Metamagnetism in frustrated ferromagnetic spin chains

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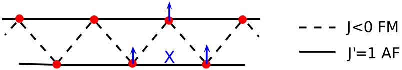
Frustrated ferromagnetic spin chain

$$H_S = \sum_{i=1}^{L} \left[\left(J\vec{S}_i \cdot \vec{S}_{i+1} + J'\vec{S}_i \cdot \vec{S}_{i+2} - hS_i^z \right) \right]$$



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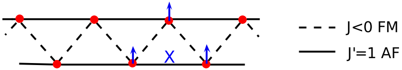

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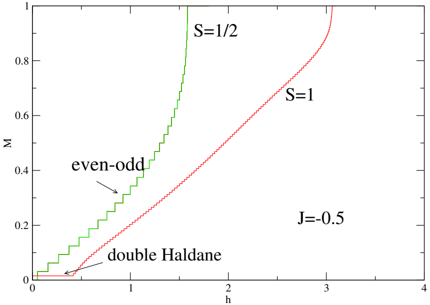

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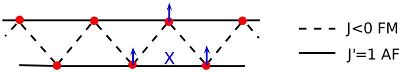

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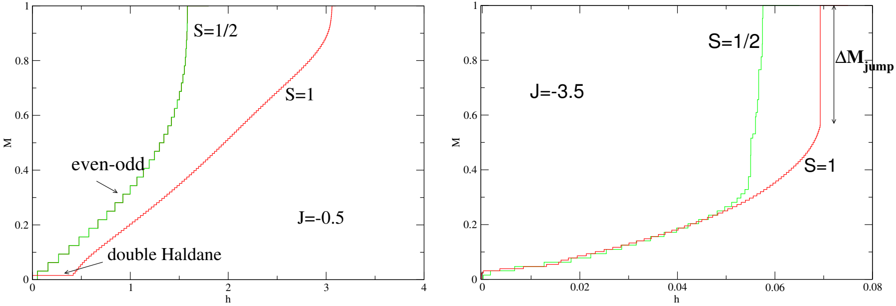
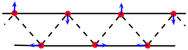


Outline

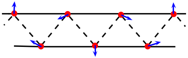
1. Classical picture
2. Two magnon problem
3. Multimagnon problem, $S \geq 1$
4. DMRG results

Limit $S \to \infty$

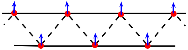
- $h = 0$: spiral order, $\theta = \arccos(-J/4)$



$J = 0, \; \theta = \pi/2$



$-4 < J < 0, \; 0 < \theta, \pi/2$



$J \leq -4, \; \theta = 0$

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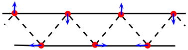

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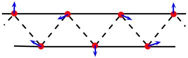

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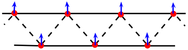

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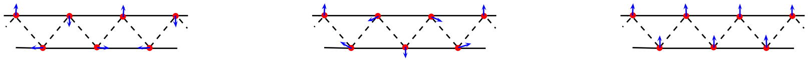

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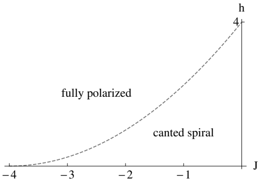
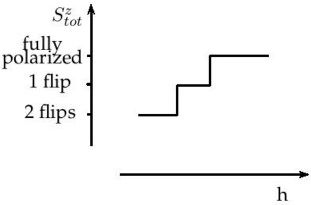
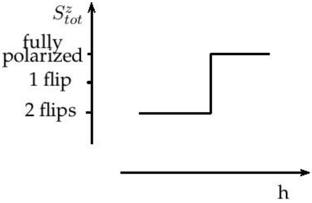
- $h \neq 0$: spiral in xy plane + canting towards z axis
- $h_{sat} = S/4(J+4)^2$



Bound states and jumps

- $H_S(h) = H_S(h=0) - hS^z_{tot}$





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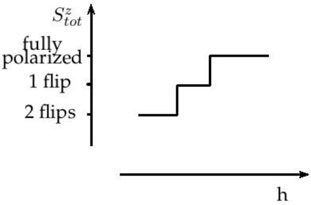

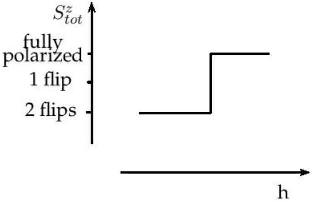

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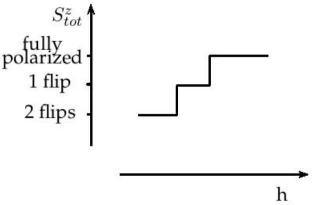
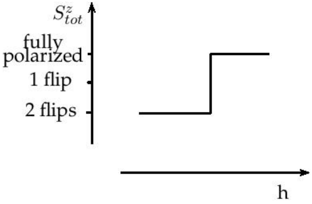
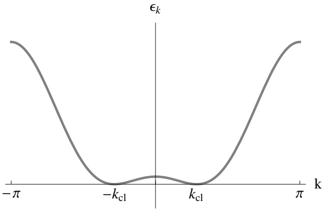




- S^z sector with especially low energy $\longrightarrow$ hides following S^z sectors in the magnetization curve
- See jumps? $\longrightarrow$ Look for bound states

One magnon dispersion

$$\epsilon_k = \frac{1}{2} V_k - \frac{1}{2} V_{k_{cl}} \geq 0 \qquad \text{with} \qquad V_k = 2J\cos(k) + 2\cos(2k)$$



$k_{cl} = \arccos(-J/4)$, incommensurate

- Two magnon states basis, with momentum K, relative coordinate r:

$$|K,r\rangle = \sum_{l=1}^{L} e^{iK(l+r/2)} S_l^- S_{l+r}^- | \uparrow\uparrow \cdots \uparrow\uparrow\rangle$$

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- Eigenstates:

$$H_S|\Psi_{2M}\rangle = E_{2M}|\Psi_{2M}\rangle$$

Two magnon eigenstates

Schrödinger equation ($S \geq 1$) for eigenstates with momentum K

$$\begin{aligned}
\Omega_0 C_0 &= \frac{S}{\sqrt{S(2S-1)}}(\zeta_1 C_1 + \zeta_2 C_2) \\
(\Omega_0 - J)C_1 &= \frac{(2S-1)^{3/2}}{S^{3/2}}\zeta_1 C_0 + \zeta_1 C_2 + \zeta_2(C_1 + C_3) \\
(\Omega_0 - 1)C_2 &= \frac{(2S-1)^{3/2}}{S^{3/2}}\zeta_2 C_0 + \zeta_2 C_4 + \zeta_1(C_1 + C_3)
\end{aligned}$$

and for $r \geq 3$:

$$\Omega_0 C_r = \zeta_1 \left(C_{r+1} + C_{r-1}\right) + \zeta_2 \left(C_{r+2} + C_{r-2}\right)$$

where Ω_0 is the eigenvalue, $\zeta_1 = 2SJ\cos(K/2)$, $\zeta_2 = 2S\cos(K)$

Two magnon bound states

- Bound state Ansatz:

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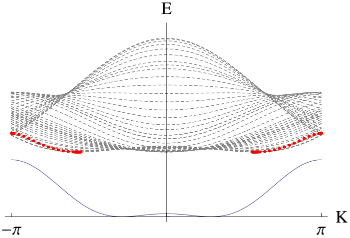
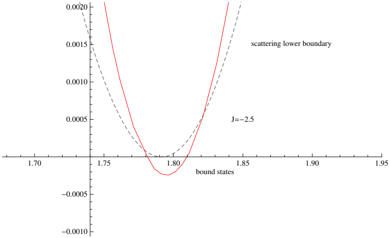
- Provides solutions for $J < J_{cr}(S)$, with width $a_s = 1/\min\{\mathrm{Re}[\kappa_\pm]\}$





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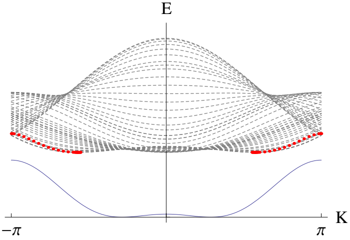
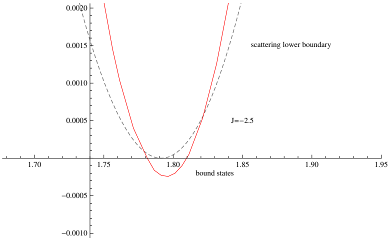
- Example: S=1, J=-2.5. Bound state solutions shown in red.





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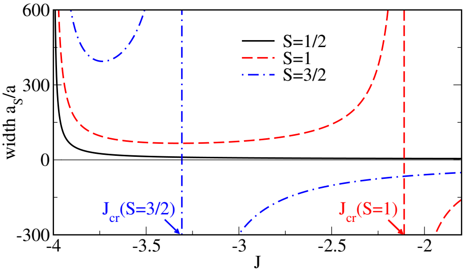

S	1/2	1	3/2	2	5/2
$J_{cr}(S)$	—	-2.11	-3.31	-3.68	-3.84

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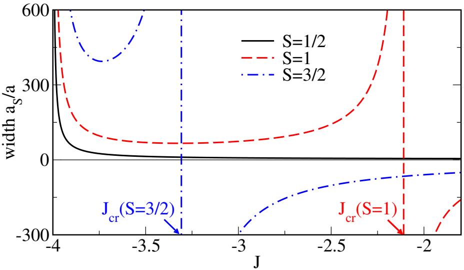

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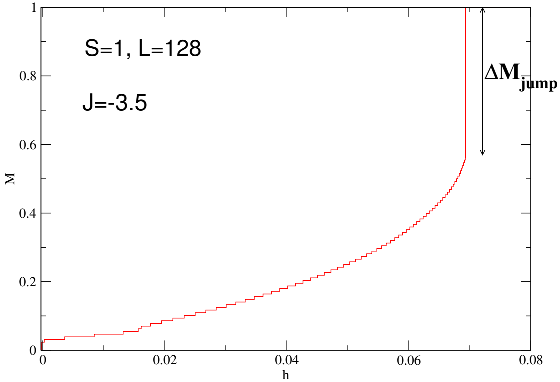
Wide bound states $\rightarrow$ *Hard coreness of magnons plays no major role in this regime!*

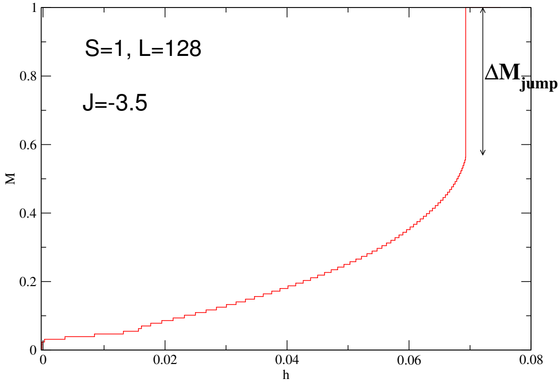
Metamagnetism

- $M = S^z_{Total}/(SL)$: normalized magnetization



Metamagnetism

- $M = S^z_{Total}/(SL)$: normalized magnetization
- Metamagnetic jump: step to saturation with finite $\Delta M \longleftrightarrow$ finite density of magnons for $L \to \infty$



Multimagnon problem

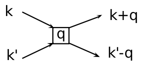
Dyson-Maleev (same results at $1/S$ using Holstein-Primakoff)

$$\begin{aligned} S_i^z &= S - a_i^\dagger a_i \,, \quad S_i^+ = \sqrt{2S} a_i \,, \\ S_i^- &= \sqrt{2S} a_i^\dagger (1 - a_i^\dagger a_i / 2S) \,, \longrightarrow \end{aligned}$$

$$\to H_S = \sum_k (2S\epsilon_k - \mu(h)) a_k^\dagger a_k + \sum_{k,k',q} \frac{\Gamma_0(q;k,k')}{2L} a_{k+q}^\dagger a_{k'-q}^\dagger a_k a_{k'}$$

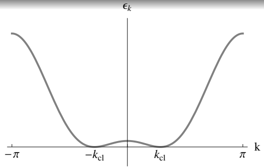
with one-magnon like dispersion $\sim 2S\epsilon_k$ close to saturation $\mu(h) = h_s^{cl} - h \to 0^+$

and bare 4-leg vertex coupling $\Gamma_0(q;k,k') = V_q - \frac{1}{2}(V_k + V_{k'})$



Low energy regime

$k \approx \pm k_{cl} \longrightarrow$ two boson species $\pm$



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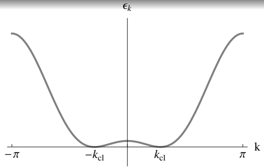

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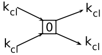

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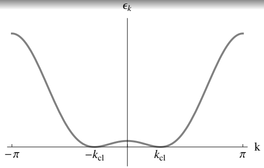

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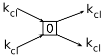
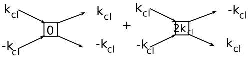
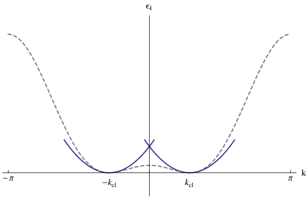


- Inter-species coupling $\tilde{g}_0 \sim \Gamma_0(0; k_{cl}, -k_{cl}) + \Gamma_0(2k_{cl}; k_{cl}, -k_{cl})$



Low density, low energy effective model

- Low magnon density + wide bound states ($S \geq 1$): *we can ignore hard core constraint and curvature*



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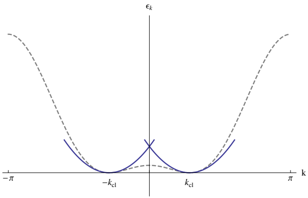
- Approximate boson modes around k_{cl} and $-k_{cl}$ by quadratic dispersion bosons $\psi_-(x)$, $\psi_+(x)$ with mass $m = \frac{2}{S(4-J)(4+J)}$



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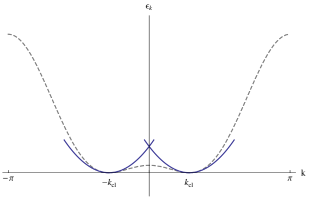


- $\cdots$ getting a two component Lieb-Liniger model, with contact density-density interactions

$$\mathcal{H}_{\text{eff}} = \sum_{\alpha=\pm} -\frac{|\nabla \psi_\alpha|^2}{2m} + \frac{g_0}{2}(n_-^2(x) + n_+^2(x)) + \tilde{g}_0\, n_-(x) n_+(x)$$

Effective coupling constants

Couplings depend on S through renormalization

$$\mathcal{H}_{\rm eff} = \sum_{\alpha=\pm} -\frac{|\nabla\psi_\alpha|^2}{2m} + \frac{g_0(S)}{2}(n_-^2(x) + n_+^2(x)) + \tilde{g}_0(S)\, n_-(x)n_+(x)$$

Effective Coupling Regimes

- $g_0(S) > 0(< 0)$: repulsive (attractive) intra-species interaction
- $\tilde{g}_0(S) > 0(< 0)$: repulsive (attractive) inter-species interaction

$g_0(S)$, $\tilde{g}_0(S)$ are obtained from the renormalized $\Gamma(q;k,k')$:

$$g(S) = \Gamma(0; k_{cl}, k_{cl})$$

$$\tilde{g}(S) = \Gamma(0; k_{cl}, -k_{cl}) + \Gamma(2k_{cl}; k_{cl}, -k_{cl})$$

Renormalization is done at the RG fixed point of a dilute, two-component Bose gas, dictated by $SU(2)$ symmetry (Kolezhuk, PRA 81 (2010))

$$\lim_{\mu\to 0} g(S) = \lim_{\mu\to 0} \tilde{g}(S) = \pi\sqrt{\mu(S)}/\sqrt{2m}$$

Then one expects

$$g(S) = \frac{g_0(S)}{1 + g_0(S)\sqrt{2m}/(\pi\sqrt{\mu(S)})}$$

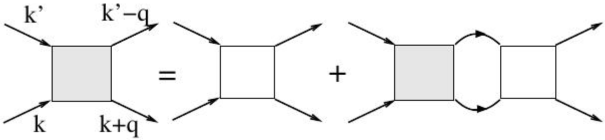
$$\tilde{g}(S) = \frac{\tilde{g}_0(S)}{1 + \tilde{g}_0(S)\sqrt{2m}/(\pi\sqrt{\mu})}$$

Extract $\Longrightarrow$ $g_0(S)$, $\tilde{g}_0(S)$

$1/S$ vertex renormalization

Bethe-Salpeter equation

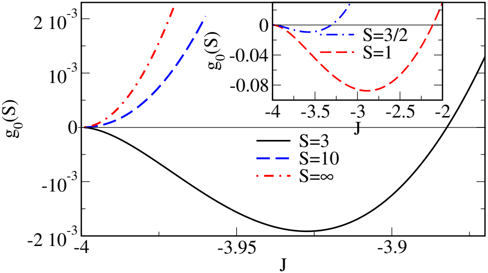
$$\Gamma(q;k,k') = \Gamma_0(q;k,k') - \frac{1}{2SL}\sum_p \frac{\Gamma_0(q-p;k+p,k'-p)}{\epsilon_{k+p}+\epsilon_{k'-p}}\Gamma(p;k,k')$$



$1/S$ expansion makes sense here, though the model is $1D$. All integrals are evaluated analytically.

Attractive regime

- $g_0(S)$ may be tuned *attractive* $\leftrightarrow$ *repulsive*
- $\tilde{g}_0(S)$ is always repulsive for $-4 < J < 0$



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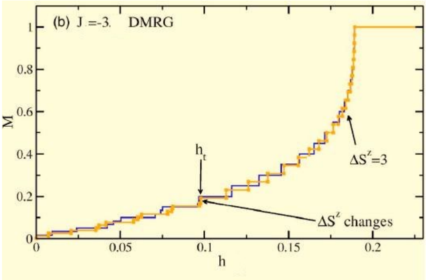
- How large ΔM? Increasing density will break the low density approximation.

$S = 1/2$

Materials like $LiCuVO_4$ realize the spin 1/2 case



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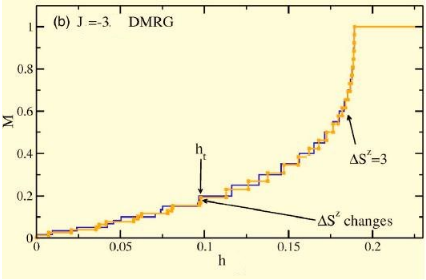

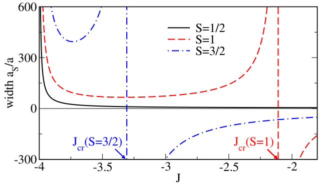

- two magnon bound states with $k = \pi$, binding magnons in different legs
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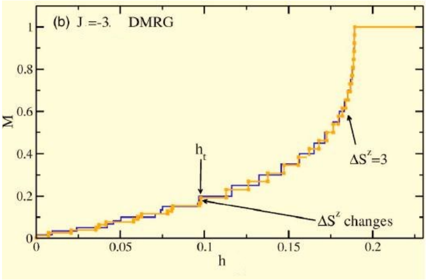

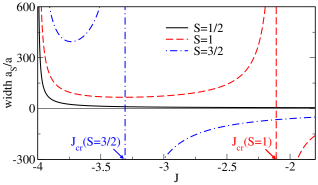

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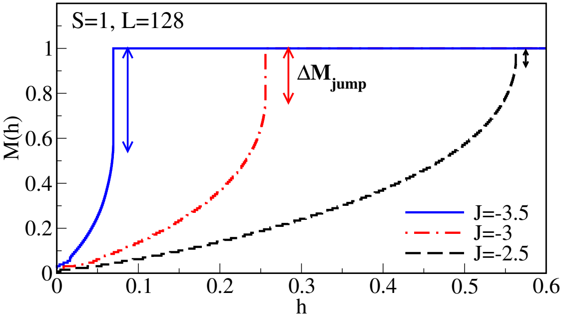
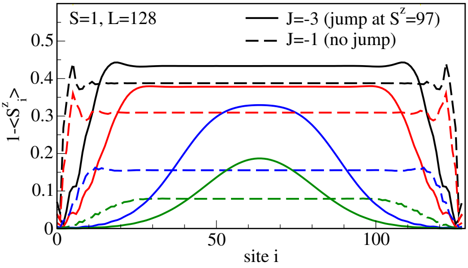
Heidrich-Meisner, Honecker, Vekua, PRB74(2006)

Sudan, Lüscher, Läuchli, PRB80(2009)

$S = 1$ magnetization curves

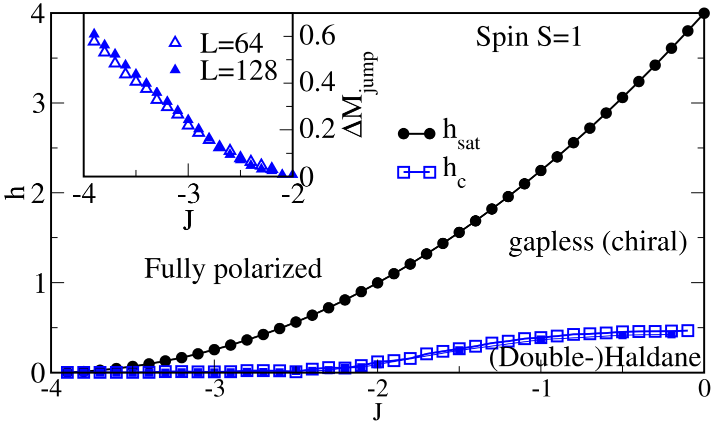


Wave function collapse

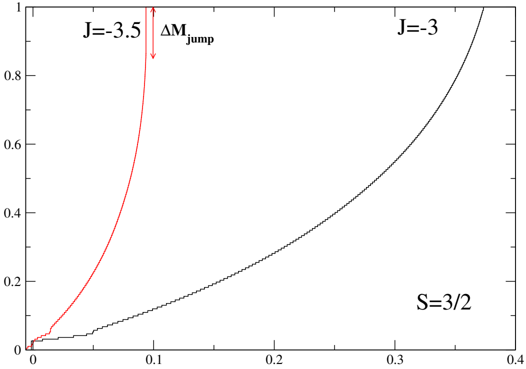


black: $S_z = 80\ /128$
red: $S_z = 90\ /128$
blue: $S_z = 110\ /128$
green: $S_z = 120\ /180$

$S = 1$ ground state phase diagram



Larger S



S	1/2	1	3/2	2	5/2
$J_{cr}(S)$	—	-2.11	-3.31	-3.68	-3.84

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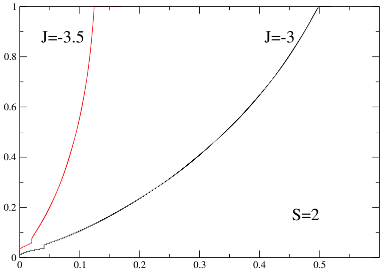

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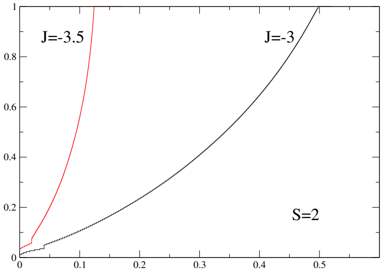

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Critical S

Critical S above which metamagnetism window dissapears: $S_{cr} = 5$ (Vekua, Kolezhuk, in progress)

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PRB 84, 224409 (2011)

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- May be some material realizes $S = 1$ ferromagnetic frustrated spin?

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