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We want to obtain information on the invariant measure (ECM93, GC95)  
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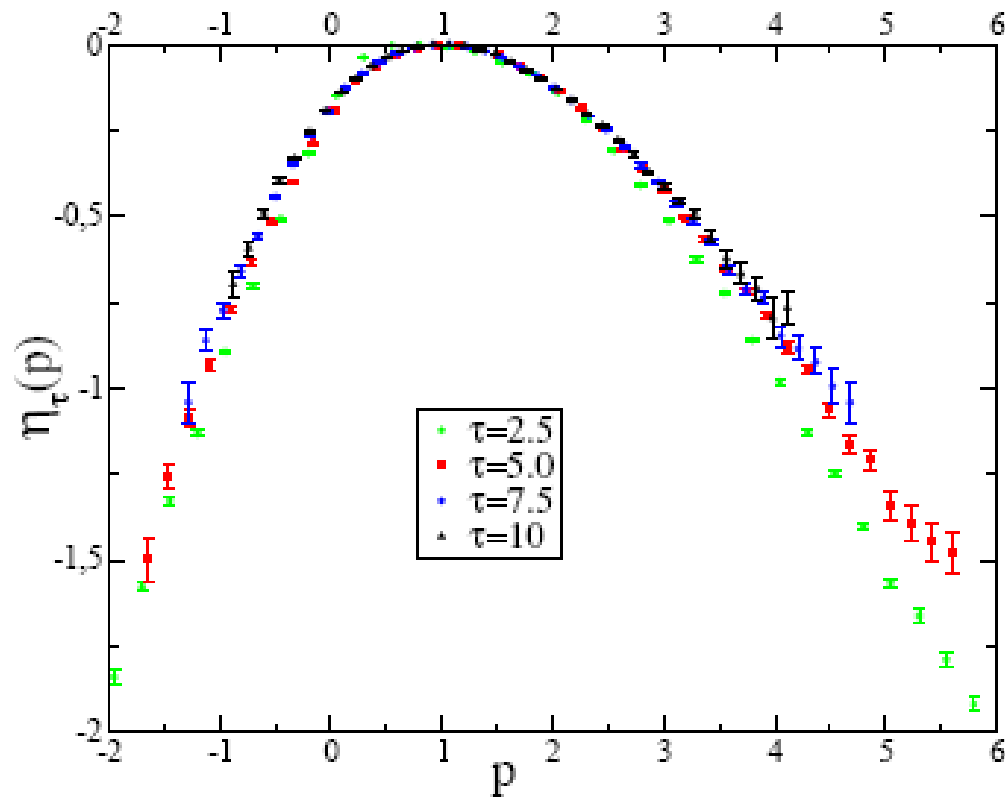
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## 4. Is it possible to measure the phase space contraction rate?

One should distinguish between the experimental system and the model.

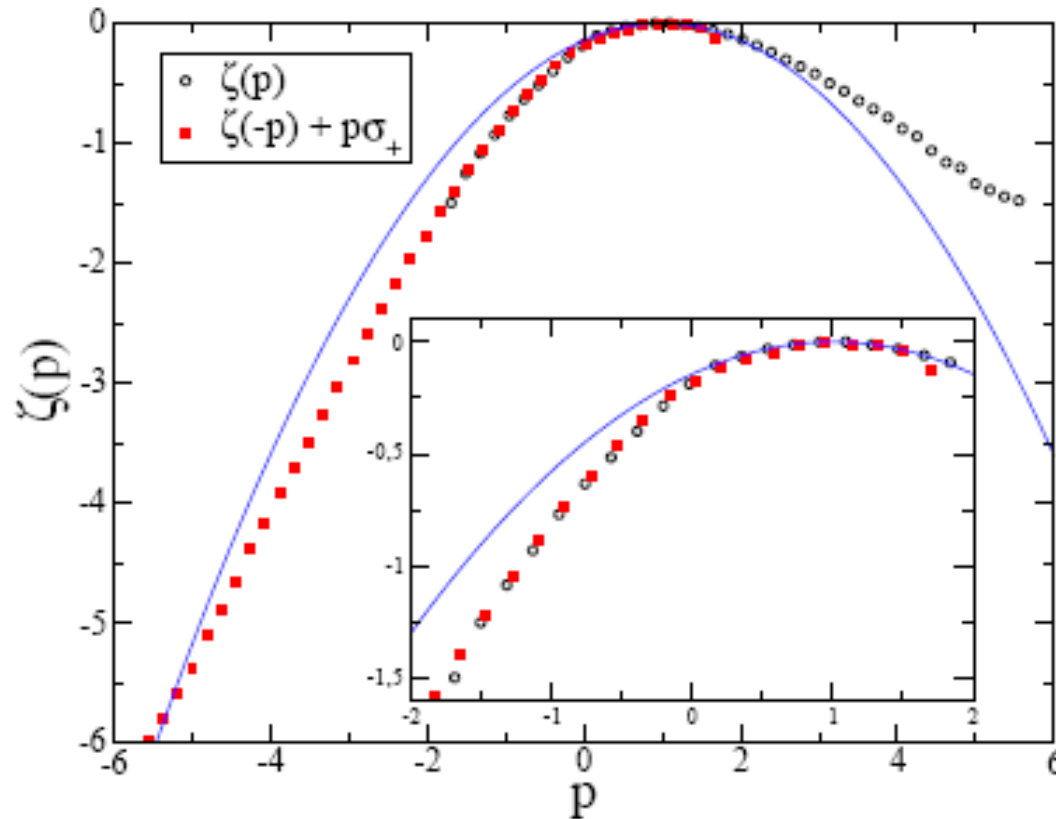
If different models are equivalent, then one can try to identify the PSCR of one of them with a measurable quantity.

If not, it is a disaster (the result of the experiment depends on all the details of the experimental setup).



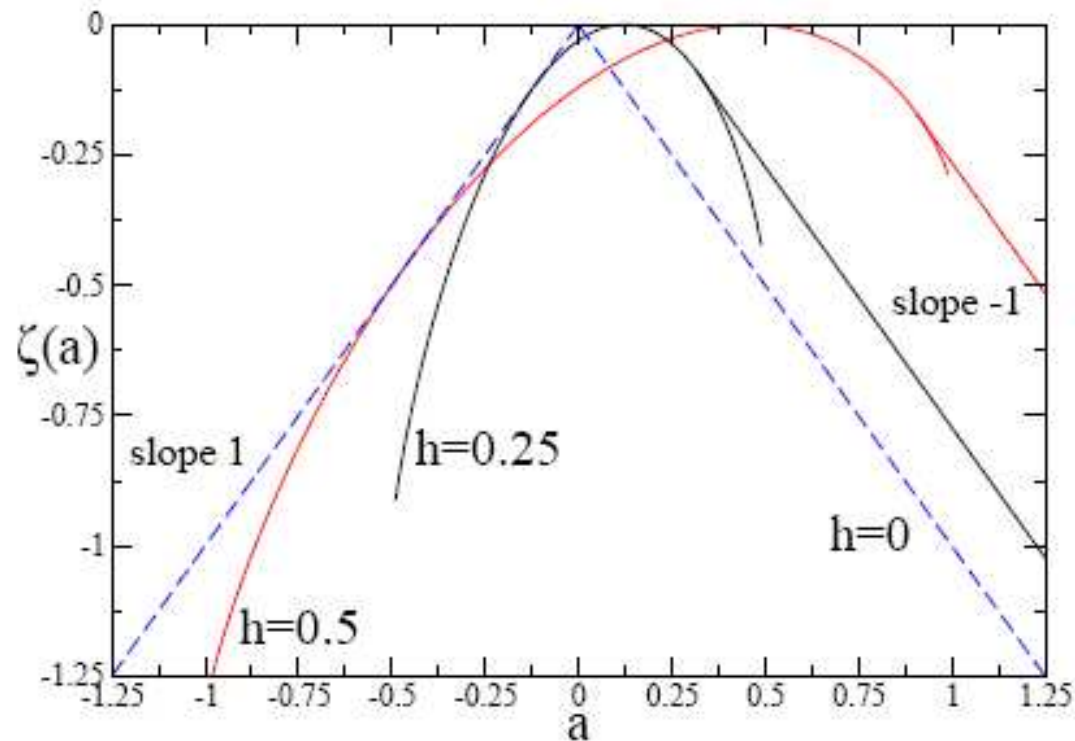
$$\tau \gg \tau_0 = 0.1$$

1. Plot the large deviation function
2. For asymmetric distribution shift the maximum in  $p=1$
3. Search for an interval of  $p$  where it does not depend on  $\tau$
4. The interval must contain the origin
  - $\tau$  not too large
  - small forcing
  - small system size



$\tau = 5.0$

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  - $\tau$  not too large
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  - small system size
5. Observable difference w.r.t. a Gaussian around  $p=0$  (otherwise linear response)
  - large forcing
  - low temperature
6. Check the slope (linearity might be due to the small interval)



### In presence of singularities:

- Non-convex LDF
- Large tails (e.g. exponentials)

### FR is for smooth and bounded (maps)

- Fix a set of timing events
  - particle collision
  - $V(t) = V_0$
  - $W(t) = W_0$
- Integrate between two timing events
- Check consistency by recomputing the original LDF

## Equivalence between reversible – irreversible thermostats

$$m\dot{q}_i = p_i \ ,$$
$$\dot{p}_i = -\partial_{q_i} V(q) + F_i(q) - \alpha(p, q)p_i \ .$$

Equivalent irreversible model:

$$\nu(E) = \langle \alpha(p, q) \rangle_E = \int dp dq \mu_E(p, q) \alpha(p, q)$$

Very different fluctuations of  $\alpha$  in the two systems -- *global quantity*  
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Equivalence of *local* observables:

$$\lim_{N, V \rightarrow \infty, \rho = N/V} \langle O_{V_0}(p, q) \rangle_{\nu(E)} = \lim_{N, V \rightarrow \infty, \rho = N/V} \langle O_{V_0}(p, q) \rangle_E$$

Local entropy production rate might satisfy a local fluctuation relation!  
(completely open problem)

- Very important for the interpretation of experiments
- What is the time scale needed to observe reversibility?

If the fluctuation relation does not hold due to irreversibility still the chaotic hypothesis might hold.

It would be very interesting to derive from the SRB measure other measurable relations not explicitly dependent on reversibility.

A very nice historical review: G.Gallavotti, [cond-mat/0606477](#)