Decomposition of graphs in given factors

Decomposition in factors

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Collaborators.

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Content

CELLST version 1.0 (1)

	3	2	1	1
3				
2				
1				
1				

	3	2	1	1
3	●	●	●	
2				
1				
1				

	3	2	1	1
3	●	●	●	
2	●	●		
1				
1				

	3	2	1	1
3	●	●	●	
2	●	●		
1	●			
1				●

CELLST version 1.0 (1)

	3	2	1	1
3				
2				
1				
1				

	3	2	1	1
3	●	●	●	
2				
1				
1				

	3	2	1	1
3	●	●	●	
2	●	●		
1				
1				

	3	2	1	1
3	●	●	●	
2	●	●		
1	●			
1				●

CELLST version 1.0 (1)

	3	2	1	1
3				
2				
1				
1				

	3	2	1	1
3	●	●	●	
2				
1				
1				

	3	2	1	1
3	●	●	●	
2	●	●		
1				
1				

	3	2	1	1
3	●	●	●	
2	●	●		
1	●			
1				●

CELLST version 1.0 (1)

	3	2	1	1
3				
2				
1				
1				

	3	2	1	1
3	●	●	●	
2				
1				
1				

	3	2	1	1
3	●	●	●	
2	●	●		
1				
1				

	3	2	1	1
3	●	●	●	
2	●	●		
1	●			
1				●

CELLST version 1.0 (2)

	1	2	3	3
1				
2				
3				
3				

	1	2	3	3
1				●
2				
3				
3				

	1	2	3	3
1				●
2			●	●
3				
4				

	1	2	3	3
1				●
2			●	●
3		●	●	●
3	●	●	●	

CELLST version 1.0 (2)

	1	2	3	3
1				
2				
3				
3				

	1	2	3	3
1				●
2				
3				
3				

	1	2	3	3
1				●
2			●	●
3				
4				

	1	2	3	3
1				●
2			●	●
3		●	●	●
3	●	●	●	

CELLST version 1.0 (2)

	1	2	3	3
1				
2				
3				
3				

	1	2	3	3
1				●
2				
3				
3				

	1	2	3	3
1				●
2			●	●
3				
4				

	1	2	3	3
1				●
2			●	●
3		●	●	●
3	●	●	●	

CELLST version 1.0 (2)

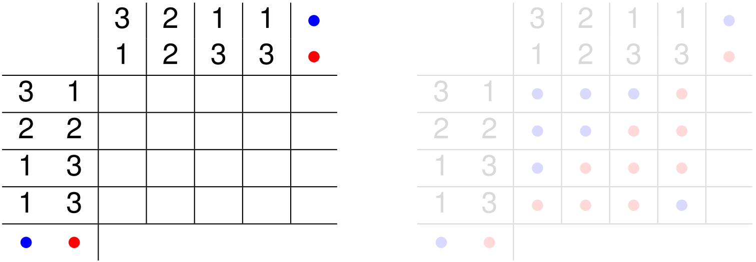
	1	2	3	3
1				
2				
3				
3				

	1	2	3	3
1				●
2				
3				
3				

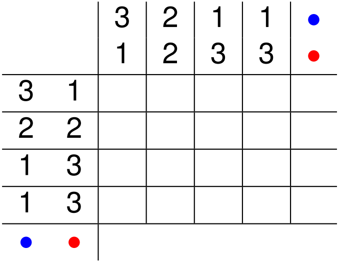
	1	2	3	3
1				●
2			●	●
3				
4				

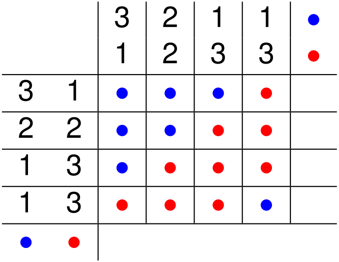
	1	2	3	3
1				●
2			●	●
3		●	●	●
3	●	●	●	

CELLST version 2.0



CELLST version 2.0





Some cells are forbidden

		3	2	1	1	●
		1	2	2	3	●
3	1					
2	2					
1	3					
1	2			*X*		
●	●					

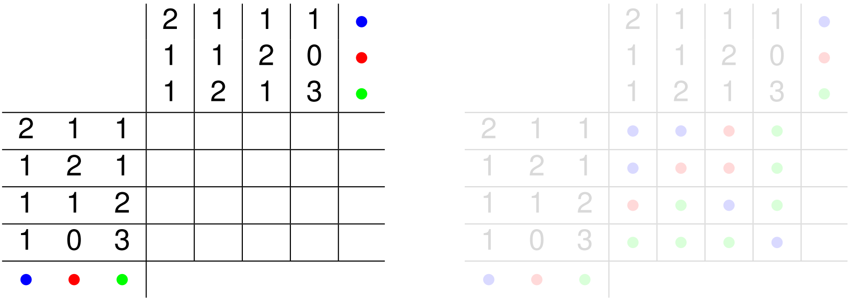
		3	2	1	1	●
		1	2	2	3	●
3	1	●	●	●	●	
2	2	●	●	●	●	
1	3	●	●	●	●	
1	2	●	●	*X*	●	
●	●					

Some cells are forbidden

		3	2	1	1	$\color{blue}\bullet$
		1	2	2	3	$\color{red}\bullet$
3	1					
2	2					
1	3					
1	2			$\color{green}X$		
$\color{blue}\bullet$	$\color{red}\bullet$					

		3	2	1	1	$\color{blue}\bullet$
		1	2	2	3	$\color{red}\bullet$
3	1	$\color{blue}\bullet$	$\color{blue}\bullet$	$\color{blue}\bullet$	$\color{red}\bullet$	
2	2	$\color{blue}\bullet$	$\color{blue}\bullet$	$\color{red}\bullet$	$\color{red}\bullet$	
1	3	$\color{blue}\bullet$	$\color{red}\bullet$	$\color{red}\bullet$	$\color{red}\bullet$	
1	2	$\color{blue}\bullet$	$\color{red}\bullet$	$\color{green}X$	$\color{red}\bullet$	
$\color{blue}\bullet$	$\color{red}\bullet$					

Three or more colors



Three or more colors

			2 1 1	1 1 2	1 2 1	1 0 3	🔵 🔴 🟢
2	1	1					
1	2	1					
1	1	2					
1	0	3					
🔵	🔴	🟢					

			2 1 1	1 1 2	1 2 1	1 0 3	🔵 🔴 🟢
2	1	1	🔵	🔵	🔴	🟢	
1	2	1	🔵	🔴	🔴	🟢	
1	1	2	🔴	🟢	🔵	🟢	
1	0	3	🟢	🟢	🟢	🔵	
🔵	🔴	🟢					

CELLSG version 1.0

- Tableaux $\to$ complete bipartite graph $(X \cup Y, E)$.
  - Rows $\to X$; Columns $\to Y$.
  - Cells $\to E$:
    cell (i, j) is the edge between vertex $i \in X$ and vertex $j \in Y$.
- Input: A function $a : X \cup Y \to \mathbb{N}$.
- Output: A set $F \subseteq E$ such that:

$$\forall v \in X \cup Y, |\{e \in F : v \in e\}| = a(v)$$

Number of edges of F incident with v are exactly $a(v)$.

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Graphs setting
Definitions

Definition (Degree function)

Let $G = (V, E)$ be a graph. The function $a : V \to \mathbb{N}$ defined by

$$a(v) := |\{e \in E : v \in e\}|$$

is the degree function of G

- The degree function is denoted by d_G.

Degree function

- The degree function of the complete graph $K_n = (V, E)$ is $n - 1$, for each vertex in V.
- The degree function of the complete bipartite graph $K_{n,m} = (X \cup Y, E)$ is m for vertex in X and n for vertex in Y.

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Graphs setting
Definitions

Definition (Factor)

Let $G = (V, E)$ be a graph and $a : V \to \mathbb{N}$.

- A a-factor of G is a subgraph (V, F) of G such that a is the degree function of (V, F).

Remark

A subgraph (V, F) is a $a-$factor of (V, E) if and only if $(V, E \setminus F)$ is a $(d_G - a)-$factor of (V, E).

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- Tableaux $\rightarrow$ bipartite graph $G = (X \cup Y, E)$.
  - Rows $\rightarrow X$; Columns $\rightarrow Y$.
  - Allowed Cells $\rightarrow E$:
- Input: A function $a : X \cup Y \rightarrow \mathbb{N}$.
- Output: A a–factor of G.

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- Output: A $a-$factor of G.

Graph setting
Three or more colors

- Tableaux $\rightarrow$ bipartite graph $G = (X \cup Y, E)$.
  - Rows $\rightarrow X$; Columns $\rightarrow Y$.
  - Allowed Cells $\rightarrow E$
- Input: A set A of three or more functions.
- Output: A partition $\mathcal{P} = \{F_a : a \in A\}$ of E.

  $(X \cup Y, F_a)$ is a a–factor of G, for each $a \in A$.

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Decomposition of graphs in factors

- Input: A graph $G = (V, E)$ and a set A of functions.
- Output: A partition $\mathcal{P} = \{F_a : a \in A\}$ of E, $\forall a \in A, (V, F_a)$ is a $a-$factor of G.

When it is possible G is called $A-$decomposable.

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Decomposition
The computational problem

- $p-$decomposition problem.

  Input: A graph $G = (V, E)$ and a set of functions A with $|A| = p$.

  Question: Does G have a $A-$decomposition?.

Decomposition
Two parts

Definition (Factors)

Let $G = (V, E)$ be a graph and $a, b : V \to \mathbb{N}$.

- A (a, b)-factor of G is a subgraph (V, F) of G such that $a \leq d_G \leq b$, where d_G is the degree function of (V, F).

Two parts: (a, b)-factors

Known results

- Tutte (1970): NSC for perfect matching and for $a-$factors.
- Lovász (1970): NSC for $(a, b)-$factors.
- Anstee (1985, 1990, 1994): S conditions and a $O(n^3)$ algorithm for $(a, b)-$factors.
- Heinrich, Hell, Kirkpatrick, Liu. (1990). *Simpler* conditions when $a < b$ or $G_{a=b}$ bipartite.
- Correa, M. (2006) *new* NSC for $(a, b)-$factors.

Three or more parts

Polynomial

- Guiñez, M. (2007), Any G and $|A| \geq 3$ such that at most two function assign to v a nonzero value.
- Kundu (1970), Kleitman, Koren and Li (1973), Guiñez, M. (2007), G a complete graph, $|A| = 3$ and at least one function a has span 1.
- Guiñez, M. (2007), G acyclic (forest) and $|A| \geq 3$.

Three or more colors

NP-completeness

- Dürr, Marek (2001), Bipartite graphs and $|A| = 3$.
- Guiñez, M. (2007), $|A| \geq 3$ and G a $|A|-$regular graph.
- Dürr, Marek (2001), $|A| = 4$ and G a complete or a complete bipartite graph.
- Open for G complete or complete bipartite and $|A| = 3$.

Scaled factors
Known scaled factors

- Lovász (1970): If $G = (V, E)$ has maximum degree k and $k_1 + k_2 = k + 1$, then E can be partitioned into a $(0, k_1)-$factor and a $(0, k_2)-$-factor.
- Tutte (1978):If $G = (V, E)$ is k-regular graph and $0 \leq r < k$, then G has a $(r, r + 1)-$factor.
- Gupta (1978):If $G = (V, E)$ has minimum degree k and $k_1 + k_2 = k - 1 \geq 1$, then E can be partitioned into a $(k_1, d_G)-$ factor and a $(k_2, d_G)-$factor.
- Thomassen (1980):If $G = (V, E)$ is such that $\deg_E(v) \in \{k, k + 1\}$ and $0 \leq r < k$, then G has a $(r, r + 1)-$factor.

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λ-factors
Definition (Correa, M)

- Let $\lambda \in (0,1)$ be. A λ-factor of $G=(V,E)$ is a subgraph (V,F) of G such that

$$\lceil \lambda \deg_E(v) \rceil - 1 \leq \deg_F(v) \leq \lfloor \lambda \deg_E(v) \rfloor + 1$$

- , that is a $(\lceil \lambda \deg_E(v) \rceil - 1, \lfloor \lambda \deg_E(v) \rfloor + 1)$-factor.

λ-factor $\equiv$ scaling the degrees of G by λ

The results of Lovász (1970), Tutte (1978), Gupta (1978), Thomassen (1980), can be formulated as the existence of a $\lambda-$factor.

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$\lambda-$factors

Results

- Theorem A: If G bipartite and $\lambda \in [0,1]$, then there exists F such that
  $\lfloor \lambda \deg_E(v) \rfloor \leq \deg_F(v) \leq \lceil \lambda \deg_E(v) \rceil$
  Hoffman (1956), using techniques of network flows.

- Theorem B: $\lambda-$factors always exists.
  Kano, Saito (1983), using the NSC of Lovasz.

  Correa, M. (2006) Direct proof of Theorem B, using alternanting paths.

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Weighted factors.

Definition

Definition (weighted factors)

Let $G = (V, E)$ be a graph, let $w : E \to [0, \infty)$ and let $a, b : V \to \mathbb{N}$. A $w-$weighted $(a, b)-$factor of G is a subset $F \subseteq E$ such that

$$a(v) \leq w(\delta_v \cap F) \leq b(v), \forall v \in V,$$

where $x(A) = \sum_{e \in A} x_e$ and $\delta_v = \{uv : uv \in E\}$.

Weighted λ-factors.

Definition

Definition

Let $G = (V, E)$ be a graph, let $w : E \to [0, \infty)$ and let $\lambda \in (0, 1)$. A $w-$weighted $\lambda-$factor of G is a subset $F \subseteq E$ such that

$$|w(\delta_v \cap F) - \lambda w(\delta_v)| \leq \max\{w_e : e \in \delta_v\} \quad \forall v \in V.$$

A $w-$weighted $\lambda-$factor always exists.
Correa, M. (2006)

Theorem C: Let $G = (V, E)$ be a graph and $l \leq \mathbf{0} \leq u : E \to \mathbb{R}$. Then, $\exists x \in \prod_{e \in E}\{l_e, u_e\}$, $\forall v \in V$

$$|x(\delta_v)| \leq \max\{u_e - l_e : e \in \delta_v\}$$

Correa, M. (2006).

Theorem D: Weighted $\lambda-$factors always exists.

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Theorem D: Weighted $\lambda-$factors always exists.

Decomposition

in several $\lambda-$factors

- Let $G = (V, E)$ be a graph and let $\lambda_1, \ldots, \lambda_k \in (0, 1)$ be such that $\sum_{i=1}^{k} \lambda_i = 1$.
- Can E be partitioned into $F_1, \ldots, F_k$ such that for all $v \in V$: $|\deg_{F_i}(v) - \lambda_i \deg_E(v)| \leq 1$?
- Combining $\lambda-$factor Theorem with Correa and Goemans' previous ideas, we show $|\deg_{F_i}(v) - \lambda_i \deg_E(v)| < 3$.

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Feasibility

A necessary condition

- G is $A-$feasible if for each $S \subseteq A$, G has a a_S-factor, where $a_s = \sum_{a \in S} a$.
- In particular, a_A must be the degree function of G.
- Feasibility can be test in polynomial time.

Feasibility

A necessary condition

- G is $A-$feasible if for each $S \subseteq A$, G has a a_S-factor, where $a_s = \sum_{a \in S} a$.
- In particular, a_A must be the degree function of G.
- Feasibility can be test in polynomial time.

Feasibility
in the complete graph

$$K_n = (V, E)$$

- Each function $a \in A$ must be a graphical function, i.e., there exists a graph G on V such that $d_G = a$.
- Can be tested in linear time: Erdös, Gallai (1960).

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Feasibility
is not sufficient

- $G = K_5$, $|A| = 3$ and A feasible but G has not $A-$decomposition.

$$A = \begin{bmatrix} 1 & 0 & 0 & 2 & 1 \\ 3 & 2 & 2 & 1 & 0 \\ 0 & 2 & 2 & 1 & 3 \end{bmatrix}$$

Feasibility and Roomy

Using feasibility

- e is forced if there is $a \in A$ such that $G - e$ has no a $a-$factor.
- A is roomy for G if no edge of e is forced.
- $p-$decomposition problem $\equiv$ its restriction to roomy matrices, in any class $\mathcal{C}$ closed under subgraphs

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- Complete graphs are not closed under subgraphs.
- Complete bipartite graphs are not closed under subgraphs.

  $\longrightarrow$ Known complexity results do not given information when A is roomy.
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  When G is complete or complete bipartite and A is roomy.

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Related open problems

Kundu's conjecture, (1973)

Definition (tree-functions)

A graphical function $a : V \to \mathbb{N}$ is a tree-function if there is a tree T on V such that $a = d_T$.

- Kundu's conjecture:

  When $|A| = n$, each $a \in A$ is a tree-function and a_A is $2n - 1$,

  then K_{2n} have a $A-$decomposition?.
- In this case A is roomy!.

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Kundu's conjecture

Partial results

- Kundu (1974), $|A| = 3$ and two functions are tree-functions.
- Kundu (1975), $|A| = 4$, three functions are tree-functions

  $+$ fourth is upper bounded by $2n - 5$.
- Kleitman, Koren, Li (1977), $|A| = 3$ and two of them are forest-functions.
- M., Zamora (2006), A is obtained by a cyclic rotation of a function a.

  Uses graceful labeling of some trees (caterpillar).

  Related to another conjecture of Rosa 1960...

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