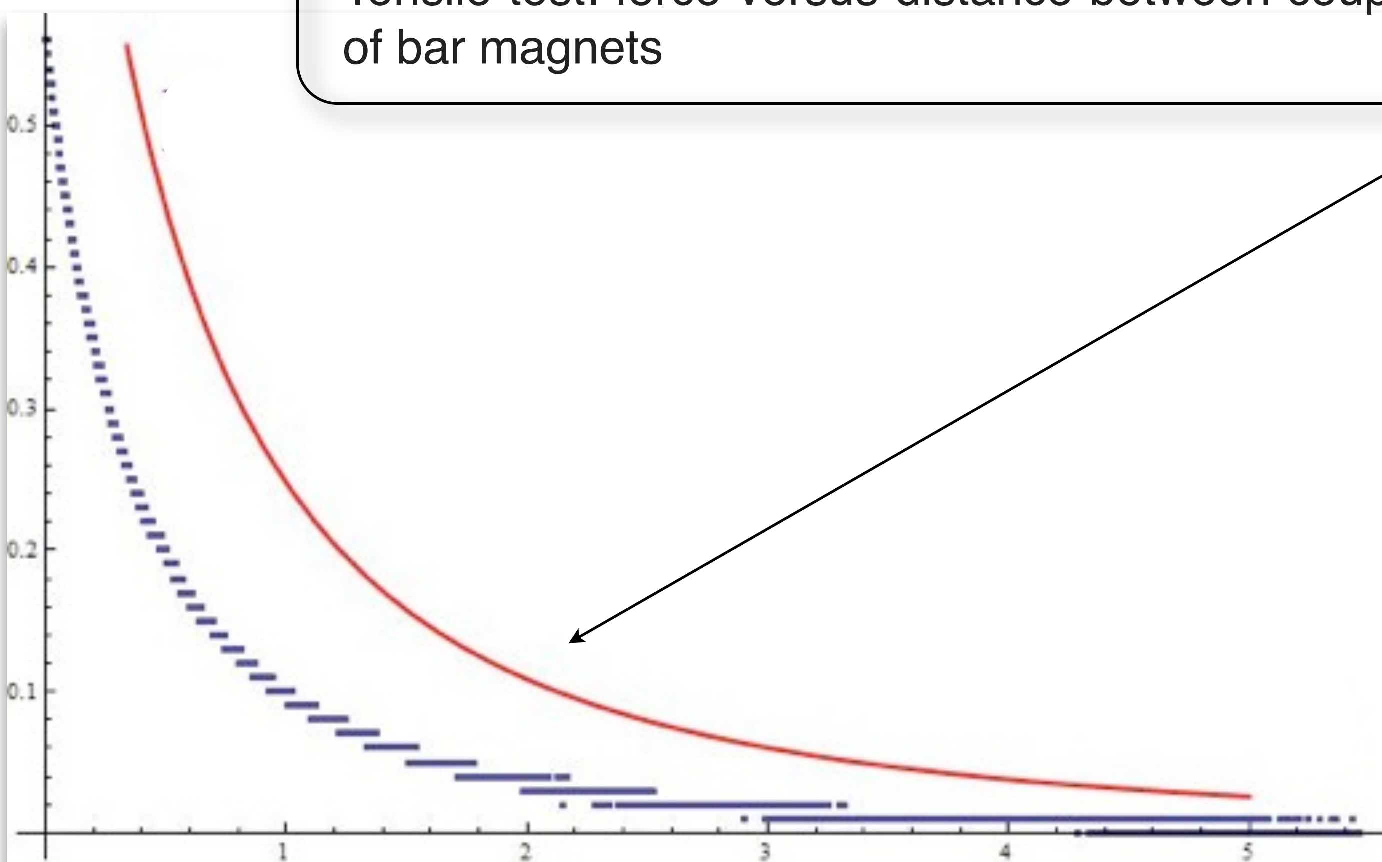


Lowest energy configuration: NNS or SSN configurations correspond to the spin-ice rules.

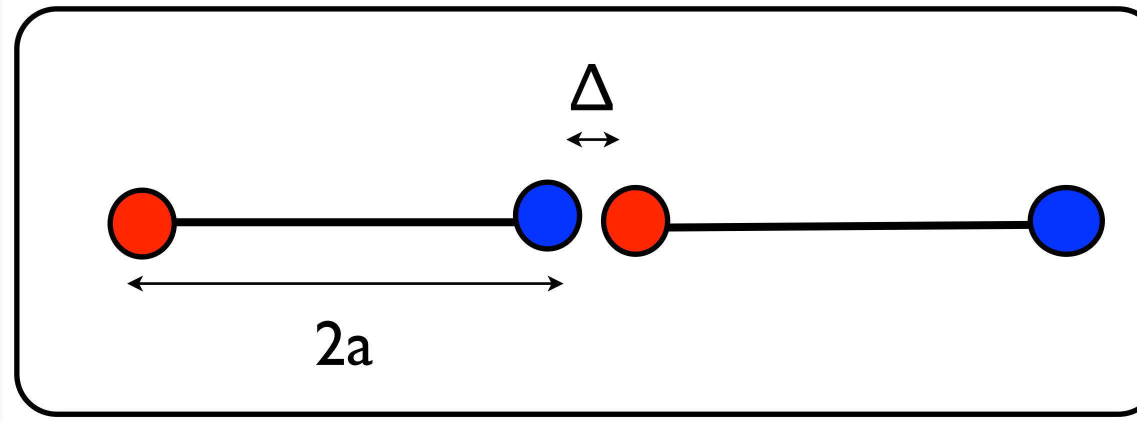
Bar magnet whose length is larger than its diameter, $2a/r_{rod}$ ~25, we defined |**m**| = 2a q, where q is the pole charge, **m** pointing from S to N pole.

Coulomb interaction between rods

Tensile test: force versus distance between couples of bar magnets



Red line (offset) represents the Coulomb **interaction between dumbbells** from where we obtained the magnitude of the magnetic charges **q = 2.38** [Am].



$$q = \pi M_s r_{rod}^2$$

Pairwise interactions decay like $1/\Delta^2$

when Δ < a/4. Beyond that the interaction becomes dipolar.

Using q, we obtained saturation magnetization *Ms* = 10^6 [A/m] in agreement with the available data for Neodymium rods, validating a Dumbbell approximation.

Damping and static friction

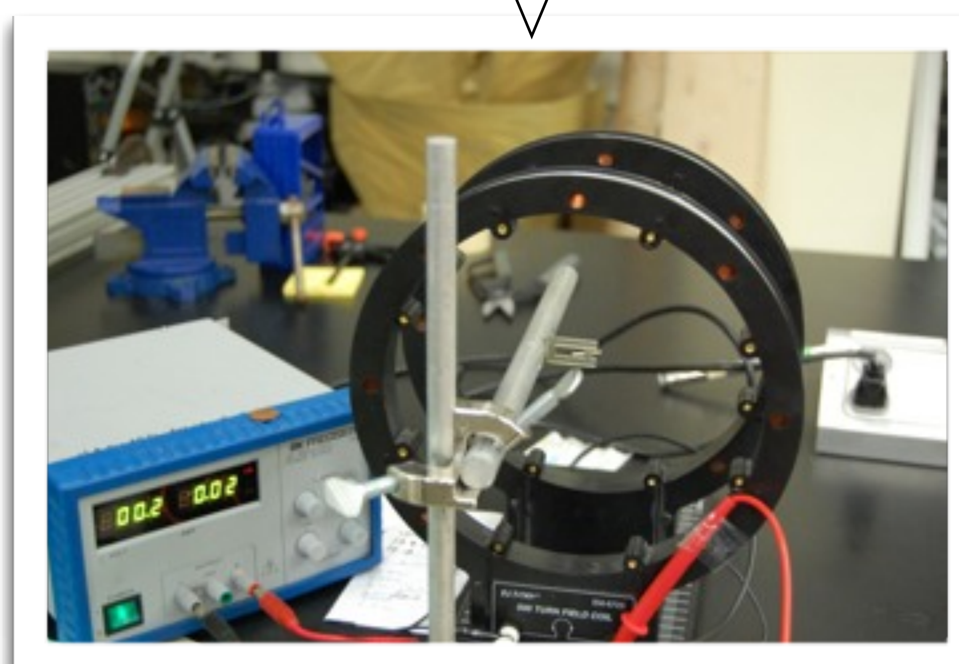
We isolated a set of single hinges and impulsively applied a torque to it. We recorded its dynamics using a high speed camera.

Using standard imaging techniques we extracted $\alpha(t)$.

The damping is computed directly by fitting the damped dynamics for the cyclic variable to the solution $\alpha(t) = \exp(-t/\tau_D)$.

For quantifying static friction we applied an external field to an individual rod using a Helmholtz coil.

1000 fps



$\tau_D = 1.0 \pm 0.01 \pm$ [sec]

$B_{critical} \sim 10$^-4 [Tesla]

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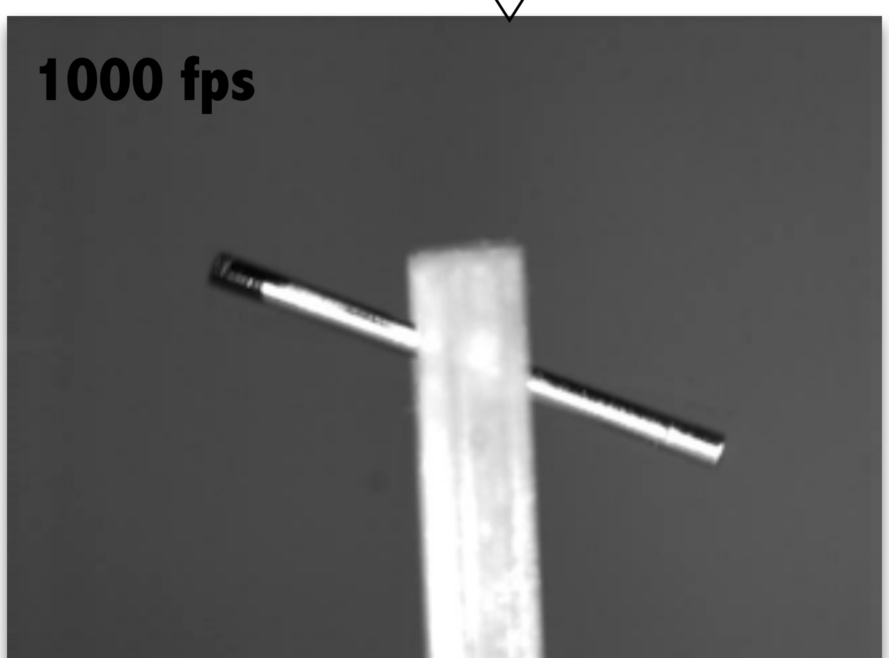

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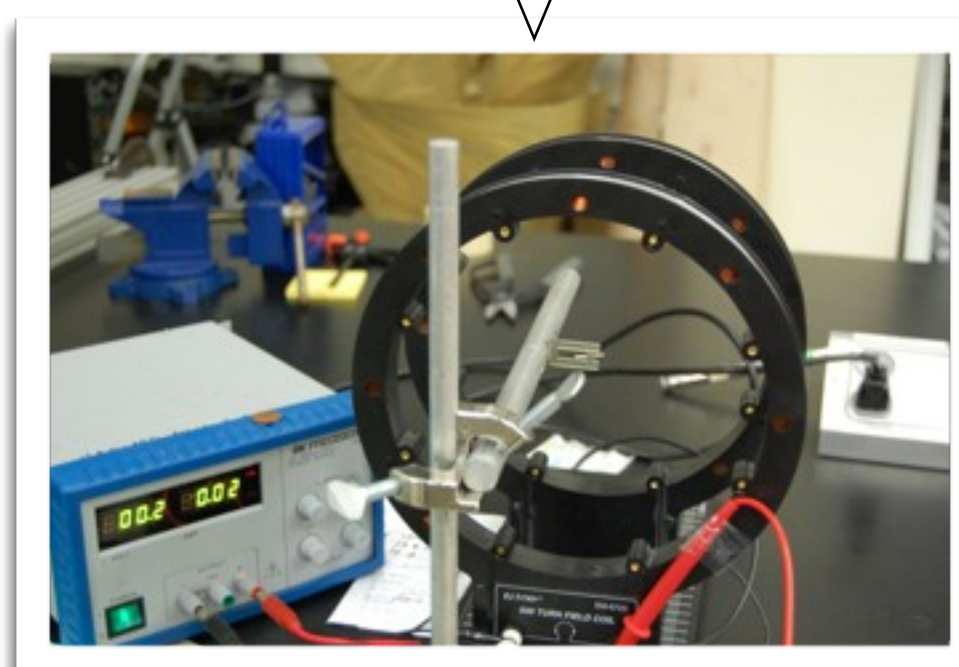

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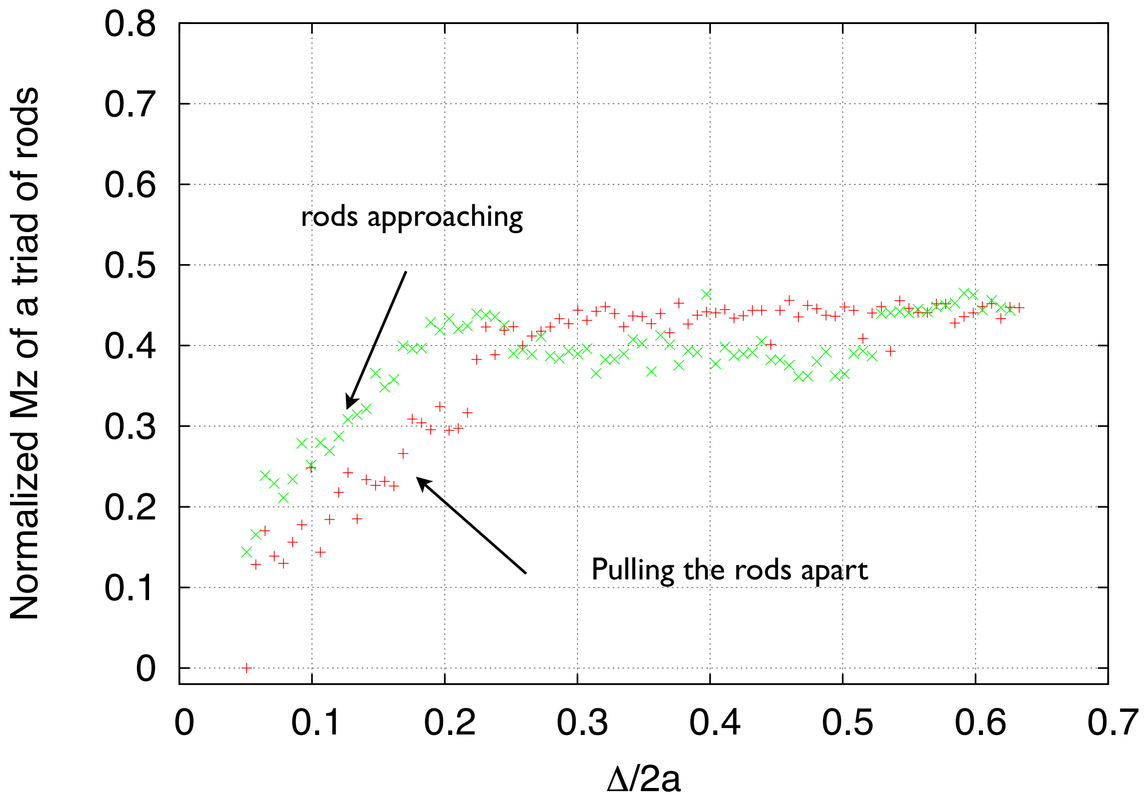
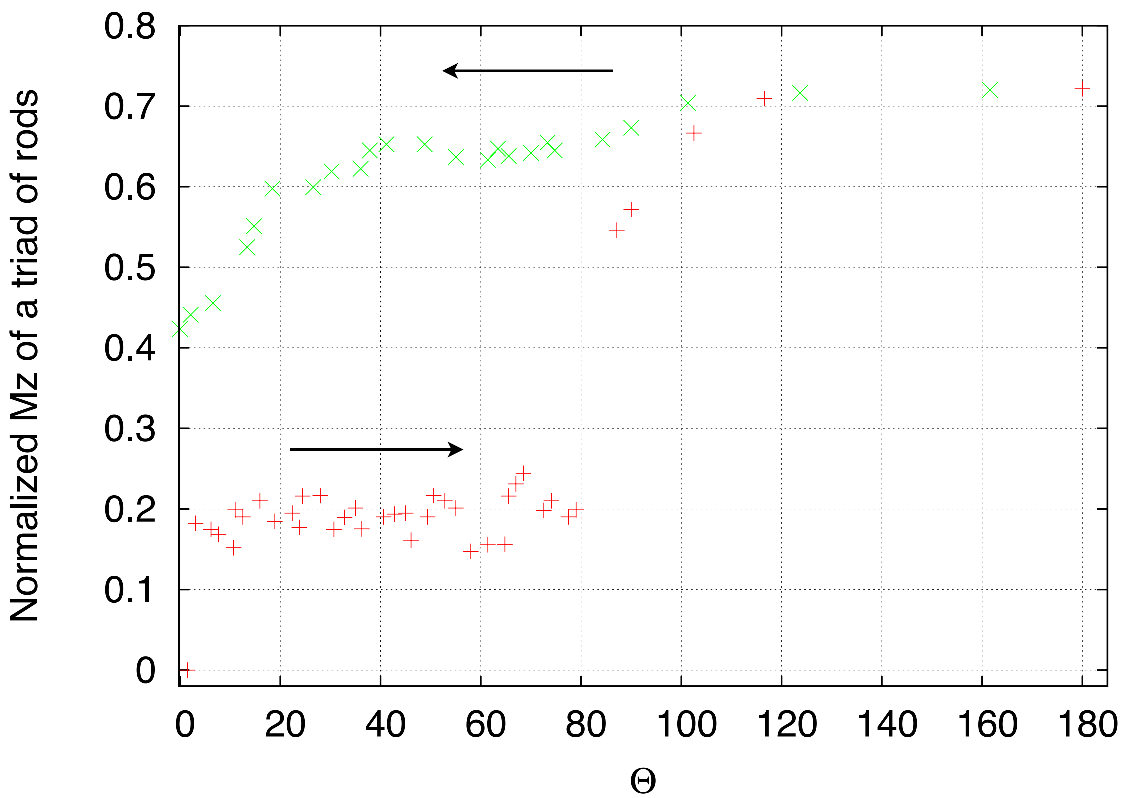
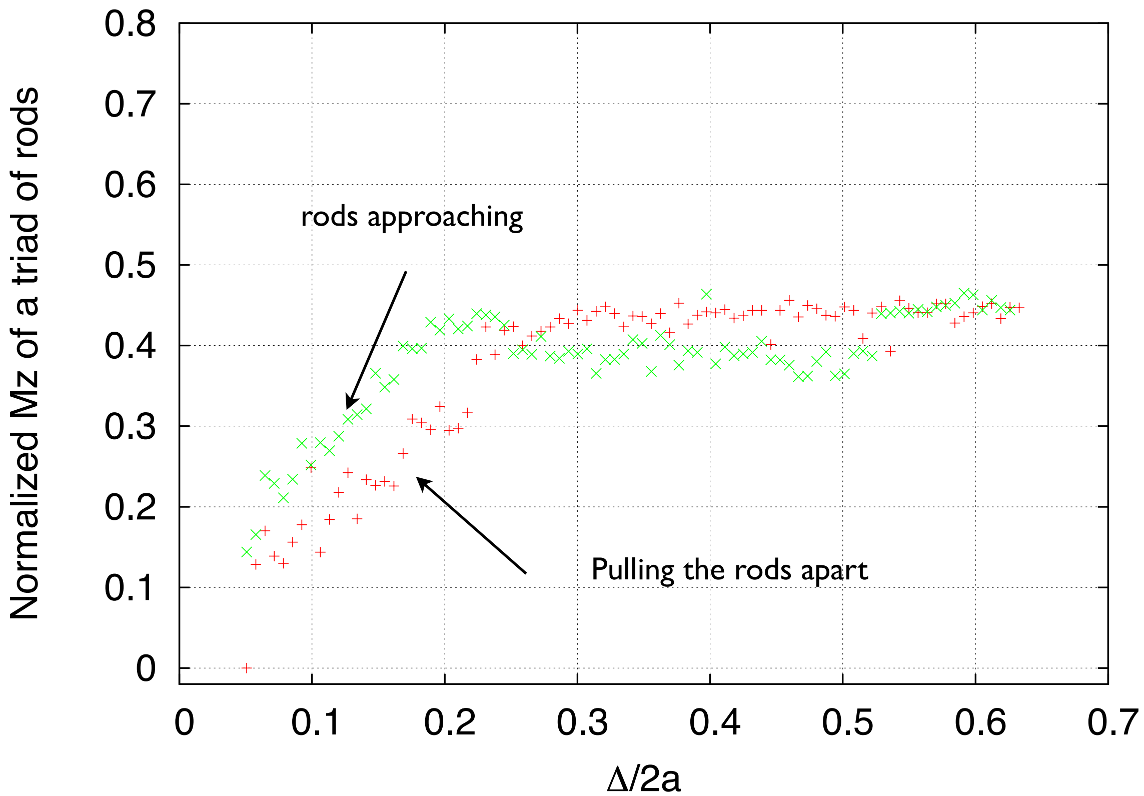
Mz v/s Δ and Mz v/s θ

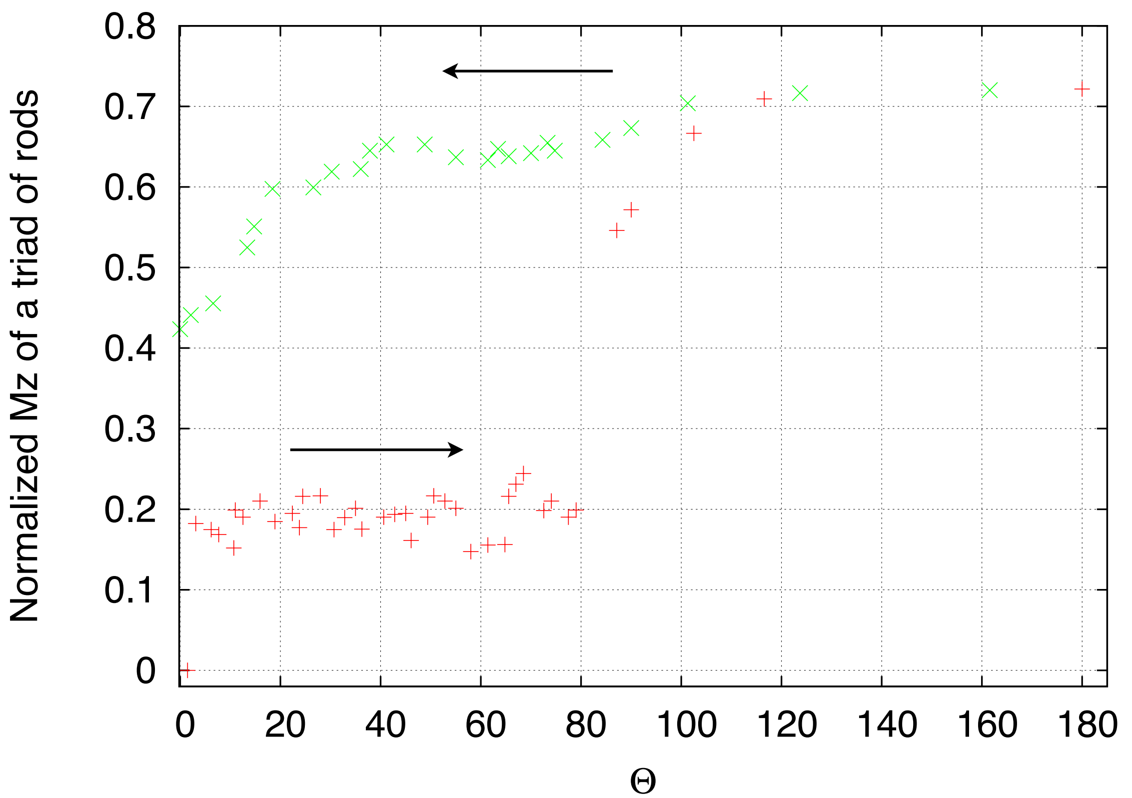




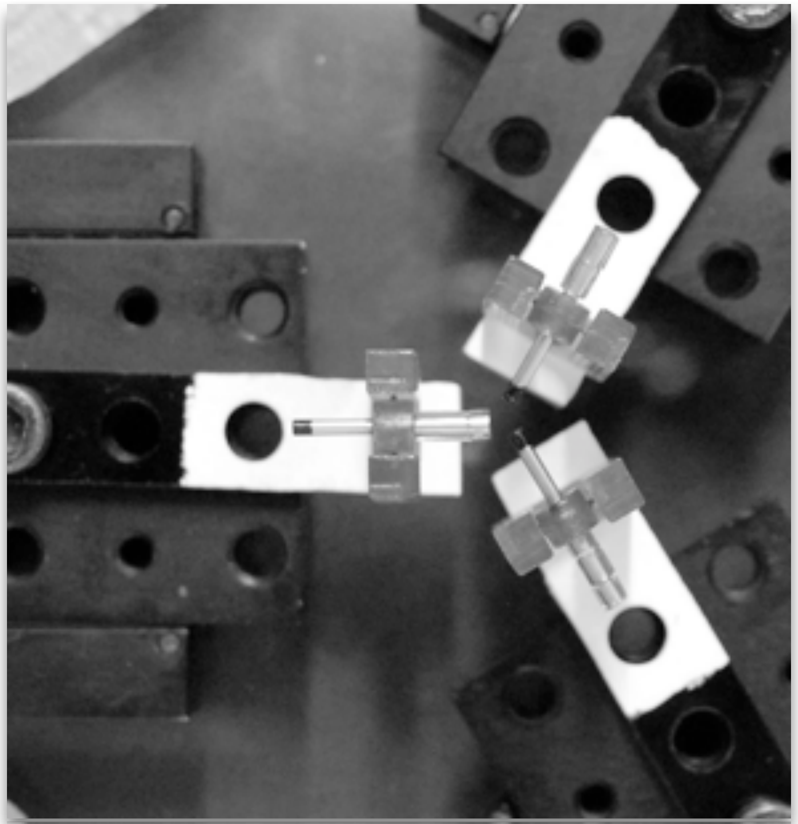
rods depart of the plane when Δ ~ 0.2 a and when θ ~ 60°

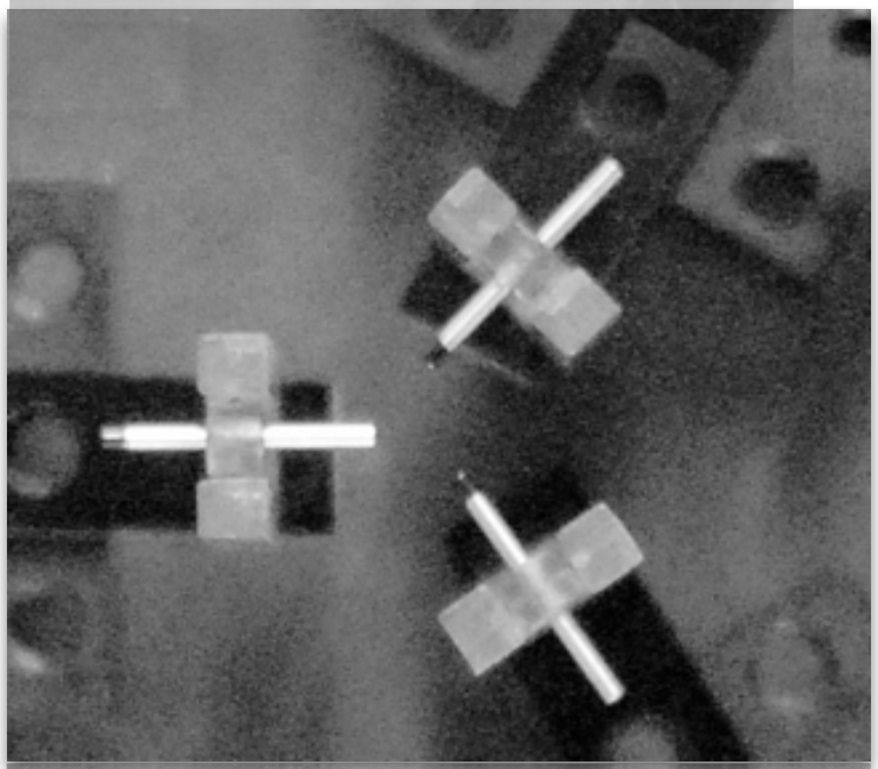
Mz v/s Δ and Mz v/s θ





rods depart of the plane when $\Delta \sim 0.2$ a and when $\theta \sim 60°$





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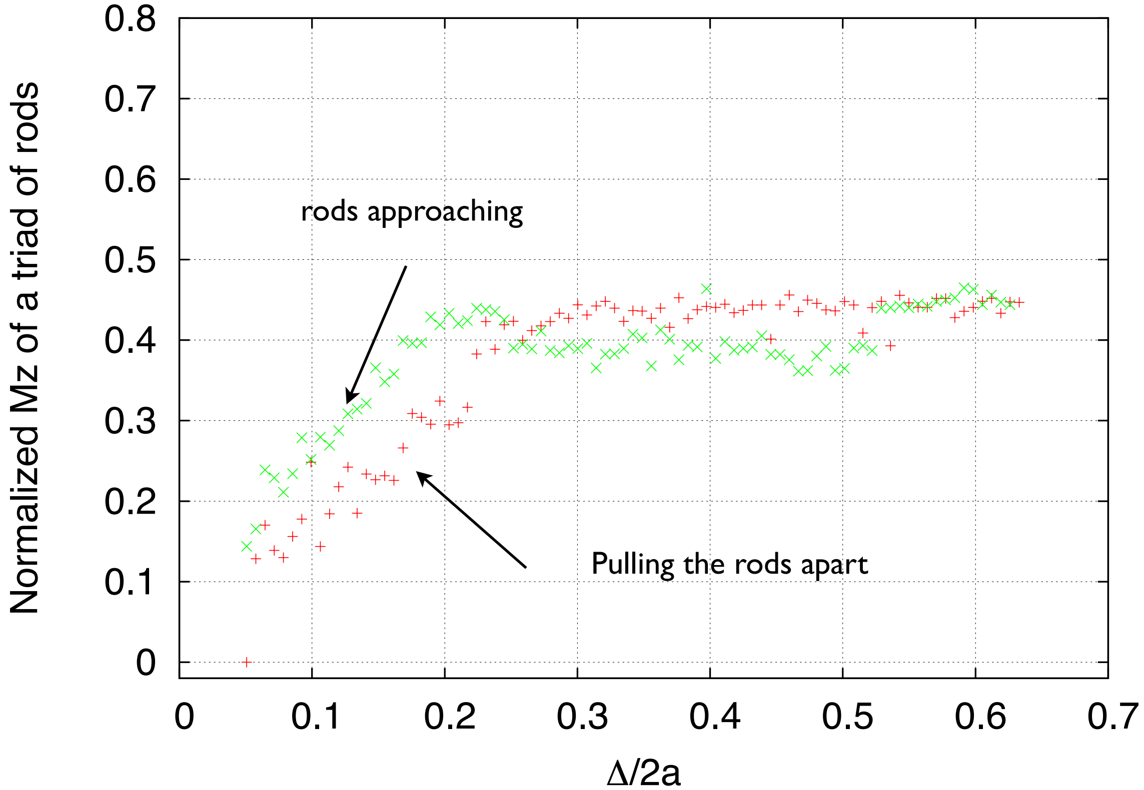

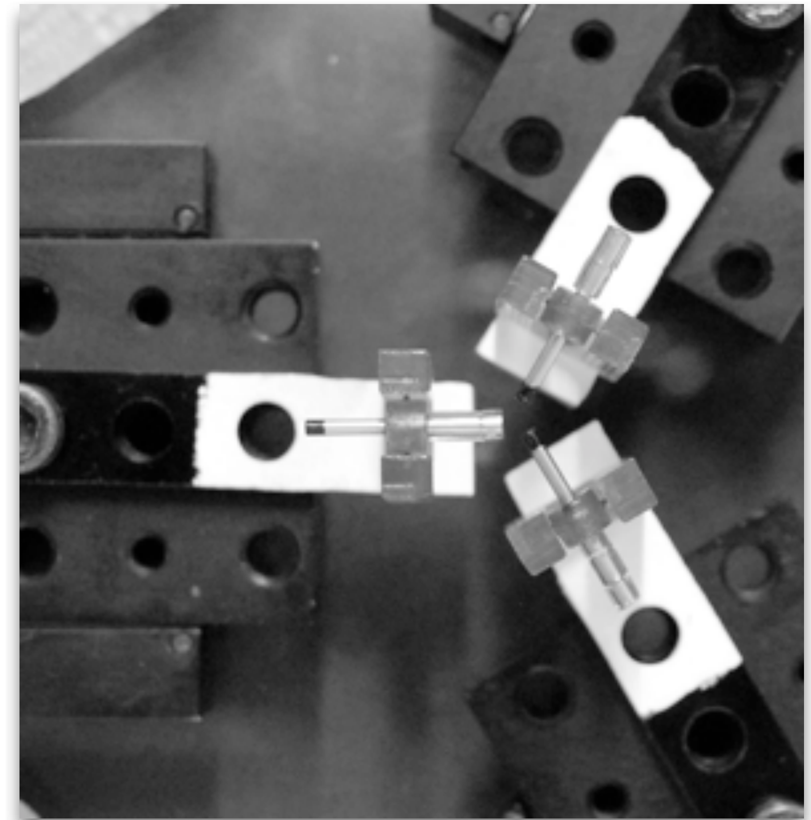

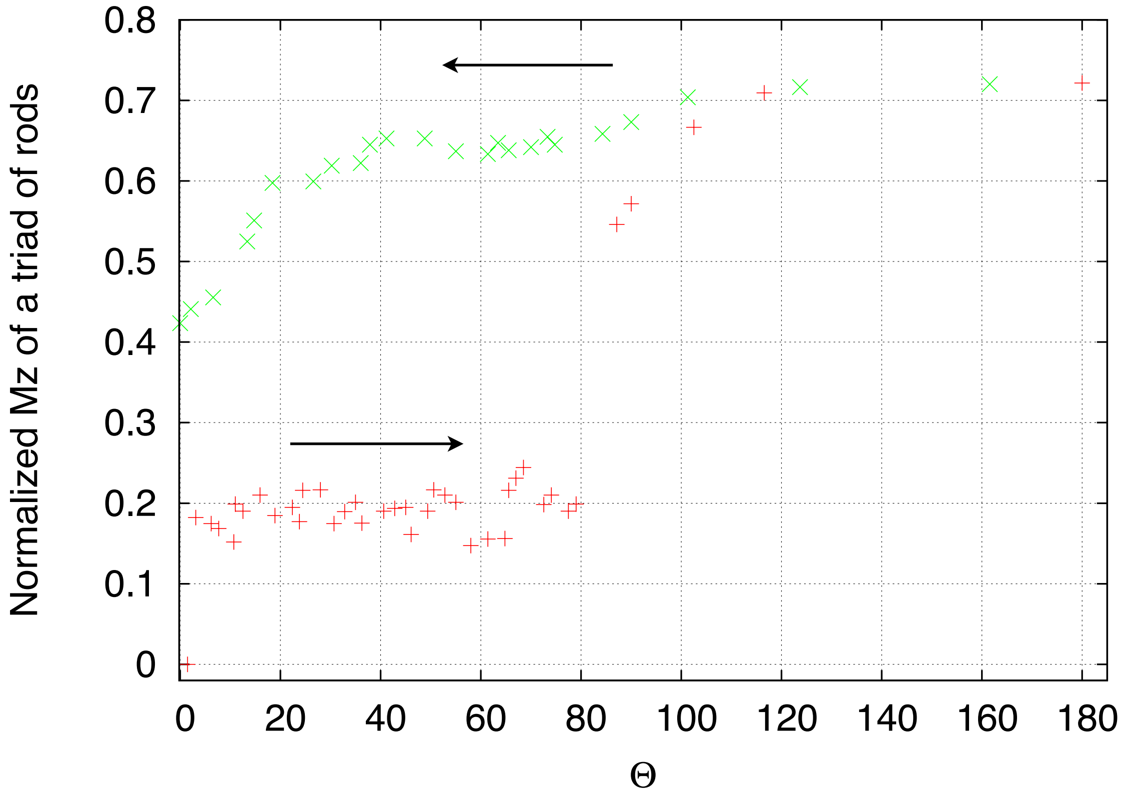

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and
when $\theta \sim 60°$

Dynamics of a single, isolated rod

External torques

axis of rotation

$$I\frac{d^2\alpha}{dt^2} = \mathbf{T}_{ext}\cdot\hat{\alpha} - \eta\frac{d\alpha}{dt}$$

Inertia moment I = 2.04*10^-8 [Kg m^2]

Damping constant

angle respect to the z axis

$\vec{B} = B_0\hat{z}$ [N/Am]

$$\mathbf{T}_{ext}\cdot\hat{\alpha} = \mathbf{m}\times\mathbf{B} = -2a|B_0 q|\sin(\alpha)$$

$$\boxed{\tau_B = \frac{2\pi}{\omega_B} \quad \omega_B = \left(\frac{2a|qB_0|}{I}\right)^{\frac{1}{2}} \quad \tau_D = I/\eta \sim 1\ \text{sec} \quad \frac{\tau_B}{\tau_D} = 10^{-3}B_0^{-\frac{1}{2}}}$$

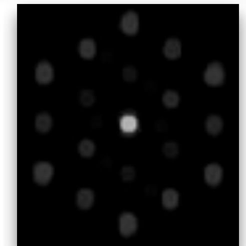
$$\alpha = \alpha_0\exp(i\omega t)$$

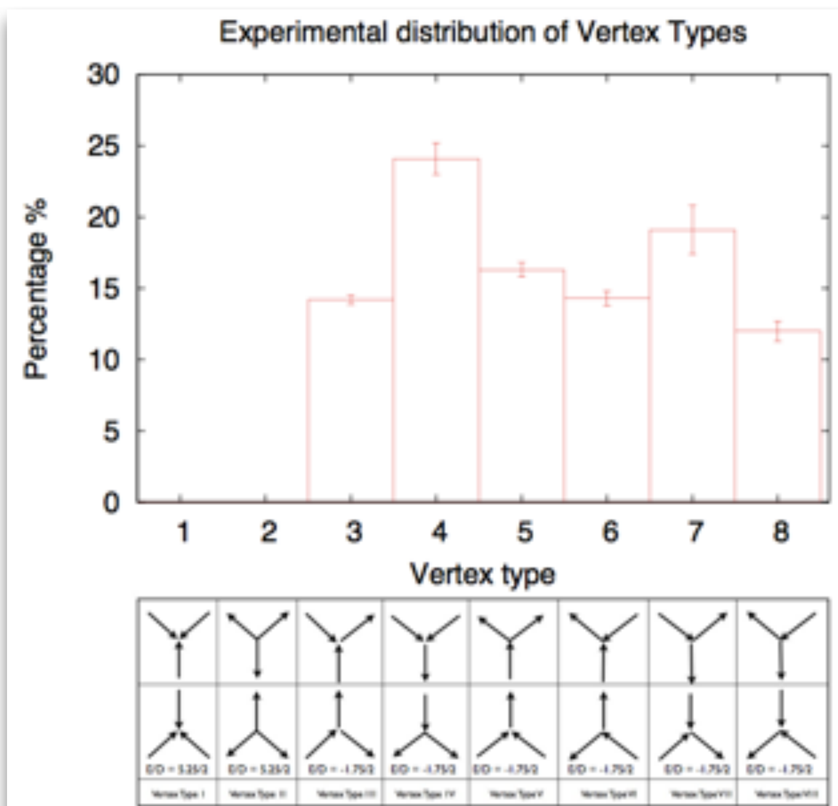
$$\omega = \frac{i}{2\tau_D} \pm \frac{1}{2}\sqrt{1/\tau_D^2 - 4/\tau_B^2}$$

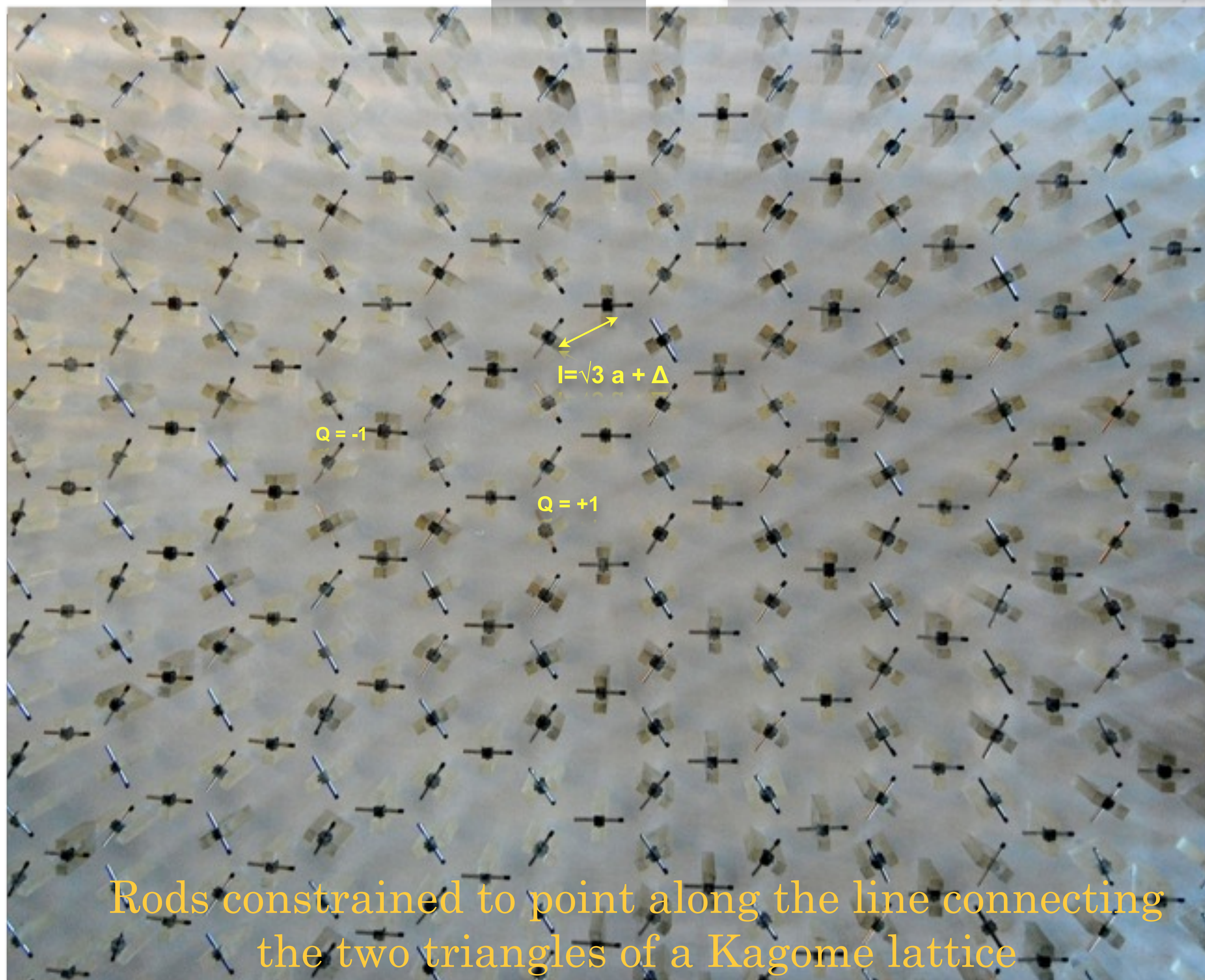
critically damped when $4\omega_B^2\tau_D^2 = 1$

The Lattice

Spin ice configurations



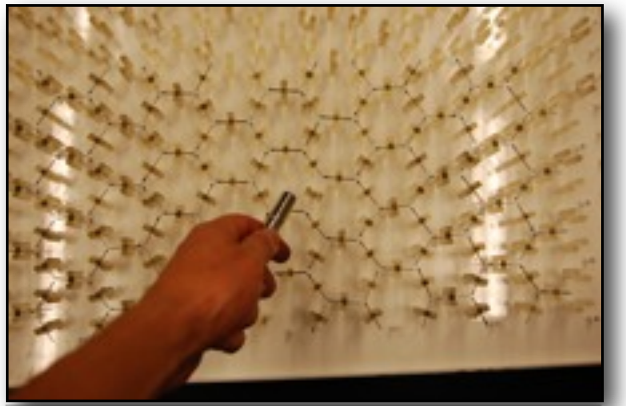




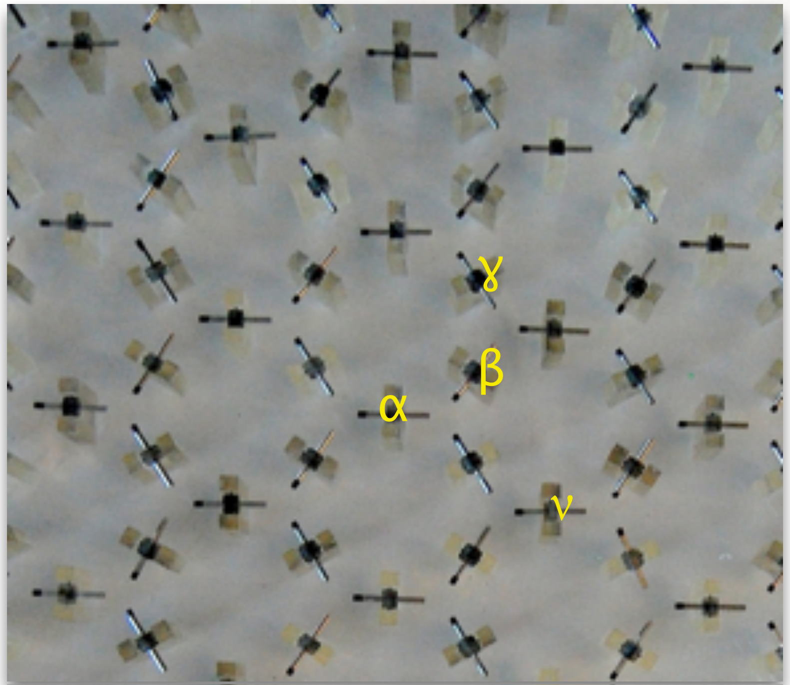
Honeycomb lattice of 150x3 rods,
(mimic Ising spins)
with lattice constant $l = \sqrt{3}\, a + \Delta$,
$\Delta/a \sim 0.2$
$a \sim 10^{-2}$ [m]

Spin and Charge Correlations

Start with random initial configuration and image the final state after equilibration (~1sec). Averaged out the spin n-n correlations $S_{\alpha\beta} = \langle m_\alpha \, m_\beta \rangle \sim 1/3$, so that our lattice satisfies the honeycomb spin ice rule.



Correlation	Average
$S_{\alpha\beta}$	0.334 +/- 0.004
$S_{\alpha\gamma}$	0.017
$S_{\alpha\nu}$	0.448
$C_{\alpha\beta}$	-0.272



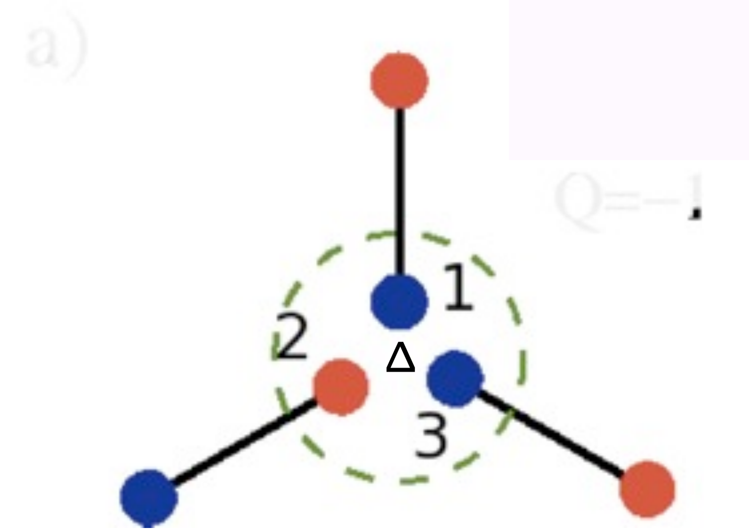
Averaged out the charge n-n correlations $C_{\alpha\beta} = \langle c_\alpha \, c_\beta \rangle$. For a totally charge disordered state in the spin ice manifold we expect $C_{\alpha\beta} \sim -1/9$. Our lattice does have a non apparent charge order.

N-N Dumbbells

<u>we can understand the basic physics by paying attention into the most divergent quantities.</u>

$$U_\Delta \sim U_{12} + U_{23} + U_{31}$$

When $\Delta \to 0$ only 3 terms contribute the divergent Coulomb potential due to the interactions between the three charges that are closest to the origin.



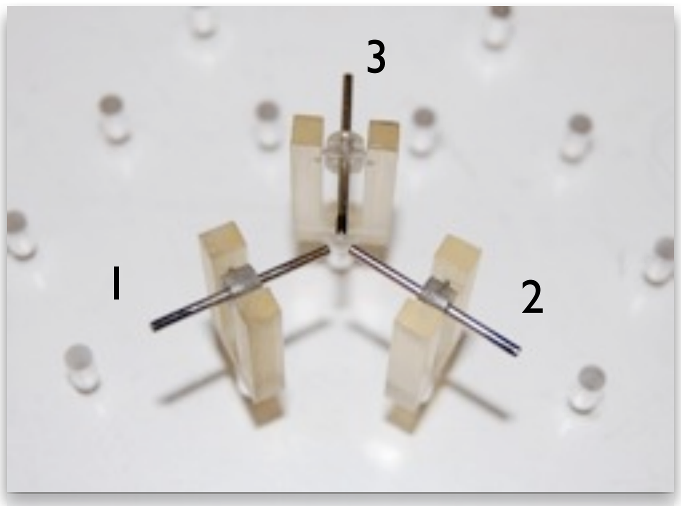
U(Δ) ~ g/Δ, we want *U* to be the less divergent possible. In the limit $\Delta \to 0$, the leading order term will only depend on the geometry.

The global minimum for $g(\alpha_1, \alpha_2, \alpha_3)$ at $(\alpha_1, \alpha_2, \alpha_3) = (\pi/2, \pi/2, -\pi/2)$ and permutations.

A system composed of three dumbbells that are arranged in a *C*3 symmetry configuration will asymptotically follow spin ice rules with dumbbells enforced to live in the X-Y plane as the distance between them decreases to zero.

Point dipoles

$$\vec{m} = 2aq(\cos\theta_i \sin\alpha_i, \sin\theta_i \sin\alpha_i, \cos\alpha_i)$$



$$H_{dip} = \frac{D}{2} \sum_{<ij>} [\hat{m}_i \hat{m}_j - 3(\hat{m}_i \hat{r_{ij}})(\hat{m}_j \hat{r_{ij}})]$$

polar angle

angle that the lattice vector r_{ij} makes with the x axis.

$$H_{dip} = -\frac{D}{2} \sum_{<ij>} [\sin\alpha_i \sin\alpha_j (\frac{3}{2}\cos(\theta_i + \theta_j - 2\phi_{ij}) + \frac{1}{2}\cos(\theta_i - \theta_j)) - \cos\alpha_i \cos\alpha_j]$$

$$\sum_{a \neq b} \cos(\alpha^a - \alpha^b) = -1$$

minimize H

plane + SI rule !

$$\sum_{a \neq b} \sin\alpha^a \sin\alpha^b = -1$$

The ground state energy in the short ranged case is degenerate, $E = -7D/8\ N_\Delta$, where N_Δ is the total number of triangles in the sample and D ~10^-5 [J].

A spin ice violation costs ºE = $7D/2$ Since,

$$D \sim (aq)^2/l^3$$

the cost of creating a spin ice violation will decrease as the ratio between the size of the rods and the lattice constant, decreases.

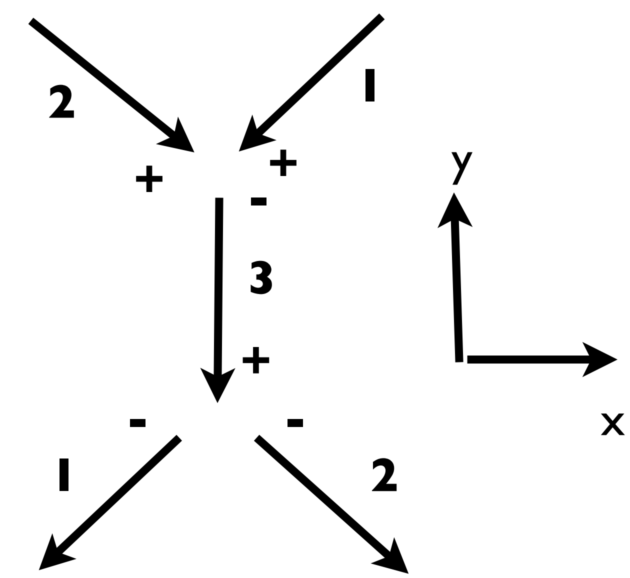
Over-damped limit: Linearized spin wave, n-n approximation

Neglecting Inertia

$$\frac{d\vec{\alpha}^1}{dt} = \frac{\vec{X}_p^1 \times (\vec{F}_p^1 - \vec{F}_n^1) \cdot \hat{z}^1}{\eta}$$

$$\vec{\phi}_1 = \vec{\alpha}_1 + \pi/2$$

small oscillations of a staggered charge order

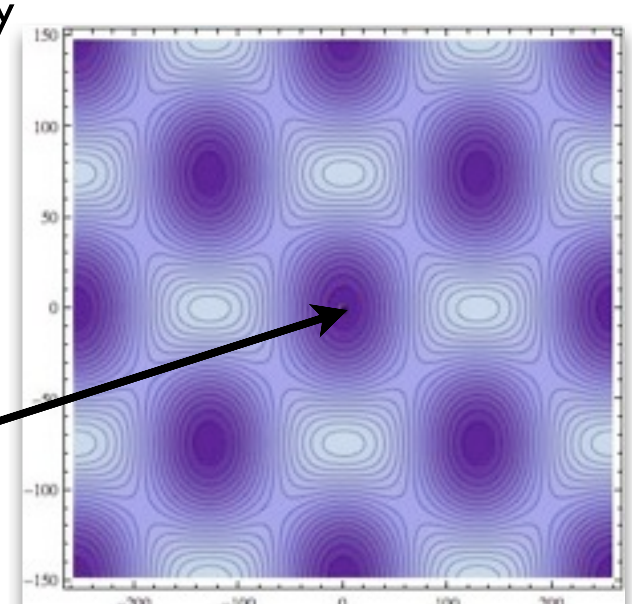


$$\boxed{\frac{d}{dt}\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \frac{2aq^2}{3\sqrt{3}\eta\Delta^3}\begin{pmatrix} 0 & -a & -a \\ -a & 0 & -a \\ -a & -a & -(2a+\Delta) \end{pmatrix}}$$

$$\phi_j^1 = \frac{1}{\sqrt{N_1}}\sum \phi^1(q)\exp\left(i\vec{q}\vec{r}_j + i\Omega t\right)$$

$$\vec{q} = (q_x, q_y)$$

ky



kx

The energy has Minimum at $\vec{q} = (q_x, q_y) = (0{,}0)$

Molecular dynamics simulations: Verlet algorithm

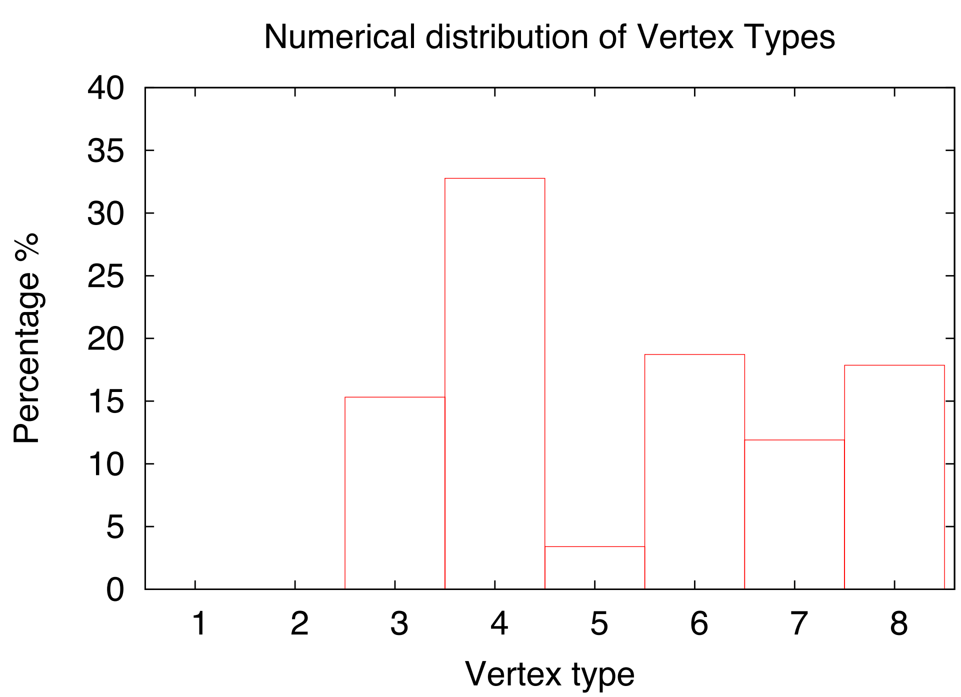
$$\alpha(t+dt) \approx f_a \alpha(t) - f_b \alpha(t-dt) + f_c T_{total}(t)$$

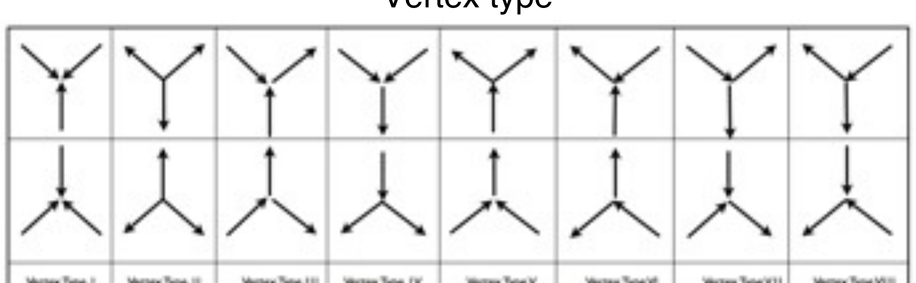
$dt \approx 10^{-4}$

$$dt/\tau_B \ll 1$$
$$dt/\tau_D \ll 1$$

torques due to internal and external magnetic field

$$f_a = \frac{2}{1+\frac{\eta}{2I}dt} \qquad f_b = \frac{1-\frac{\eta}{2I}dt}{1+\frac{\eta}{2I}dt} \qquad f_c = \frac{dt^2}{I(1+\frac{\eta}{2I}dt)}$$





We neglect for the moment static friction due to the hinges, we focus in the dynamics

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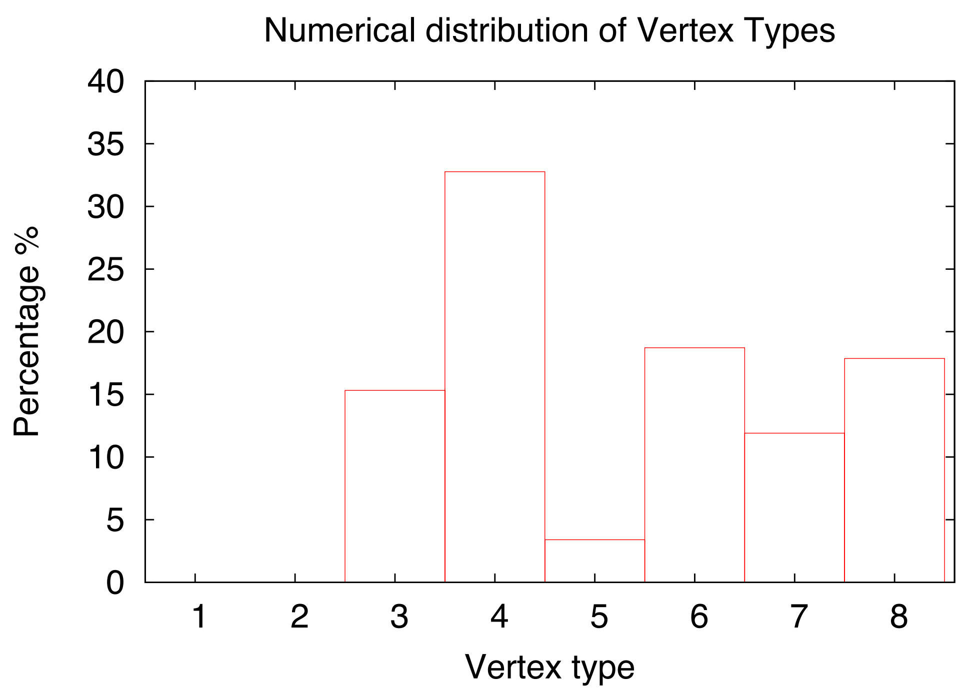

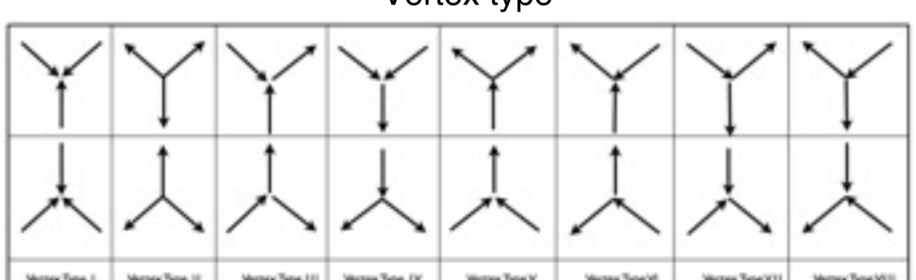

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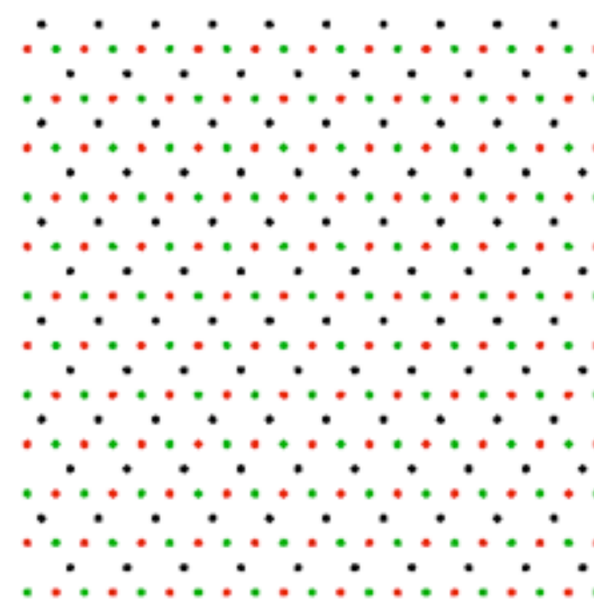
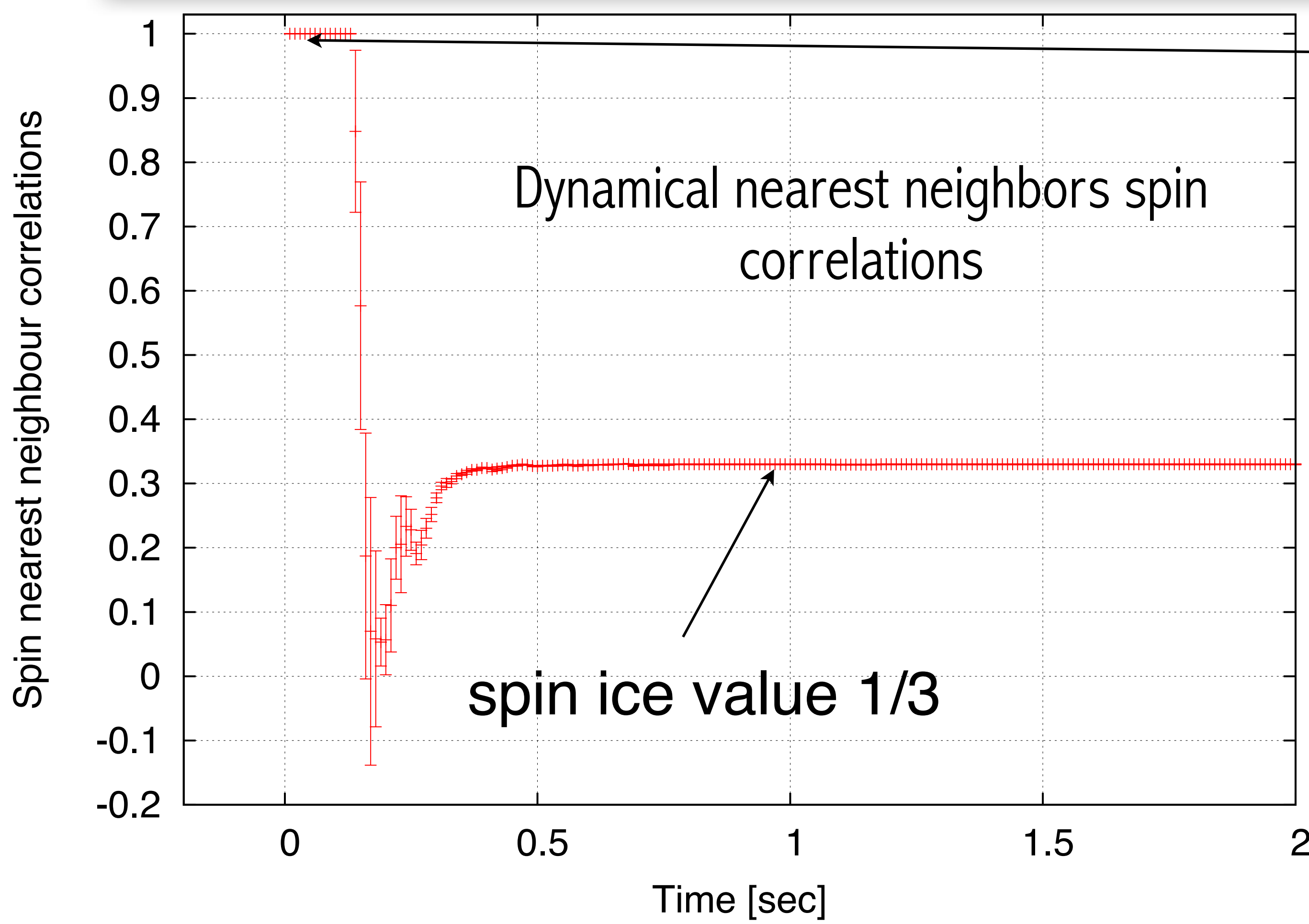
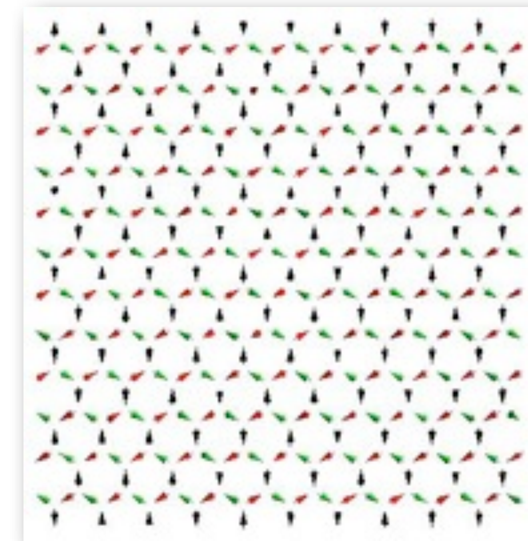
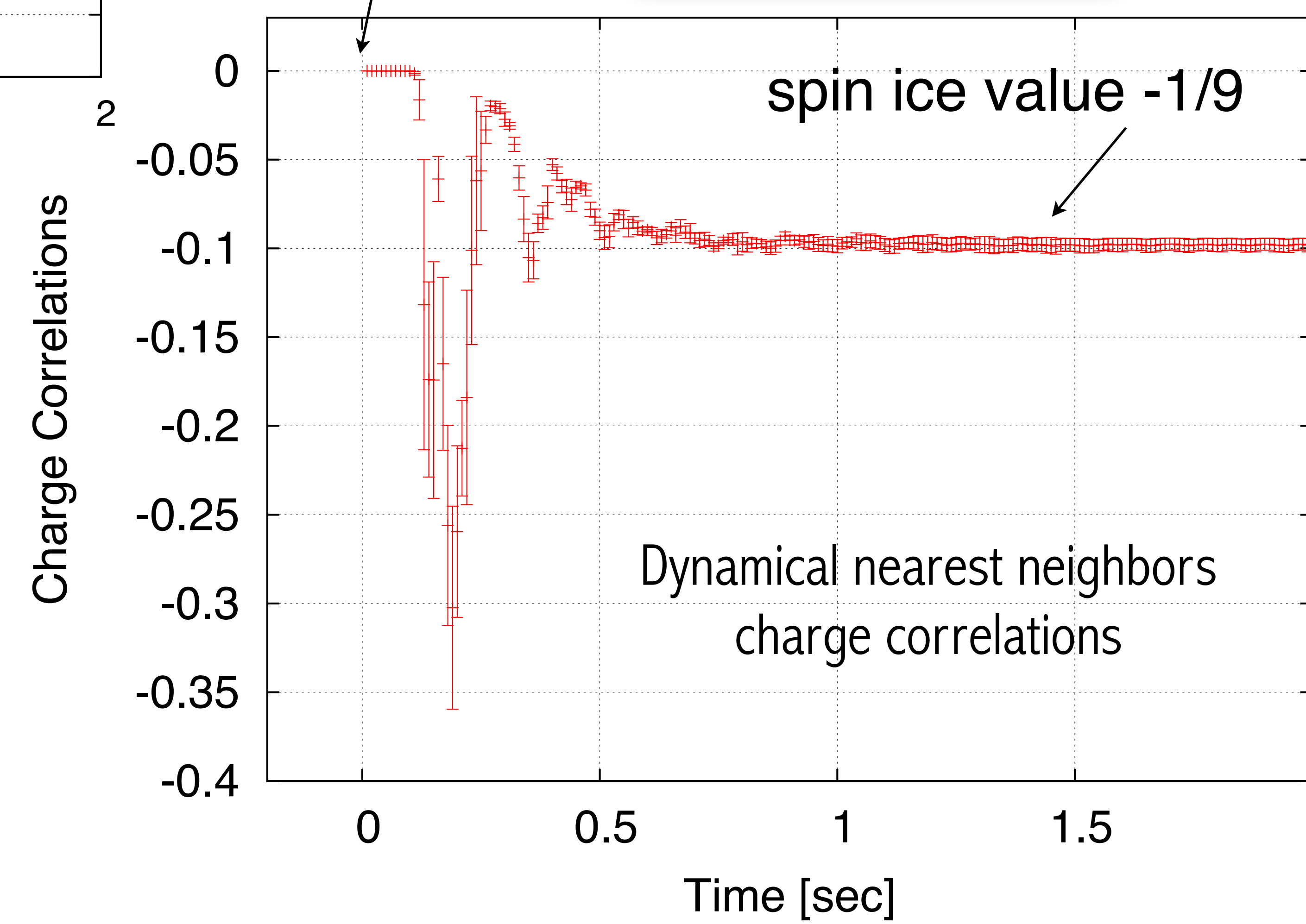


Correlations, numerical results



Simulations 135 x 3 rods, at t=0 they are all pointing along +z





correlation	Average from all spin up	Average from disordered state
$S_{\alpha\beta}$	0.3298 +/- 0.0006	0.323
$S_{\alpha\gamma}$	0.232 +/- 0.005	-0.168
$S_{\alpha\nu}$	0.71+/-0.01	0.235
$C_{\alpha\beta}$	-0.098 +/- 0.003	-0.076

Dynamics of the lattice

Time scales and parameter space

Δ/a fixed in experiments

$$\tau_{ph} = \sqrt{I\Delta^2/aq^2}$$ ~ 0.01 [sec]

due to lattice oscillations, B_{int} $\tau_{ph}/\tau_D \sim 0.01$

due to external uniform magnetic field

$$\tau_B/\tau_{ph} = \frac{1}{\Delta}\sqrt{\frac{q}{2B_0}}$$

Useful dimensionless when a monopole of charge Q is located at h from the lattice

$$\frac{qh^2}{Q\Delta^2}$$

$$\frac{v\tau_{ph}}{a}$$

when the monopole Q moves at speed v

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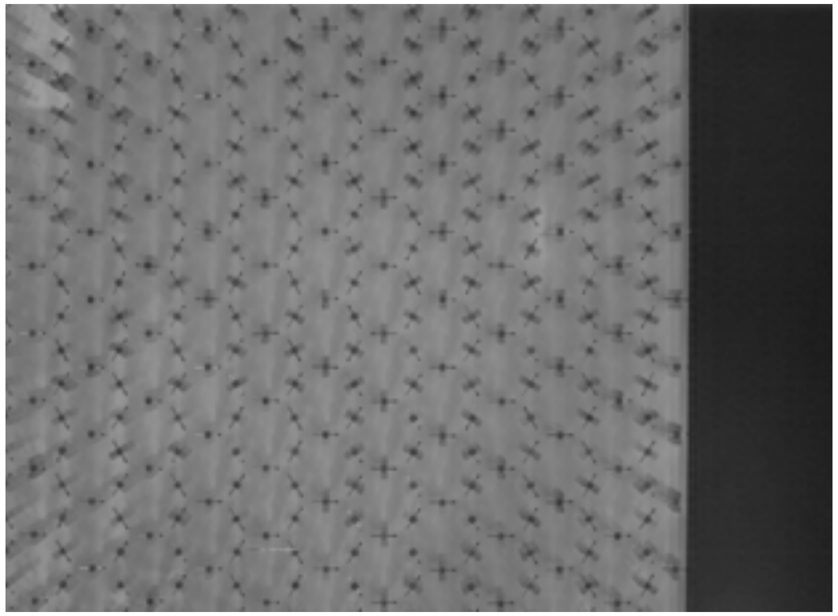

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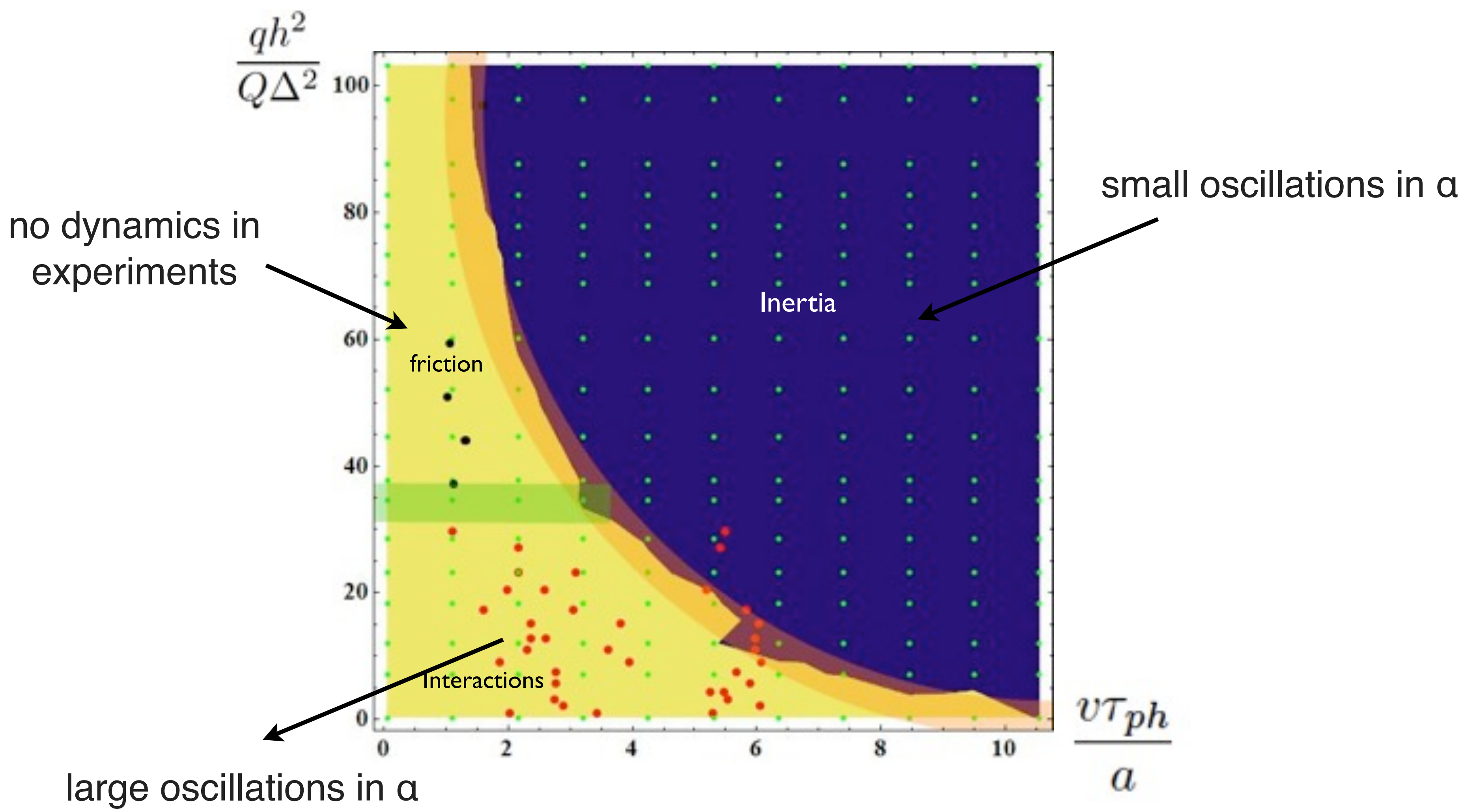
Ongoing work: Interactions versus Inertia

We examined the role of inertia of the rods when an external dipole moving at distance **h** from the lattice at speed **v** excite the system.

Phase Diagram: **x** axis is the ratio between the phononic and the inertial time scale. The **y** axis is the ratio between the internal and the external magnetic force. We start from a disordered state.

Experimentally **h** varied from 3.0 to 35 [cm] and **v** from 0.7 to 6 [m/s]. The dipole charge **Q** = 64q. Simulations **v** varied between 0.05 and 10.05 [m/s], **h** between 2 and 38 [cm].



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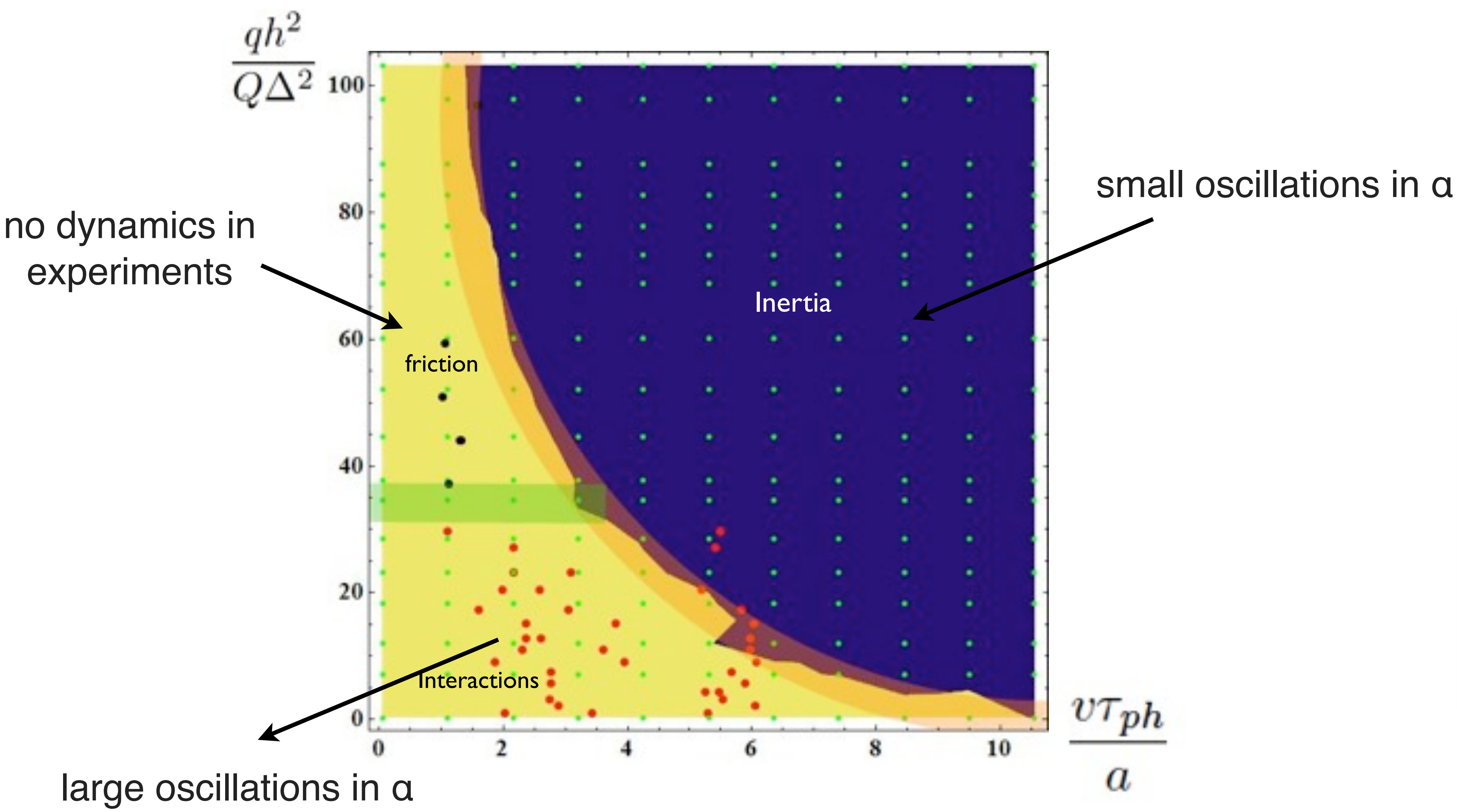

Summary

We have built a macroscopic version of a frustrated classical system which shows spin ice correlations.

The macroscopic lattice is an ideal scenario to prove in a controllable way the role of vacancies, geometrical disorder and inertia in a frustrated system.

The viscous dynamic observed is a result of the interplay between inertia of the magnets, friction and the Coulomb interactions between monopoles at the end of ferromagnetic rods.

We have built a phase diagram where an interaction dominated regime distinguishes from an inertia dominated one, when the speed of an external dipole and its distance relative to the lattice are varied.