Liquid metal dynamo experiments

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Dynamics and turbulent transport in plasmas and conducting fluids
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Bibliography

• A.Gailitis et al, Rev. Mod. Phys. 74 (2002)
• S. Fauve & D. Lathrop, *Laboratory experiments on liquid metal dynamos and liquid metal MHD turbulence* in Fluid dynamics and dynamo in astrophysics and geophysics (2005)
Outlook

• Background & Motivation

• Constrained lab dynamos
  – Riga & Karlsruhe exp.

• Unconstrained lab dynamo: the VKS experiment
  – Structure and dynamics of the Magnetic field
  – Elements for dynamo action in VKS.
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Dynamo action

Def: Instability that transfers mechanical energy into magnetic energy

Example: the Bullard dynamo

\[ L \frac{dI}{dt} = (M\omega - R)I \]

- \( L \) the self inductance of the spire
- \( M \) the mutual inductance spire-disk
- \( R \) the circuit resistivity

More realistic solid dynamo:
Tools for Liquid dynamos

\[ \partial_t \vec{B} + \vec{u} \cdot \nabla \vec{B} = \vec{B} \cdot \nabla \vec{u} + \frac{1}{\mu_0 \sigma} \Delta \vec{B} \]

\[ \rho [\partial_t \vec{u} + \vec{u} \cdot \nabla \vec{u}] = -\nabla P + \vec{J} \times \vec{B} + \eta \Delta \vec{u} \quad ; \quad \nabla \times \vec{B} = \mu_0 \vec{J} \]

\[ Rm = \sigma \mu_0 UL \quad ; \quad Re = \frac{UL}{\nu} \quad ; \quad N = \frac{\sigma B^2 L}{\rho U} \]

NB : \( Pm = \frac{Rm}{Re} = \mu_0 \sigma \nu << 1 \quad \Rightarrow \) The dynamo grows on a turbulent flow

\( Pm = 10^{-5} \) for Na

\[ P \propto \rho L^2 U^3 \iff Rm = \mu_0 \sigma \left( \frac{P}{\rho L} \right)^{1/3} \]

\[ Rm \leftarrow 2 \times Rm \Rightarrow P \leftarrow 8 \times P \]
Tools for Liquid dynamos.

Transfer mechanism in the homogeneous dynamos:

Ω-effect

\[ U \]

\[ B_{\perp} \]

\[ B_{\parallel} \]

α-effect

\[ \alpha \rightarrow \text{Dynamo} \]

\[ \alpha^2, \alpha - \Omega \Rightarrow \text{Dynamo} \]
The oldest experienced Dynamo: The Earth.

1st compass ~ 1040 in China
1600 in Europe → Magnetic dipole almost aligned with rotation axis

Since 1979 measurement with satellite

Oersted 1 (1999—2002)

From CHAMP (2000-2010)
Characteristics of the Earth dynamo

Larmor 1919: Earth magnetic field = dynamo due to the convective motion of the liquid core.

Magnetic Field $B \sim 10^{-4} T$

Magnetic Energy $B^2 L^3 / 2 \mu_0 \sim 2 \times 10^{17} J$

($L \sim 3 \times 10^6$)

Diffusion time $\mu_0 \sigma L^2 \sim 10^4$ years

Reynold Number $\sim 10^9$

Magnetic Reynolds Number $\sim 500$

Rossby number: $U/L \Omega \sim 10^{-6}$

Paleomagnetisme and Earth reversal

Inversion of the earth polarity $T \sim 10^6$

[Glatzmaier Roberts Nature 377 (1999)]
Other example: the sun

Magnetic Field $B \sim 10^{-4}T$

Magnetic Energy $B^2 L^3/2 \mu_0 \sim 4 \times 10^{22} \text{ J}$

$(L \sim 2 \times 10^8)$

Diffusion time $\mu_0 \sigma L^2 \sim 10^6 \text{ years}$

Reynold Number $\sim 10^{14}$

Magnetic Reynolds Number $\sim 10^8$

Dynamics with a period of 11 years
Motivations for controlled Dynamo experiments

• Dynamo with small $Pm$ ($\ll$ simulation) → closer to geo & astrophysical object
• Instability on turbulent flow
  – Role on dynamo onset?
  – Instability with multiplicative noise
    • Intermittency?
    • Scaling for saturation $B \propto (Rm - Rmc)^\beta$; $\beta$?
• Energy partition
• Dynamical regime
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First dynamos generation

• Constrained flows reproducing some theoretical prediction
  
  – The Riga Dynamo and the Ponomarenko flow
  
  – The Karlsruhe dynamo and the Robert flow
The Ponomarenko flow and the Riga Experiment

Ponomarenko flow: helicoidal flow + fluid at rest
→ $Rm_c = 18$ oscillating (propagating) dynamo


$\Omega < 40 \text{ Hz; } P < 300 \text{kW}$
The Riga Dynamo

[Gailitis et al PRL 86-14 (2001); PoP 11-5 (2004)]

Growth of an oscillating magnetic field at almost the expected value

Ohmic lost about 10%

\[ B \propto (\Omega - \Omega_c)^\beta \quad ; \quad \beta = f(z) \]
The Robert flow and the Karlsruhe experiment

The Robert’s flow

Prediction [Busse et al Magnetohydrodynamics 32 (1996)]

\[ \text{Re}_C \cdot \text{Re}_H > 4\pi \{ 1 + \left[ \left( \frac{3.83}{\pi} \right) \cdot \left( \frac{d}{r_o} \right) \right] \} \]

\[ \Rightarrow \text{Electromagnetic Pump with a flow rate up to 150 m}^3/\text{h} \]
The Karlsruhe dynamo

[Stieglitz & Müller PoF 13-3 (2001)]

Stationary imperfect bifurcation
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The von Karman flow

Poloidal & Toroidal (P/T)
The VKS experiment


• Diameter \( \sim 500 \text{ mm}, 150 \text{ l of liquid sodium (120}^\circ \text{ C)} \Rightarrow Pm = 10^{-5} \)

• 4 motors \( \Rightarrow 300 \text{ kW} \)

• Sensors: 40 Hall probes (3 components), 2 torquemeters, 1 potentiel velocity probe (2 components)

• \( Rm = \mu_0 \sigma R_{cell} R_{disk} (2 \pi F) \leq 62; \text{ Re} \sim 10^6 \)

• Fluctuation rate:

\[
\frac{\langle u^2 \rangle - \langle u \rangle^2}{\langle u \rangle^2} \approx 50 - 100\%
\]
VKS In its 1st Dynamo configuration

10 years of Optimization ➔ Aspect ratio
+ Blades curvatures
+ Sodium at rest;
+ Ring in the equatorial plane
+ Soft-Iron impellers

⇒ Steel imp.
⇒ No Dyna.

⇒ Dyna

⇒ Dynamo !!! (September 2006)
At $F_1 = F_2$ : Stationnary dynamo

[Monchaux et al PRL 98 2007]

Ohmic lost of few % compared to stainless steel impellers
Dynamo onset

![Graph showing $P_0$ vs. $R_m - R_m^c$ with data points for demagnetized and magnetized impellers.]

Imperfect bifurcation due to iron impellers magnetization Earth magnetic field …

$$B \propto (R_m - R_{mc})^\beta \ ; \ 1/2 \leq \beta < 1$$

- Demagnetized impellers
- Magnetized impellers
Scaling for the saturated field

[Turbulent flow $\Rightarrow$ balance:

$$\rho \bar{u} \cdot \nabla \bar{u} \approx \frac{1}{\mu_o} \left( \nabla \times \bar{B} \right) \times \bar{B}$$

$$u \sim u_c \Rightarrow B^2 \propto \rho \mu_o \cdot u \cdot u_c$$

$$\Rightarrow \frac{B^2}{\left( \frac{\rho}{\sigma^2 \mu_o L^2} \right)} \propto Rm_c \cdot Rm$$

[Petrelis Mordant Fauve GAFD 101 (2007)]

- VKS
- Riga
- Karlsruhe
Correlation length

Instantaneous correlations on scales of order $R$

[monchaux et al PoF 21 (2009)]
The magnetic mode ($F_1 = F_2$)

Mainly an axial dipole
- Numerical simulations (mean flow) → Equatorial dipole
- Cowling Theorem: Axisymmetrical flow CANNOT generates an axial dipole.

No-axisymmetrical fluctuations are necessary to get this dynamo
The magnetic mode ($F_1 = F_2$)

The shape of the mode does not depend on $Rm$

- $Rm=34$
- $Rm=36$
- $Rm=41$
Dynamical regimes for $F_1 \neq F_2$

Breaks the $R_{\pi}$ symmetry
~Adding a global rotation
Reversal (Earth)

\[ \frac{\tau_{\text{exp}}}{T_{\text{exp}}} = \frac{\tau_{\text{Earth}}}{T_{\text{Earth}}} \]

\[ F_1/F_2=16/22 \]

[Berhanu et al. EPL 77 5900 (2007)]
Similarity with Earth reversal

[From F. Petrelis, S. Fauve, E. Dormy, J. P. Valet *PRL* 102 (2009) ]
Limit cycle

F$_1$/F$_2$=18.5/22

[Ravelet et al PRL 101, 074502 (2008)]
Bursts

[Image of data plots]

[15/21

$B_\theta(t)$ (G)

15/22

$B_\theta$ (G)

[Ravelet et al PRL 101, 074502 (2008)]
Oscillations

→Sun?
Dynamo with one impeller: Bistability

[Berhanu et al JFM 641 (2009)]

Two kinds of dynamo in the same range but not by the same path
$F_2 = 26 \text{ Hz}$

$F_2 = 26.5 \text{ Hz}$
Low dimensional model


• Near the onset only 2 interacting modes \( D(t).d(r) \) symmetric and \( Q(t).q(r) \) anti-symmetric
  – Time evolution for this 2 modes \( D(t) \) and \( Q(t) \)?
  – \( A(t) = D(t) + iQ(t) \)

• Symmetry considerations:
  \[
  B \leftarrow -B \Rightarrow D \leftarrow -D; Q \leftarrow -Q \quad \text{and} \quad A \leftarrow -A
  \]

\[
\dot{A} = \mu A + vA + \beta_1 A^3 + \beta_2 |A|^2 A + \beta_3 |A|^2 \overline{A} + \beta_4 \overline{A}^3 + o(A^5)
\]

For \( F_1 = F_2 \) \quad \( R_\pi : D \leftarrow -D; Q \leftarrow Q \) and \( A \leftarrow -\overline{A} \)

⇒ All coefficient real

• Multiplicative noise \( \xi \)
Low dimensional model

\[ \dot{D} = (\mu_r + \nu_r)D + (\nu_i - \mu_i)Q + \xi_1 D - \left( aD^3 + bD^2Q + cQ^2D + dQ^3 \right) \]
\[ \dot{Q} = (\mu_r - \nu_r)Q + (\nu_i + \mu_i)D + \xi_2 Q - \left( a'Q^3 + b'Q^2D + c'D^2Q + d'D^3 \right) \]

\( \mu_r \) and \( \nu_r \) even function of \( F_1 - F_2 \)
\( \mu_i \) and \( \nu_i \) odd function of \( F_1 - F_2 \)

At \( F_1 = F_2 \): D and Q decoupled
Dipole grows first: \( \nu_r > 0 \) for \( F_1 \cong F_2 \)

For: \( 0 < \mu_r - \nu_r < \mu_r + \nu_r \) \( Q(t) \) is solution but instable via a dipolar perturbation
\[ A = D + iQ = R \cdot \exp(i \theta) \]
Path in phase space for limit cycle
Path in phase space for limit cycle
Reversal

Noise strong enough to cross the unstable mode and trigger reversal

Model can describe bi-stability by a codimension-2 bifurcation point
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Ingredients for dynamo Action

• Reminder: Dynamo reaches with
  – A ring in the mid-plane
  – A optimized curvature of the blade (ratio P/T)
  – A given aspect ratio
  – Continuous electrical BC
  – Soft iron impellers

• Question: Which ingredients are really necessary and why??
Role of the ring in the equatorial plane

- No dynamo
- Stationary
- Dynamical
Curvature of the blades

The threshold is increased in the reverse sense

Without ring in the equatorial plane, no dynamo

\[
\frac{P_r^+}{T_r^+} < \frac{P_r^-}{T_r^-} < \frac{P_{wr}^-}{T_{wr}^-}
\]

The sense of rotation not essential but pumping and shear act on the threshold
Sodium at rest and Aspect Ratio

Without the inner cylinder → dynamo at same $Rm = \mu_0 \sigma R_{cell} \cdot R_{disk} \cdot (2 \pi F)$

Conclusion: Magnetic boundary conditions on the impellers is the main element that improves dynamo action
**Possible mechanism**

[Petrelis et al GAFD. 101 (2007)]

Requirement: axial magnetic field necessitates non axis-symmetric fluctuations of the flow

vortices at the edge of the blades: $B_\theta \rightarrow B_{axe}$ by $\alpha$-effect

Action of the Soft-Iron impellers?

Shear: $B_{axe} \rightarrow B_\theta$ by $\Omega$–effect
Kinematic simulation of non-axis-symmetric flow

[CJP Gissinger EPL 87 2009]

Synthetic mean flow + 8 rotating vortices

Axial magnetic field at the threshold
Axial dipole and quadrupole threshold similar $\Rightarrow$ dynamical regime if
$R_{\pi}$ symmetry is broken

Suggest that : Soft-Iron boundary conditions (at rest) decrease the threshold,
Localized alpha-effect and periodic boundary condition


Local $\alpha$-effect $\Rightarrow$ axial dipole

Realistic impellers $\Rightarrow$ realistic $\alpha$-effect
Periodic boundary conditions help to decrease the intensity of the $\alpha$-effect by Busse & Wicht mechanism

Suggest that the periodic permittivity due to blades are important
Omega effect the moving disk

Herzenberg et Lowes Phil. Trans. R. Soc. A 249 (1957)]

Omega effect due to the jump of the magnetic permeability of the moving disks

\[ B_\theta \approx B_{ax} R m_{disk} \frac{V_{disk}}{r^3} \]

Role of the soft iron blades?
Distance to the threshold

• Decay time Below the threshold of an applied B (axial or equatorial)

\[
\lim_{\tau(Rm) \to 0} \frac{1}{\tau(Rm)}\left\langle B^2 \right\rangle_c = 0
\]

• Form induction

\[
\lim_{Rm \to Rmc} \frac{|B_{app}|^2}{|B(Rm)|^2} = 0
\]
Copper impellers

Role of electrical conductivity of the impeller?

$$\sigma_{Cu} > \sigma_{Iron} > \sigma_{Steel}$$
Copper impellers

Role of electrical conductivity of the impeller?

\[ \sigma_{Cu} > \sigma_{Iron} > \sigma_{Steel} \]

Higher conductivity \( \implies \) dynamo action
No flow behind the impellers

Idea: soft-iron impellers decouple the magnetic field from the unfavorable flow behind them

No flow behind the impellers

Idea: soft-iron impellers decouple the magnetic field from the unfavorable flow behind them


: No dynamo but the induction is improved and decay time \( \Rightarrow Rmc \sim 70-80 \)

(which mode ?)
Motionless soft-iron disks

Motion of the iron necessary?

- Copper
- Soft iron
- Stainless steel
Motionless soft-iron disks

Motion of the iron necessary?

Answer: YES
High permittivity BC

Full soft iron the most favorable (following C. Gissinger EPL) ?

- Copper
- Soft iron
- Stainless steel
High permittivity BC

Full soft iron the most favorable (following C. Gissinger EPL)

Answer: No but the threshold them closer than without (by study of the decay time)
Mixed configuration

Question: rotation of the iron impeller necessary?
Question: rotation of the iron impeller necessary?

Answer: Yes, if the soft-iron impeller does not move: no dynamo.
Mixed impellers I

Question : Improvement due to the soft-iron blades ?
• Gieseck *et al* suggestion ;
• Constrain on field-line (S. Fauve) ?

<table>
<thead>
<tr>
<th>Copper</th>
<th>Soft iron</th>
<th>Stainless steel</th>
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B
Mixed impellers I

Question: Improvement due to the soft-iron blades?
• Gieseck et al suggestion;
• Constrain on field-line (perpendicular to the blades at $\mu \to \infty$) ?

Answer: NO decay time and induction gives a threshold above $R_m \sim 100$
Mixed impellers II

Question: Dynamo due to the disk?
- Verhille et al. Suggestion
- Most of the iron masse

| Copper |
| Soft iron |
| Stainless steel |
Mixed impellers II

Question: Dynamo due to the disk?
- Verhille et al. Suggestion
- Most of the iron masse

Answer: NO !!!

- Copper
- Soft iron
- Stainless steel
Conclusions

• **Generation of the dynamo**: no-axisymmetrical fluctuations $\rightarrow$ axial magnetic dipole
  - No exact understanding of the role of BC on field generation

• **Dynamical Regimes**: Few modes involved in the dynamics, fluctuations act as noise triggering the reversals
  - Share similarities with Earth reversal

• **Accurate velocity and power measurements** (feedback $u \leftrightarrow B$)

• **Unconstrained dynamo without soft iron**