High-Level Outline

- FEAST as filtered subspace iteration, for Hermitian problems
- FEAST for non-Hermitian problems
- SS method as filtered Krylov subspace method
- Function approximation and computer arithmetic studies related to the filter

Subspace Iteration:

Pick $Q_0 = [y_1, y_2, \ldots, y_p]$, $p \ll n$

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$$y_k = (z_k I - A)^{-1} y, \qquad k = 1, 2, \ldots, q$$

$$\rho_\ell(A) y = \text{linear combination of } \{y_1, y_2, \ldots, y_q\}$$

$$M = X \begin{pmatrix} \lambda_{\text{smallest}} & & \\ & \ddots & \\ & & \lambda_{\text{largest}} \end{pmatrix} X^{-1}$$

$$\Pi_0(M) = X \begin{pmatrix} 0 & & \\ & 1 & \\ & & 0 \end{pmatrix} X^{-1}$$

$$\Pi_\ell(M) = X \begin{pmatrix} 0 & & \\ & \lambda_1^\ell \; \lambda_2^\ell \; \lambda_m^\ell & \\ & & 0 \end{pmatrix} X^{-1}$$