Two Applications of Kolmogorov complexity

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Outline

1. Multi-head Finite Automaton

2. Repitionons-free sequences

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MFA hierarchy theorem

Definition MFA_n is the set of all languages that can be recognized by an n-heads finite automaton.

MFA hierarchy Theorem $MFA_n \subsetneq MFA_{n+1}$

Idea of the proof:

$$L_m = \{w_1\#w_2\#\ldots\#w_m\#w_m\#\ldots\#w_1\},\ \ w_i \in \{0,1\}^*$$

L_m is recognized by an MFA with n heads if $m = (n-1)+(n-2)+\ldots+1 = \frac{n(n-1)}{2}$

Claim: L_m cannot be recognized by an m-heads MFA if $m > \frac{n(n-1)}{2}$

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Lemma [Levin] $\forall \alpha < 1\ \exists \omega = \omega_0\omega_1 \ldots \omega_n \ldots,\ \exists C > 0$ such that $K(\omega_k\omega_{k+1} \ldots \omega_{k+n-1}) > \alpha n - C$

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That's wrong!!

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Too weak...

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Lemma [Rumyantsev] $\forall\alpha < 1\ \exists\omega = \omega_0\omega_1\ldots\omega_n\ldots,\ \exists C$ such that for every $X = \{i_1, i_2, \ldots, i_n\}$ it holds $K(\omega_{i_1}\omega_{i_2}\ldots\omega_{i_n}, X | i_1) > \alpha n - C$

Theorem $\forall \gamma > 1\ \exists \omega = \omega_0 \omega_1 \ldots \omega_n \ldots$ (ω_i is from a finite alphabet A) such that

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2. ω does not contain γ'-repetitions $x^{\gamma'}$ for any $\gamma' > \gamma$

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1. ω contains γ'-repetions for every rational $\gamma' < \gamma$

2. (weak form) ω does not contain γ'-repetitions $x^{\gamma'}$ for $\gamma' > \gamma$ for any word x of length at least N