A General Church-Rosser Theorem

Corrections

p1. replace 'contraction' by 'contractum' in line 2 from bottom.

p2. replace lines 5-9 by:

1.3 When is a contraction operation consistent in the sense that two definitionally equal expressions that are not redexes or variables must have the same form and their corresponding parts must be definitionally equal (after matching up the bound variables)? The

p2. replace line 17 by:

PROOF. If an expression a, that is neither a redex nor a variable, reduces to an expression c, then a and c must have the same form and each part of a must reduce to the corresponding part of c (after matching up the bound variables). Hence consistency

p3. replace lines 18, 19 by:

contractum a, $a_1 > b_1, \ldots, a_m > b_m$ implies that $a > F(b_1, \ldots, b_m)^*$ where b^* is the contractum of b if b is a redex and b itself otherwise.

p4. replace lines 7-13 by:

for $i = 1, \ldots, m$. Let $d = F(d_1, \ldots, d_m)^*$. If $F(b_1, \ldots, b_m) = b$ then $b > d$ as $F(d_1, \ldots, d_m) \doteq d$. If $F(b_1, \ldots, b_m) \neq b$ then $F(b_1, \ldots, b_m)$ is a redex with contractum b and hence by coherence $b > d$. So $b > d$ and $c > d$ follows similarily.

p4. insert before 'variables' in line 15 'pairwise distinct'.

p5. insert before the theorem of 2.1.:

A contraction operation is strongly coherent if, for every redex $F(a_1, \ldots, a_m)$ with contractum a, $a_1 > b_1, \ldots, a_m > b_m$ implies that $F(b_1, \ldots, b_m)$ is a redex with contractum b such that $a > b$. Notice that every strongly coherent contraction operation is coherent.

- over -

p5. insert 'strongly' before 'coherent' in the theorem of 2.1. and also 'strong'
before 'coherence' in line 15.

p5. replace $a_1[a_2/x]$ by $((\lambda x)a_1)(a_2)$ in line 4 from the bottom.

p6. replace lines 1-6 by:

coherent relative to >'. To see this, note that a formal system for >' can be set up using the axioms and rules of the formal system for > obtained from β-contraction and the additional <u>η-rules</u> $\dfrac{a >' b}{(\lambda x)(a(x)) >' b}$ if x is not free in a. Now if $((\lambda x)a)(b)$ is a β-redex and $(\lambda x)a >' a'$; $b >' b_1$ we must show that $a[b/x] >' a'(b_1)^*$. Without loss we may assume that a proof of $(\lambda x)a >' a'$ ends with an inference having one of the forms $\dfrac{a >' a_1}{(\lambda x)a >' a'}$ where $(\lambda x)a_1 \doteq a'$ or $\dfrac{a_1 >' a'}{(\lambda x)a >' a'}$ where $a = a_1(x)$ and x is not free in a_1. In the first case we may argue as in the proof of strong coherence for β-contraction. In the second case $a[b/x]$ is $a_1(b)$ and as $a_1 >' a'$, $b >' b_1$ and $a'(b_1) \doteq a'(b_1)^*$ we get $a[b/x] >' a'(b_1)^*$ as desired.

p7. erase 'by' in line 11.

p8. insert 'strongly' before 'coherent' in line 2 from bottom.

p10 ~~p9~~. in line 3 replace 'constraction' by 'contraction'.

p10 ~~p9~~. replace 'contracti' by 'contracta' in lines 3, 9, 16 and also line 7 of p5.

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