

# GALAAD

## Geometry, Algebra & Algorithms

INRIA, BP93, 06902 Sophia Antipolis

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## Permanent staff:

- L. Busé (CR2 INRIA)
- M. Elkadi (MC UNSA)
- A. Galligo (Prof. UNSA)
- B. Mourrain (DR2 INRIA)

## Ph. D. Student:

- L. Alberti (2005-2008) (resp. B. Mourrain, G. Comtes);
- E. Ba (2006-2009) (resp. M. Elkadi, A. Galligo);
- S. Chau (2004-2007) (resp. M. Elkadi, A. Galligo);
- D. N'Diatta (2006-2009) (resp. B. Mourrain, O. Ruatta);
- M. Dohm (2005-2008) (resp. A. Galligo, M. Elkadi, L. Busé);
- H. Khalil (2004-2007) (resp. B. Mourrain, M. Schatzmann);
- Le Thi Ha (2003-2006) (resp. M. Elkadi, A. Galligo);
- J. Wintz (2005-2008) (resp. B. Mourrain);

**Expert engineer:** J.P. Pavone (PhD. 2001-2004, resp. A. Galligo, B. Mourrain).

**Post-Doctorat:** V. Sharma (NYU) 2006-2007.

## Algebraic Geometry:

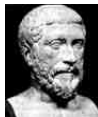
Ubiquitous polynomial models:

- for **representation**: geometric modeling, robotics, computer vision, signal processing, computer biology, ...
- for **approximation**: compact encoding of functions, higher order approximation ...



## Symbolic Numeric:

- Approximation of output (and input) vs. control of errors.
- Exact implicit representation vs. intrinsic complexity obstacles.  
(Pythagoras dilemma.)

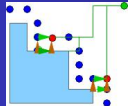


- Dedicated methods to solve **algebraic problems**:
  - Resultant, normal form, duality, residue, structured matrices, factorisation.
- Efficient **geometric algorithms** for curves and surfaces:
  - Computational topology, arrangements, singularity analysis, classification of specific families of surfaces, real semi-algebraic varieties.
- Develop **symbolic-numeric computation**:
  - Certification, output sensitive algorithms, analysis of degeneracies, discriminant varieties, robustness issues, approximation level.

# Algebra

*Find condition(s) on parameters  $\mathbf{c}$  for a overdetermined system  $\mathbf{f}_{\mathbf{c}}(\mathbf{x}) = 0$  to have a solution in a variety  $X$ .*

- 👉 New resultants for systems with **base points**, called **residual resultants** ( $X$  is the blow-up of the base-point variety).  
[B'01,BCD'03]
- Their construction, based on matrix formulation, provides dedicated solvers for a class of problems.
- Application to implicit equation of polynomial maps.
  - Previous methods: use syzygies between polynomials (moving plane [Sederberg'90]).
  - Generalisation based on approximation complexes [BJ'05,BC'05].
- 👉 New resultants for **determinantal systems** giving conditions on matrix of polynomials to drop its rank [B'03, BEG'06].



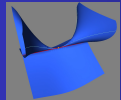
# Border basis

(Ph.D. thesis of Ph. Trébuchet)

Resolution a polynomial system  $f_1 = 0, \dots, f_m = 0 \Leftrightarrow$  Compute the structure of  $\mathcal{A} = \mathbb{K}[\mathbf{x}]/(f_1, \dots, f_n)$ .

- Structural instability of Gröbner basis computation with approximate coefficients.
- ☞ Generalisation of the normal form criterion for the reduction onto a fixed generating set of  $\mathcal{A}$ : involves commutator polynomials [T'02, MT'02, MT'05].
- Linear algebra which exploits column pivoting and sparsity in triangulation of the coefficient matrices (SYNAPS) [T'02].
- More freedom to choose a *good basis* and representation of  $\mathcal{A}$ , from a numerical point of view [T'02, MT'05].

# Geometry



# Topology of algebraic curves and surfaces (Ph.D. thesis of J.P. T  court)

*Sweeping methods choose a generic direction of projection, analyse the critical fibers of this projection and deduce the topology.*

- ➡ New algorithm for curves in  $\mathbb{R}^3$  which uses plane projections [GLMT'05].
- New algorithm for the arrangement of quadrics [MTT'05].
- ➡ New algorithm for an algebraic surface  $S$  in  $\mathbb{R}^3$ , based on Morse stratified theory [MT'05]:
  - Compute an explicit Whitney stratification of  $S$ , based on resultant computation.
  - Compute the topology of the polar variety in a generic direction.
  - Use it to deduce the topology of the surface.

# Symbolic-Numeric computation

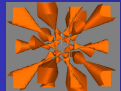


# Factorisation

(Ph.D. thesis of G. Chèze)

*Given a polynomial  $F \in \mathbb{Q}[x, y]$ , find (an algebraic extension  $\mathbb{Q}[\alpha][x, y]$  and) a decomposition into (absolute) irreducible factors.*

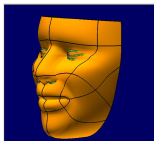
- Use monodromy around a  $x$ -coordinate to deduce the recombination of series expansion factors [C'04, CG'05].
- 👉 Optimal analysis of the order of approximation and numerical precision; improve substantially the previous complexity bound [CG'06, CL'06].
- Implementation world record (degree 200; mod  $p$ , degree 1000).



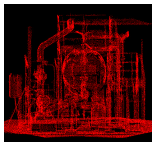
# Subdivision solvers (Ph.D. thesis of J.P. Pavone)

*Fast methods to localise real roots, to exclude domains with no root, to filter computation, to approximate and certify.*

- Exploit the property of Bernstein representation.
- ☞ Reduction of multivariate to univariate root finding, and using preconditioning strategy [MP05].
- Extension to the topology of implicit [curves](#) and [surfaces](#) [ACM05], [CMP06].
- ☞ Detailed complexity analysis [MP05, ACM05] based on entropy.
- Very good performance in practice (SYNAPS).



**CAGD:** collaboration with Think 3 on intersection problems, identification of key problems, integration of SYNAPS auto-intersection algorithm (subdivision) in THINK DESIGN.



**Reconstruction:** collaboration with Th. Chaperon (MENSI), on dedicated solver for fast reconstruction of cylinders, based on resultant construction.



**Virtual plants:** collaboration with Franck Aries (INRA Avignon) and C. Godin team (CIRAD) on modeling leaves and trees, for agronomic analysis.

❑ MULTIRES maple package for resultant, resolution, residues;

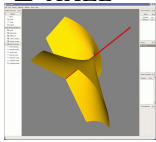
❑ SYNAPS



- template C++ library for Symbolic and Numeric computation; univariate, multivariate polynomials; resultants; solvers; topology of algebraic curves and surfaces.

- **Open source project:** GPL; collaborative work; 200 000 l; distributed in source code (.tgz, rpm); 410 dwl for 2006.

❑ AXEL



- geometric modeler (J. Wintz); manipulation of implicit, parameterised algebraic curves and surfaces.

- intersection, arrangements, CSG operations; integration of external modules (dynamic libraries); algebraic computation by SYNAPS.

**Contribution:** MATHEMAGIX; high-level interpreter; play-plug-play.

- ANR DECOTES (Tensor Decomposition) 2006-2009.
- ANR GECKO (Geometry and Complexity) 2005-2008.
- ACS (Algorithm for Complex Shapes) 2005-2008.
- AIM@SHAPE (Shapes and Semantics) 2003-2007.
- CALAMATA (Associate team with Athens, Patras) 2003-2006.
- GAIA (Intersection algorithms for geometry based IT-applications using approximate algebraic methods) 2002-2005.
- NSF-INRIA (with R. Goldman, Rice Univ. USA) 2004-2006.
- SIMPLES (COLORS with F. Aries, INRA Avignon) 2002-2003.
- ECG (Effective Computational Geometry for Curves and Surfaces) 2001-2004.
- ECos-Sud (with A. Dickenstein, Buenos Aires) 2001-2003.
- Distance Geometry and Structural Molecular Biology (with Montpellier, Paris 6) 2001-2003.

- 5 PhD.
- Publications: 29 journals; 32 conf. proc.; 7 book chapters; 3 books (editors).
- Organisation of workshops and conferences: 11.
- Implication in the main conf.: MEGA, ISSAC, SNC, ACA.

*Consolidation and improvement of our activities.*

## **Algebra:**

- more on polynomial solvers,
- resultants, discriminants, resultant systems,
- factorisation and decomposition of varieties,

## **Geometry:**

- singularity and topology,
- classification and algorithms for small degree algebraic models,
- intersection, arrangement of algebraic curves and surfaces.

## **Symbolic-numeric:**

- certification of subdivision methods,
- numerical structured linear algebra,
- numeric computation for singularities.