Systèmes ``Elastiques'' désordonnés

P. Le Doussal (ENS)

D. Carpentier (ENS Lyon)

R. Chitra (Jussieu)

E. Orignac (ENS)

P. Chauve (Orsay/ENS)

G. Schehr (ENS)

A. Rosso (Geneva)

A. Kolton (Geneva)

S. Lemerle (Orsay)
J. Ferré (Orsay)

T. Klein (Grenoble)
I. Joumard (Grenoble)

J.M. Triscone (Geneva)
P. Paruch (Geneva)
T. Tybell (Trondheim)

General References on DES

- Classical systems :

G. Blatter et al. Rev. Mod. Phys 66 1125 (1994).

T.G. + P. Le Doussal, In ``Spin Glasses and Random Fields'', ed. A.P. Young, World Scientific 1998, cond-mat/9705096.

T. Nattermann and S. Scheidl Adv. Phys. 49 607 (2000)

TG + S. Bhattacharya, In ``High Magnetic Fields'', ed. C. Berthier et al., Springer 2002, cond-mat/0111052.

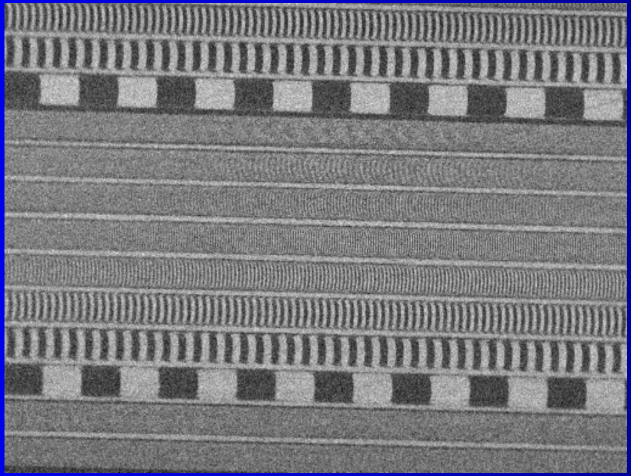
- Quantum systems :

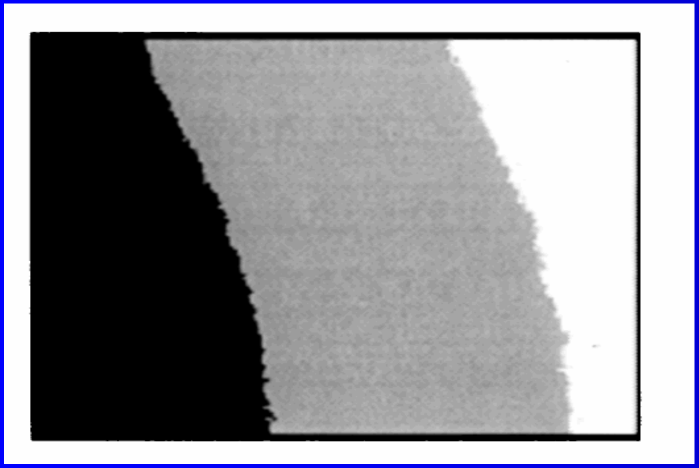
T.G. + E. Orignac In ``Theoretical Methods for Strongly Correlated Electrons'', D. Senechal et al. ed, Springer (2004), cond-mat/0005220.

T.G. In ``Quantum phenomena in mesoscopic systems'' (Varenna school CLI) IOS Press (2003), cond-mat/0403531.

And references therein..

Magnetic domain wall



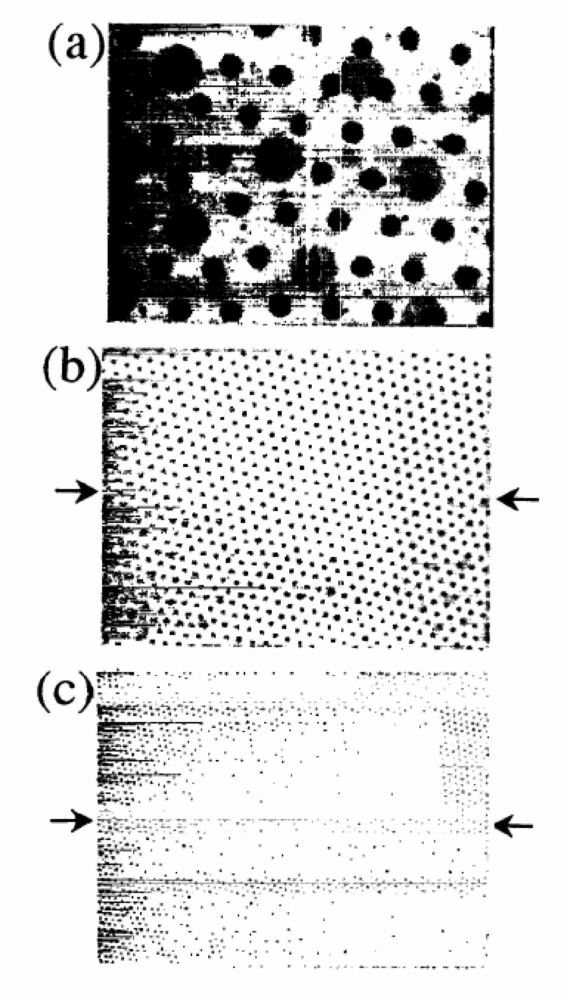


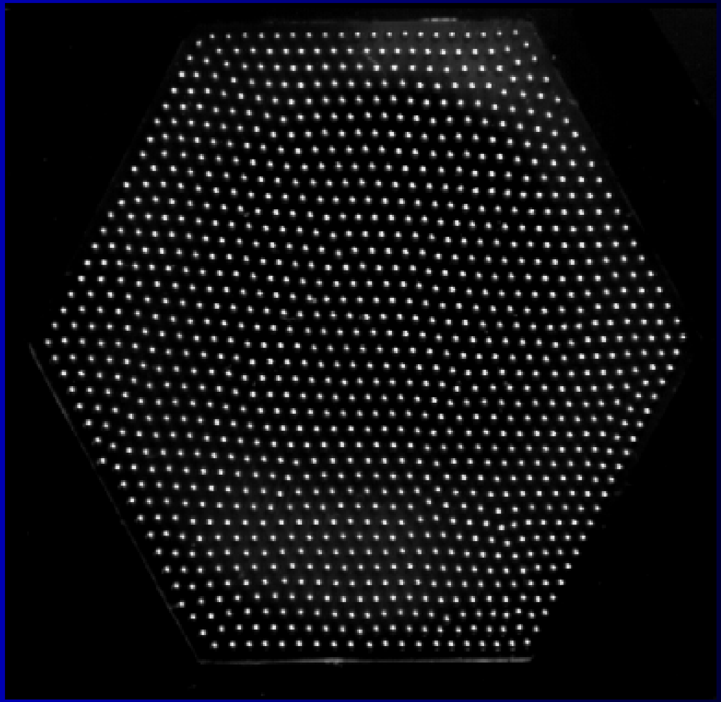
S. Lemerle et al. PRL 80 849 (98)

Other Interfaces

- Contact line [E. Rolley]
- Ferroelectrics [P. Paruch]
- Epitaxial growth

Classical crystals





Charged spheres: M. Saint Jean, GPS (Jussieu), 2000

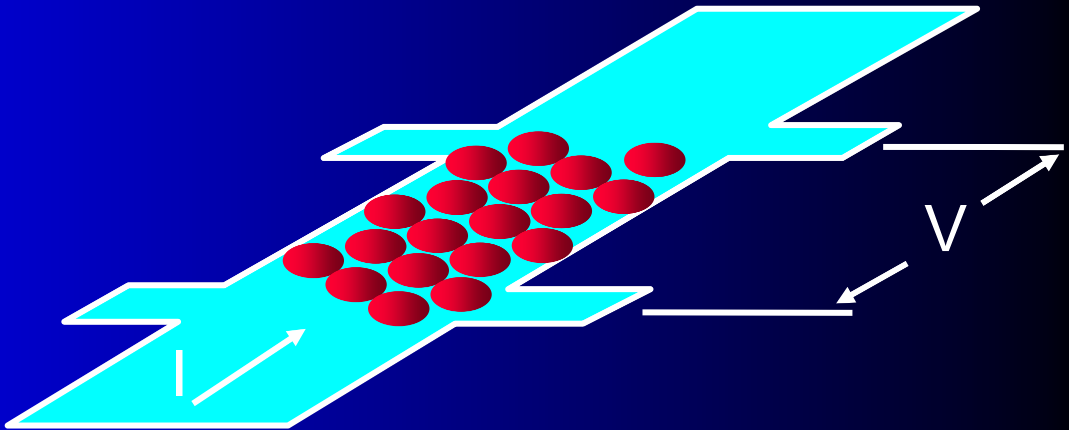
Magnetic Bubbles: R. Seshadri et al.

Other ‘classical’ Crystals

- Vortex Lattice [K. van der Beek]

- Charge density waves

Quantum systems



• Strong repulsion : Wigner crystal

• Quantum fluctuations instead (in addition to) thermal fluctuations

Wigner Crystal

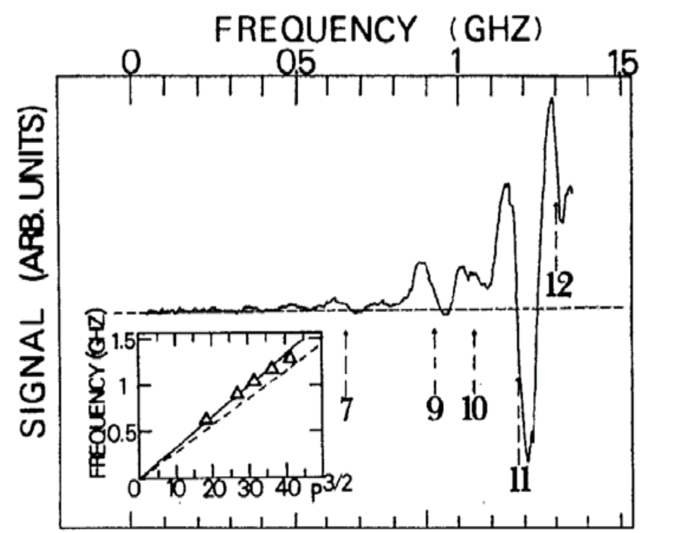


FIG. 1. Absorption spectrum at 28 T and 60 mK for density 0.77×10^{11} cm^{-2} (filling factor $\nu = 1/8.7$, reduced temperature $t = 0.33$) showing successive resonances and their identification as pth spatial harmonics ($q = pq_0$) of the exciting structure. The values of p are chosen for the best alignment with the origin (full line) on the accompanying plot of f_p vs $p^{3/2}$; the dashed line is the zero-order *a priori* calculation of the frequency of the lower hybrid mode of the solid.

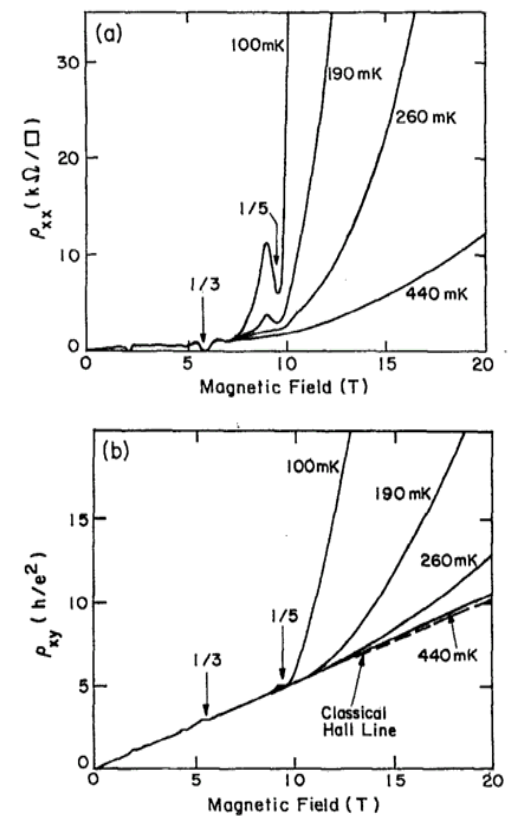


FIG. 1. (a) Diagonal resistivity ρ_{xx} and (b) Hall resistance ρ_{xy} of a low-density ($n = 4.8\times10^{10}$ cm^{-2}) high-mobility ($\mu = 1.7\times10^{6}$ cm^2/V sec) two-dimensional electron system at various temperatures.

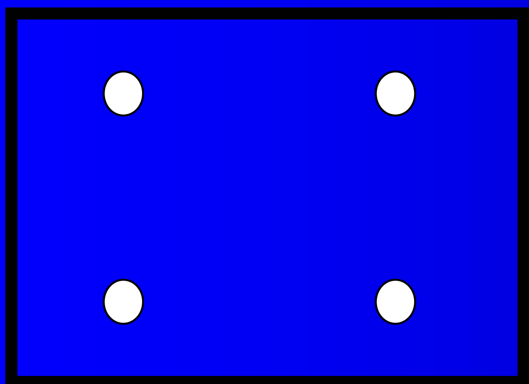
E.Y. Andrei, et al PRL 60 2765 (1988)

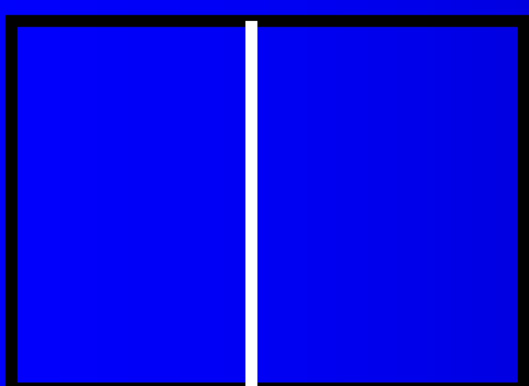
R.L. Willett, et al. PRB 38 R7881 (1989)

Other quantum Crystals

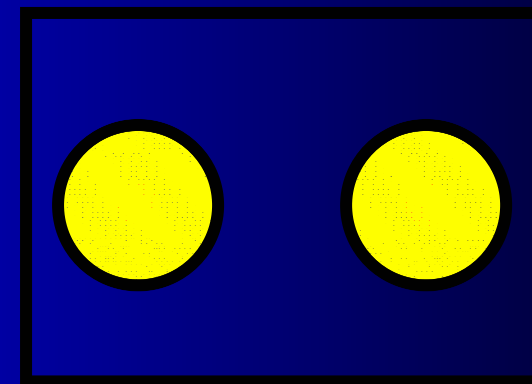
- Spin density waves
- Luttinger liquids (1D interacting electrons)

- Basic Features :

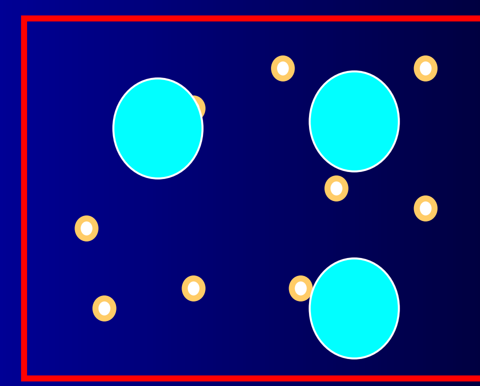




`Elasticity'



(Thermal, quantum) fluctuations



Disorder

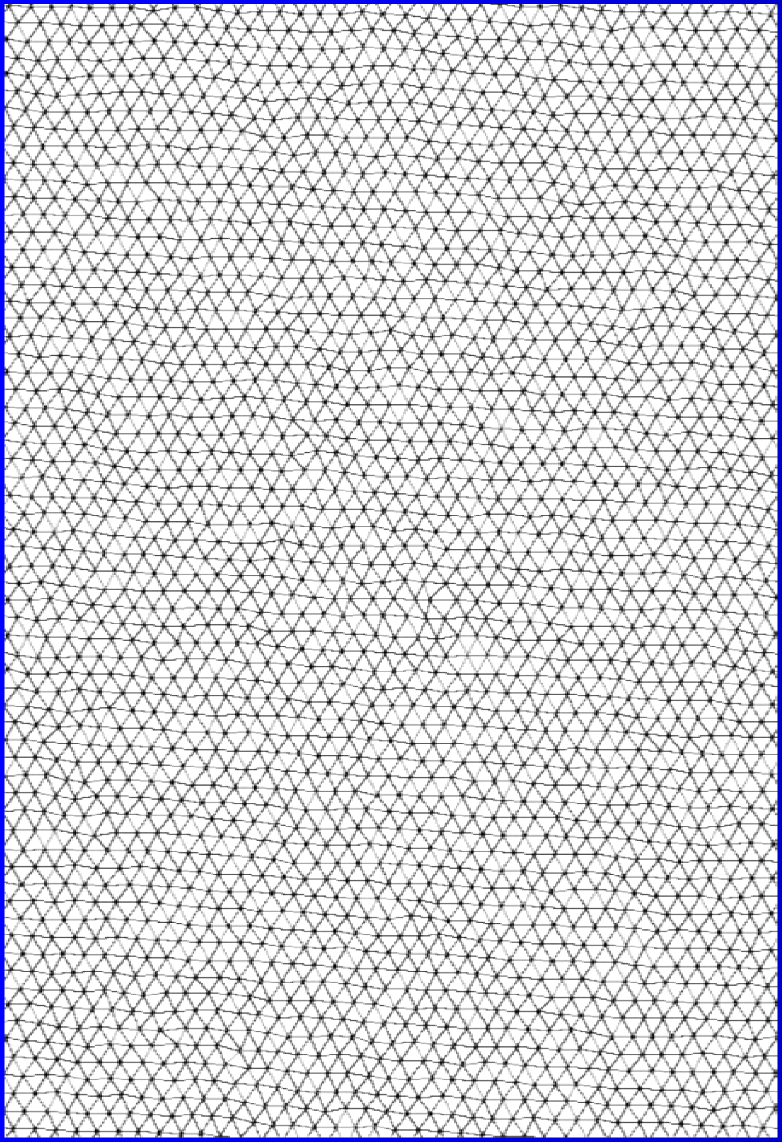
Questions

Competition ``Order'' / ``Disorder''

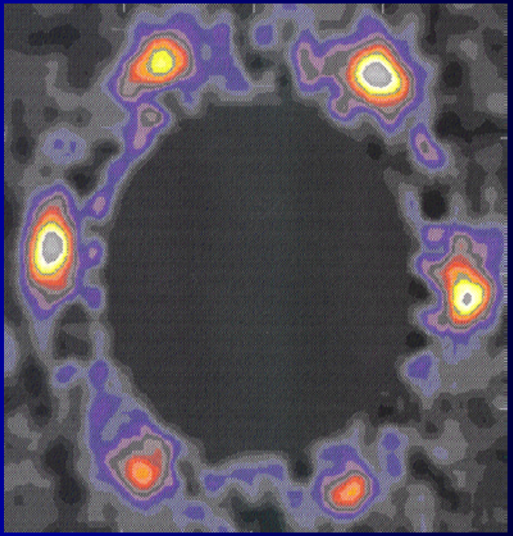
- Melting
- Glassy phases

- Statics
- Dynamics

Statics



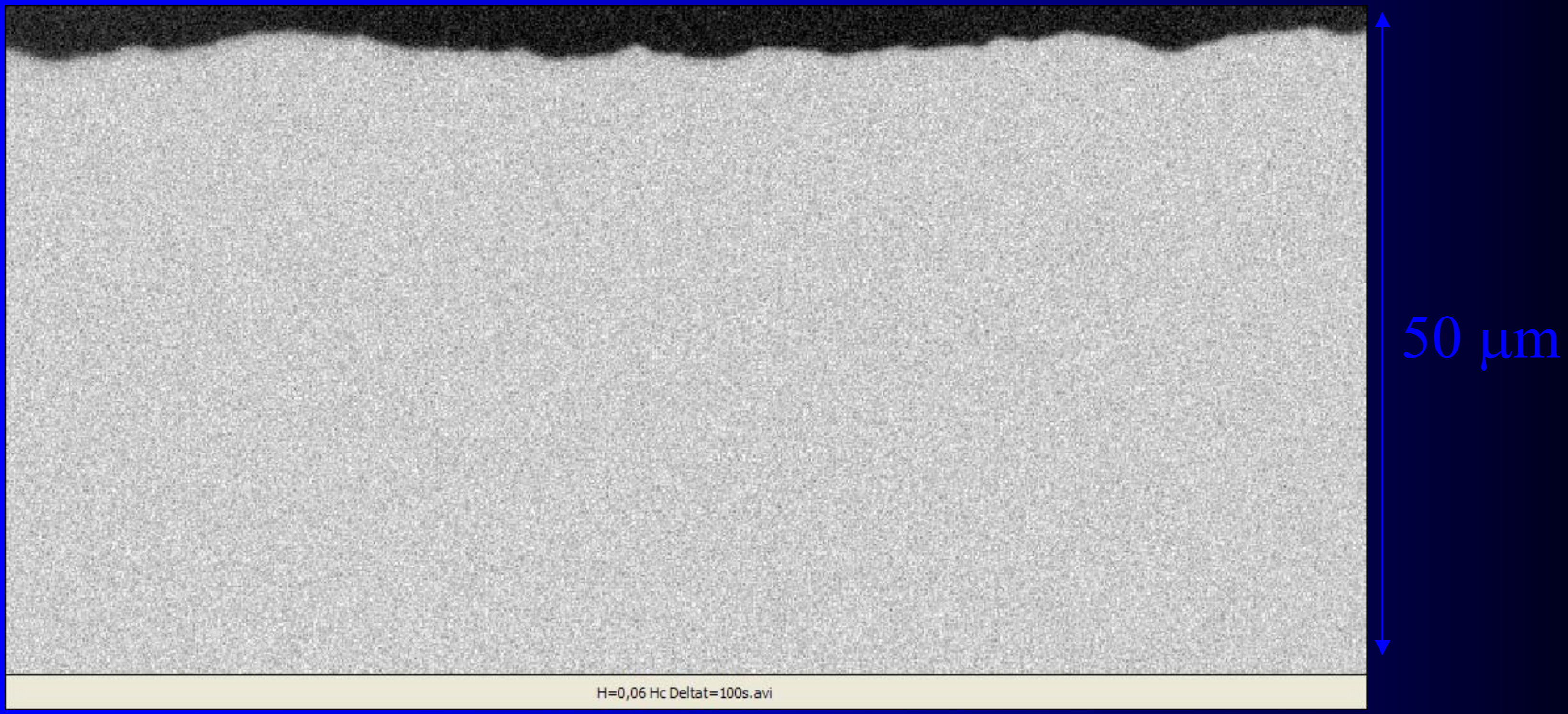
P. Kim PRB 60 R12589 (99)



T. Klein et al. Nature 413, 404 (2001)

Dynamics

+



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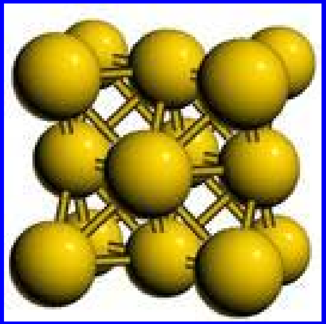
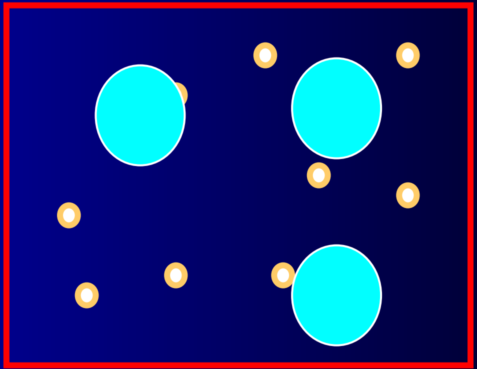
(V. Repain et al. (Orsay))

New type of physics

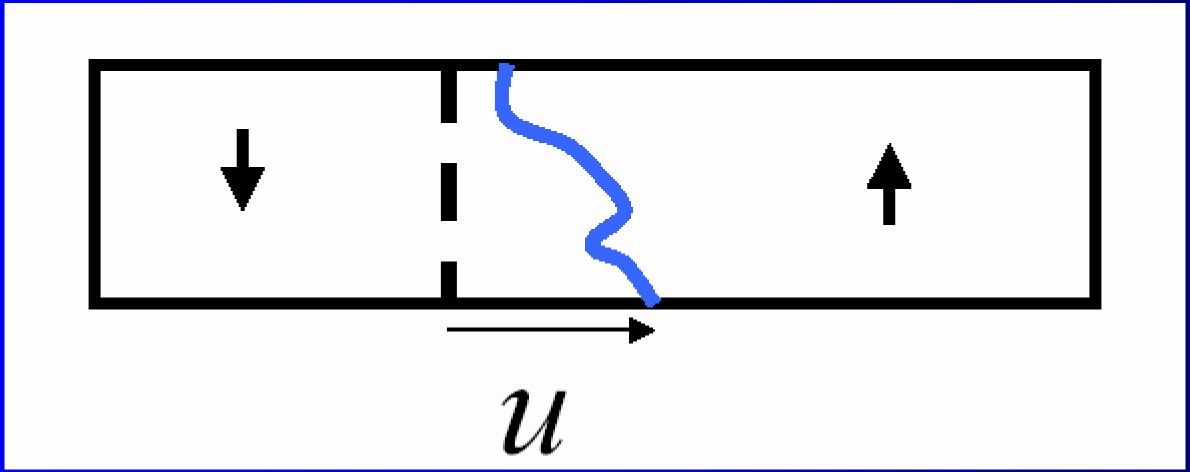
- Very controlled (e.g. magnetic field)
- Can pull on on the system
- Plunged in an external disorder





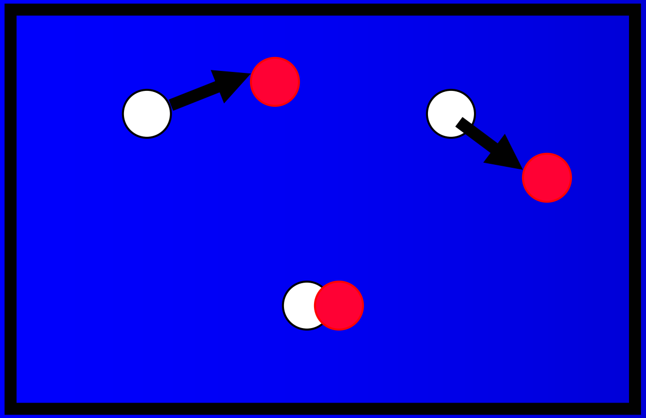
How to model

- Elastic description



$$H = \frac{c}{2}\int dx (\nabla u(x))^2 = \frac{c}{2}\sum_q q^2 u^*(q) u(q)$$

Elastic description of crystals



R^0_i : crystal

u_i : displacements

n=2 d=3 vortices

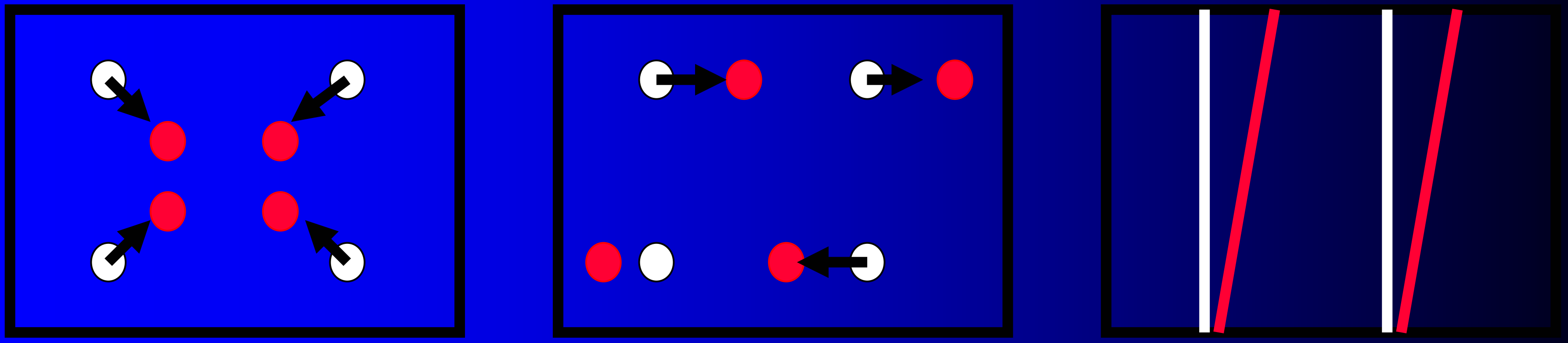
Elastic hamiltonian

$$H = \frac{1}{2}\sum_{\alpha\beta}\int c_{\alpha\beta}(q)u_\alpha(q)u_\beta(-q)dq$$

Simplest elastic hamiltonian : c(q) = c q^2

$$H = \frac{1}{2}\int c(\partial_\alpha u_\beta(x))^2$$

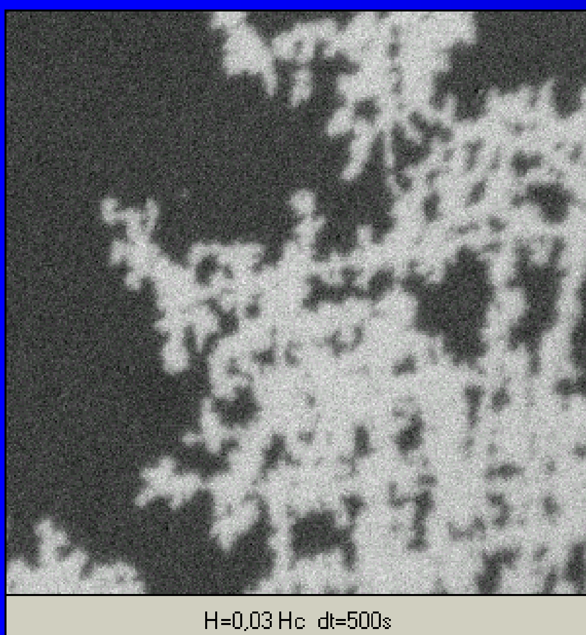
Long range forces ; bulk, shear and tilt



$\partial_\alpha u_\alpha(x,y,z)$ $\partial_y u_x(x,y,z)$ $\partial_z u(x,y,z)$

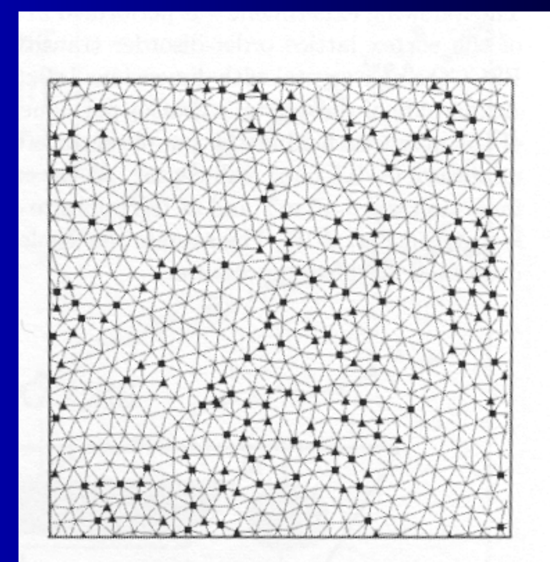
Limitations

- Interfaces (overhangs, bubbles)



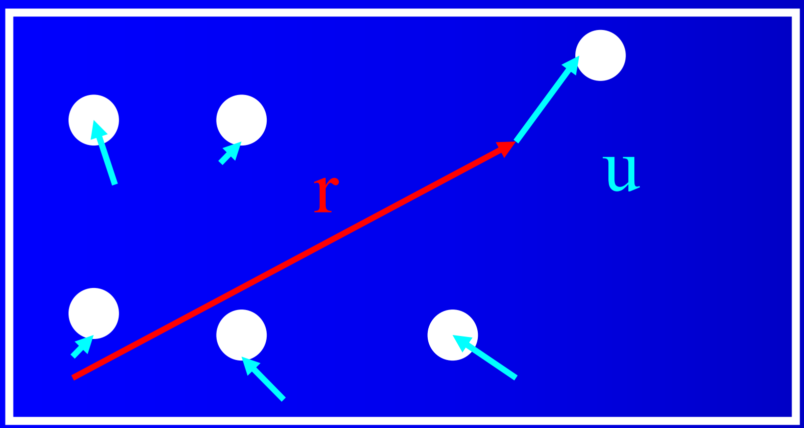
J. P Jamet, V. Repain

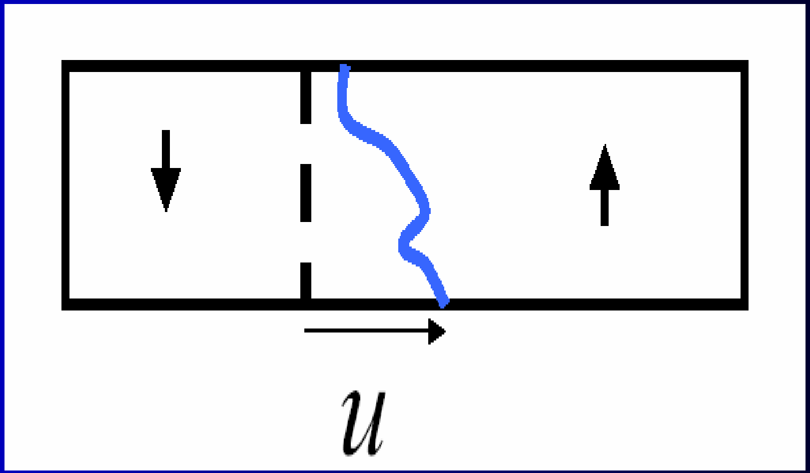
- Periodic dislocations, etc.



M. Marchevsky, J. Aarts, P.H. Kes

What to measure (statics)



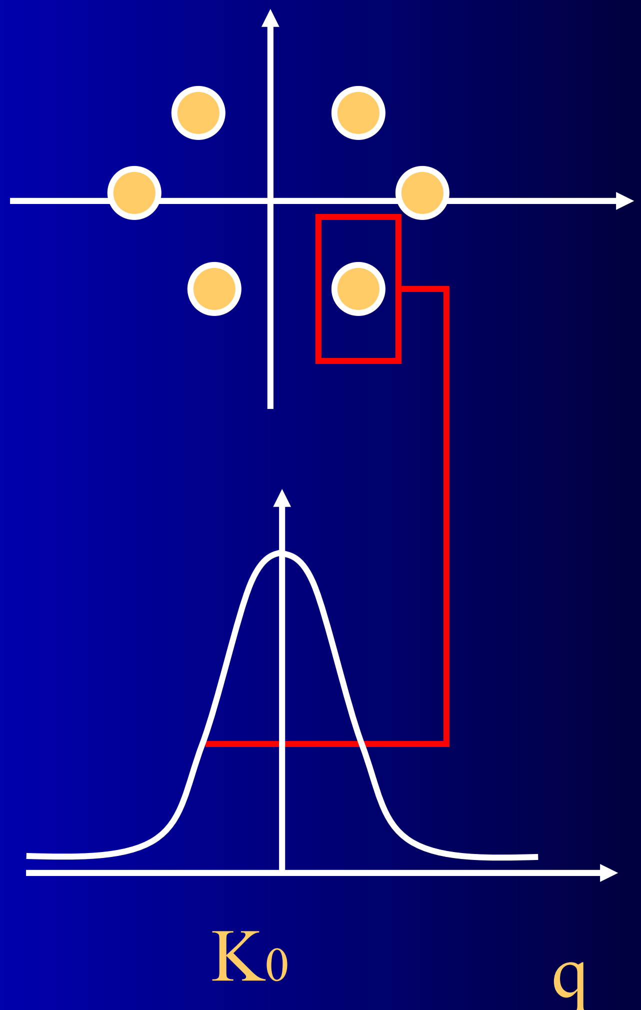


$$B(r) = \overline{\left\langle [u(r) - u(0)]^2 \right\rangle}$$

Positional order

Structure Factor

$$S(q) = \langle \rho(q)\rho(-q) \rangle$$



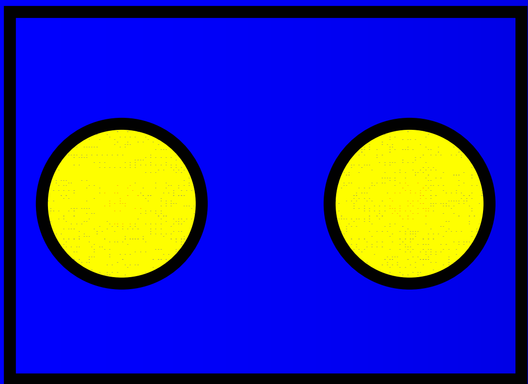
Fourier transform of:

$$C(x) = \overline{\left\langle e^{iK_0 u(r)} e^{-iK_0 u(0)} \right\rangle}$$

Decorations

Neutrons

Thermal fluctuations : Melting

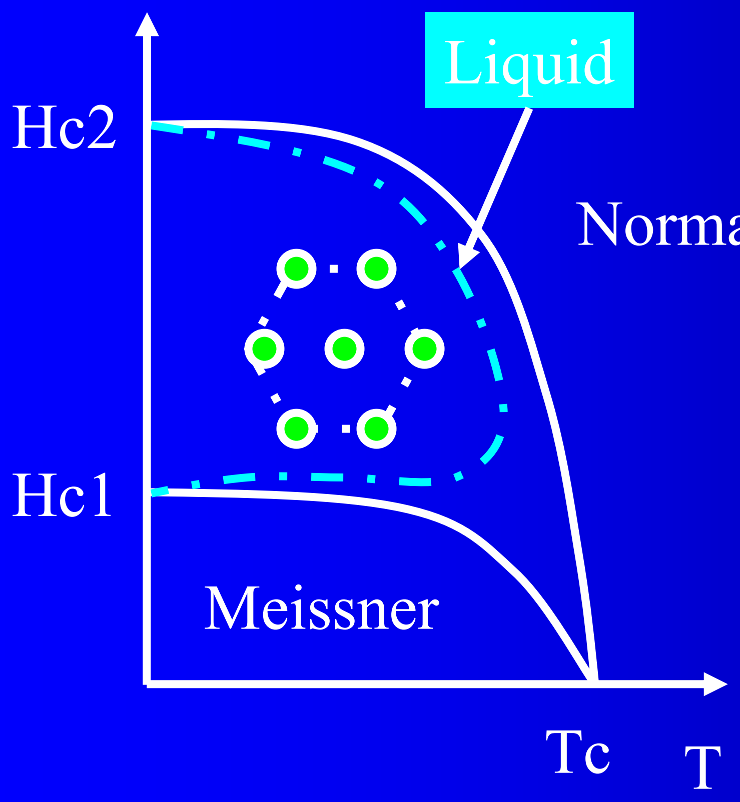


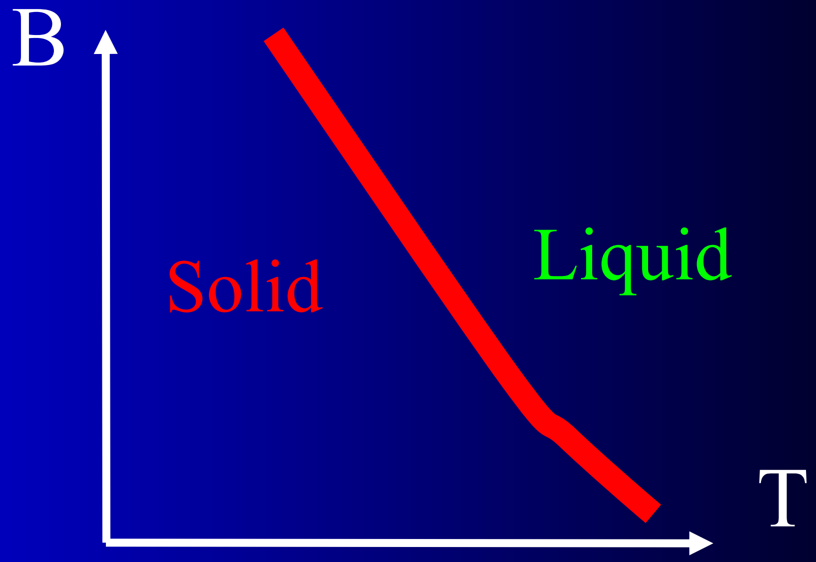
$$\langle u^2 \rangle = l_T^2 \propto \frac{T}{c}$$

Lindemann criterion of melting :

$$\langle u^2 \rangle = l_T^2 = C_L^2 a^2 \qquad C_L \approx 0.1 - 0.2$$

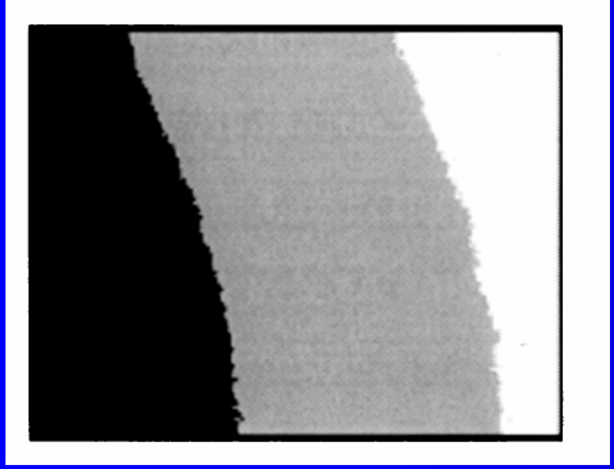
Vortices

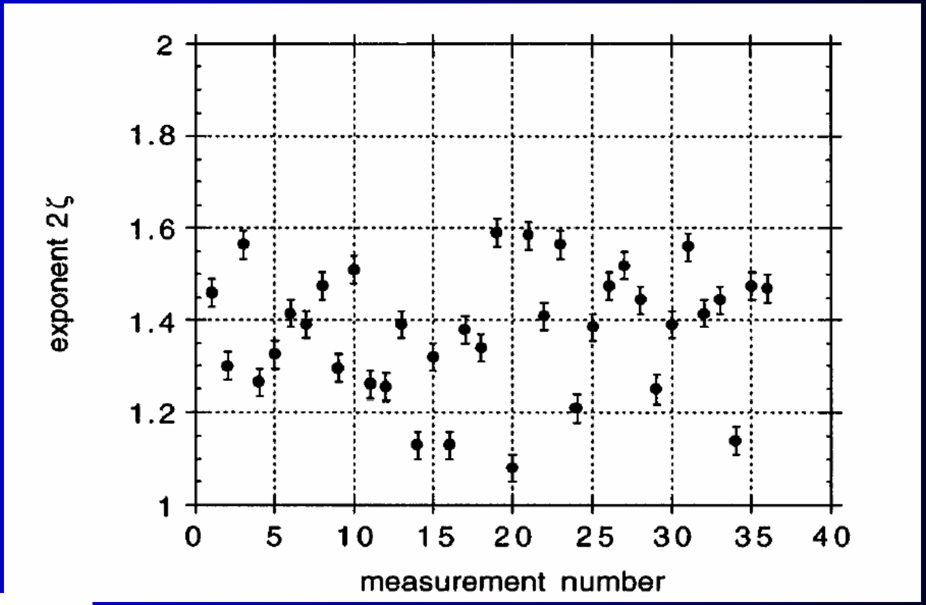


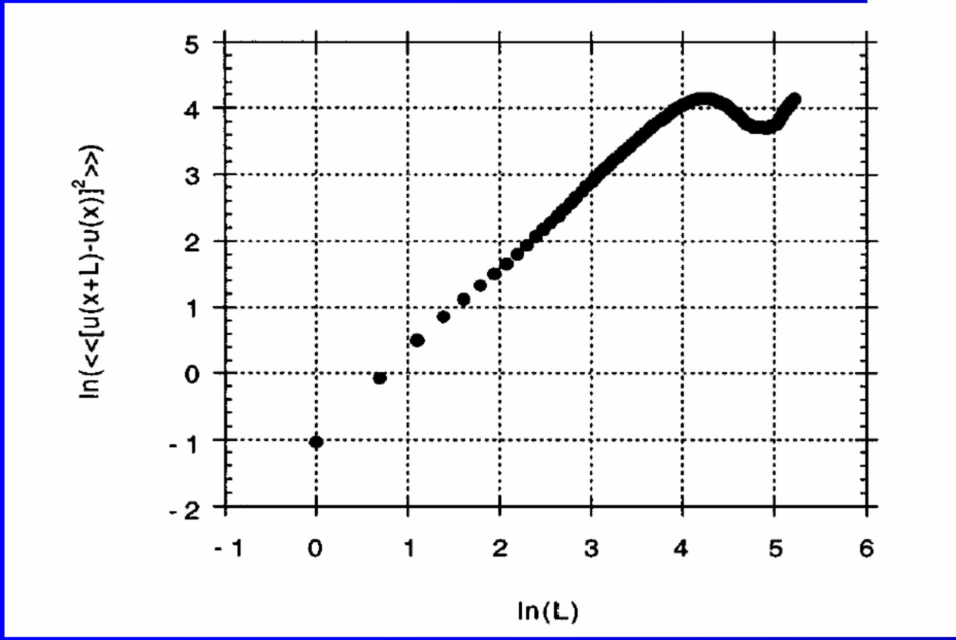


Should we care about disorder?

S. Lemerle et al. PRL 80 849 (98)

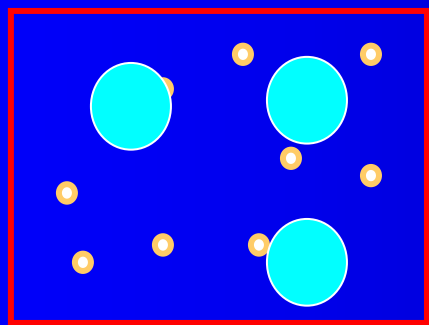






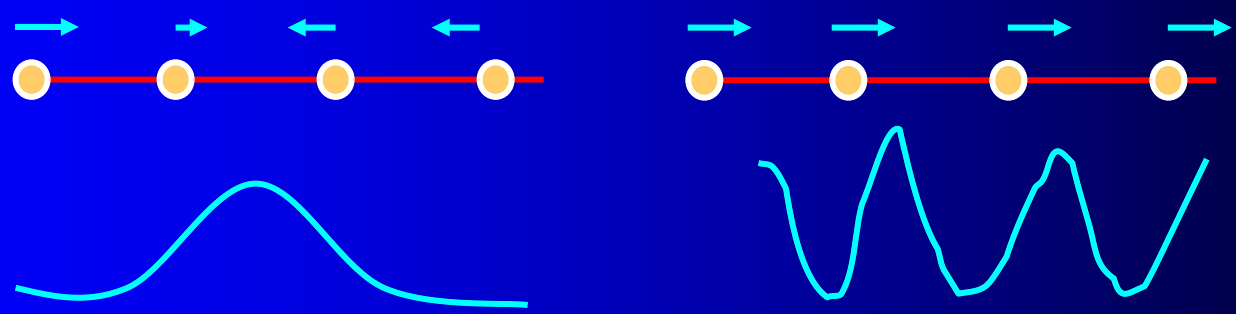
$$u \propto L^{\varsigma}$$

Disorder (point like defects)



$$H = \int V(x)\rho(x)dx$$

$$\rho(x) = \sum_i \delta(x - R_i^0 - u_i)$$



$$\rho(x) = \rho_0 - \rho_0 \nabla u(x) + \rho_0 \sum_K e^{iK(x-u(x))}$$

Loss of translational order (Larkin)

$$u(R_a) \approx a$$

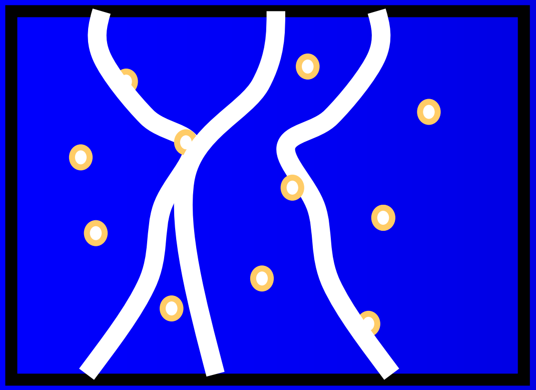
$$H_{el} = \frac{c}{2}\int (\nabla u(r))^2 d^d r \qquad H_{dis} = \int V(r)\rho(r) d^d r$$

$$cR_a^{d-2}a^2 \qquad VR_a^{d/2}\rho_0$$

$$R_a \propto a\left(\frac{c^2 a^d}{V^2 \rho_0^2}\right)^{1/(4-d)}$$

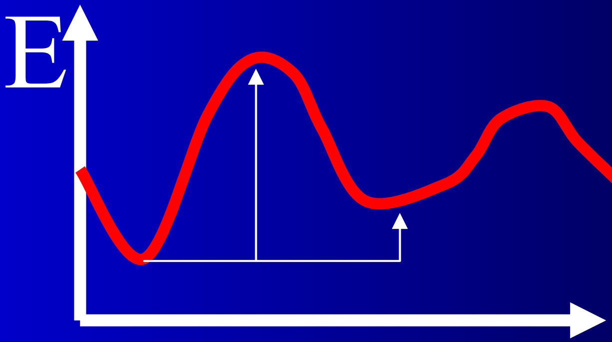
No crystal below four spatial dimensions

Very difficult stat-mech problem



- Glass

- Optimization : many solutions



Larkin Model

$$H_{el} = \frac{c}{2}\int (\nabla u(r))^2 d^d r$$

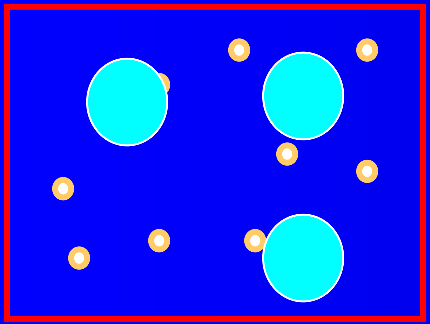
$$H_{dis} = \int f(r)u(r)d^d r$$

- Exactly solvable

$$B(r) = B_{th} + \frac{\Delta}{c^2} r^{4-d}$$

Exponential loss of translational order $C(r) \approx e^{-r^{4-d}}$

- Not valid at large distance



$$\rho_0 \sum_K e^{iK(x-u(x))} V(x) \approx f(x) u(x)$$

Not valid when : $K_{MAX} u \approx 1$ $u(R_c) \approx \xi$

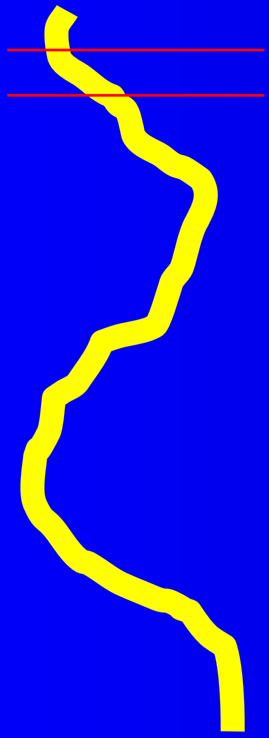
- New length Rc

- Larkin model has no metastable states and pinning

- Rc is related to pinning $J_c \propto \dfrac{c\xi}{R_c^2}$

Interfaces: only one length

- Larkin length $u(R_c) \approx \xi$



$R < R_c$; $u(R) = R^{(4-d)/2}$

$R > R_c$; $u(R) =$?????

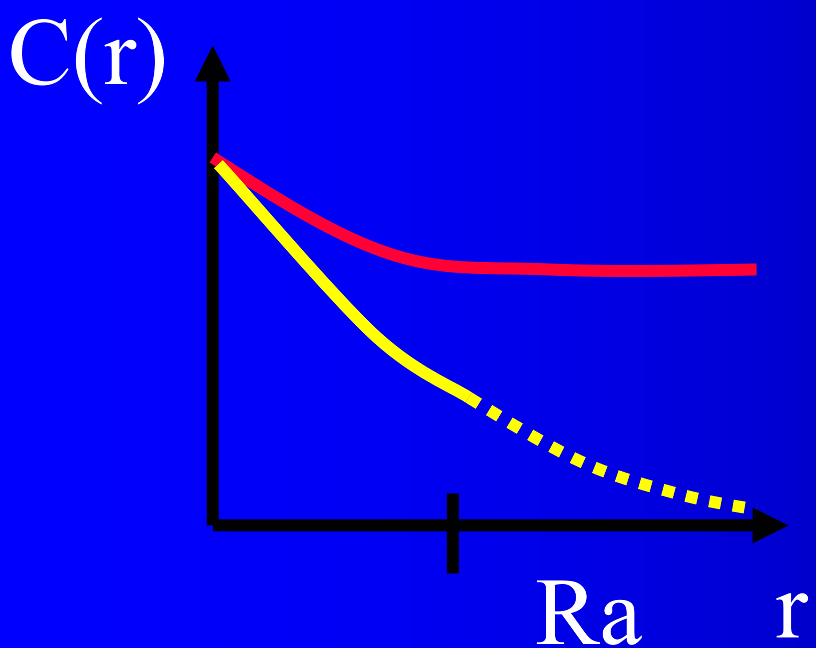
Crystals: Two crucial lengthscales

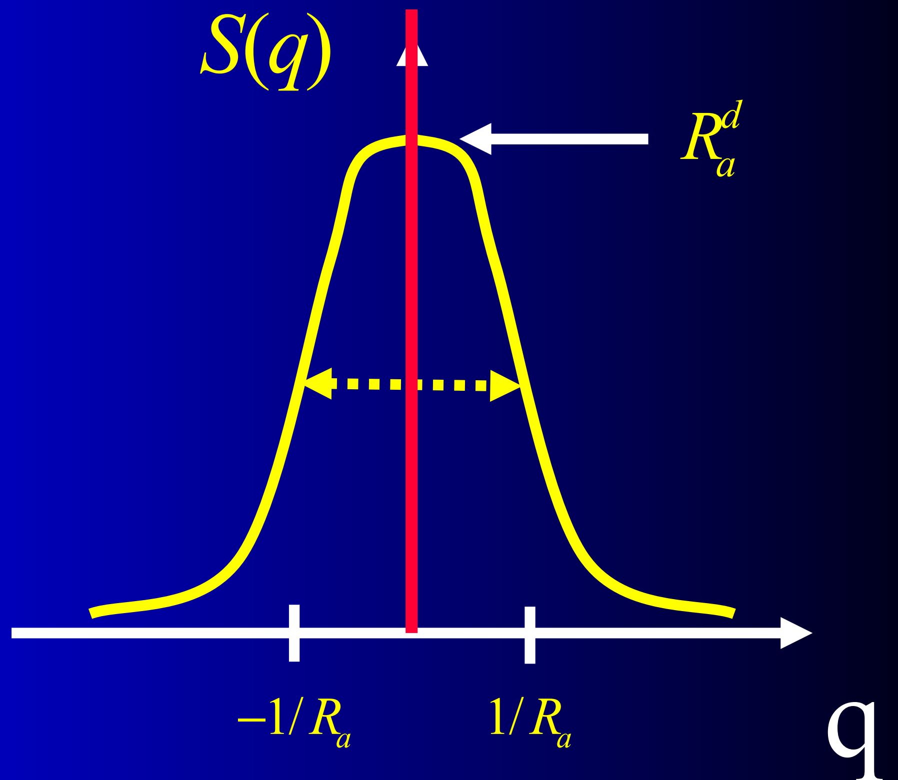
- Positional order

$$u(R_a) \approx a$$

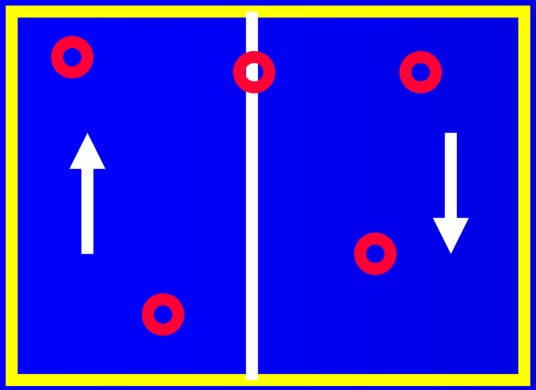
- Larkin length

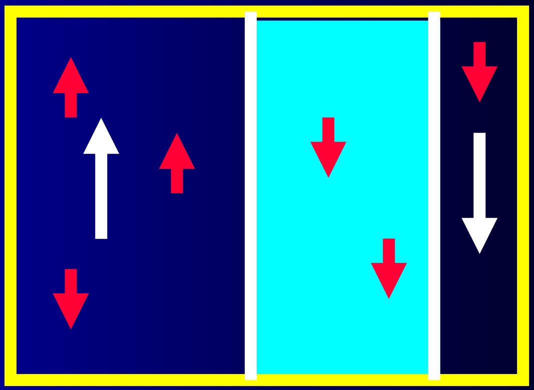
$$u(R_c) \approx \xi$$





Two types of disorder





Random bond

$$\int dx dz V(x,z)\rho(x,z)$$
$$= \int dz V(u(z),z)$$

Random field

$$\int dz \int_0^{u(z)} V(x,z)$$

Interfaces

$$H_{el} = \frac{c}{2}\int (\nabla u(r))^2 d^d r \qquad H_{dis} = \int V(r,u(r)) d^d r$$

$$\overline{V(z,x)V(z',x')} = D\delta(x-x')\delta(z-z')$$

$$cu^2 L^{d-2} \qquad D^{1/2} L^{d/2} u^{-m/2}$$

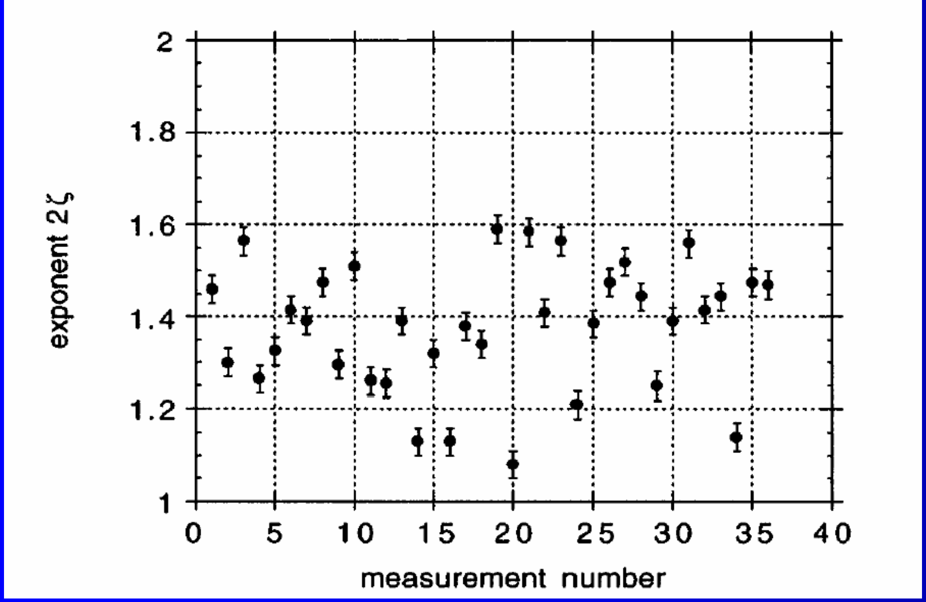
$$u_{RB} \propto L^{\frac{4-d}{4+m}}$$

$$u_{RF} \propto L^{\frac{4-d}{4-m}}$$

Flory argument (mean field)

$u_{RB} \propto L^{\varsigma}$ ζ : roughness exponent

d = 1 ; ζ = 2/3 (random bond)



Crystals

- Identical to interfaces ? $u \sim L^{\zeta}$

- Above R_c

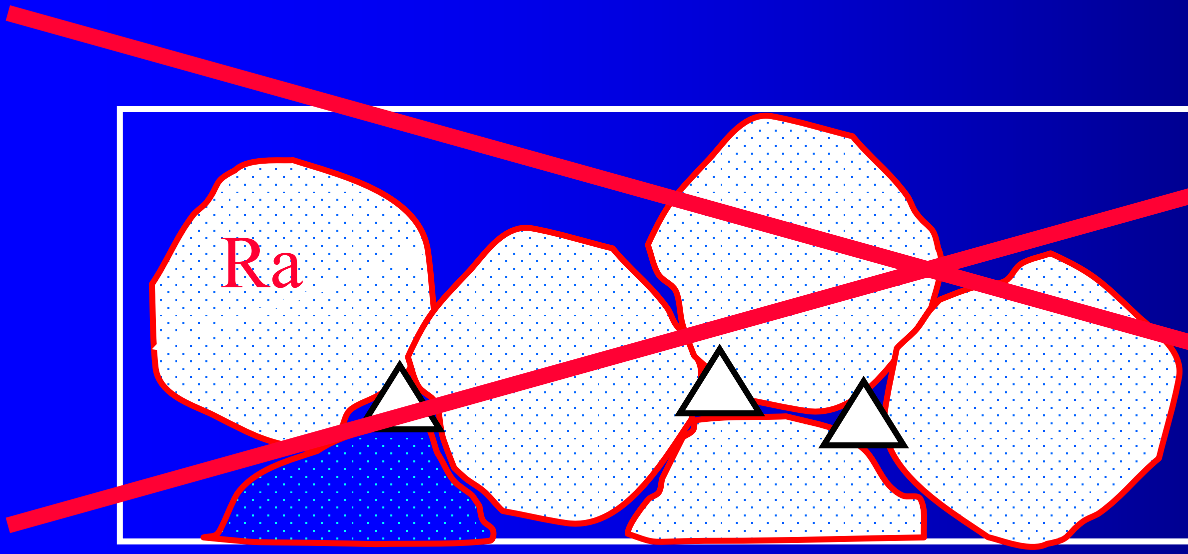
$$C(r) \propto e^{-L^{2\zeta}}$$

- Expontential loss of positional order ??

Naive vision of a D.E. crystal

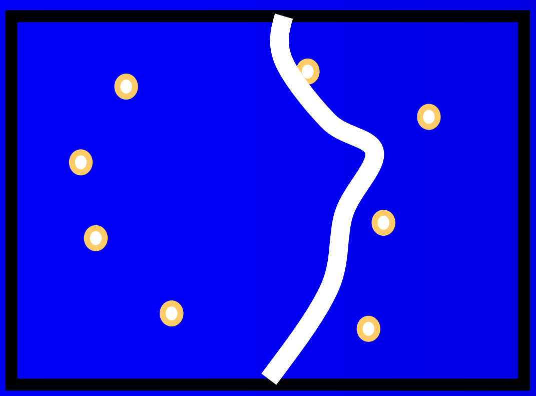
- Loss of translational order beyond Ra

- (Wrong) argument: disorder induces dislocations at Ra

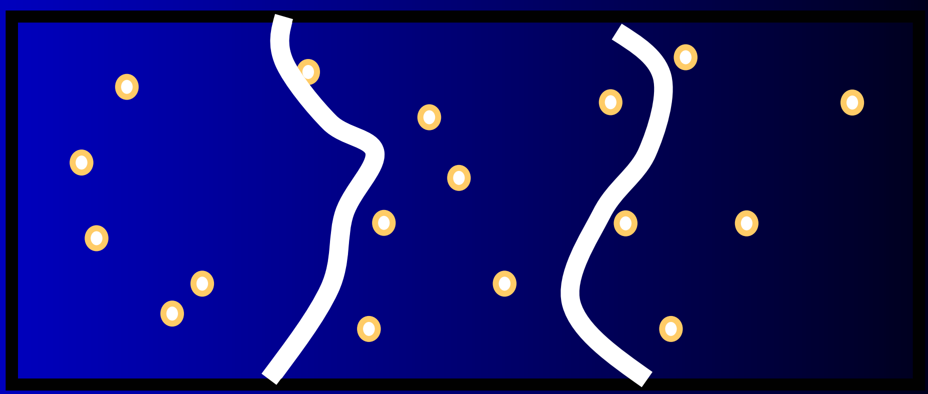


Crystal broken in crystallites of size Ra

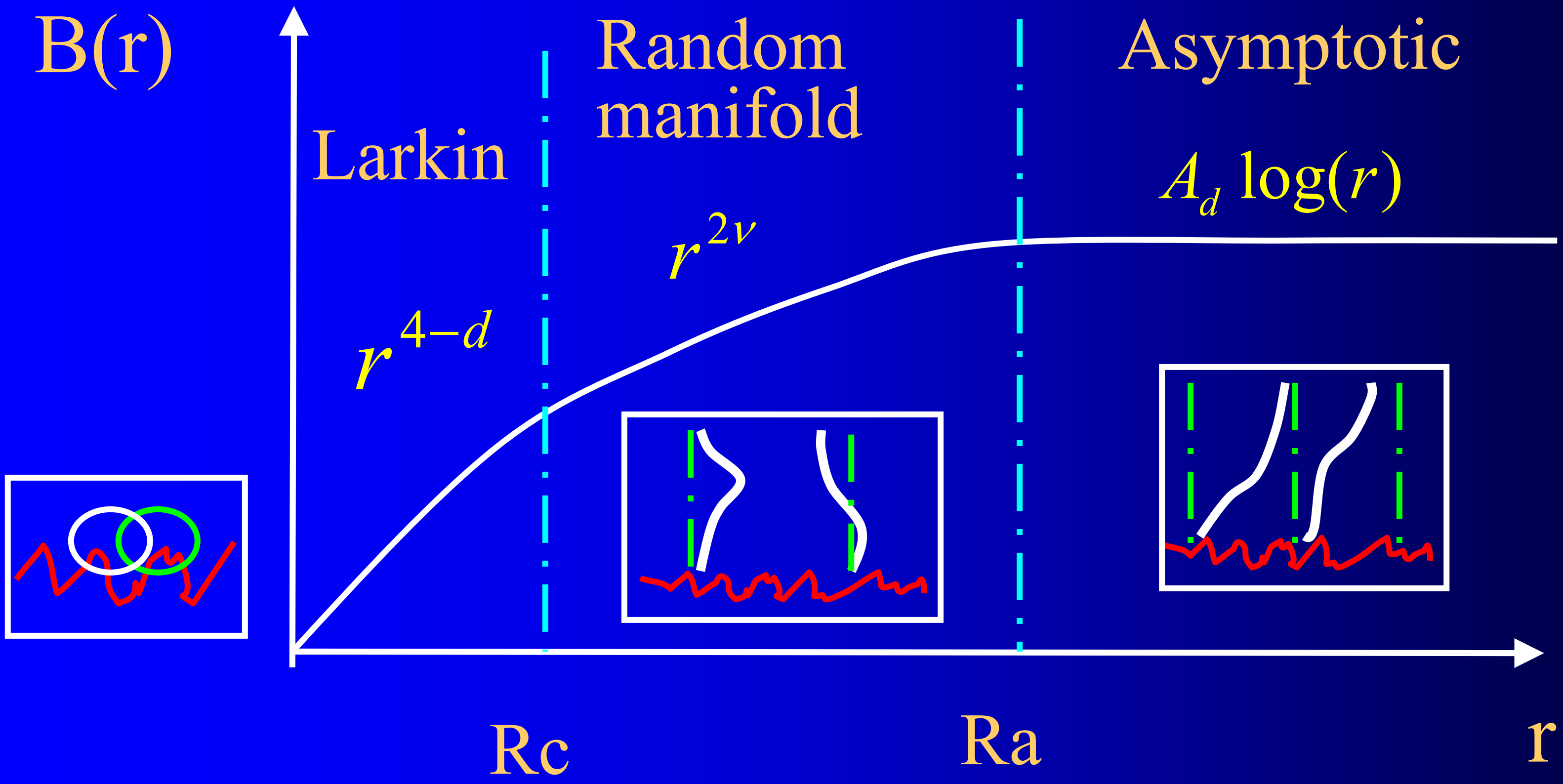
Periodic systems: new universality class



$$u \sim L^{\varsigma}$$

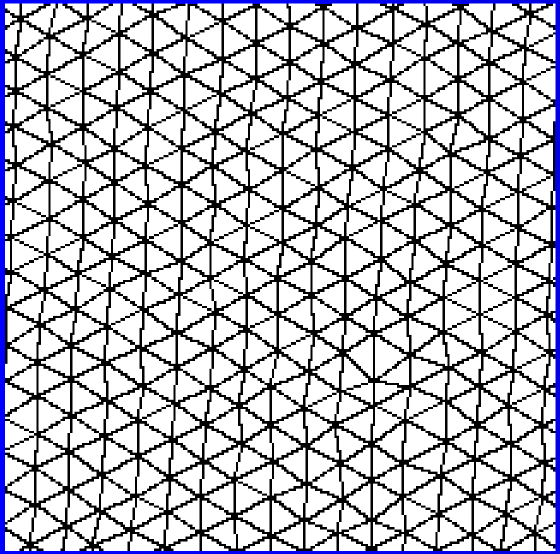


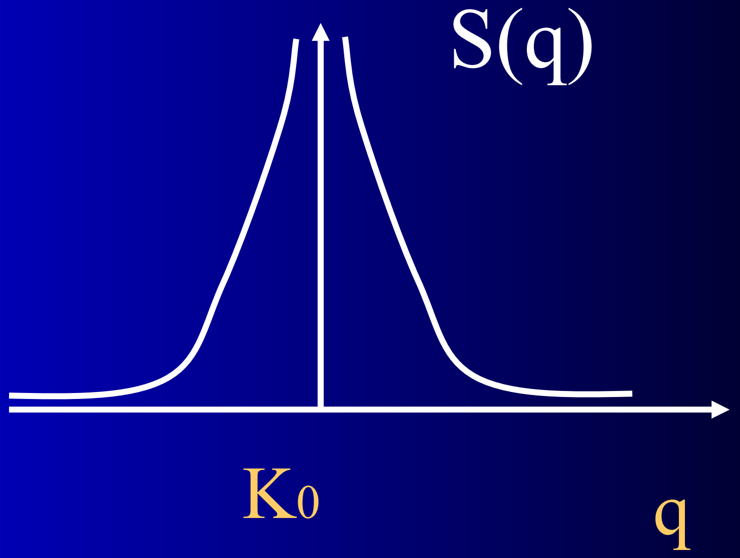
$$u \sim Log(L)^{1/2}$$



- many metastable states ï glass !
- Quasi long range translational order !
- Power law Bragg peaks; Ad = 4-d

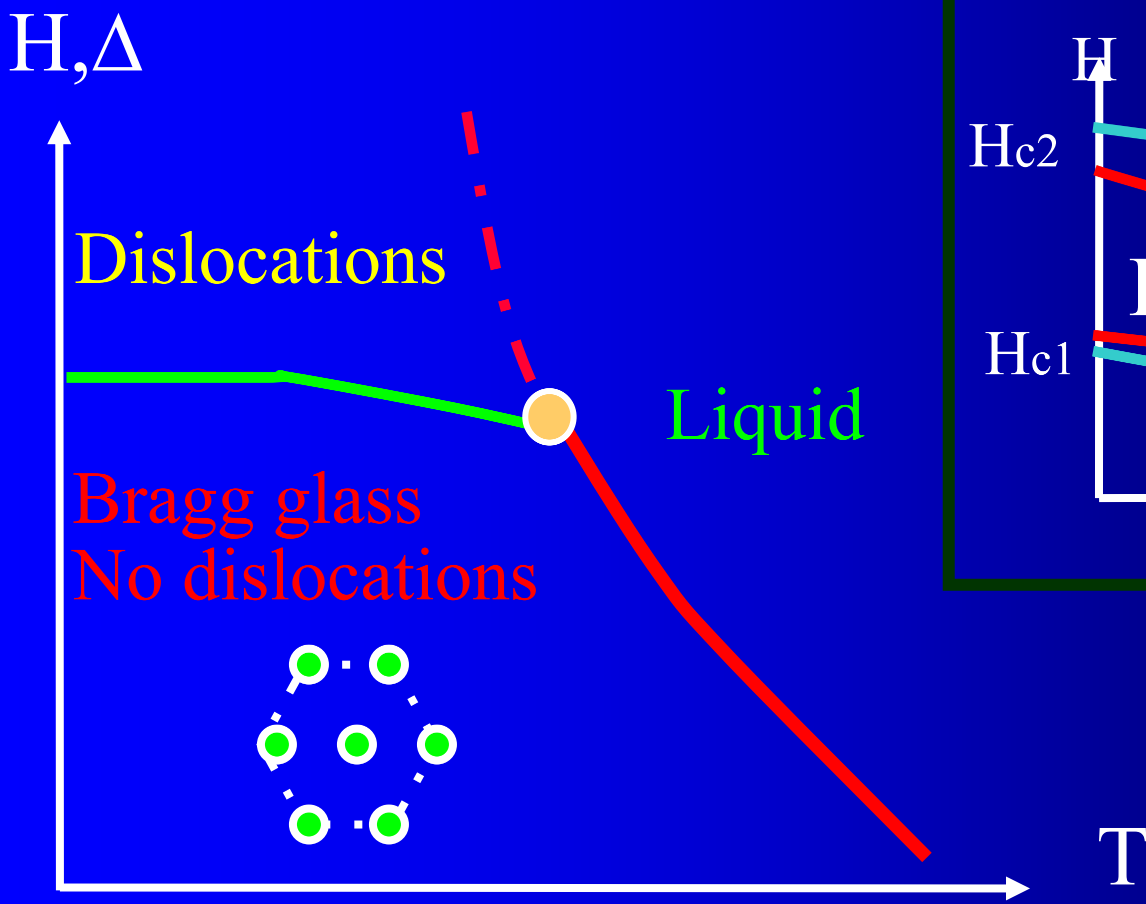
Bragg Glass

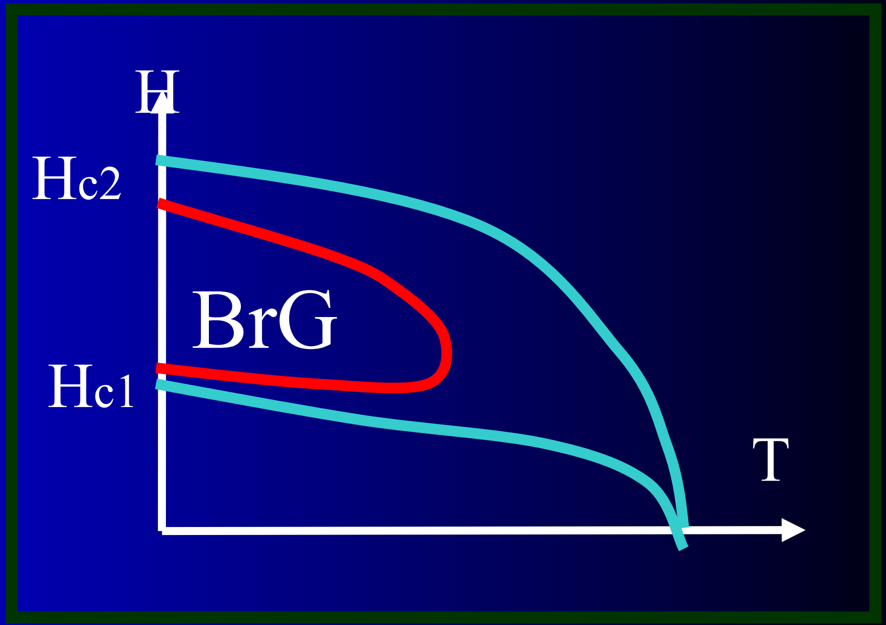


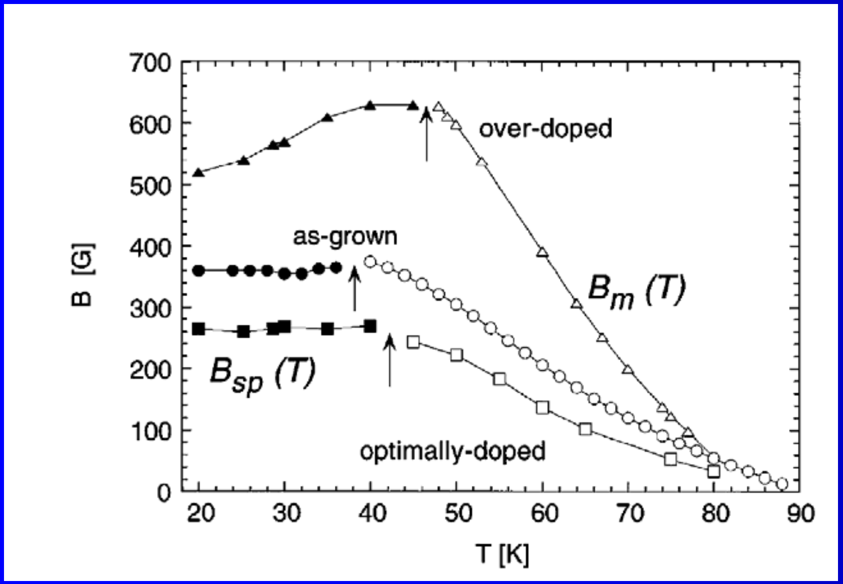


•Existence of a **thermodynamically** stable glassy phase with quasi long range translational order (power law Bragg peaks) and perfect topological order (no defects)

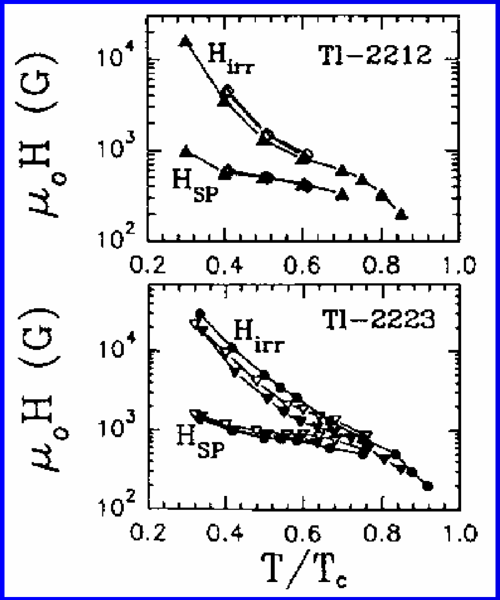
Unified phase diagram



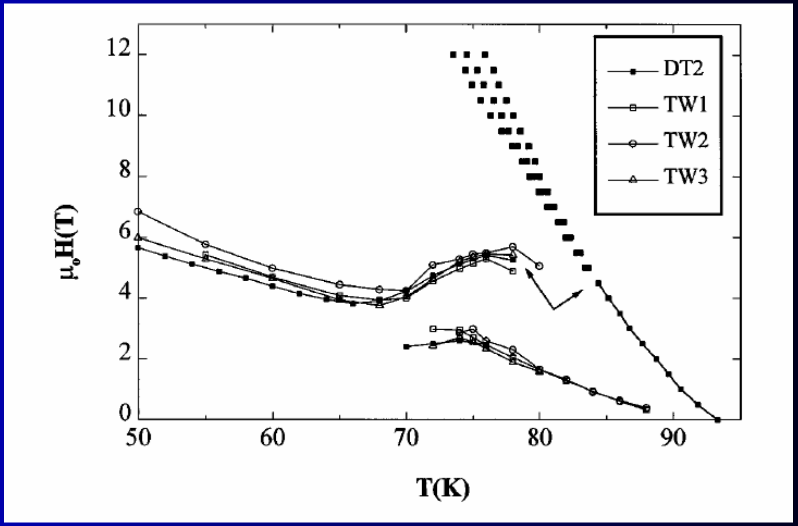


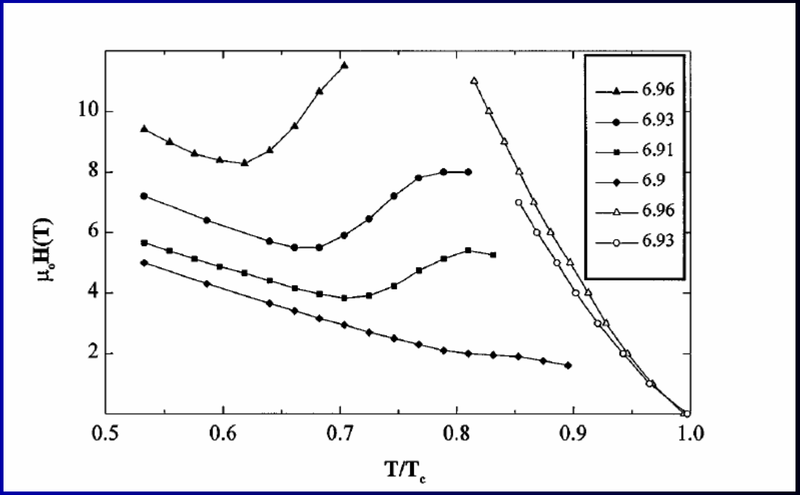


B. Khaykovich et al. PRL 76 2555 (96)

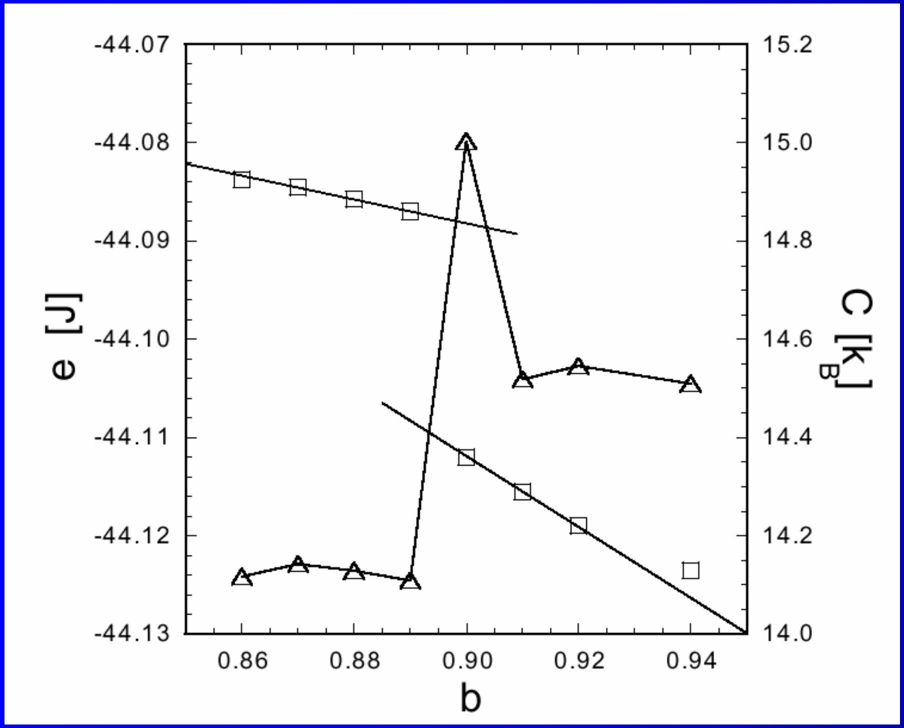


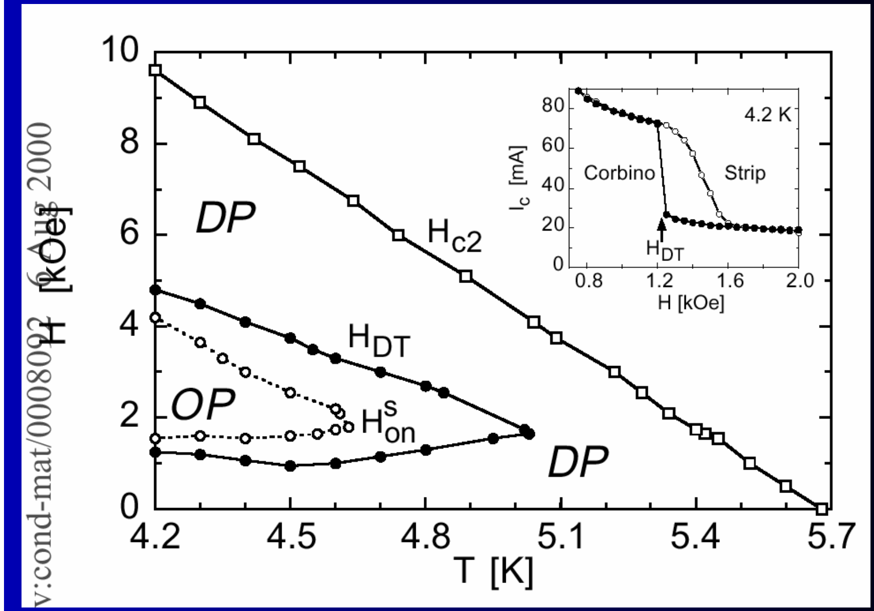
Hardy et al. Physica C 232 347 (94)





K. Deligiannis et al. PRL 79 2121 (97)

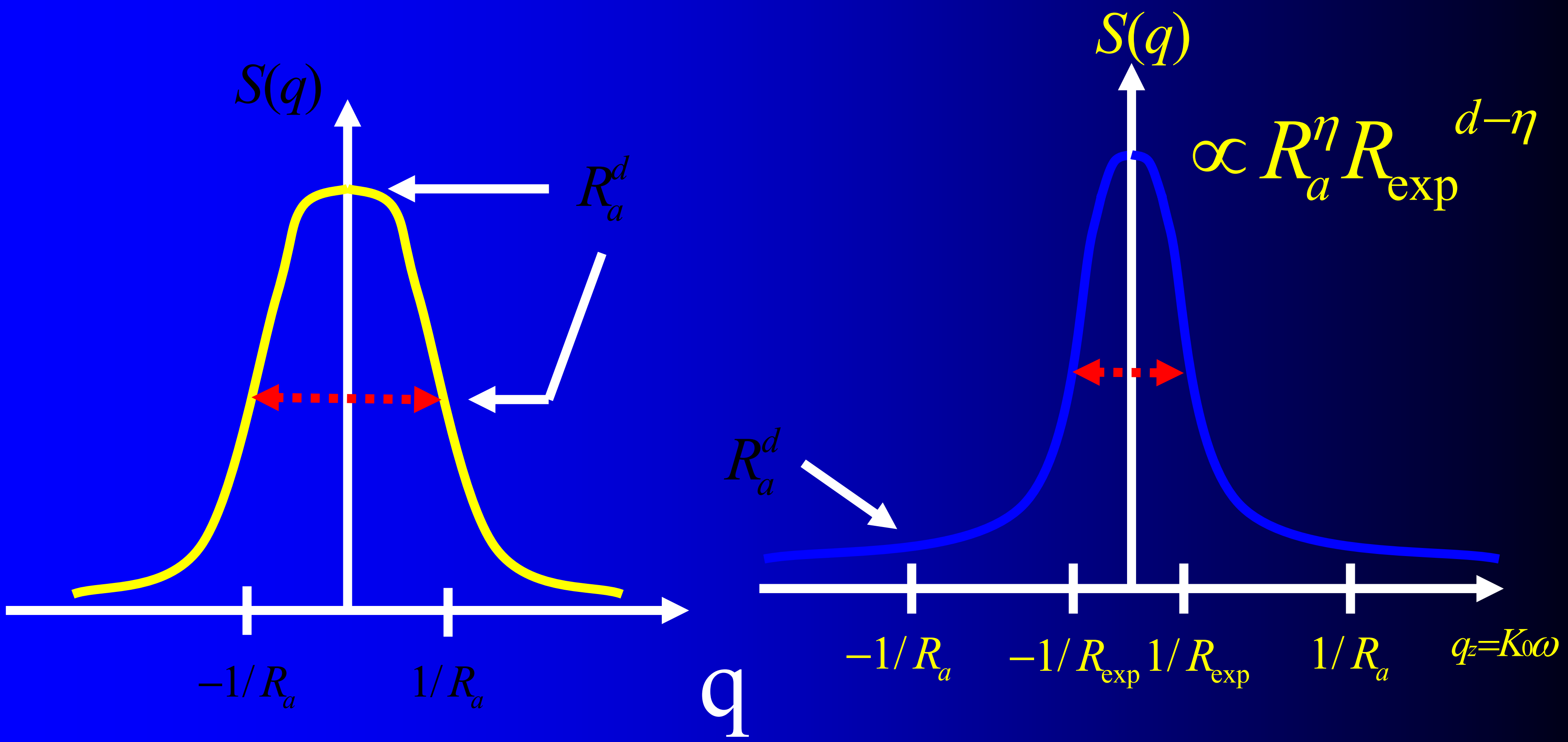




Y. Nonomura and X. Hu cond-mat 0002263

Y. Paltiel et al. cond-mat 0008092

Neutrons

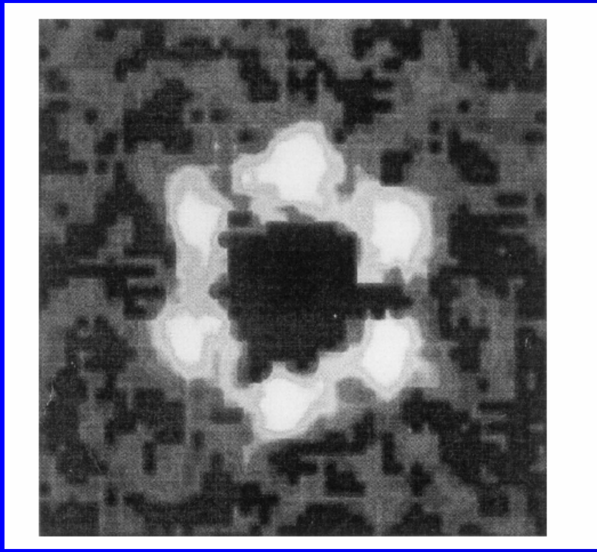


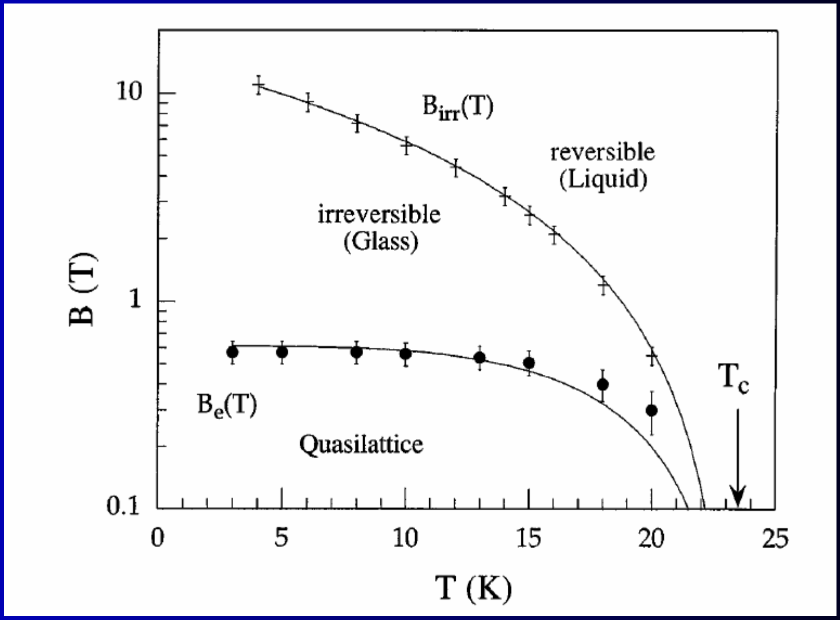
No positional order

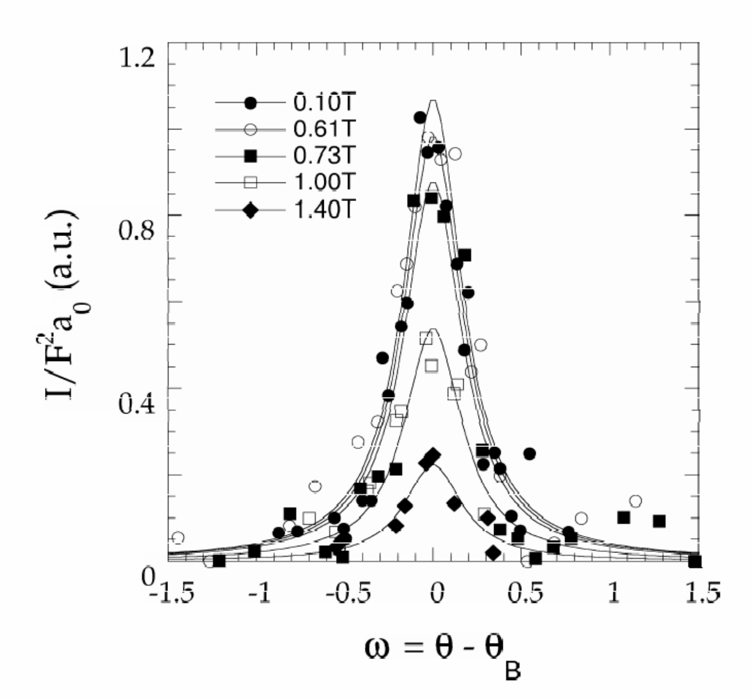
Bragg Glass

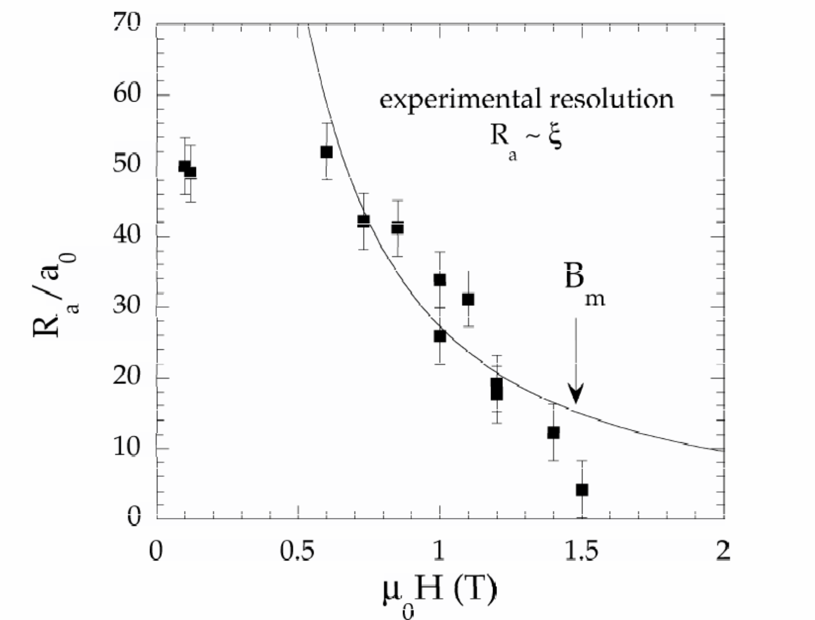
- Collapse of intensity without broadening

I. Joumard et al. PRL 82 4930 (99); T. Klein et al. nature (01)

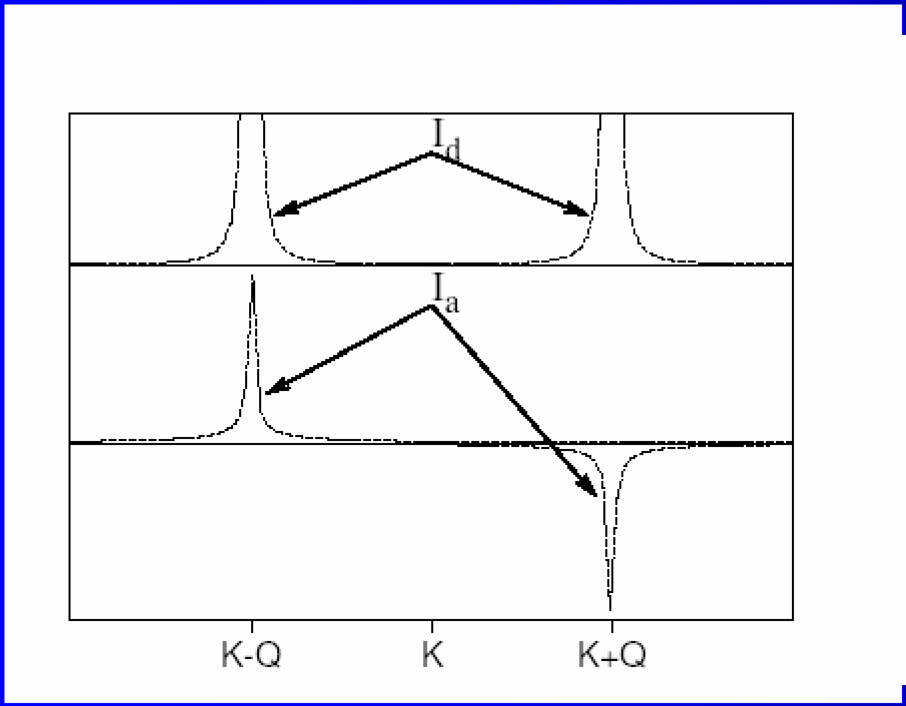


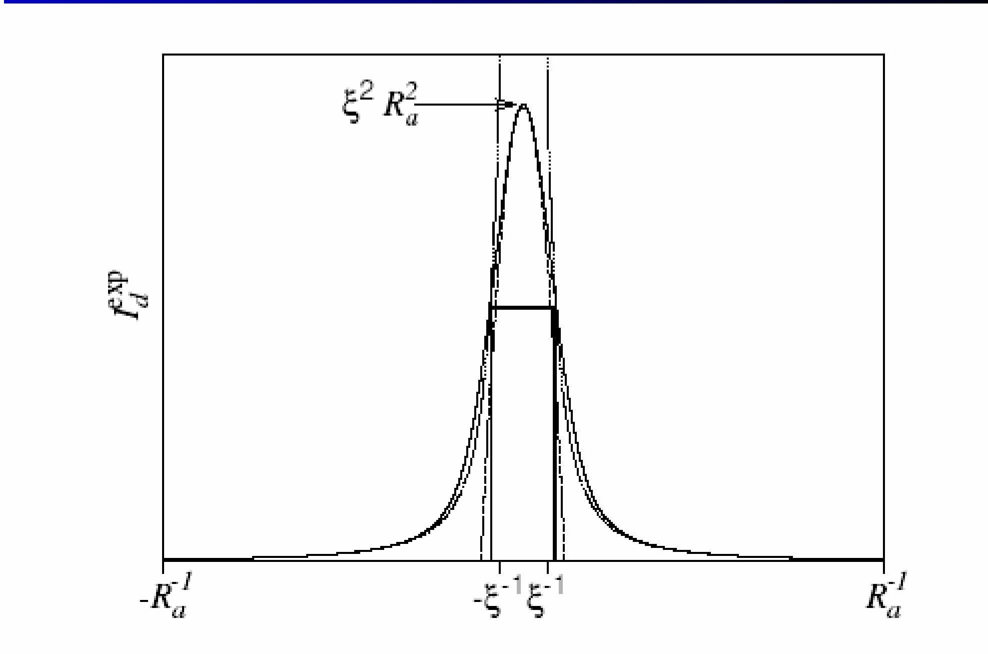






CDW





A. Rosso + TG PRB 68 140201(R) (2003)
+ cond-mat

How to solve ?

- Average over disorder (replica trick)

- Two main methods :

  Variational approach

  Renormalization (functional RG)

Replicas

$$\overline{\langle O\rangle} = \int \mathcal{D}V p(V)\langle O\rangle_V = \int DV p(V)\frac{\int \mathcal{D}\phi O[\phi]e^{-S_V[\phi]}}{\int D\phi e^{-S_V[\phi]}}$$

$$\int D\phi_1 D\phi_2 \ldots D\phi_n O[\phi_1]e^{-\sum_{i=1}^n S_V[\phi_i]} =$$

$$\int D\phi O[\phi]e^{-S_V[\phi]}\left[\int D\phi e^{-S_V[\phi]}\right]^{n-1}$$

Average over disorder

$$H = \frac{c}{2}\int (\nabla u)^2 d^d x + \rho_0 \sum_K \int e^{iK(x-u(x))} V(x)$$

- Classical systems

$$H = \sum_a c\int (\nabla u_a)^2 d^d x - \rho_0 \Delta \sum_{a,b} \sum_K \int \cos(K(u_a(x) - u_b(x))) d^d x$$

- Quantum problem (disorder is time independent)

$$S = \sum_a c\int (\nabla u_a)^2 d^{d+1} x$$

$$- \rho_0 \Delta \sum_{a,b} \sum_K \int \cos(K(u_a(x,\tau) - u_b(x,\tau'))) d^d x d\tau d\tau'$$

Variational Method

Find the best quadratic Hamiltonian

$$S_0 = \sum_{ab} \sum_{i\omega_n} \int G_{ab}^{-1}(i\omega_n, q) \Phi^a_{i\omega_n, q} \Phi^b_{-i\omega_n, -q} d^d q$$

Minimize

G_{ab} is a 0x0 matrix

$$F_{\text{var}} = F_0 + \left\langle (S - S_0) \right\rangle_{S_0}$$

$$\frac{dF_{\text{var}}}{dG_{ab}(i\omega_n, q)} = 0$$

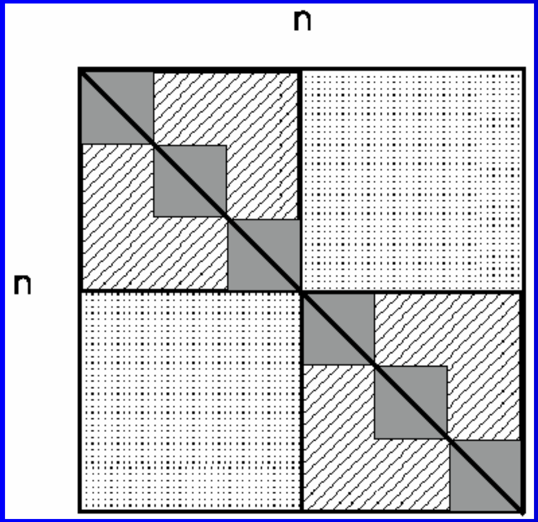
$$G_{ab}^{-1}(i\omega_n, q) = \omega_n^2 + q^2 + \int ..e^{\sum \int G_{ab} \cdots\cdots}$$

0x0 limit

- Replica symetric

G_{aa} $G_{a \neq b}$ Unstable

- Replica symetry broken solution



Hierarchical structure

a,b continuous in [0,1]

RSB

- Signals metastability and Glassy properties

- Disordered elastic system = glass

- RSB from d=4 to d=2 (or 1+1) above a lengthscale Rc

FRG method

$$\beta H = \sum_a \tfrac{c}{T} \int (\nabla u_a)^2 d^d x$$

$$- \rho_0 \tfrac{\Delta}{T^2} \sum_{a,b} \sum_K \int \cos(K(u_a(x) - u_b(x))) d^d x$$

In usual RG :

$$\Delta(u) \approx a + bu^2 + cu^4 + \ldots.$$

Needs only to keep b and c (higher powers are irrelevant)

Disordered System

$$\sum_a \tfrac{c}{T} \int (\nabla u_a)^2 d^d x$$

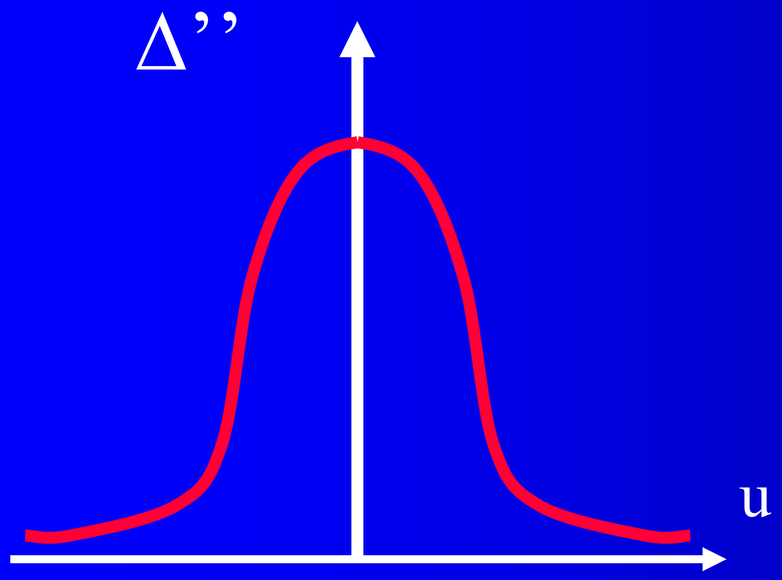
$$u \propto L^0$$

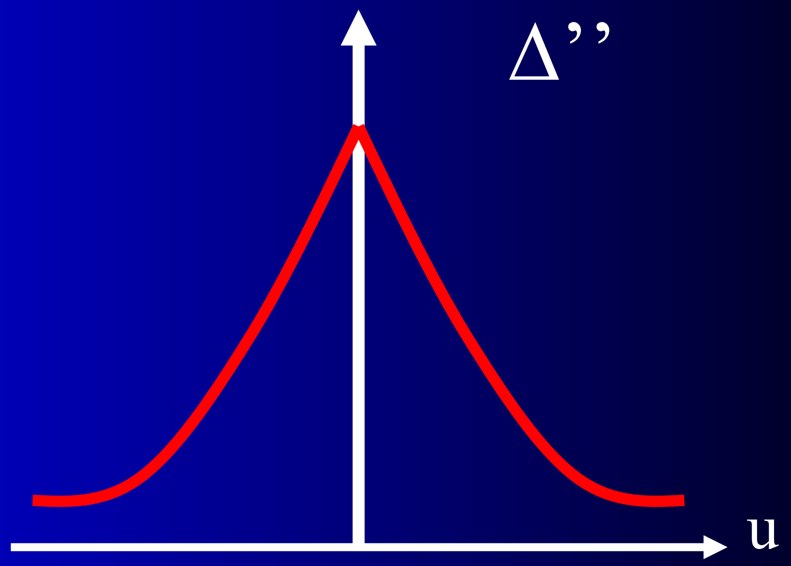
$$T \propto L^{2-d}$$

$$-\tfrac{1}{T^2} \sum_{a,b} \sum_K \int \Delta(K(u_a(x) - u_b(x))) d^d x$$

$$\Delta \propto L^{4-d}$$

Needs to keep the *whole* function



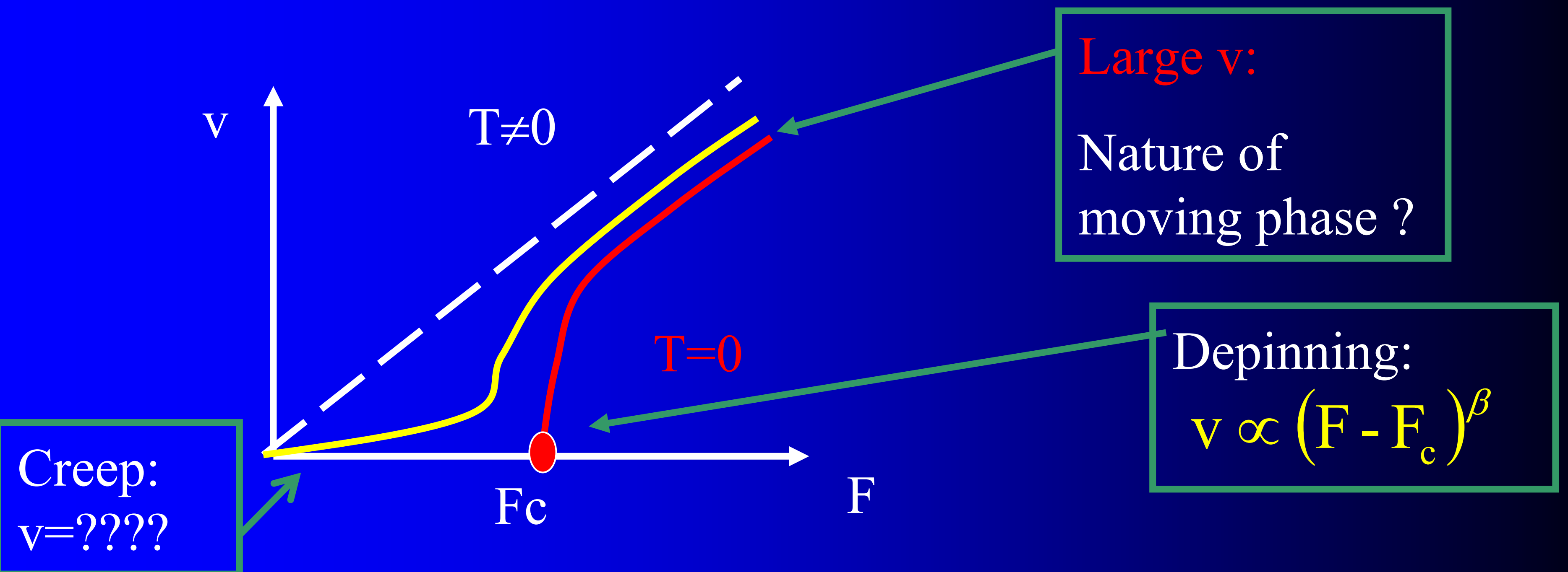


- Nonanalyticity at a finite lengthscale Rc such that u(Rc) ~lc
- Cusp signals metastability and glassy states

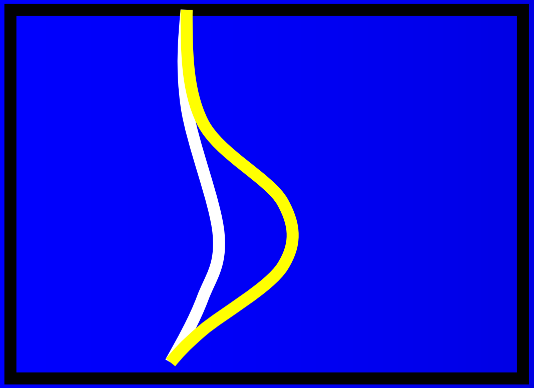
Dynamics

- Competition between disorder and elasticity: glassy properties

- Dynamics ?



Pinning (Fc) and Larkin length (Rc)



$$F_c = \frac{c\xi}{R_c^2}$$

$$H_{el} = \frac{c}{2}\int (\nabla u(r))^2 d^d r$$

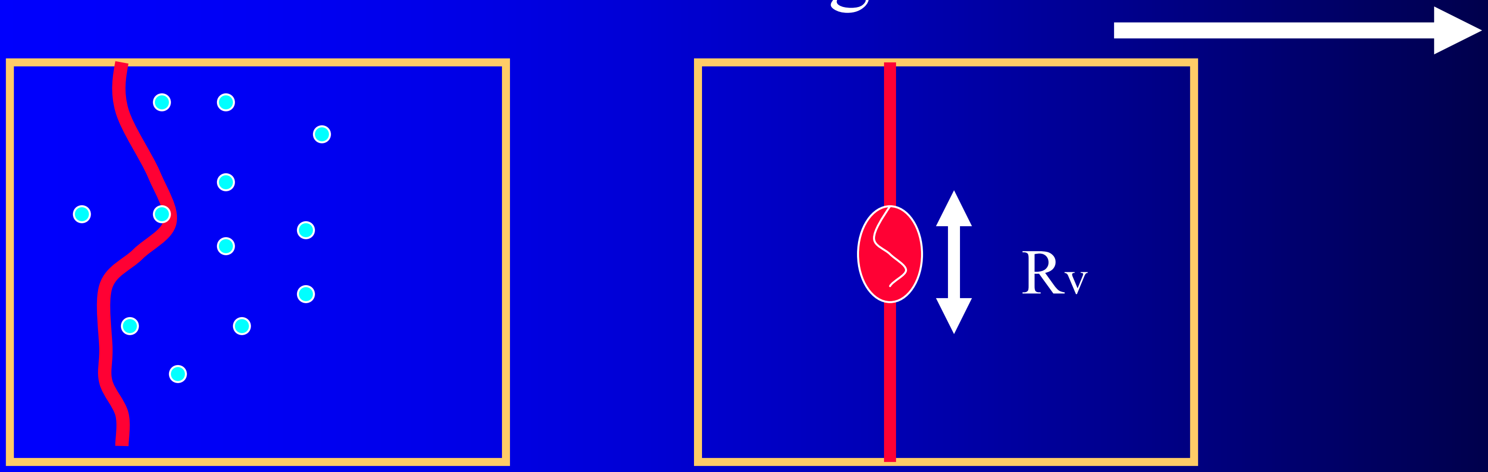
$$H_{el} = \int F u(r) d^d r$$

$$c R_c^{d-2} \xi^2$$

$$F R_c^d \xi$$

Large V

Interfaces reorder at large V



Thermal Roughening for $R > R_V$

Depinning

$$\overline{(u_{r,t}-u_{0,0})^2}=r^{2\zeta}\mathcal{C}(t/r^z),$$

ζ for F $\sim$ F_c differs from ζ for F=0

$$v\sim(f-f_c)^{\beta},$$

$$\xi\sim(f-f_c)^{-\nu}.$$

$$\nu=\frac{1}{2-\zeta}=\frac{\beta}{(z-\zeta)}.$$

Only RF universality class

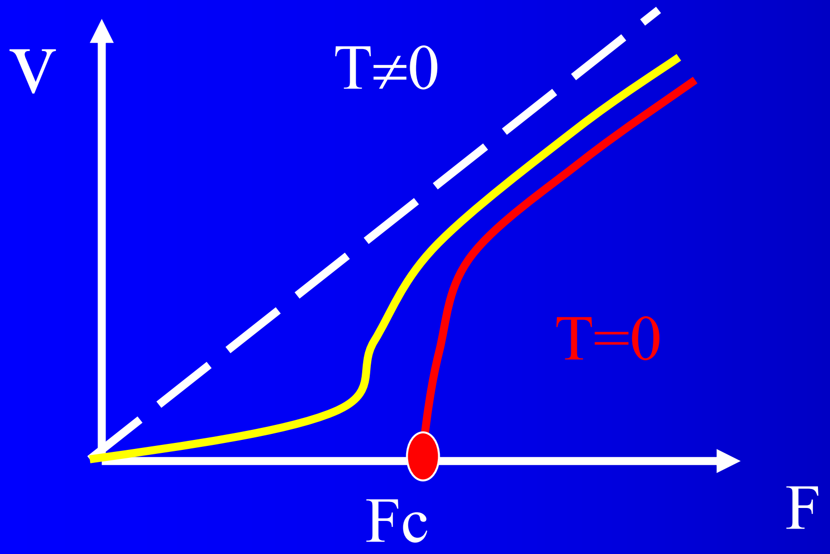
How to study

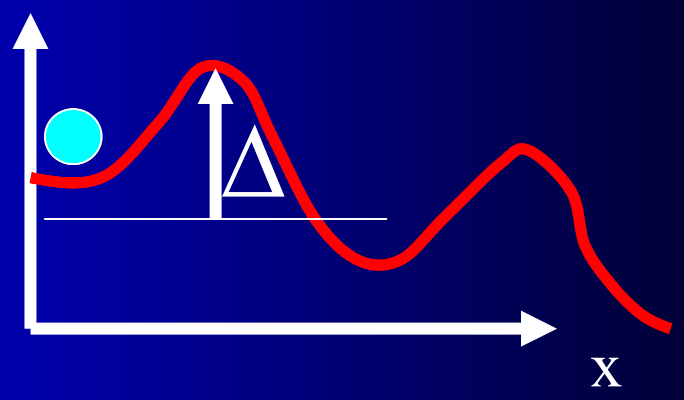
$$\eta \partial_t u_{rt} = c\nabla^2 u_{rt} + F(r, u_{rt}) + \zeta_{rt} + f,$$

$$S_{\text{uns}}(u,\hat{u}) = \int_{rt} i\hat{u}_{rt}(\eta\partial_t - c\nabla^2)u_{rt} - \eta T \int_{rt} i\hat{u}_{rt} i\hat{u}_{rt} - f \int_{rt} i\hat{u}_{rt} - \frac{1}{2}\int_{rtt'} i\hat{u}_{rt} i\hat{u}_{rt'} \Delta(u_{rt} - u_{rt'}). \qquad (4.1)$$

Martin-Siggia-Rose; Keldysh

TAFF vs Creep





- TAFF : typical barrier
- Linear response

$$v \propto e^{-\beta\Delta} F$$

Creep

- Glassy system
- Slow dynamics determined by statics qty

$$H_{el} = \frac{c}{2}\int (\nabla u(r))^2 d^d r$$

$$H_{el} = \int F u(r) d^d r$$

$$cR^{d-2+2\varsigma}$$

$$FR^{d+\zeta}$$

$$L_{opt} \approx F^{\varsigma-2}$$

$$v \approx e^{-\beta U(L_{opt})}$$

$$U(L_{opt}) \approx F^{\frac{d+2\varsigma-2}{\varsigma-2}}$$

S. Lemerle et al. PRL 80 849 (98)

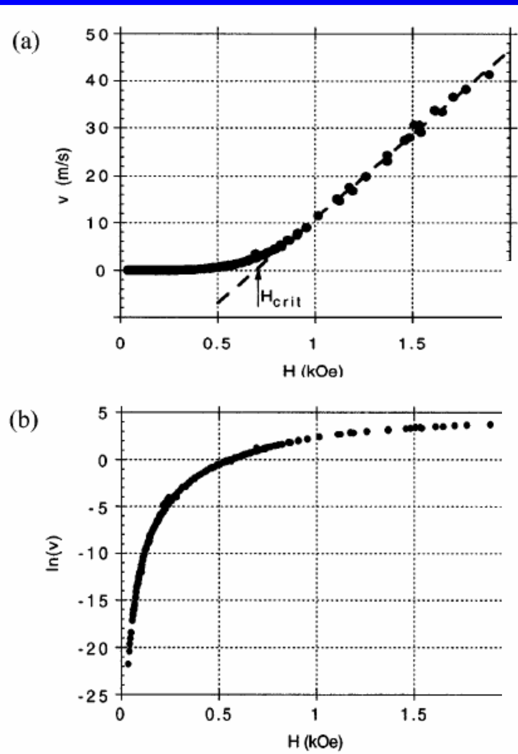


FIG. 2. (a),(b): MDW velocity versus applied magnetic field at room temperature (v in m/s). The dashed line in (a) is the linear fit of the high field part ($H > 0.86$ kOe) and the arrow marks its intersection with the line $v(H) = 0$. This is the definition of H_{crit}.

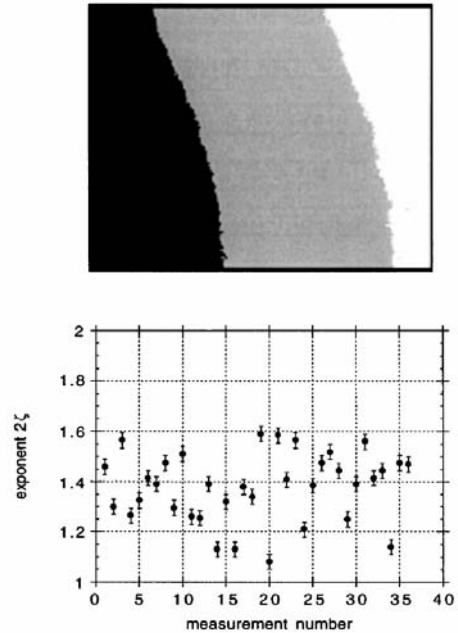


FIG. 5. Wandering exponent 2ζ. Measurements on diffe
MDW driven at $H = 50$ Oe during 20–45 min and then fr
($T = 300$ K, estimated error on 2ζ for a given image: $\pm 0.$

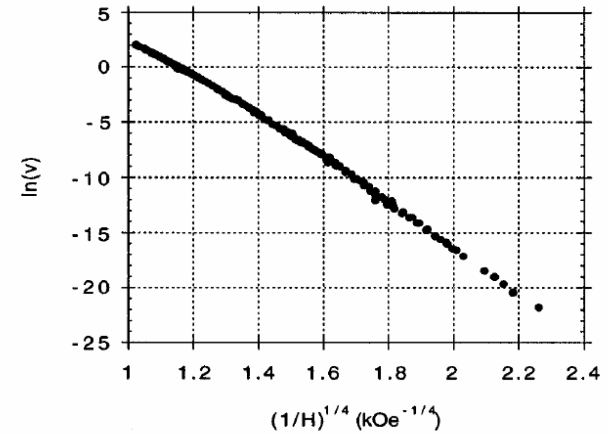
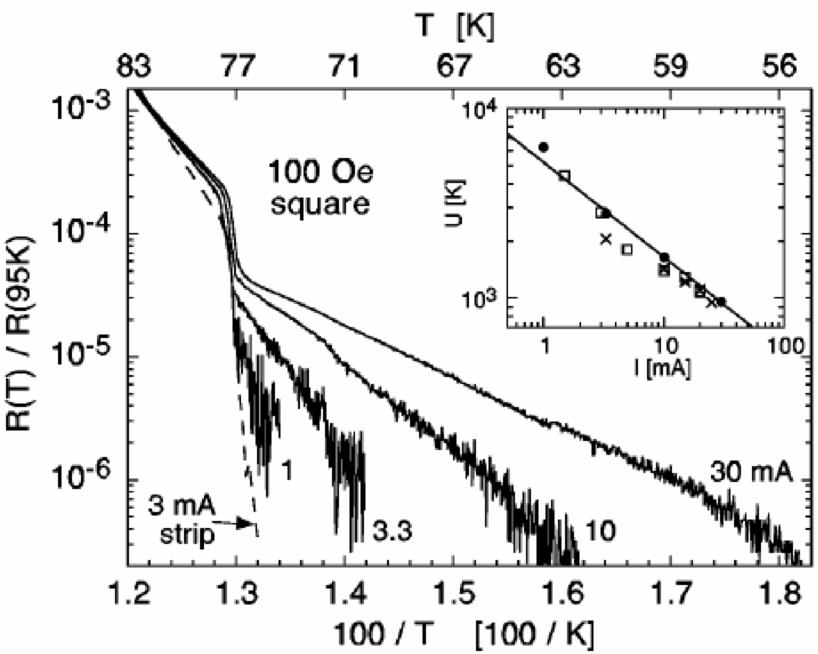
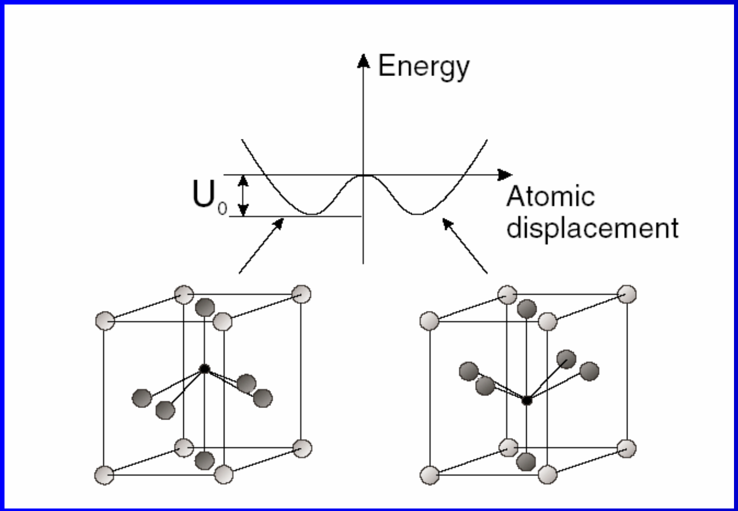


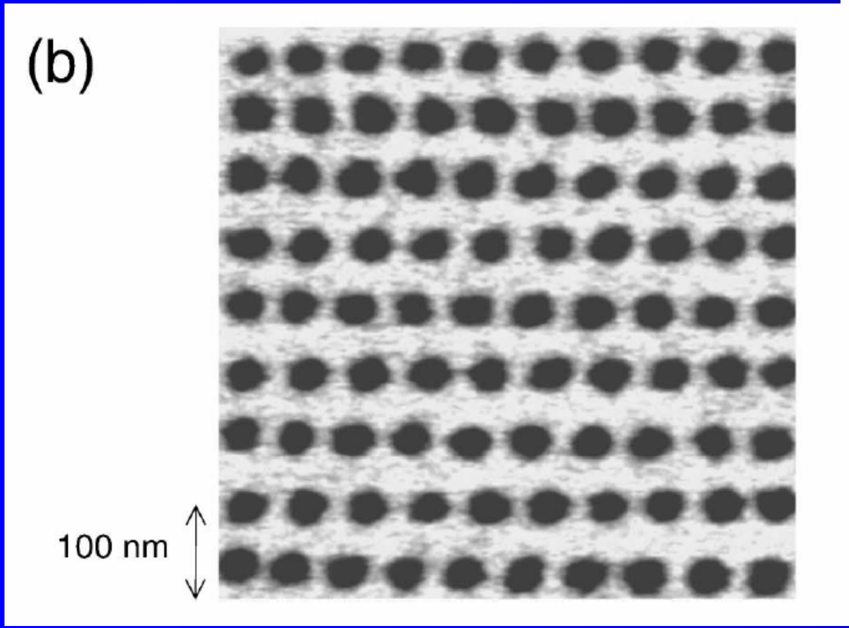
FIG. 3. Natural logarithm of MDW velocity as a function of $(1/H)^{1/4}$ (room temperature, $H \leq 955$ Oe).

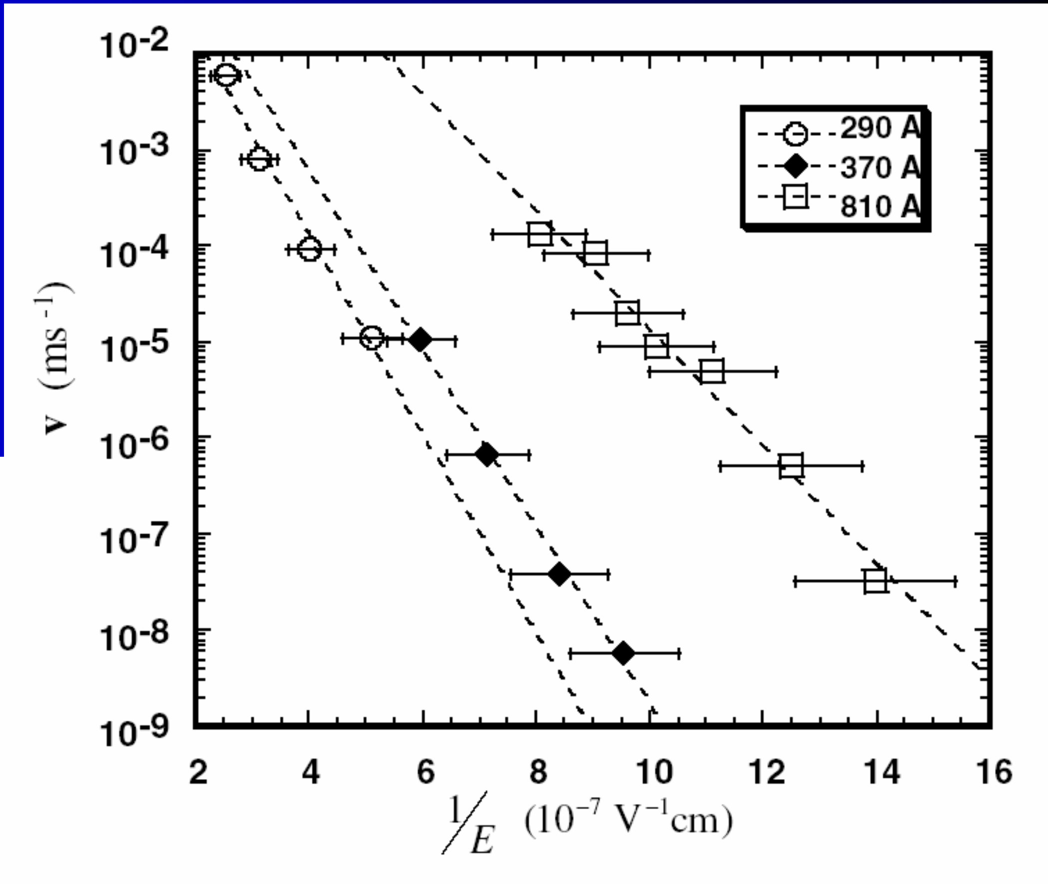


D.T. Fuchs et al. PRL 81 3944 (98)

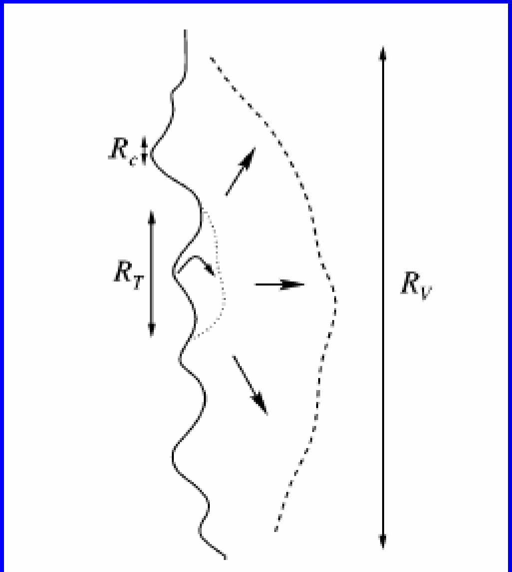
Ferroelectics







T. Tybell et al. PRL 89 097601 (02)

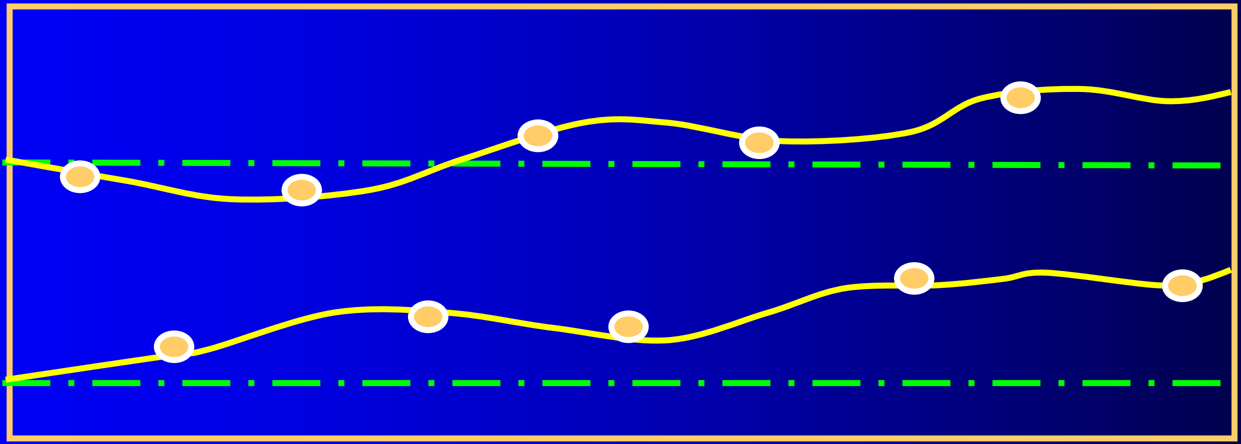


FRG: other scales than naive version of creep

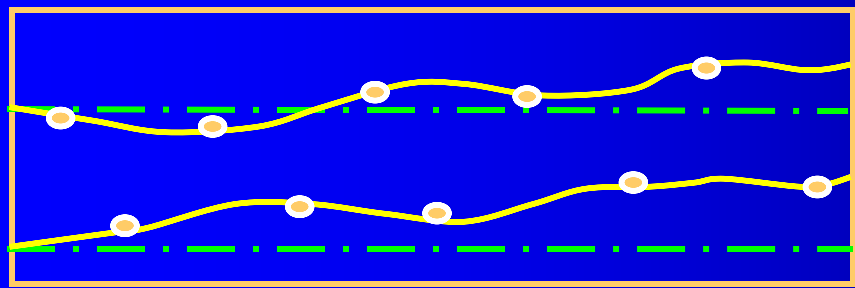
Molecular dynamics;
(A. Kolton)

Crystal vs Interfaces

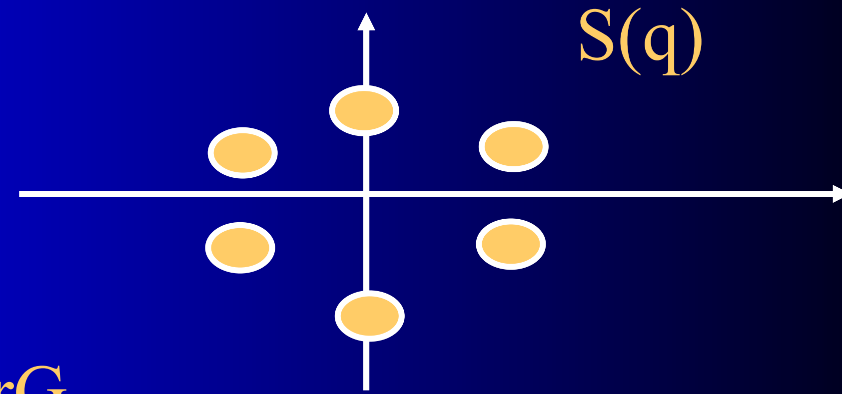
- Disorder remains in perp. direction
- Motion via static channels

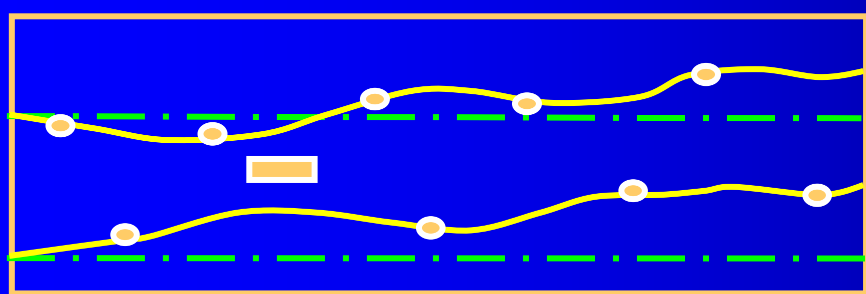


- Moving glass

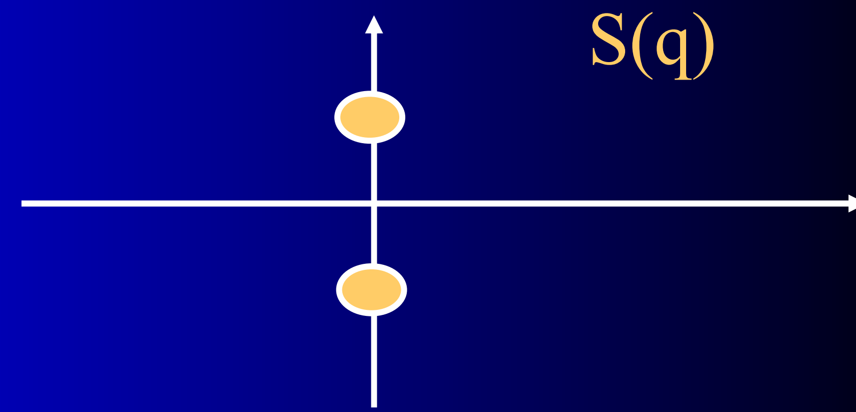


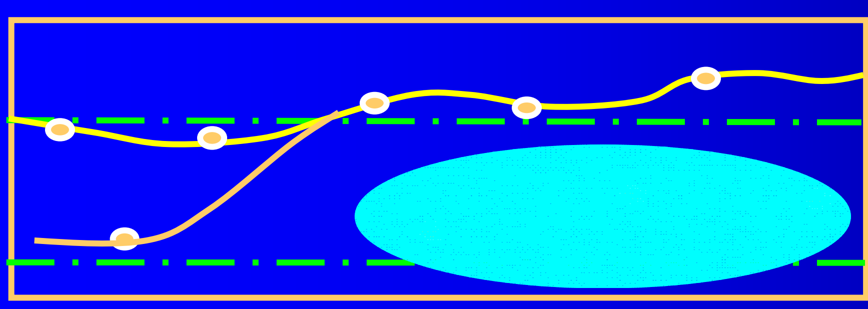
Coupled channels: Moving BrG



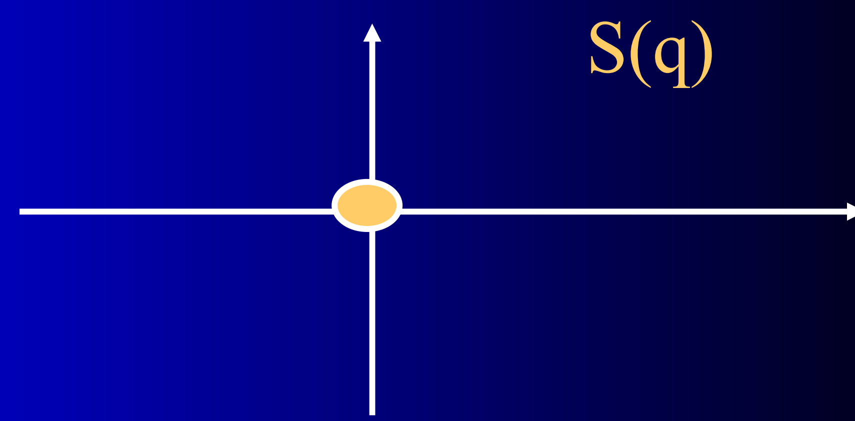


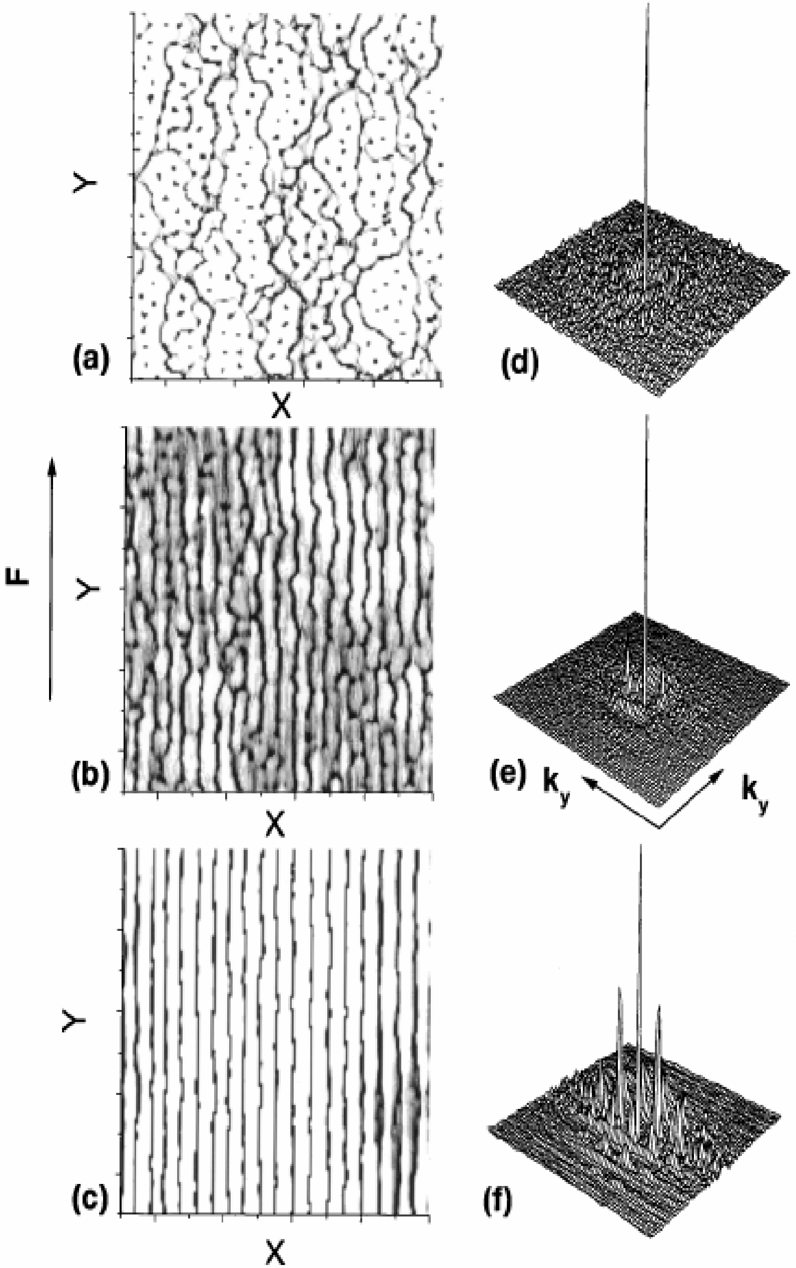
Decoupled channels:Smectic



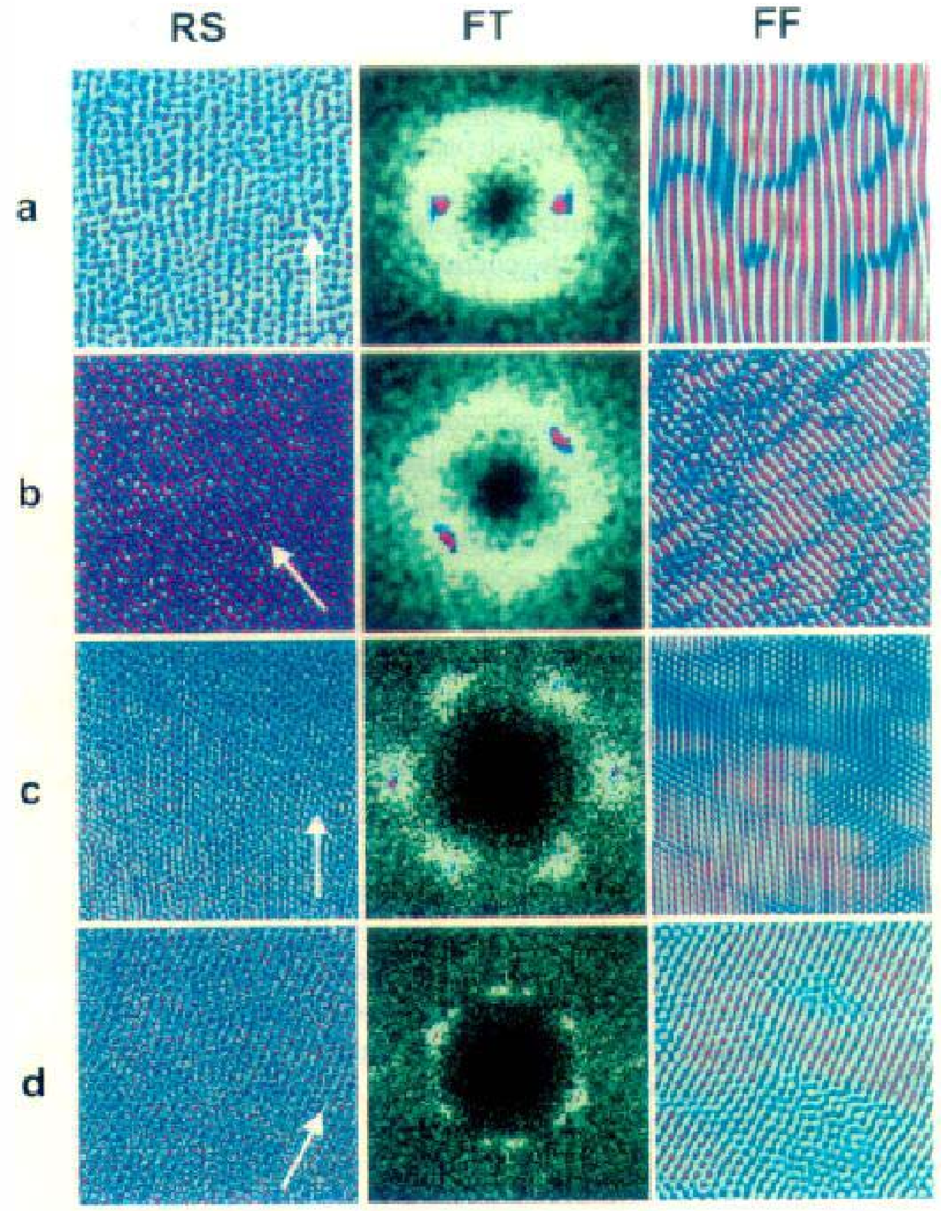


No channels:Plastic





A. Kolton et al PRL 83 3061 (1999)



F. Pardo et al. Nature, 396 348 (1998)

Molecular dynamics (A. Kolton)

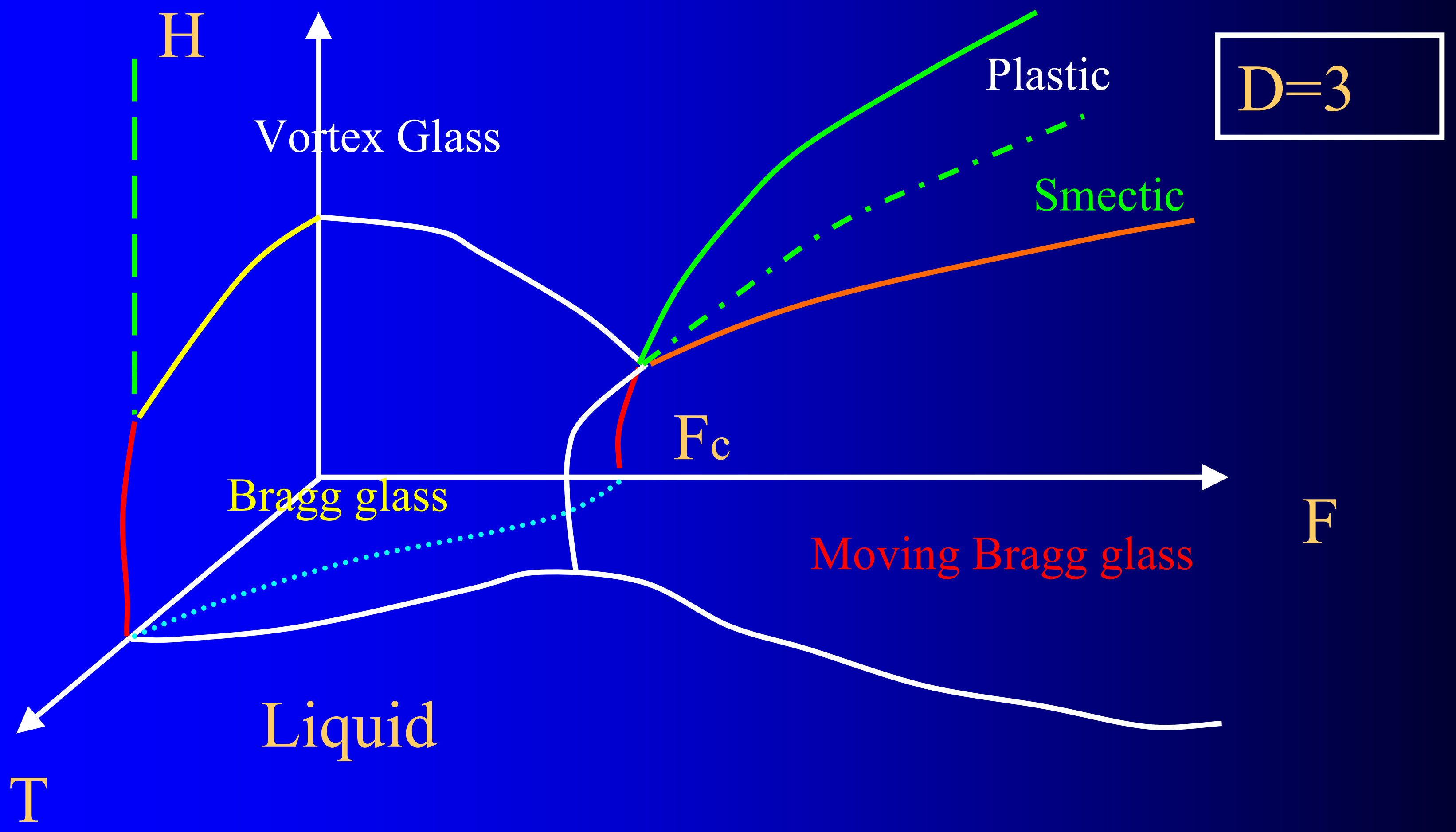
canales.mpeg

plastic.mpeg

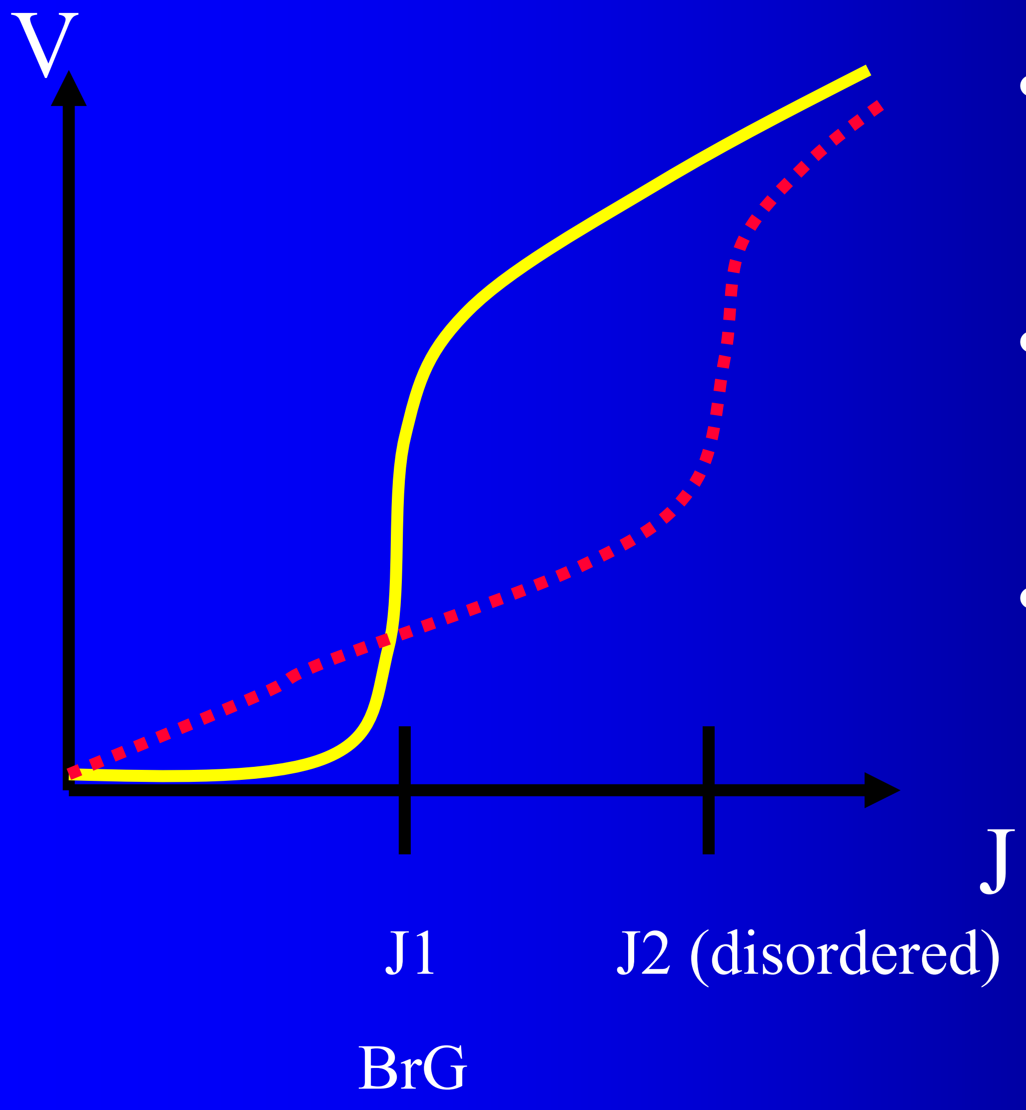
smectic.mpeg

transol.mpeg

Dynamical Phase Diagram



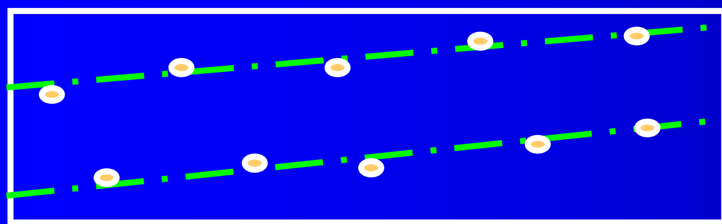
Peak Effect

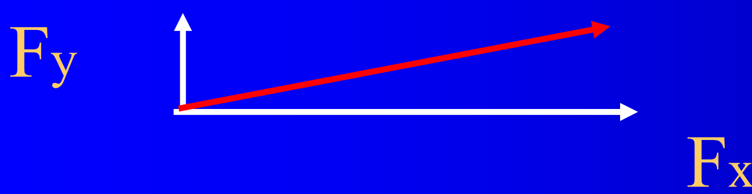


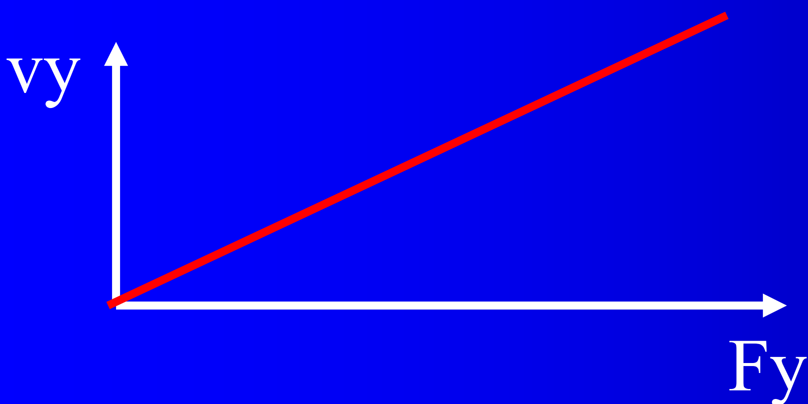
- Peak at « melting » of the Bragg glass :
- second peak in magnetization
- Peak effect in transport

Transverse critical force

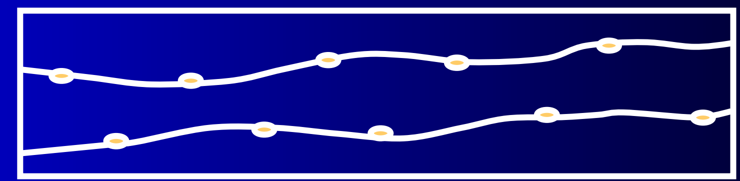
Crystal without disorder

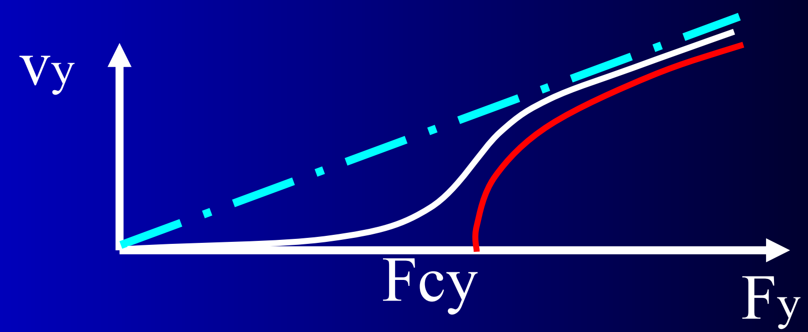


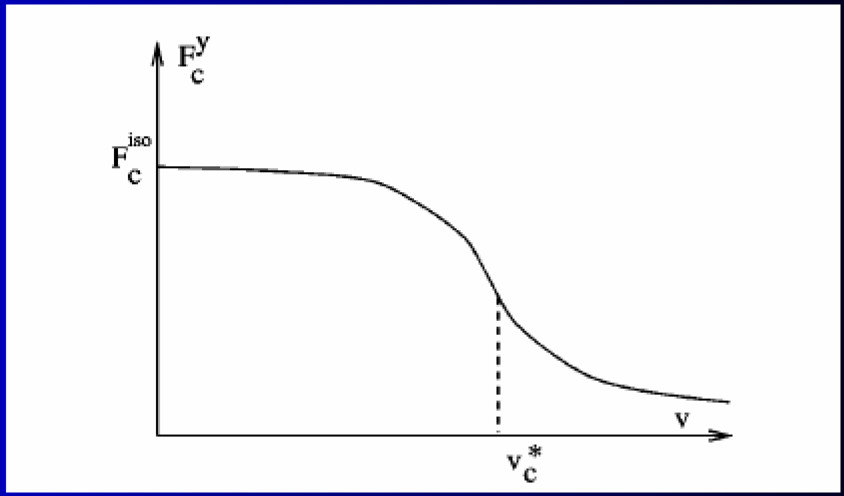




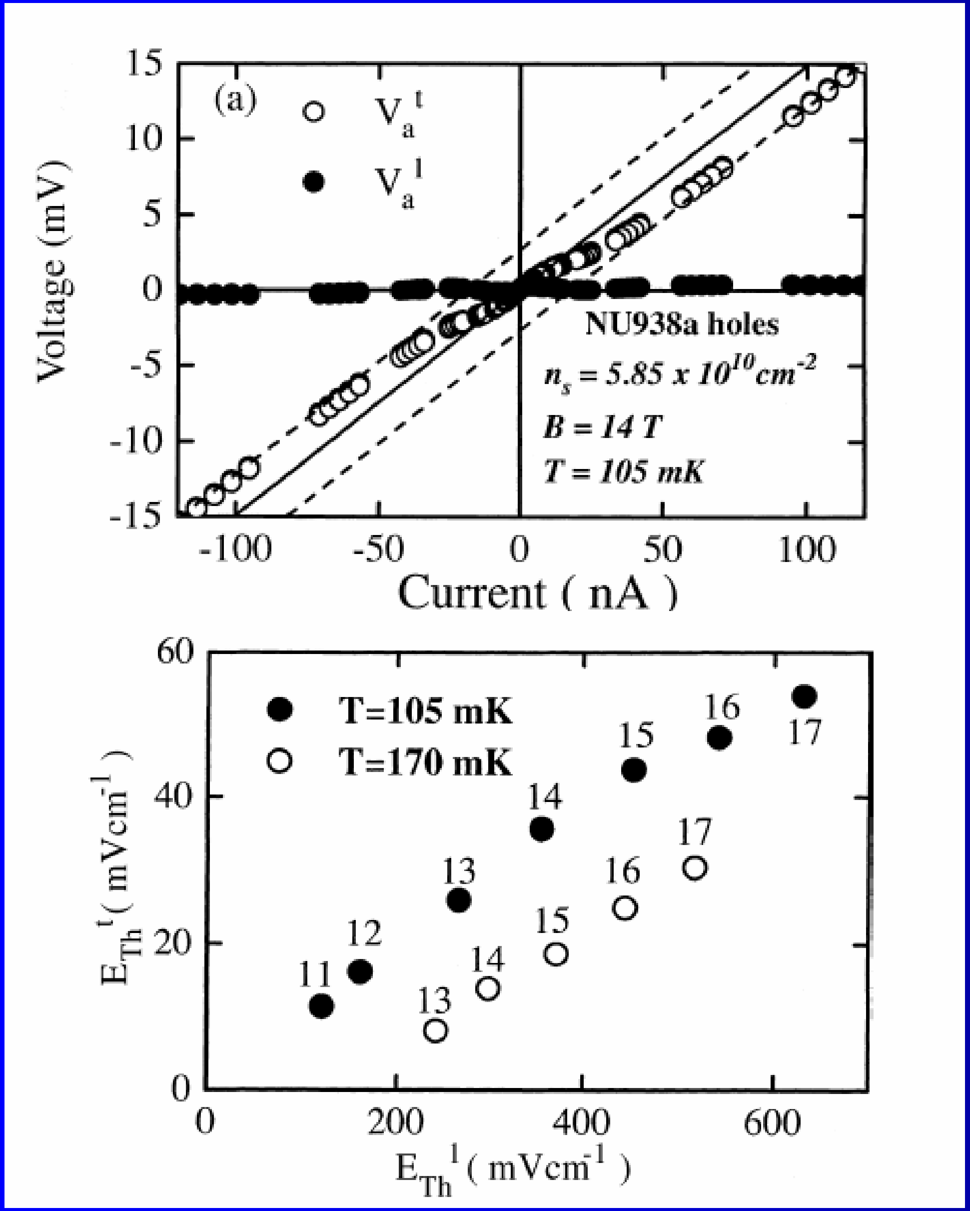
Moving glass







Absence of Hall voltage



- $F_{lor} < F_{tran}$: no hall voltage
- Compatible with the existence of a transverse threshold

F. Perruchot et al. Physica B 256 587 (1998)

Pandora Box

- Dislocations

    3D: lucky (Bragg glass)

    2D: dislocations control dc transport

    Melting

- Dynamics ??????
- Aging and other glassy goodies

Specialized Refs.

•Bragg glass :

T.G. + P. Le Doussal Phys. Rev. B 52 1242 (1995).
T.G. + P. Le Doussal Phys. Rev. B 55 6577 (1997).
P. Le Doussal + T.G. Physica C 331 233 (2000).

Moving glass:

T.G. + P. Le Doussal Phys. Rev. Lett. 76 3408 (1996).
P. Le Doussal + T.G. Phys. Rev. B 57 11356 (1998).

• Creep from RG:

P. Chauve + T.G. + P. Le Doussal Phys. Rev. B 62 624 (2000).

•Quantum problems:

T.G. + P. Le Doussal Phys. Rev. B 53 15206 (1996).
R. Chitra + T.G. + P. Le Doussal Phys. Rev. B 65 035312 (2001)
T.G. + P. Le Doussal + E. Orignac Phys. Rev. B 64 245119 (2001)