Frustration index of signed graphs

Research internship

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Signed graphs are graphs whose arcs are either positive or negative. They appear naturally in many applications. For instance, in the context of gene networks, experiments reveal the existence of activation or inhibition between genes, which are represented under the form of a signed graph G. This signed graph G imposes some constraints on the underlying dynamics of the network. A very useful operation to analyze these constraints is the switch operation: switching a vertex in G means that we switch the sign of all the arcs incident to this vertex (negative arcs become positive, and vice versa). The nice fact is that the switch preserves the underlying dynamics (up to isomorphism) while it can make the signed graph much more simple. For instance, if every cycle of G contains an even number of negative arcs, there is a sequence of switches which make all arcs positive, and the analyzis of the underlying dynamics is then much more simple. More generally, one might think that the analysis of the underlying dynamics is simpler when the number of negative arcs is minimal. Two signed graphs are said switch equivalent if one can be obtained the other one by a sequence of switches. For a signed graph G, the minimum number of negative arcs in a graph which is switch equivalent to G is called the frustration index of G. Given an unsigned graph H, the frustration index of H is the maximum frustration index of the signed graphs having H as unsigned underlying graph.

Many simple questions about this parameter remain unsolved, both in the directed and undirected case. For H being an unsigned clique K, it is known that its frustration index f is the frustration index of the all-negative signed version K^- of K. Moreover, any signed version of K that has frustration f is switch equivalent to K^-. It has been conjectured that the same may hold not only for H being a clique but more generally for H being a chordal graph. The main purpose of this internship is to prove or disprove this conjecture. Another related question that may be investigated during the internship is the complexity of computing the frustration index of a signed chordal graph.

The internship will take place in the I3S laboratory in Sophia Antipolis, near Nice, advised by Adrien Richard (https://www.i3s.unice.fr/~richard/), Florian Bridoux (bridoux@i3s.unice.fr) and Christophe Crespelle (https://www.i3s.unice.fr/~ccrespelle/).