with $\omega \in \Omega_p$ and $d\psi = k - 1$. Since $d\psi = \sum d(\psi i)$ $i = 1, \cdots, p$ we must have $d(\psi i) > 0$ for some i. Without loss of generality, assume that $d(\psi p) > 0$.

Finding $\bar{P}$ is equivalent to finding the minimal solution of the system of equations

$$X = XP \tag{17.1}$$

in subsets $X = (X_1, \cdots, X_n)$ of A_0. The equation (17.1) may be rewritten as

$$X = XR + (X\psi_1, \cdots, X\psi_p)\omega M, \tag{17.2}$$

where $\psi_i = \psi i$.

We now consider the system of $n + 1$ equations with $n + 1$ unknowns as follows:

$$\begin{aligned} X &= XR + (X\psi_1, \cdots, X\psi_{p-1}, X_{n+1})\omega M \\ X_{n+1} &= X\psi_p, \end{aligned} \tag{17.3}$$

where $X = (X_1, \cdots, X_n)$. It is clear that if $X = (X_1, \cdots, X_n)$ is the minimal solution of (17.2), then $(X_1, \cdots, X_n, X_{n+1})$ with $X_{n+1} = X\psi_p$ is the minimal solution of (17.3).

The right-hand side of (17.3) yields a polynomial $Q: [n + 1] \to [n + 1]$ whose constituents are: (1°) compositions

$$I \xrightarrow{\gamma} [n] \xrightarrow{f} [n + 1],$$

where γ is a constituent of P different from ϕ and f is an inclusion; (2°) morphisms $\tau: I \to [n + 1]$ satisfying $0 < d\tau < k$. By the above, $\bar{Q}_i = \bar{P}_i$ for $i = 1, \cdots, n$.

An iteration of the above procedure yields the conclusion of Proposition 2.

18. PROOF OF THEOREM 2

Let

$$Q_i = \{\phi \mid \phi: I \to \emptyset, \quad i \in \zeta_A\phi\}.$$

Then $Q_i \subset A_0$ and we wish to show that $Q = \bar{P}$ where $Q = (Q_1, \cdots, Q_n)$.

We first show that $\bar{P} \subset Q$. For this, it suffices to show that $QP \subset Q$; i.e., that

$$(Q_1, \cdots, Q_n)\phi \subset Q_i \quad \text{whenever} \quad \phi \in P_i.$$

Let $\phi \in P_i$. Then ϕ is the composition

$$I \xrightarrow{\omega} [p] \xrightarrow{x} [n]$$

with $\omega \in \Omega_p$, x is a mapping in S_0 and

$$i \in (x1, \cdots, xp)\omega.$$

Thus we must show that

$$(Q_{x1}, \cdots, Q_{xp})\omega \subset Q_i. \tag{18.1}$$

Let then

$$\psi_j \in Q_{xj} \qquad j = 1, \cdots, p$$

$$\psi = (\psi_1, \cdots, \psi_p): [p] \to \emptyset.$$

Then $xj \in \zeta_A\psi_j$ for $j = 1, \cdots, p$, and

$$i \in (x1, \cdots, xp)\omega \subset (\zeta_A\psi_1, \cdots, \zeta_A\psi_p)\omega = \zeta_A(\psi\omega).$$

Consequently, $\psi\omega \in Q_i$, so that (18.1) holds.

To show the opposite inclusion, we must prove that

(18.2) if $\phi: I \to \emptyset$ and $i \in \zeta_A\phi$, then $\phi \in \bar{P}_i$.

This will be done by induction with respect to the degree of ϕ. First let $d\phi = 1$. Then $\phi \in \Omega_0$ and $i \in \phi_A$. Then the composition

$$I \xrightarrow{\phi} \emptyset \to [n]$$

is in P_i so that $\phi = \phi_{A_0} \in \emptyset P_i$. Thus $\phi \in \bar{P}_i$ as required.

Now assume $d\phi = k > 1$. Let

$$I \xrightarrow{\omega} [p] \xrightarrow{\psi} [n]$$

be the factorization of ϕ with $\omega \in \Omega_p$ and $d\psi = k - 1 > 0$. Then $p > 0$. We have

$$i \in \zeta_A\phi = \zeta_A\psi\omega = \zeta_A(\psi1, \cdots, \psi p)\omega = (\zeta_A\psi_1, \cdots, \zeta_A\psi p)\omega.$$

Let then

$$x: [p] \to [n] = A \quad \text{in} \quad S_0$$

be such that

$$xj \in \zeta_A\psi j \quad j = 1, \cdots, p \tag{18.3}$$

$$i \in (x_1, \cdots, xp)\omega. \tag{18.4}$$