Statistical transmutation in Quantum dimer models

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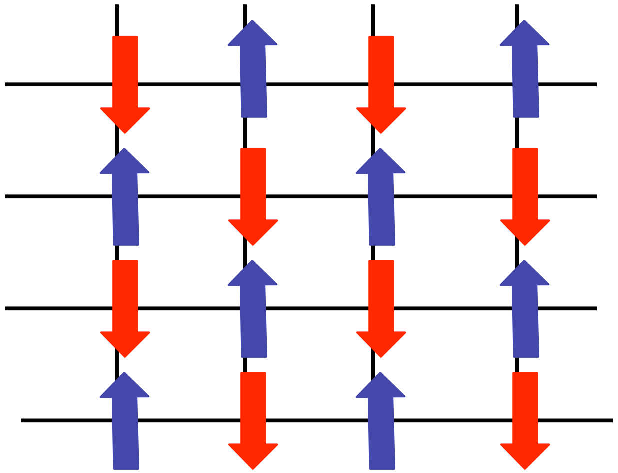
Quantum dimer models as a low effective theory of frustrated magnets

Non frustrated AF:

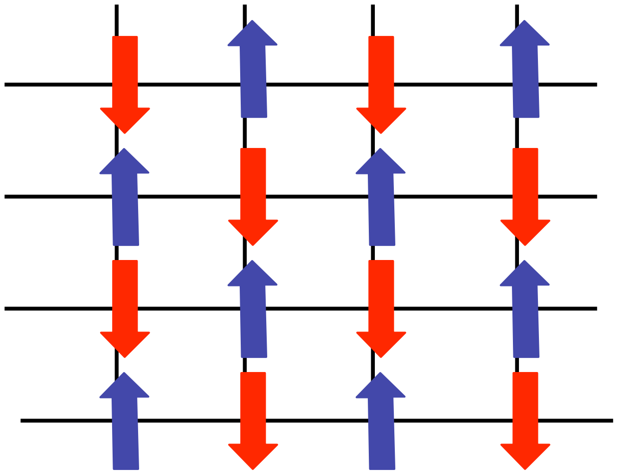


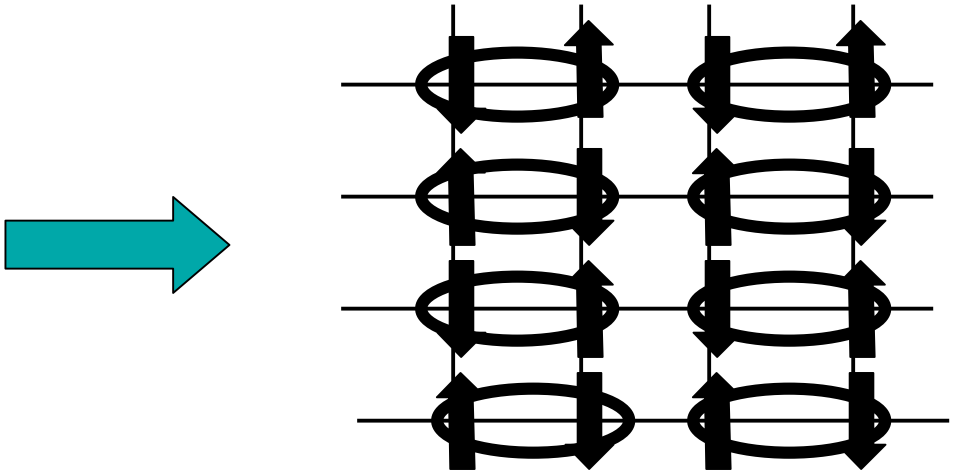
(anti-ferro) magnetic order

Quantum dimer models as a low effective theory of frustrated magnets

Non frustrated AF:

Frustrated AF:





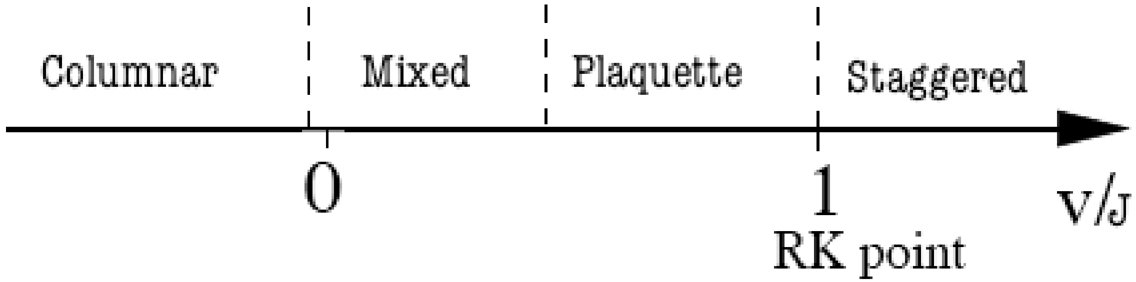
(anti-ferro) magnetic order

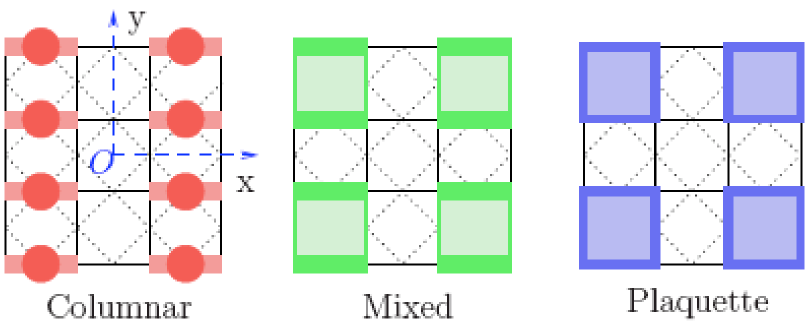
Non magnetic order

The Rokhsar-Kivelson Model

$$H_J = -J\sum_{\square}\left\{\left|\,\mathbf{I}\,\mathbf{I}\,\right\rangle\left\langle\,=\,\right| + \mathrm{H.C}\right\}$$

$$H_V = V\sum_{\square}\left\{\left|\,\mathbf{I}\,\mathbf{I}\,\right\rangle\left\langle\,\mathbf{I}\,\mathbf{I}\,\right| + \left|\,=\,\right\rangle\left\langle\,=\,\right|\right\}$$

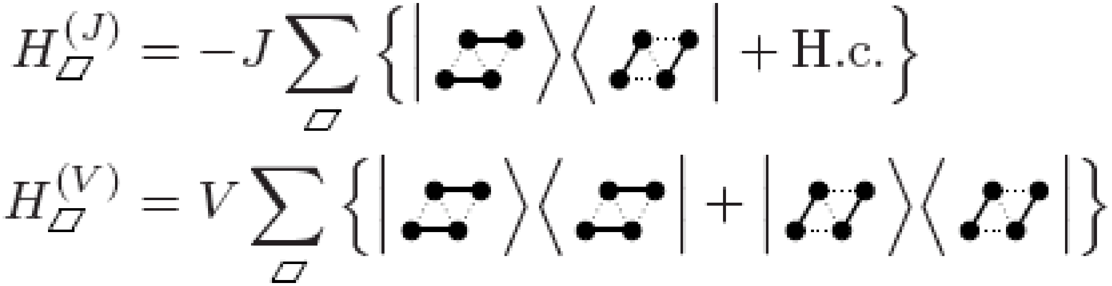


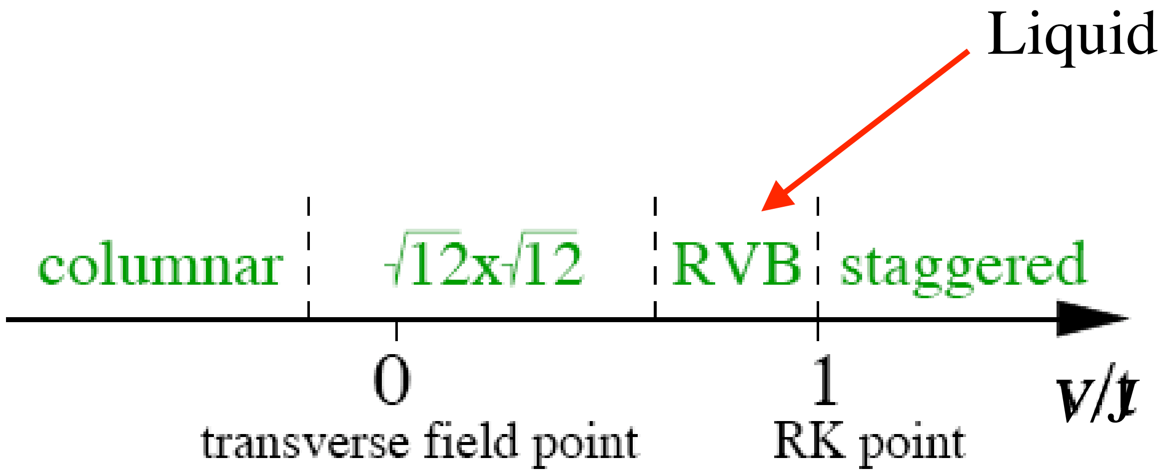


Last update of the phase diagram in 2007 by:
Ralko, Poilblanc, Moessner

A very interesting case: the Triangular lattice

(Moessner and Sondhi)



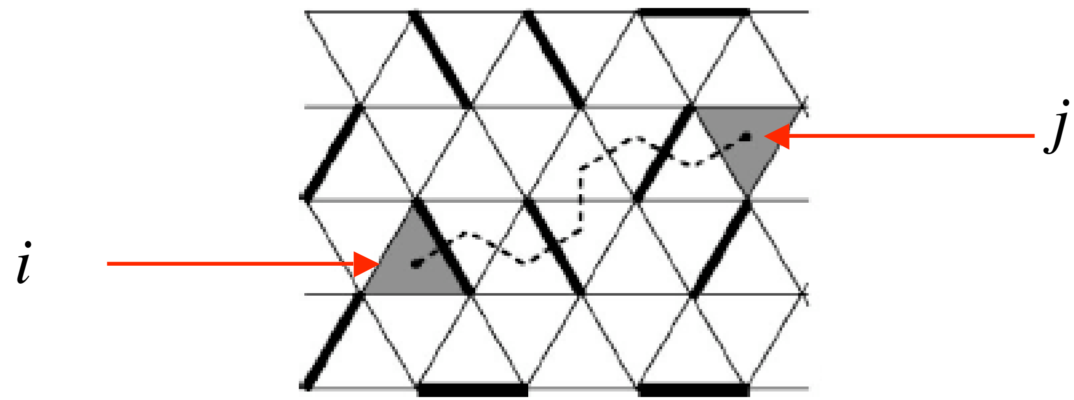


Playing with gauge transformations

Square lattice: $\left| = \right\rangle \to e^{i\frac{\pi}{4}} \left| = \right\rangle \; ; \; \left| \parallel \right\rangle \to e^{-i\frac{\pi}{4}} \left| \parallel \right\rangle$

The change in the hamiltonain is *J -> -J*

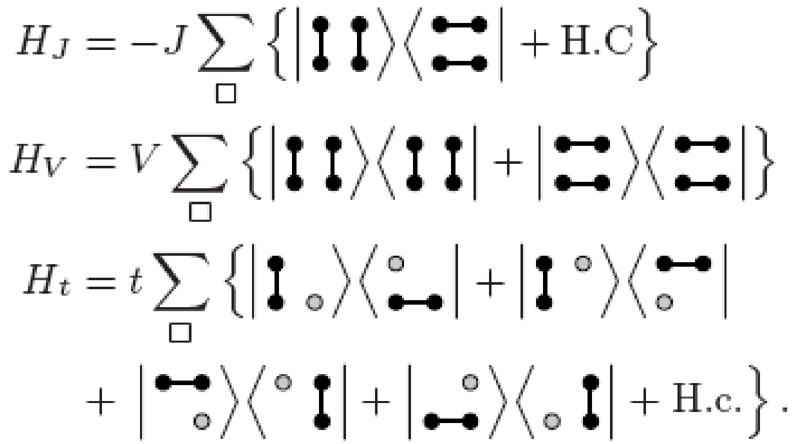
Excitations: visons



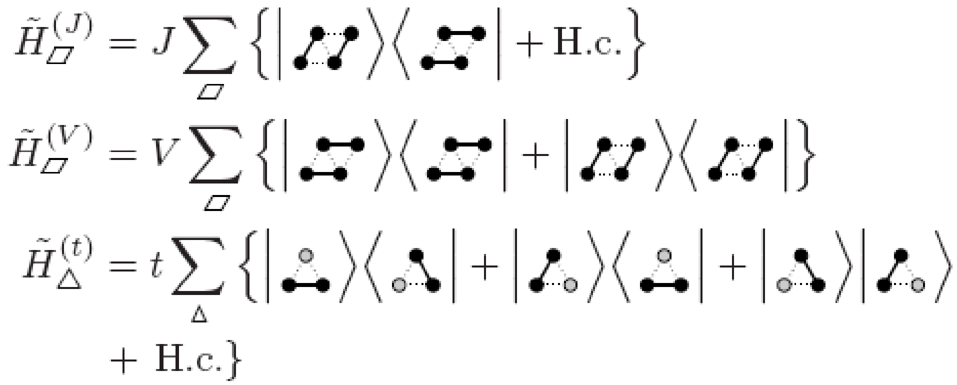
$$|\Psi_{i,j}\rangle = \sum_c (-1)^{\Gamma_{i,j}} (-1)^{\tilde{\Gamma}_{i,j}} |\Psi^c_{GS}\rangle$$

… and we can add doping (holons)

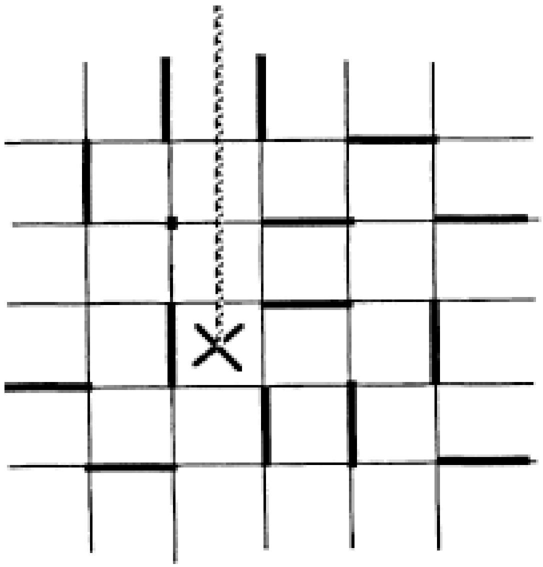
Square lattice:



Triangular lattice:



Non-trivial braiding between holons and visons



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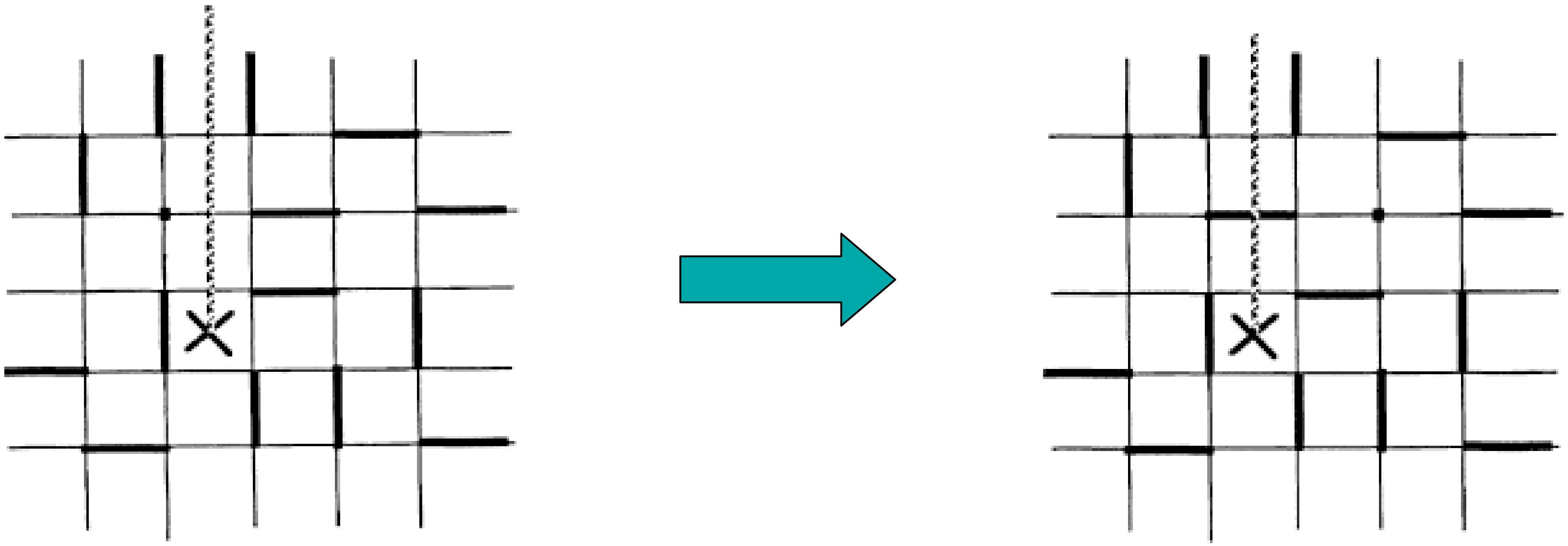

Like a π flux vortex

And… what is the statistics of holons?

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…See below

What holons do to the phase diagram of the QDM ?

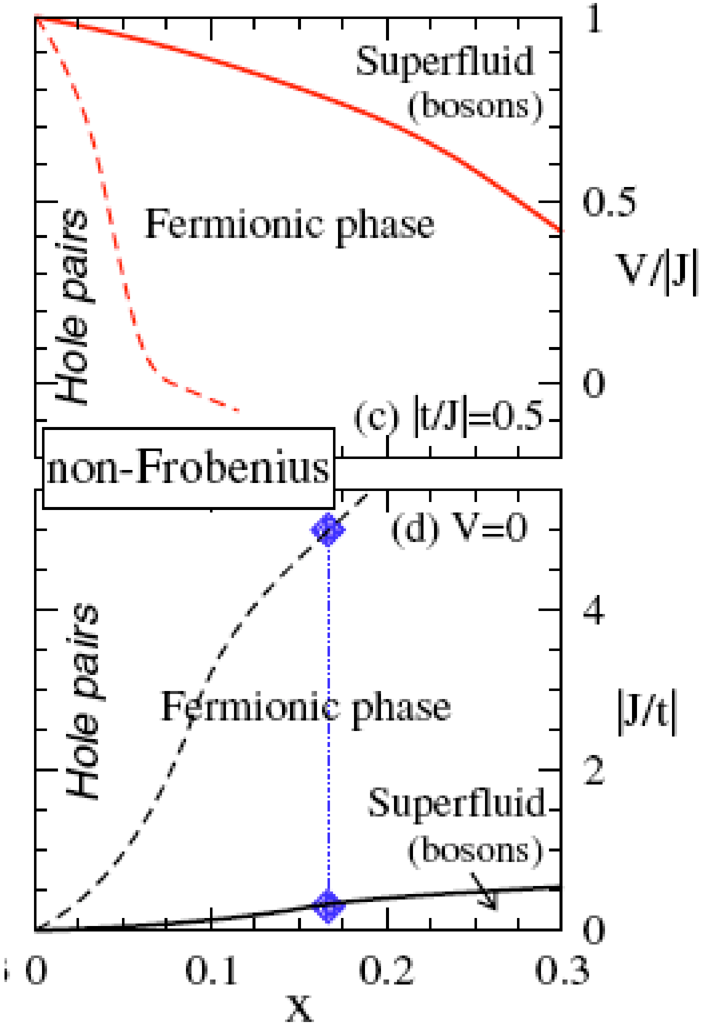
- *H(J)* and *H(-J)* are no longer equivalent
- For weak doping, the crystalline phase seems to give:
  - Phase separation for $J>0$
  - Supersolid for $J<0$

(Poilblanc et al. 2006, Ralko, Mila, Poilblanc 2007, Poilblanc 2008)

And…holon-vison binding: a statistical transmutation

(Read and Chakraborty, Kivelson, Poilblanc)

Square lattice:
(Poilblanc 2008)



Holon-vison binding: a statistical transmutation

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Square lattice:

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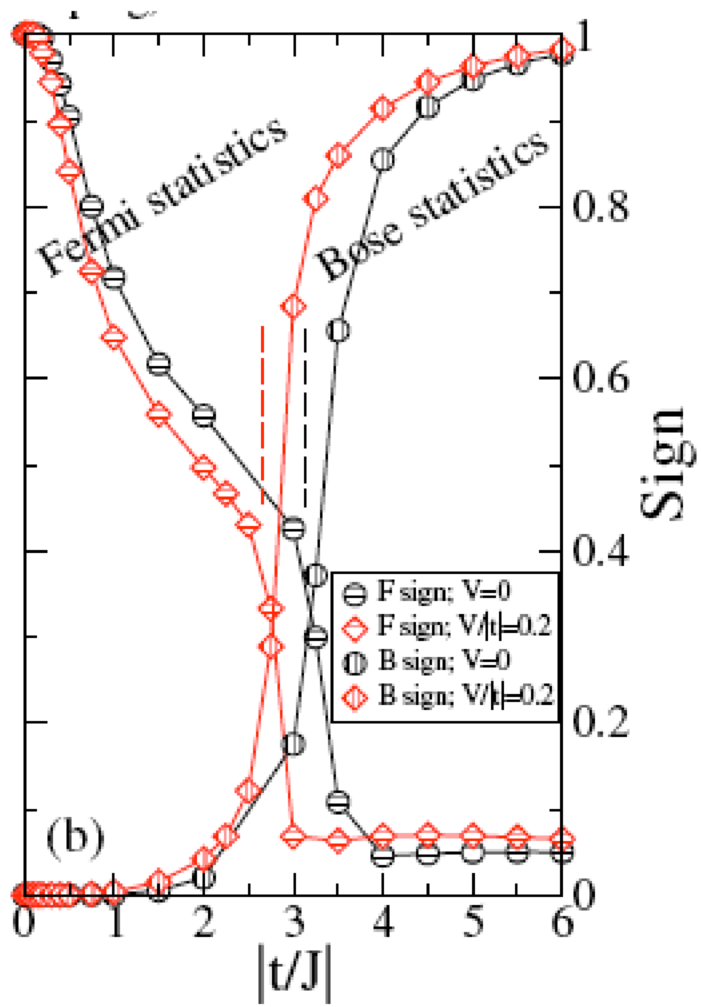

A duality transformation

Let us use the square lattice to illustrate the method.
Write down the hamiltonian in a second quantized form:

Build creation an annihilation operators for dimers and holons

$$\left[b_{i,j}, b^\dagger_{k,l}\right] = \delta_{i,k}\delta_{j,l} + \delta_{i,l}\delta_{j,k}$$
$$[b_{i,j}, b_{k,l}] = \left[b^\dagger_{i,j}, b^\dagger_{k,l}\right] = 0.$$

$$\left[a_i, a^\dagger_j\right] = \delta_{i,j}$$
$$[a_i, a_j] = \left[a^\dagger_i, a^\dagger_j\right] = 0,$$

with

$$[a_i, b_j] = \left[a^\dagger_i, b^\dagger_j\right] = \left[a^\dagger_i, b_j\right] = 0.$$

and

$$a^\dagger_i a_i + \sum_z b^\dagger_{i,i+z} b_{i,i+z} = 1.$$

A duality transformation

Let us use the square lattice to illustrate the method.
Write down the hamiltonian in a second quantized form:

$$H_J = -J \sum_i \left\{ \tilde{b}^\dagger_{i,i+e_1} \tilde{b}^\dagger_{i+e_2,i+e_1+e_2} \tilde{b}_{i,i+e_2} \tilde{b}_{i+e_1,i+e_1+e_2} \right\}$$

$$H_V = V \sum_i \left\{ \tilde{b}^\dagger_{i,i+e_2} \tilde{b}^\dagger_{i+e_1,i+e_1+e_2} \tilde{b}_{i,i+e_2} \tilde{b}_{i+e_1,i+e_1+e_2} \right\} .$$

$$H_t = \sum h^{(t)}_{(ijk)}$$

$$h^{(t)}_{(ijk)} = t \; \hat{\mathcal{P}} \; b^\dagger_{i,j} b_{j,k} a^\dagger_k a_i \; \hat{\mathcal{P}}.$$

$\hat{\mathcal{P}}$ Projector into the subspace spanned by the constraint:

$$a^\dagger_i a_i + \sum_z b^\dagger_{i,i+z} b_{i,i+z} = 1.$$

A duality transformation

Then use the 2-D Jordan-Wigner transformation (a highly non-local transformation) (Fradkin, Semenov, Wang)

$$a_i = e^{-i\phi_i} f_i \qquad \phi_i = \sum_{j \neq i} f_j^\dagger f_j \arg(\vec{\tau}_j - \vec{\tau}_i)$$

$$e^{i\phi_i} f_j = f_j e^{i\phi_i} e^{-i \arg(\vec{\tau}_j - \vec{\tau}_i))} \qquad h^{(t)}_{(ijk)} = t\, e^{i \arg(\vec{\tau}_k - \vec{\tau}_i)} \hat{\mathcal{P}} e^{i\phi_k} e^{-i\phi_i}\, b^\dagger_{i,j} b_{j,k} f^\dagger_k f_i\, \hat{\mathcal{P}}$$

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$$\phi_i = \sum_{j \neq i} f_j^\dagger f_j \arg(\vec{\tau}_j - \vec{\tau}_i) \;\longrightarrow\; \tilde{\phi}_i = \sum_{r \neq i} \left[1 - \sum_z b^\dagger_{r,r+z} b_{r,r+z} \right] \arg(\vec{\tau}_r - \vec{\tau}_i).$$

And define new dimer operators, still bosonic:

$$\tilde{b}^\dagger_{i,j} = b^\dagger_{i,j}\, e^{-i(\tilde{\phi}_i + \tilde{\phi}_j)}$$

$$\tilde{b}_{i,j} = e^{i(\tilde{\phi}_i + \tilde{\phi}_j)}\, b_{i,j}$$

A duality transformation

After a few gauge transformations like: $f_j \to e^{i\vec{Q}\cdot\vec{r}_j} f_j$ with $\vec{Q} = (\frac{\pi}{2}, \frac{\pi}{2})$

We get the same hamiltonian but with the interchange :

Bosons <-> Fermions and *J* <-> -*J*

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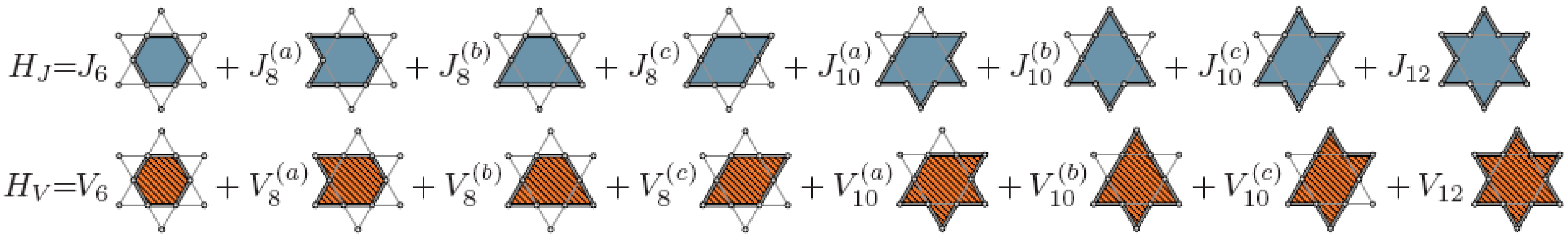
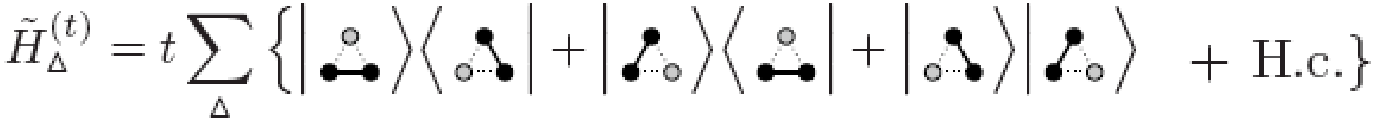
Same result for the triangular lattice

And also for a recent model on the kagome lattice



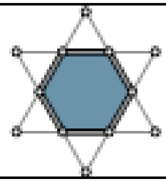
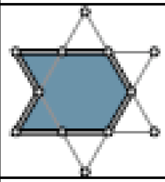
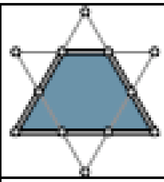
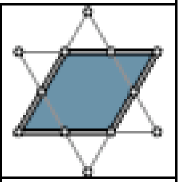
$$H_t = H_{\vartriangle}^{(t)} + H_{\triangledown}^{(t)}$$

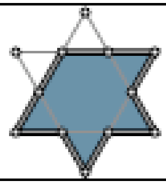
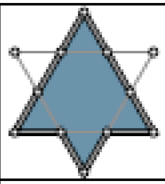
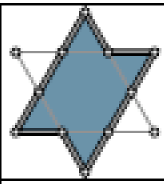
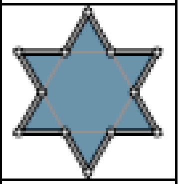


(Schwandt, Mambrini, Poilblanc, Ralko)

And also for a recent model on the kagome lattice

How H looked before a few gauge transformations:

Length	6	8		
				
ψ	π	0	π	0
$\tilde{J}/J$	1	-1	1	-1

Length	10			12
				
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	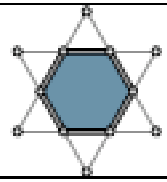	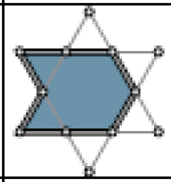	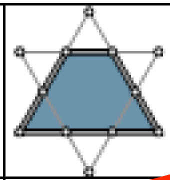	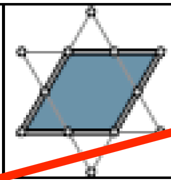
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	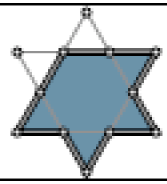	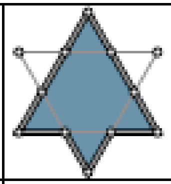	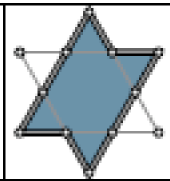	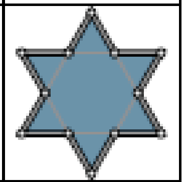
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!

Conclusions

Equivalence classes of hamiltonians:

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$J > 0$, Fermions

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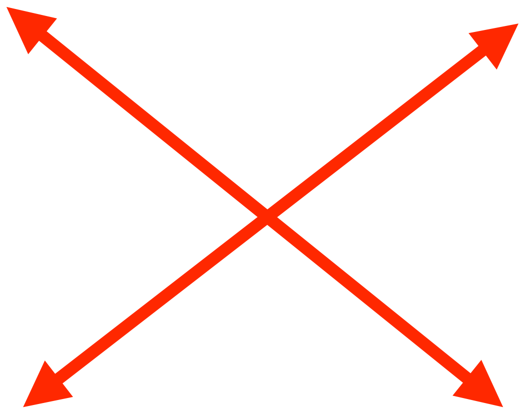

$J < 0$, Fermions

$J < 0$, Bosons

Square and triangular lattices

Conclusions

Phenomenological field theory description in terms of a Chern-Simons + matter field theory

QDM as a toy model for anyons dynamics

$$a_i = e^{-i\phi_i} f_i \qquad \phi_i = \upsilon \sum_{j \neq i} f_j^\dagger f_j \arg(\vec{\tau}_j - \vec{\tau}_i)$$

Conclusions

Statistical transmutation a generic feature in QDM?

Triangular lattice
phase diagram ?