Energy spectrum and low temperature properties of the Triangular AF:
An interpretation from a bosonic spinon theory

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- **Motivations to come back to the Triangular AF**

  - Breakdown of linear spin wave spectrum
  - high energy roton-like excitations found with series expansions

- **Possible explanations**

  - Interacting magnon picture: Spin wave Theory
  - Bosonic spinon picture: Schwinger boson Theory
  - Anomalous low temperature behavior

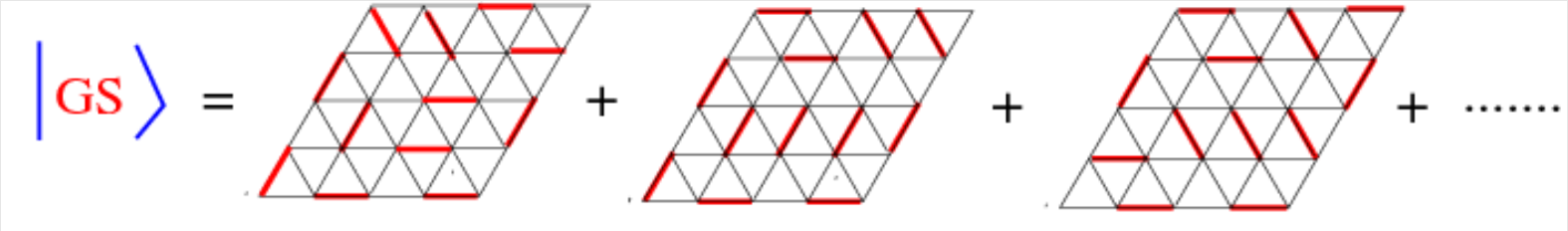
- **Conclusions**

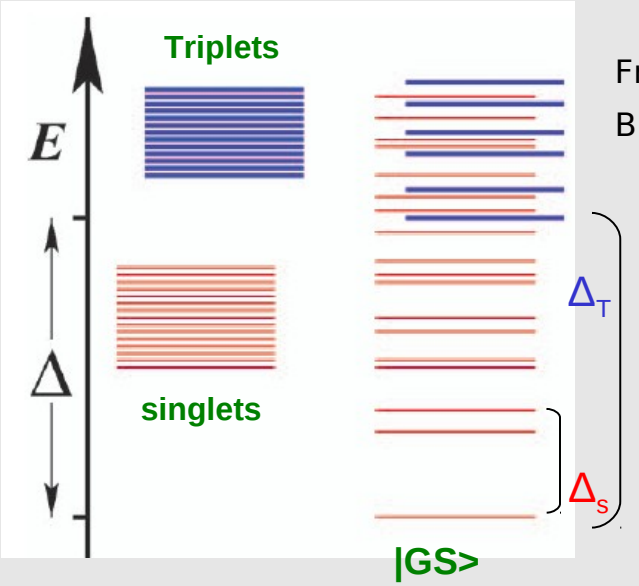
The original motivation of the Triangular AF

The Resonating Valence Bond State (RVB) proposed by

P. W. Anderson and P. Fazekas (1973)

RVB Short range version





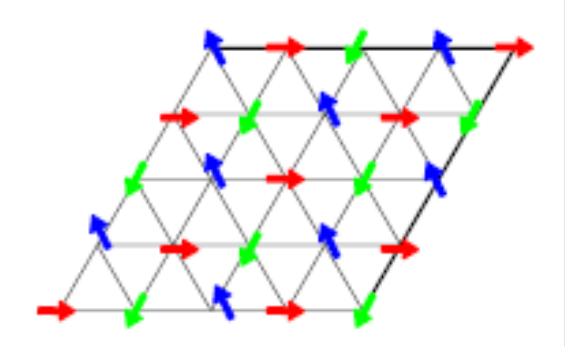
From B. Normand 2009

Main properties

- SU(2) symmetry is not broken
- Translational symmetry is restored due to resonance

Theoretical works on triangular AF

Frustration is not enough → robust 120° Neel order



Finally, $\Delta_T=0$ $\Delta_s \neq 0$
m~ 0.205

Capriotti, A.E.T., Sorella PRL (1999)

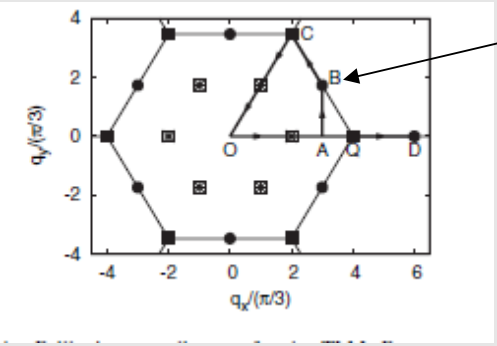
Zheng et al. Phys. Rev. B 74, 224420 (2006)

Method	Ref.	N	E_0/N	M	ρ_s
Series	this work	∞	−0.5502(4)	0.19(2)	
ED	5 and 70	12	−0.6103		
		36	−0.5604	0.40	
V SRVB	9	12	−0.6096	0	0
		36	−0.5579	0	0
ED	62	36			0.06
DMRG	40	∞	−0.5442		
GFQMC	41	∞	−0.5458(1)	0.205(10)	
VQMC, SRVB	10	∞	−0.5123	0	0
VQMC, RVB	10	∞	−0.5357	0	0
VQMC, BCS+Néel	42	∞	−0.532(1)	0.36	
SWT+1/S	43	∞	−0.5466	0.2497	
SWT+1/S	44	∞		0.266	0.087
$d+id$ RVB GA	60	∞	−0.484(2)	0	0
Coupled cluster	55	∞		0.2134	$\rho_{\parallel}=0.056$
SB+1/N	56	∞	−0.5533		$\rho_{\parallel}=0.09$

Anomalous spectrum of the triangular Heisenberg AF

Linear spin-wave (LSWT)

Series (Ising) expansions (SE)
Zheng et al, PRL 96, 057201 (2006)



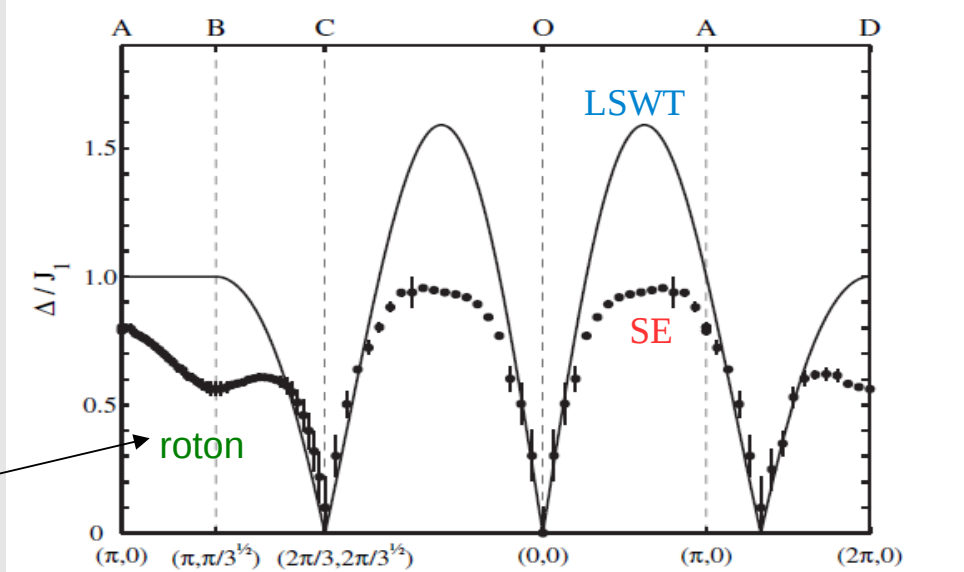


FIG. 1. Excitation spectrum for the TLM ($J_1 = J_2$) along the path ABCOAQD shown in Fig. 4. The high-energy spectrum is strongly renormalized downwards compared to the LSWT prediction (solid line). Note the roton minima at B and D and the flat dispersion in the middle parts of CO and OQ.

are magnons valid?

Main features of the spectrum

- strong downward renormalization with respect to free magnons (LSW theory)
- Roton minima at midpoints of BZ edges

Interacting magnon S=1 picture

LSW + 1/S corrections

Chernyshev et al. Phys.Rev. Lett (2006)

However, magnons are not always well defined

Chernyshev et al. Phys.Rev. Lett (2006)

Can this spectrum be described with spin- ½ spinon excitations?

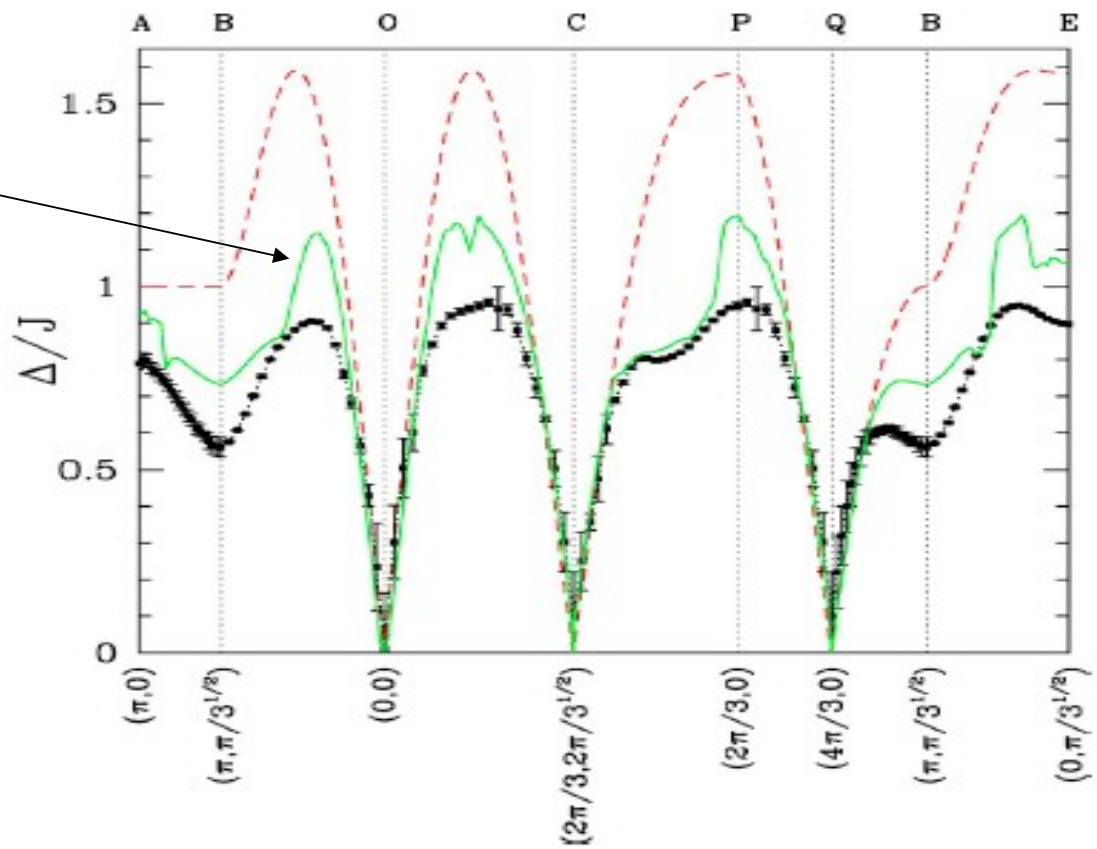


FIG. 5. (Color online) Magnon spectra along *ABOCPQBE* from series expansions compared with LSWT (dashed red line) and SWT+1/*S* (solid green line).

Bosonic spinons S=1/2 picture

$$\hat{\mathbf{S}}_i = \frac{1}{2}\hat{b}^\dagger_{i\sigma}\sigma_{\sigma\sigma'}\hat{b}_{i\sigma'} \qquad \hat{b}^\dagger_{i\sigma}\hat{b}_{i\sigma} = 2S$$

Schwinger boson

Arovas and Auebach PRL(1988)

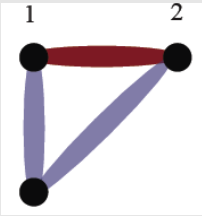
Link operators

$$\hat{A}^\dagger_{ij} = \frac{1}{2}\sum_\sigma \sigma \hat{b}^\dagger_{i\sigma}\hat{b}^\dagger_{j\bar{\sigma}}$$

creates singlets

$$\hat{B}^\dagger_{ij} = \frac{1}{2}\sum_\sigma \hat{b}^\dagger_{i\sigma}\hat{b}_{j\sigma}$$

moves singlets



$$\hat{A}^\dagger_{ij}\hat{A}_{ij} = 2(\hat{\mathbf{S}}_i - \hat{\mathbf{S}}_j)^2$$

AF

$$\hat{B}^\dagger_{ij}\hat{B}_{ij} = 2(\hat{\mathbf{S}}_i + \hat{\mathbf{S}}_j)^2$$

Ferro

Two singlet scheme

$$\hat{S}_i \cdot \hat{S}_j =: \hat{B}^\dagger_{ij}\hat{B}_{ij} : - \hat{A}^\dagger_{ij}\hat{A}_{ij}$$

Ideal to take frustration into account

Mila, Poilblanc, Bruder PRB (1991)
Ceccatto Gazza and A. E. T PRB (1993)
Flint and Coleman PRB (2009)

One singlet scheme

$$\hat{S}_i \,.\, \hat{S}_j = -2\hat{A}^\dagger_{ij}\hat{A}_{ij} + S^2$$

Arovas and Auerbach PRL (1988)
Yoshioka and Miyazaki Phys. Soc Jpn (1991)
Read and Sachdev PRB (1992)

Static properties : mean field approximation

|GS> is a

singlet collection

$$|\mathrm{gs}\rangle = \exp\left[\sum_{ij} f_{ij}\hat{A}^{\dagger}_{ij}\right]|0\rangle_b,$$

magnetism as a quantum fluid

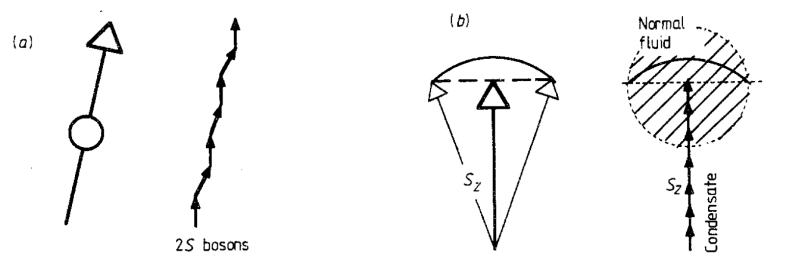


Figure 1. The semi-classical versus the quantum fluids approach to magnetism. (*a*) Illustration of how a spin S is built with $2S$ bosons. (*b*) Schematic diagram of the two fluid picture; the normal fluid describes the spin fluctuations while classical magnetism is the condensate.

Chandra, Coleman, Larkin,
J. Condens. Matt2 7933 (1990)

Schwinger results: triangular AF

	Mean field	Fluct 1/N	QMC
Q (order)	(4/3π,0)	(4/3π,0)	(4/3π,0)
energy	-0.5697	-0.5533	-0.545(1)
mag	0-.275	?	0.205(10)
stiffness	0.085	0.066	0.05 (ED)

Manuel, A.E.T., Ceccatto PRB (1998)

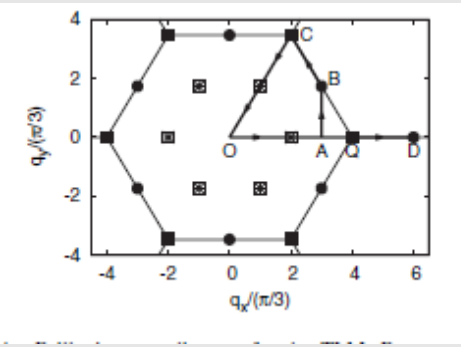
A.E.T., Manuel, Gazza, Ceccatto PRL (1997)

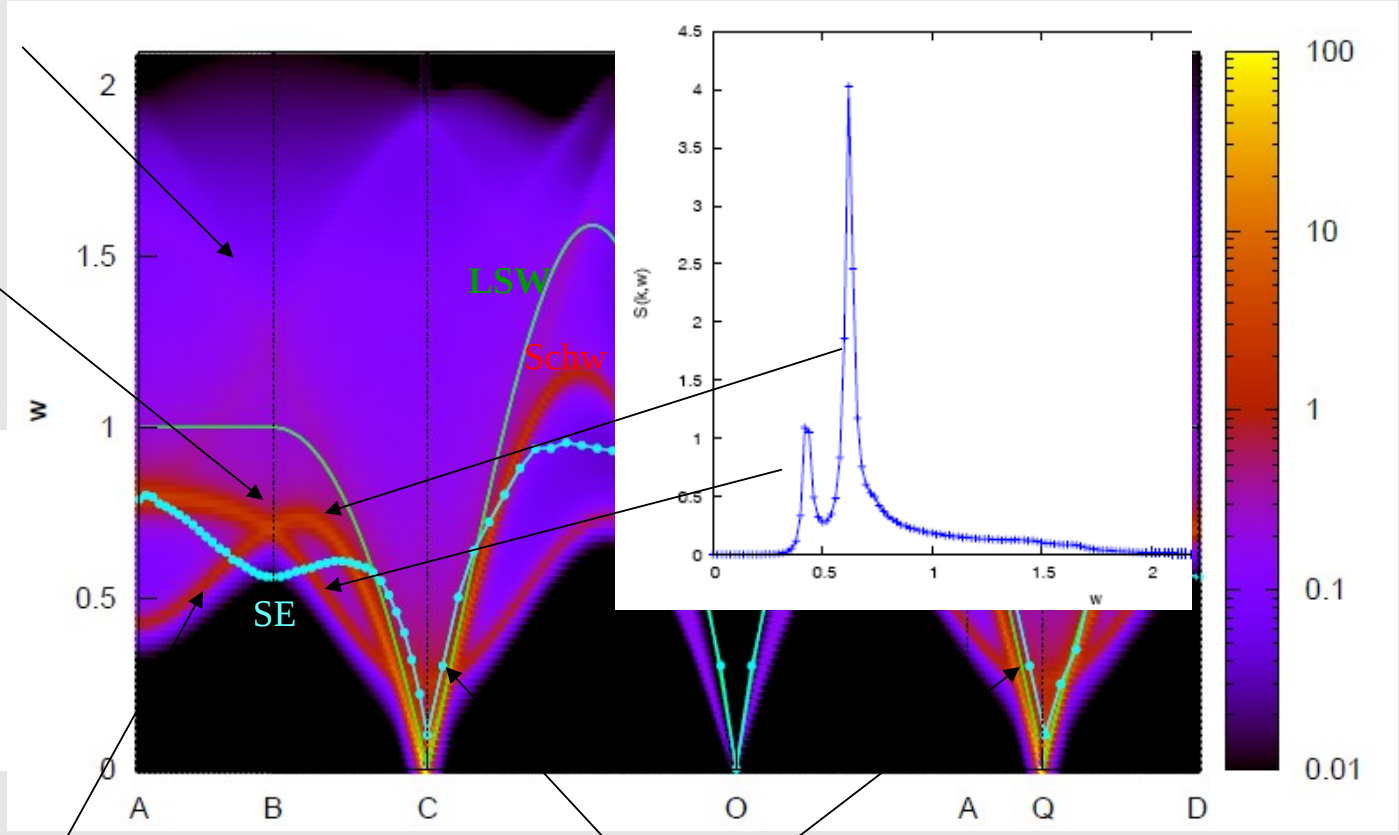
Dynamical magnetic structure factor at T=0 : mean field theory

$$S^{zz}(\mathbf{k},\omega) = \sum_{\mathbf{q}} |u_{\mathbf{k}+\mathbf{q}} v_{\mathbf{q}} - u_{\mathbf{q}} v_{\mathbf{q}+\mathbf{k}}|^2 \delta(\omega - (\omega_{-\mathbf{q}\uparrow} + \omega_{\mathbf{k}+\mathbf{q}\downarrow}))$$

Two spinons continuum
Normal fluid excitations

Roton at the crossing of the spinon bands





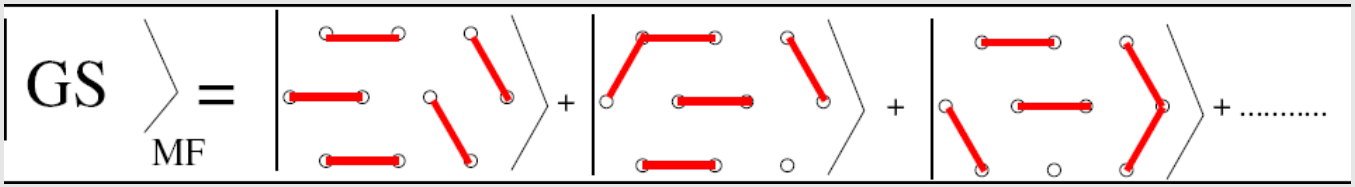
What about these excitations ?

Low energy spin wave-like bands

The mean field ground state

$$|\mathrm{gs}\rangle = \exp\left[\sum_{ij} f_{ij}\hat{A}^{\dagger}_{ij}\right]|0\rangle_b,$$

Looks like…….

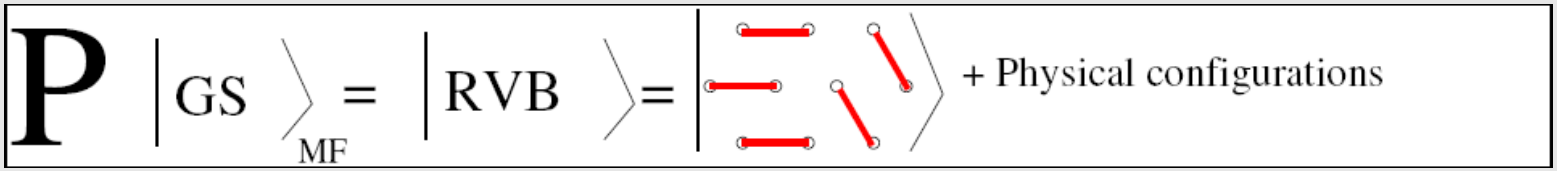


Unphysical configurations

remember

$$\hat{\mathbf{S}}_i^2 = \frac{n_i}{2}\left(\frac{n_i}{2}+1\right)$$

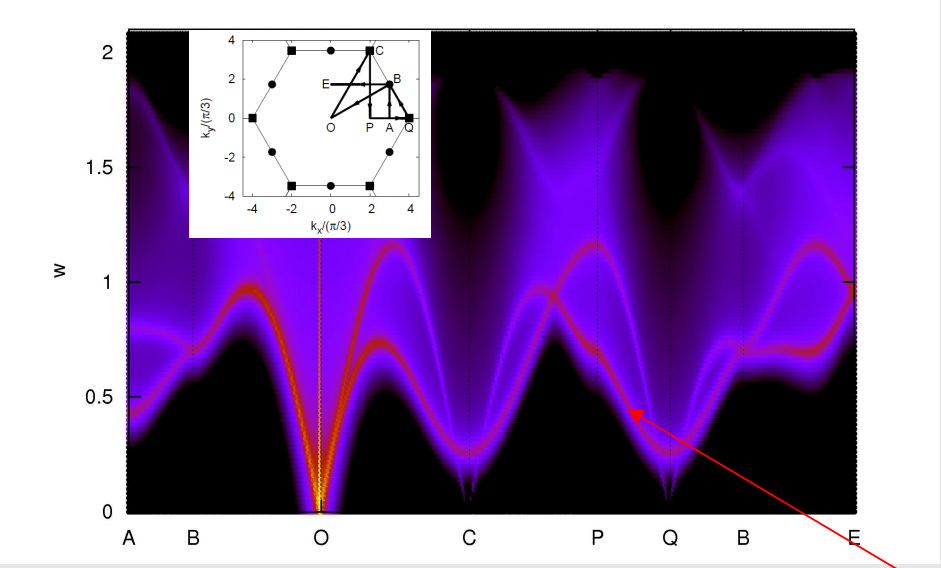
After projection to the space of one boson per site…….



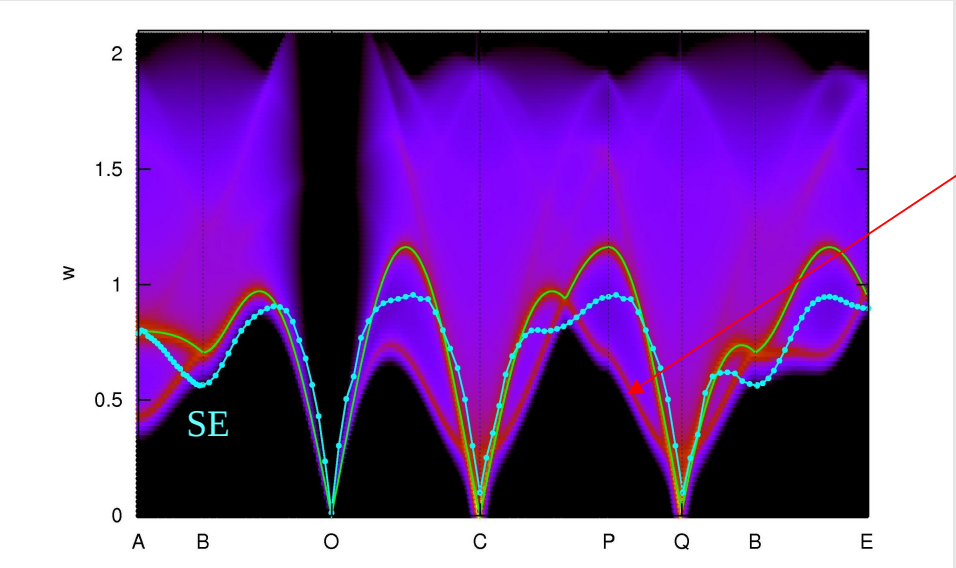
Then, unphysical configs are related to boson density n_i fluctuations

Density-density dynamical structure factor T=0

$$N(\mathbf{k},\omega) = \sum_n |\langle \mathrm{gs}|\hat{n}_{\mathbf{k}}(0)|n\rangle|^2 \delta(\omega - (\epsilon_n - E_{\mathrm{gs}}))$$



$$\mathcal{N}(\mathbf{k},\omega) = \sum_{\mathbf{q}} |u_{\mathbf{k}+\mathbf{q}} v_{\mathbf{q}} + u_{\mathbf{q}} v_{\mathbf{q}+\mathbf{k}}|^2 \; \delta\left(\omega - (\omega_{\mathbf{k}} + \omega_{\mathbf{k}+\mathbf{q}})\right)$$



We argue that these bands are unphysical

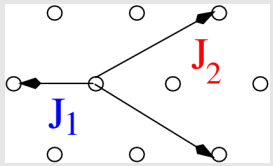
Due to violation of the local constraint

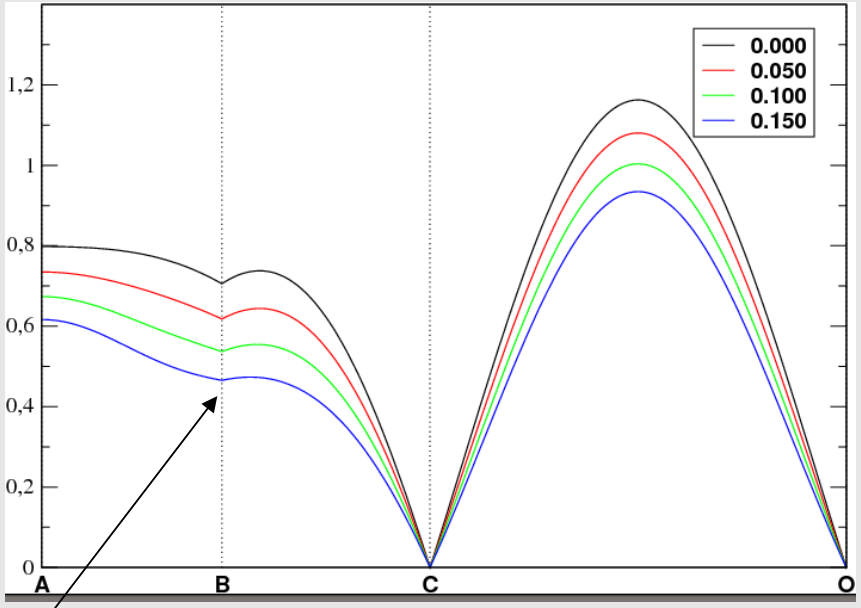
Mezio, Sposetti Manuel and A.E.T. EPL (2011)

$$S^{\alpha\alpha}(\mathbf{k},\omega) = \sum_n |\langle \mathrm{gs}|\hat{\mathbf{S}}^{\alpha}_{\mathbf{k}}(0)|n\rangle|^2 \delta(\omega - (\epsilon_n - E_{\mathrm{gs}}))$$

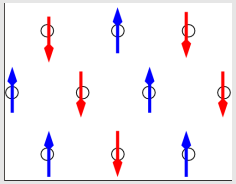
$$S(\mathbf{k},\omega) = \frac{1}{4N}\sum_{\mathbf{q}} |u_{\mathbf{k}+\mathbf{q}} v_{\mathbf{q}} - u_{\mathbf{q}} v_{\mathbf{k}+\mathbf{q}}|^2 \delta(\omega - (\omega_{-\mathbf{q}} + \omega_{\mathbf{k}+\mathbf{q}}))$$

An interpretation of the rotonic excitations





Softening at $B = (\pi, \pi/\sqrt{3})$



We interprete rotonic excitations as

Collinear AF fluctuations

Above the 120 Nèel order

Entropy

HTE, Misguich and Bernu PRB (2001)

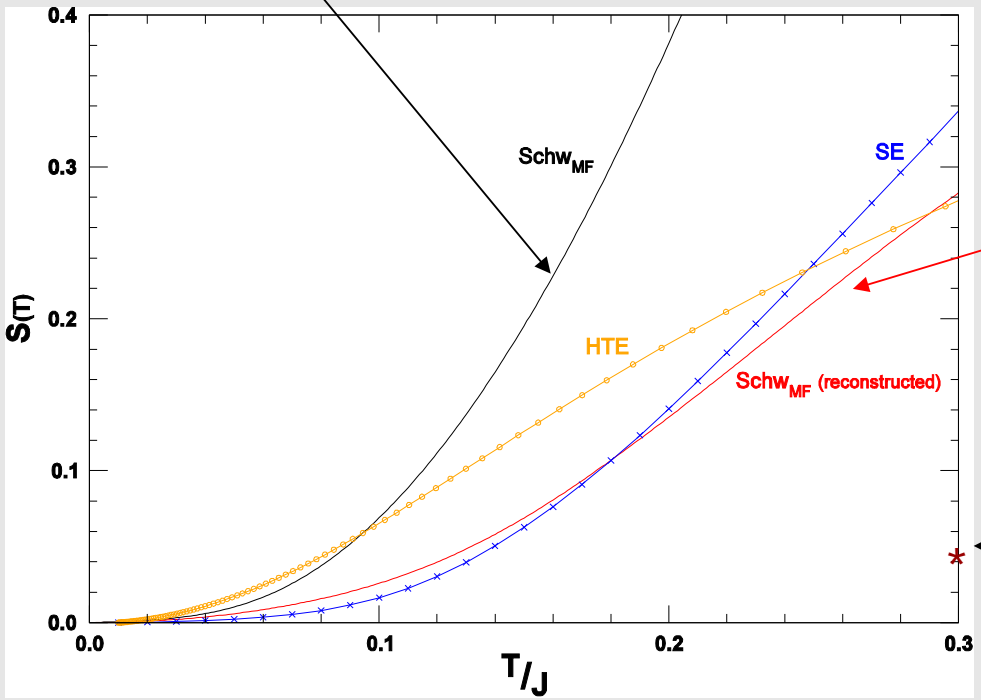
SE , Zheng et al. PRB (2006)

Schwinger mean field

$$S_{MF} = \sum_{\mathbf{k}\sigma} [\, (n_{\mathbf{k}\sigma} + 1)\, \ln (n_{\mathbf{k}\sigma} + 1) - n_{\mathbf{k}\sigma}\ \ln n_{\mathbf{k}\sigma}\,]$$

Reconstructed Schwinger mean field

$$S^{rec}_{MF} = \sum_{\mathbf{k}} [\, (n_{\mathbf{k}} + 1)\, \ln (n_{\mathbf{k}} + 1) - n_{\mathbf{k}}\ \ln n_{\mathbf{k}}\,]$$



$$\frac{\xi_{NLSM}}{\xi_{HTE}} \sim 10$$

0.05 (Non linear sigma model)

$$s(T) = \mathcal{A}\left(\frac{1}{c_{\parallel}^2} + \frac{2}{c_{\perp}^2}\right) T^2 + O(T^4)$$

$$\xi(T) = 0.021 \left(\frac{c}{\rho_s}\right) \left(\frac{4\pi\rho_s}{T}\right)^{1/2} \exp\left(\frac{4\pi\rho_s}{T}\right)$$

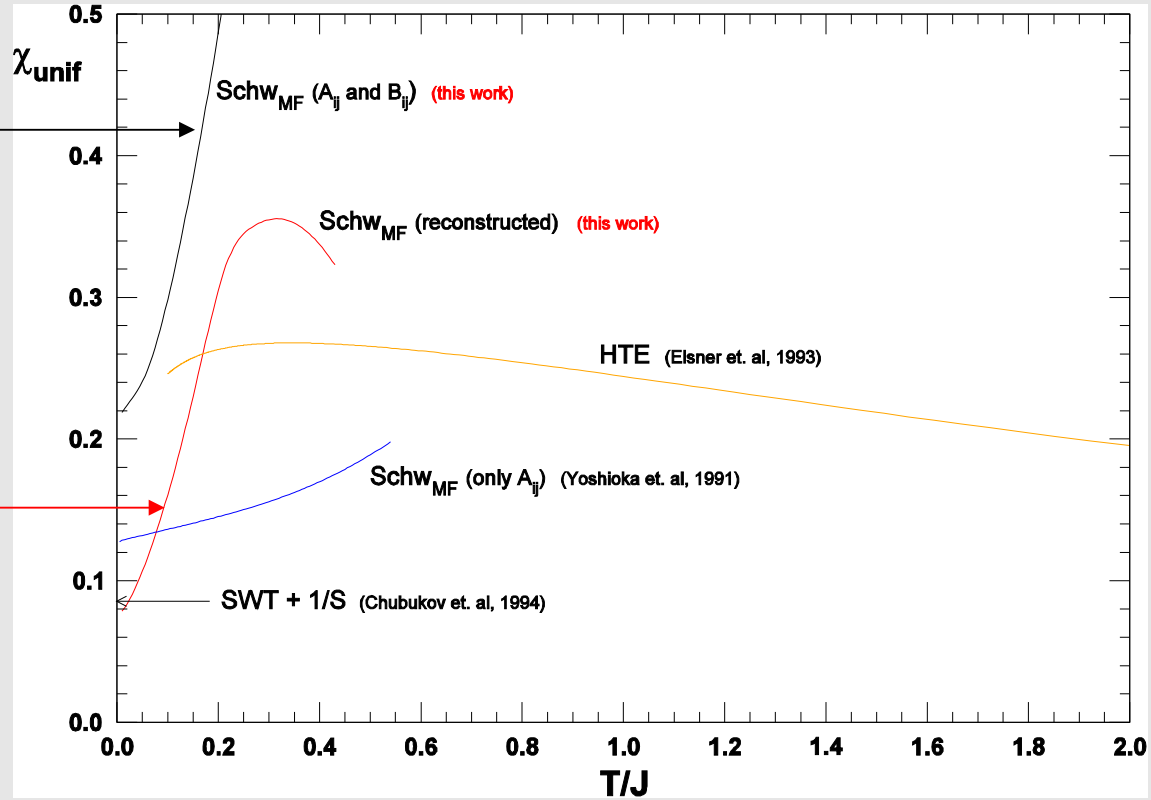
Uniform susceptibility

Schwinger mean field

$$\chi_{MF} = \frac{1}{T} S(\mathbf{q}=0) = \frac{1}{T} \sum_{\mathbf{k}} n_{\mathbf{k}\sigma}(n_{\mathbf{k}\sigma}+1)$$

Reconstructed Schwinger mean field

$$\chi_{MF}^{rec} = \frac{1}{T} S(\mathbf{q}=0) = \frac{1}{T} \sum_{\mathbf{k}} n_{\mathbf{k}}(n_{\mathbf{k}}+1)$$



Conclusions

Alternatively to spin wave theory

- **the spectrum of the triangular AF can be described in terms of bosonic ½ spinon excitations**

- **roton like features of the spectra: collinear Fluct**

By including the physical excitations

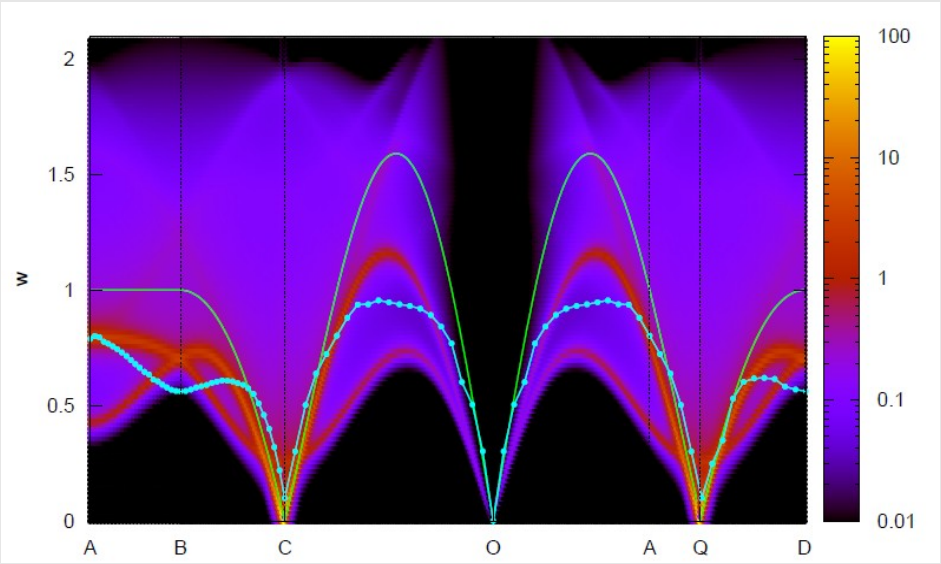
- **entropy and uniform suspectibility is well described at low T**

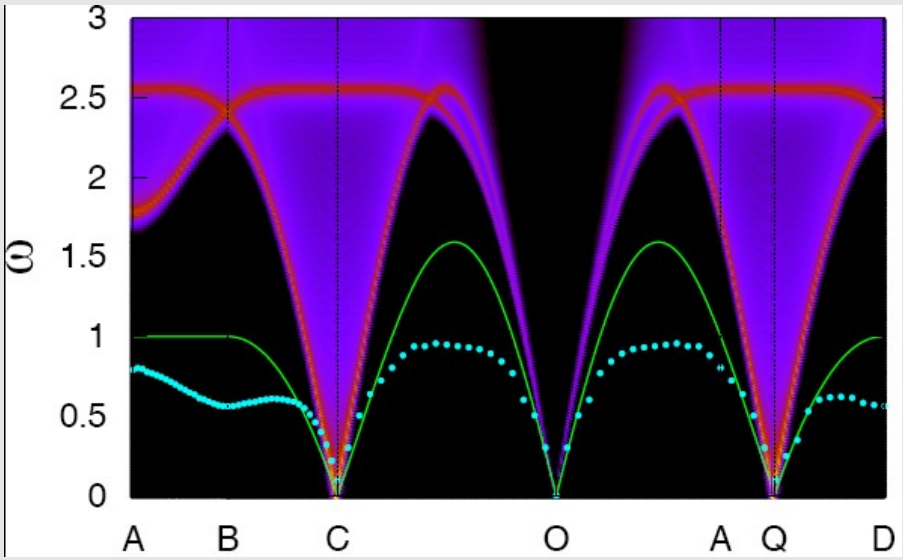
Future work: to go beyond mean field theory

- **to include spinon interaction**
- **a better implementation of the local constraint**
- **to include gaussian corrections in Partition function**

Two singlet scheme

One singlet scheme





Mezio, Sposetti Manuel and A.E.T. EPL (2011)

Spin excitations

$$\mathbf{S}^z_{\mathbf{k}}(0)|GS> \equiv \mathbf{S}^z_{\mathbf{k}}(0)|0>_\alpha = \frac{1}{2}\sum_{\mathbf{q},\sigma} -\sigma u_{\mathbf{k}+\mathbf{q}} v_{\mathbf{q}} e^{i\theta_{\sigma\mathbf{q}}} \alpha^\dagger_{\mathbf{k}+\mathbf{q},\sigma} \alpha^\dagger_{-\mathbf{q},\bar{\sigma}} |0>_\alpha$$

Two spinons continuum

(Good) Features of the Schwinger-boson theory

- **Mermin-Wagner theorem is satisfied exactly.**
  Arovas and Auerbach PRL 61, 617 (1988)

- **Ferro- and antiferromagnetic channels take into account frustration properly.**
  Ceccatto, Gazza, Trumper, PRB 47, 12329 (1993). Flint and Coleman PRB 79, 14424 (2009).

- **Ordered and disordered phases can be studied on equal footing.**
  Read and Sachdev PRL 62 1694 (1989).

- **Magnetization is interpreted as a Bose condensation.**
  Hirsch and Tang PRB 39, 2850 (1989). Sarker et al PRB 40 50 28 (1989).

- **A large N expansion can be formulated.**
  Read and Sachdev PRL 66 1773 (1991); Flint and Coleman PRB 79, 14424 (2009)

- **Gaussian fluctuations above mean field can be computed**
  Trumper, LOM, Gazza , Ceccatto, PRL 78, 2216 (1997)

- **The agreement with exact diagonalization on finite clusters is very good.**
  Trumper, LOM, et al. PRL 78, 2216 (1997); PRB 57, 8348 (1998).