ASTRES – *ENS de Lyon* (SISYPHE)

- N. Pustelnik, P. Borgnat, P. Flandrin : « Empirical Mode Decomposition revisited by multicomponent non smooth convex optimization », *Signal Proc.*, 2014
- J. Schmitt, N. Pustelnik, P. Borgnat, P. Flandrin : « 2D Hilbert-Huang transform », *ICASSP-14*, Florence, 2014
- J. Schmitt, N. Pustelnik, P. Borgnat, P. Flandrin, L. Condat : « A 2-D Prony-Huang Transform: A New Tool for 2-D Spectral Analysis, » *IEEE Trans. on Image Proc.*, à paraître
- N. Tremblay, P. Borgnat, P. Flandrin : « Graph Empirical Mode Decomposition, » *EUSIPCO-14*, Lisbonne, 2014
- R. Fontugne, P. Borgnat, P. Flandrin : « On-line Empirical Mode Decomposition, » *IEEE Signal Proc. Lett.*, soumis
- P. Flandrin : « On the maxima of white Gaussian noise spectrograms, » *Found. of Comp. Math. FoCM-14*, Montevideo, 2014

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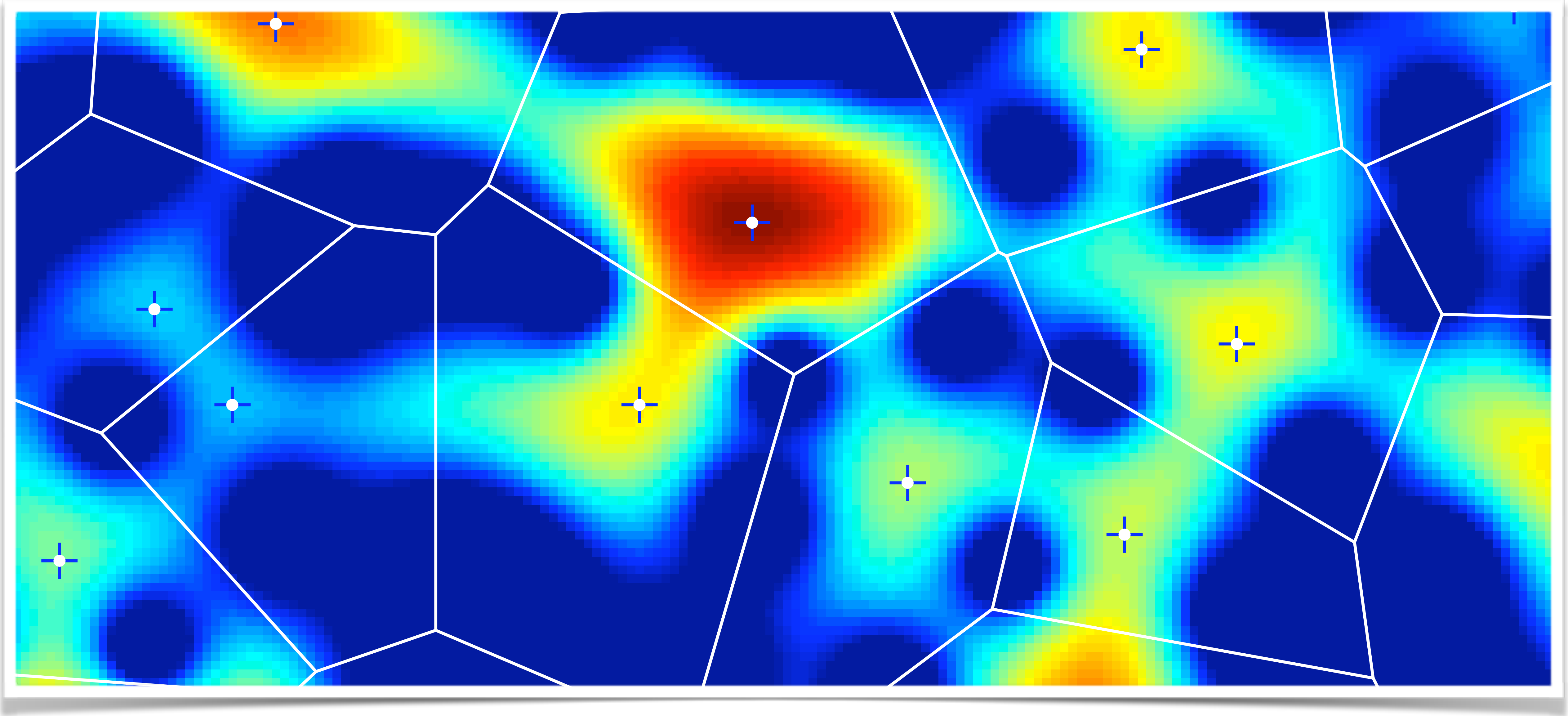

On the maxima of white Gaussian noise spectrograms

Patrick Flandrin

Spectrogram extrema?

- Built-in redundancy (Heisenberg uncertainty) [Daubechies, ’90]
- Full characterization by zeros in the case of Gaussian windows (Hadamard-Weierstrass factorization) [Korsch *et al.*, ’97]
- Reassignment vector field driven by local maxima [Chassande-Mottin *et al.*, ’97][F. *et al.*, ’03]

(follow-up of [F. *et al.*, ’12])

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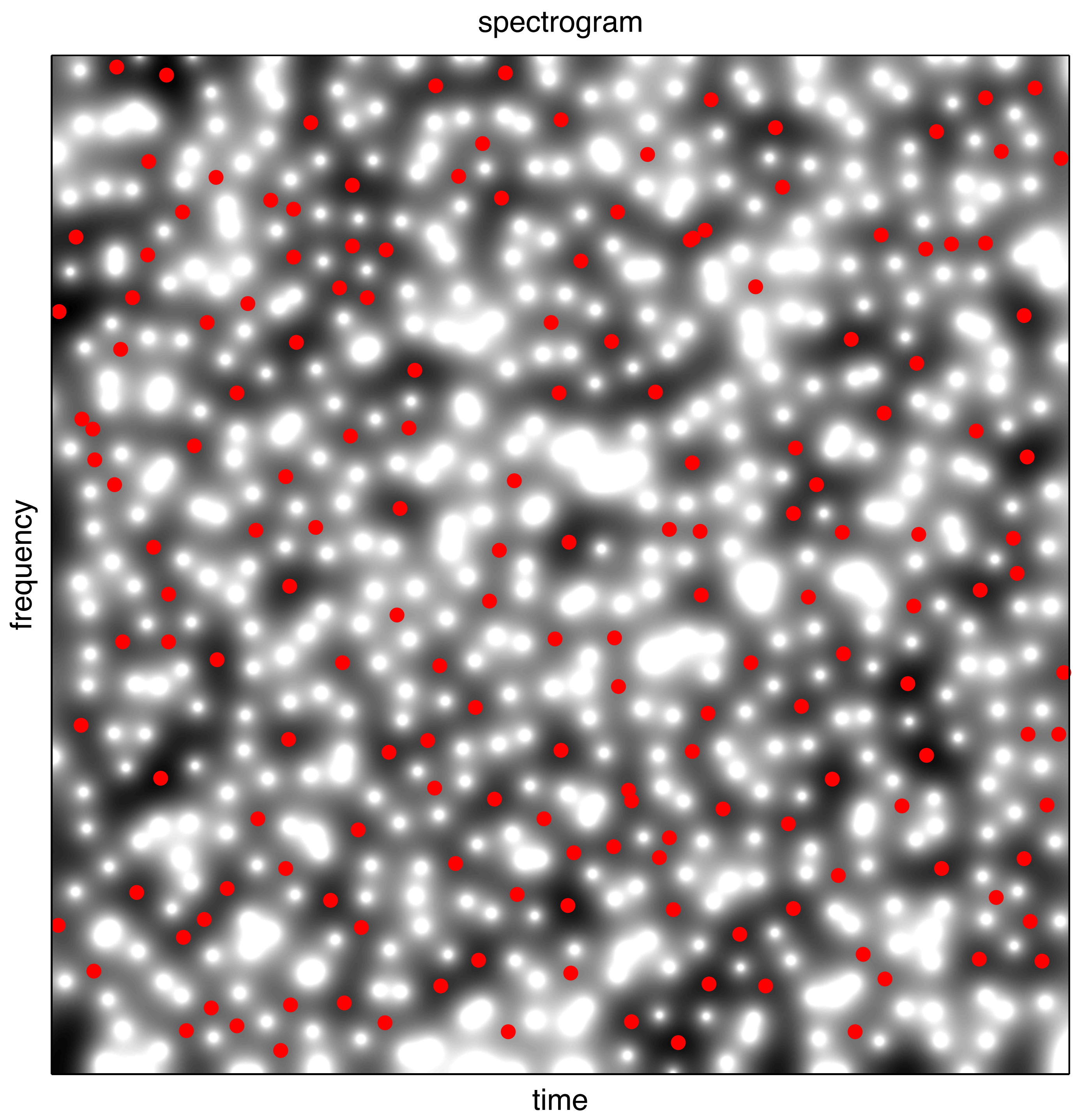
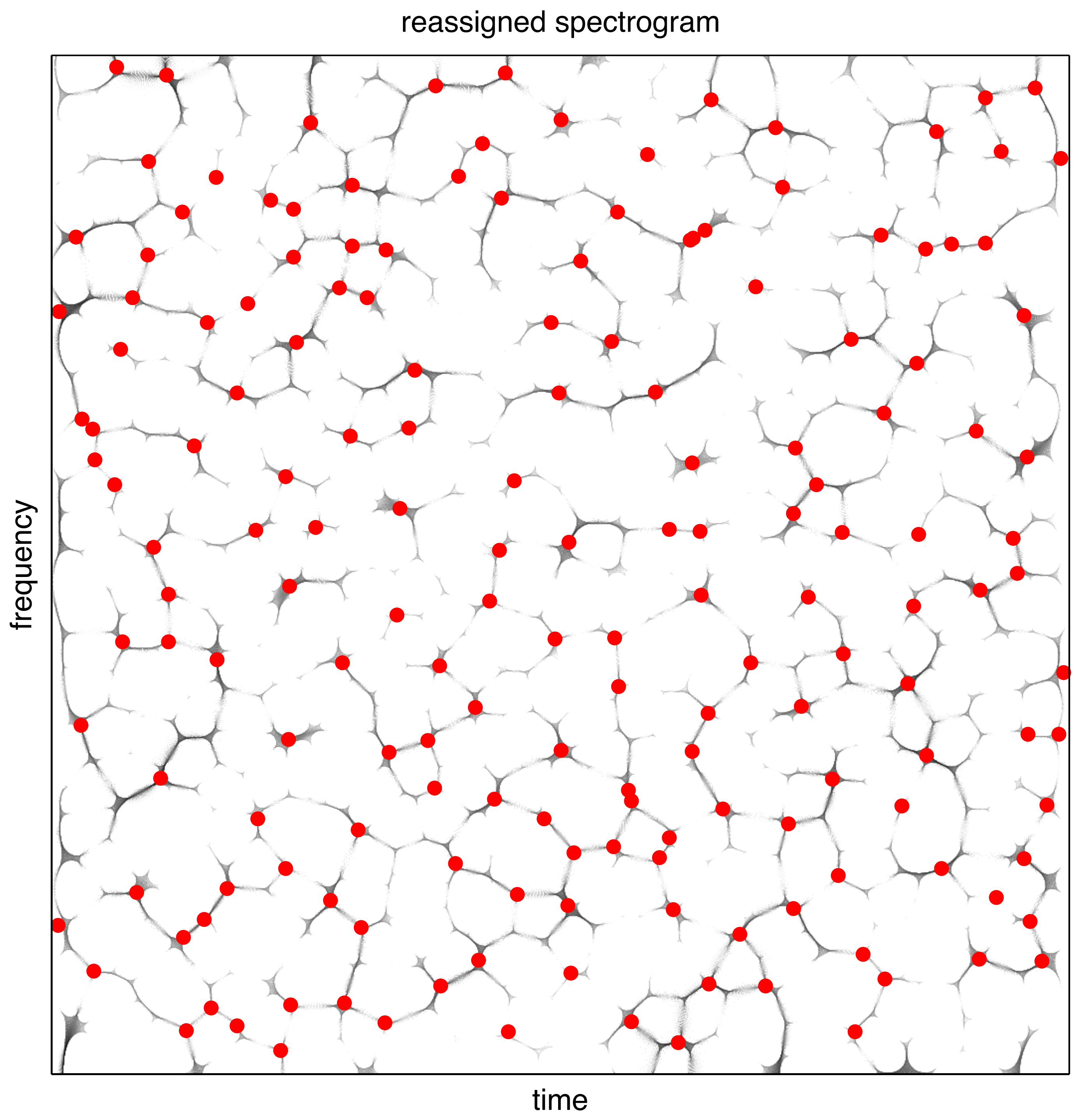
The generic example of white Gaussian noise



The generic example of white Gaussian noise



Reproducing kernel

Spectrogram reads $S_x^{(h)}(t,\omega) = \left|F_x^{(h)}(t,\omega)\right|^2$, with

$$F_x^{(h)}(t,\omega) = \int_{-\infty}^{+\infty} x(s)\,\overline{h(s-t)}\,\exp\left\{-i\omega(s-\frac{t}{2})\right\}\,ds$$

such that

$$F_x^{(h)}(t',\omega') = \iint_{-\infty}^{+\infty} K(t',\omega';t,\omega)\,F_x^{(h)}(t,\omega)\,dt\,\frac{d\omega}{2\pi}$$

with

$$K(t',\omega';t,\omega) = \frac{1}{\|h\|_2^2}\exp\left\{\frac{i}{2}(\omega t' - \omega' t)\right\}\,F_h^{(h)}(t'-t,\omega'-\omega)$$

Uncertainty

Inequalities in terms of

- volume [Lieb, ’90]

$$\begin{cases} \|F_x^{(x)}\|_p \geq (2/p)^{1/p}\|x\|_2^2 & \text{for} \quad p < 2 \\ \|F_x^{(x)}\|_p \leq (2/p)^{1/p}\|x\|_2^2 & \text{for} \quad p > 2 \end{cases}$$

- support [Gröchenig, ’11]

$$\iint_\Omega |F_x^{(x)}(\xi,\tau)|^2 \, d\tau \, \frac{d\xi}{2\pi} \geq (1-\epsilon)\|x\|_2^2 \Rightarrow |\Omega| \geq 1-\epsilon$$

Spectrogram of white Gaussian noise

With a « circular » Gaussian window

$$g(t) = \pi^{-1/4} \exp\{-t^2/2\} \Rightarrow S_g^{(g)}(t,\omega) = \exp\left\{-\frac{1}{2}(t^2+\omega^2)\right\}$$

the spectrogram of wGn is an homogeneous field [F., ’99]:

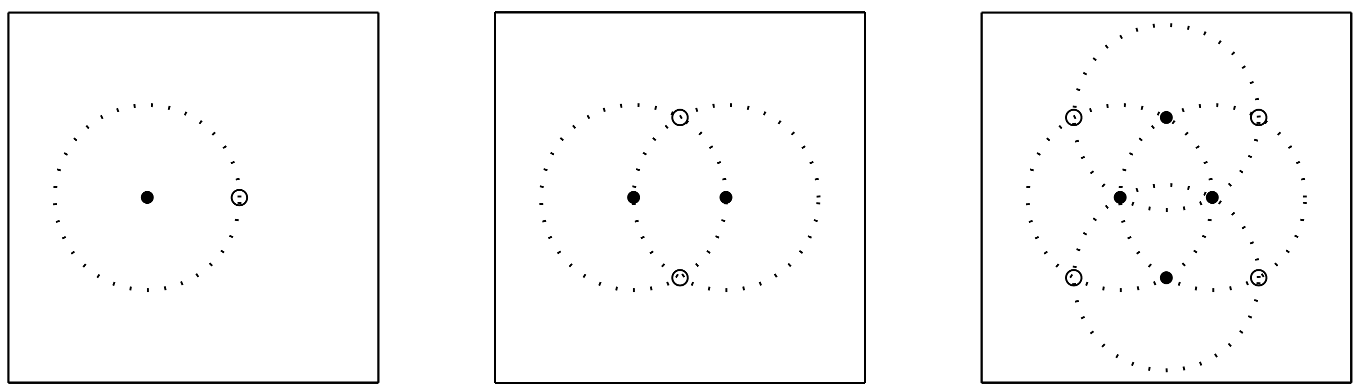
$$\operatorname{cov}\left\{S_n^{(g)}(t,\omega), S_n^{(g)}(t',\omega')\right\} = \gamma_0^2 \exp\left\{-\frac{1}{2}d^2((t,\omega),(t',\omega'))\right\}$$

with

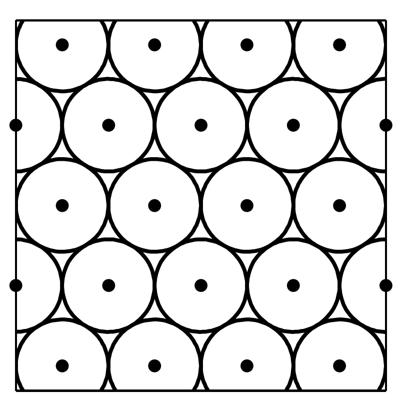
$$d((t,\omega),(t',\omega')) = \sqrt{(t-t')^2 + (\omega-\omega')^2}$$

A hexagonal lattice model for logons

- Gabor logons as minimum uncertainty building blocks [Gabor,'46]
- Circular symmetry and maximum packing [Conway & Sloane, '99]

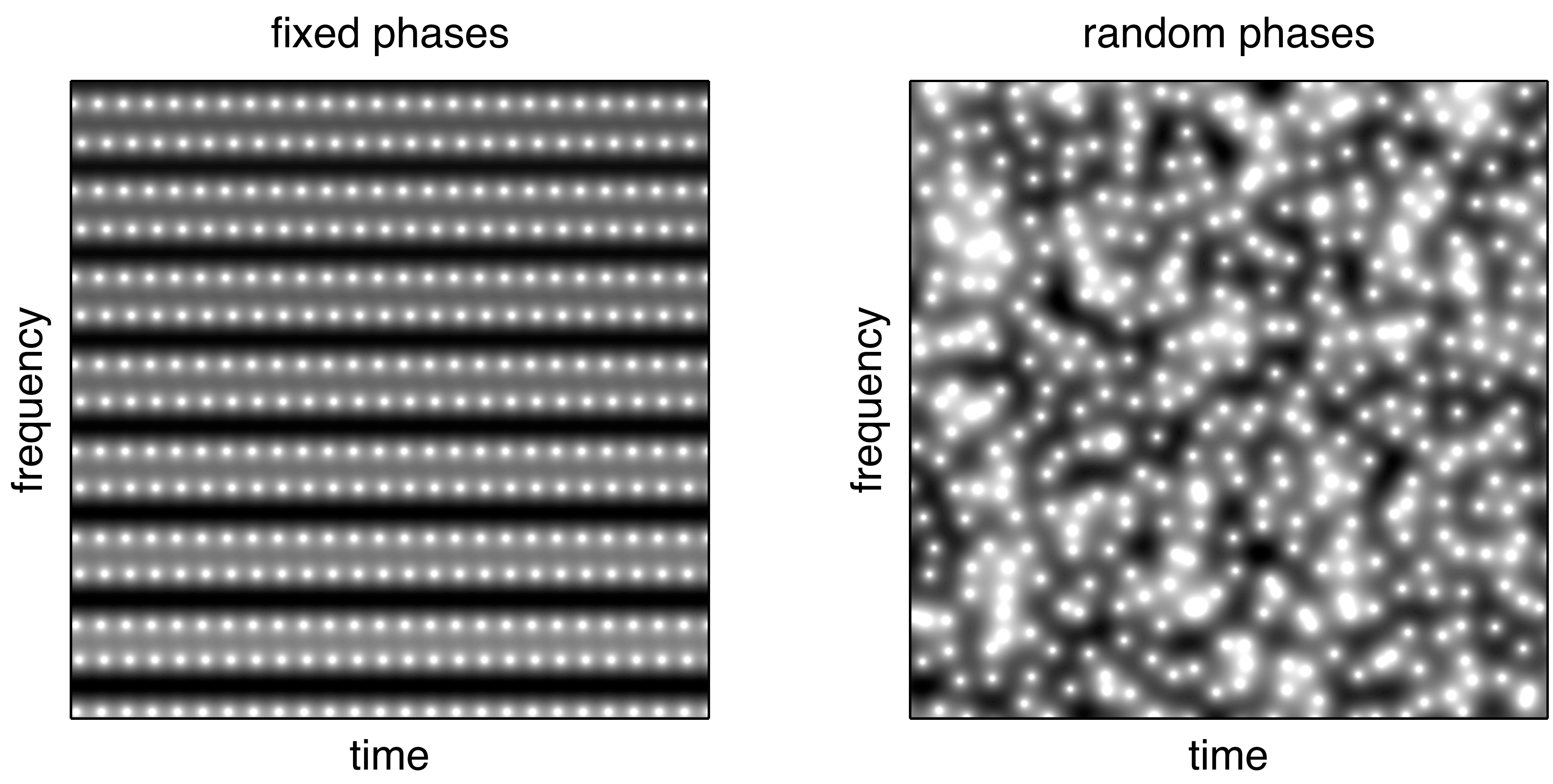


(a) (b) (c)



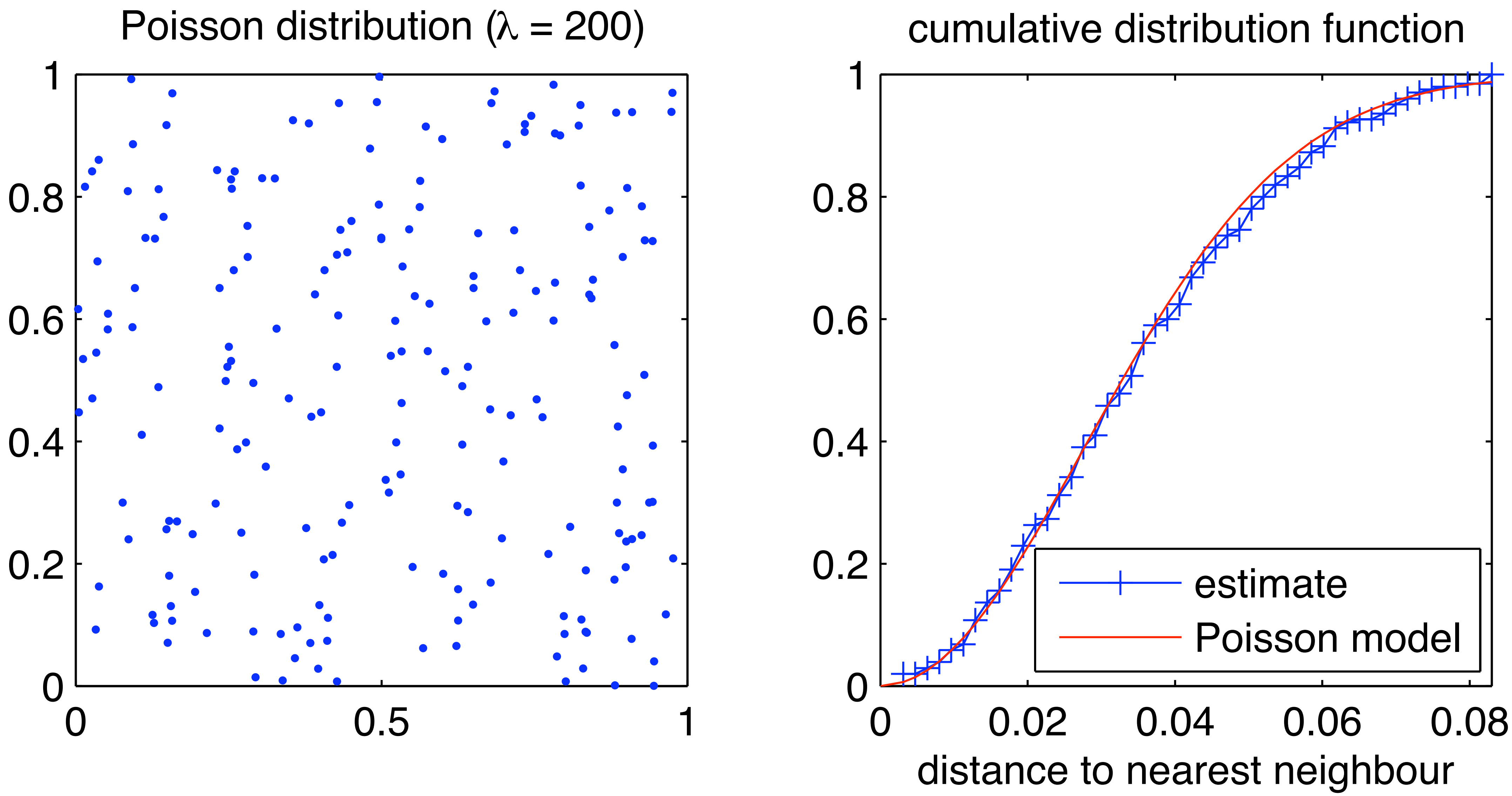
Random patches from random phases

$$x(t) = \sum_m \sum_n x_{mn}\, g(t - t_m)\, e^{i(\omega_n t + \varphi_{mn})}$$

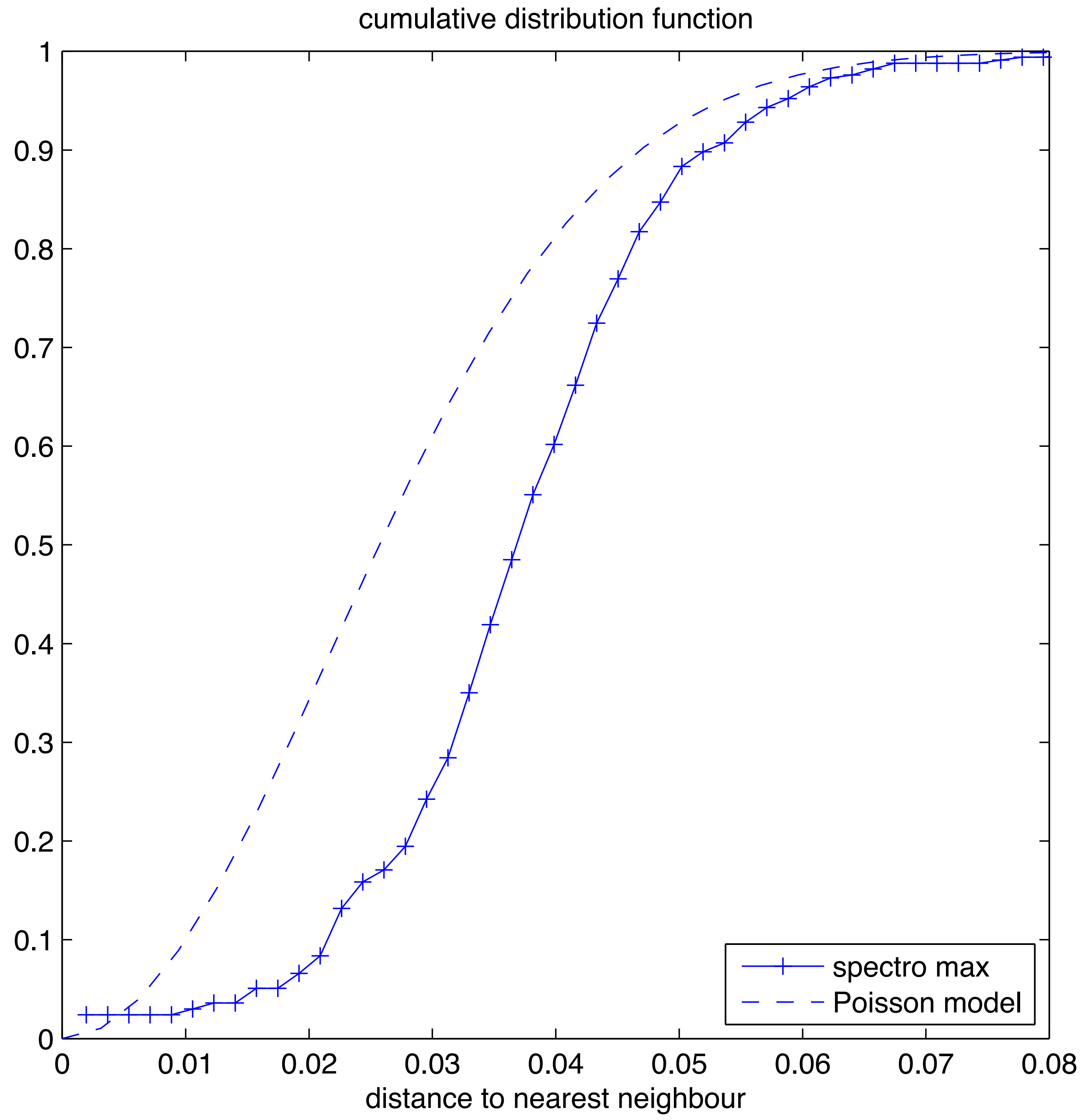


Local maxima: Poisson or not Poisson?

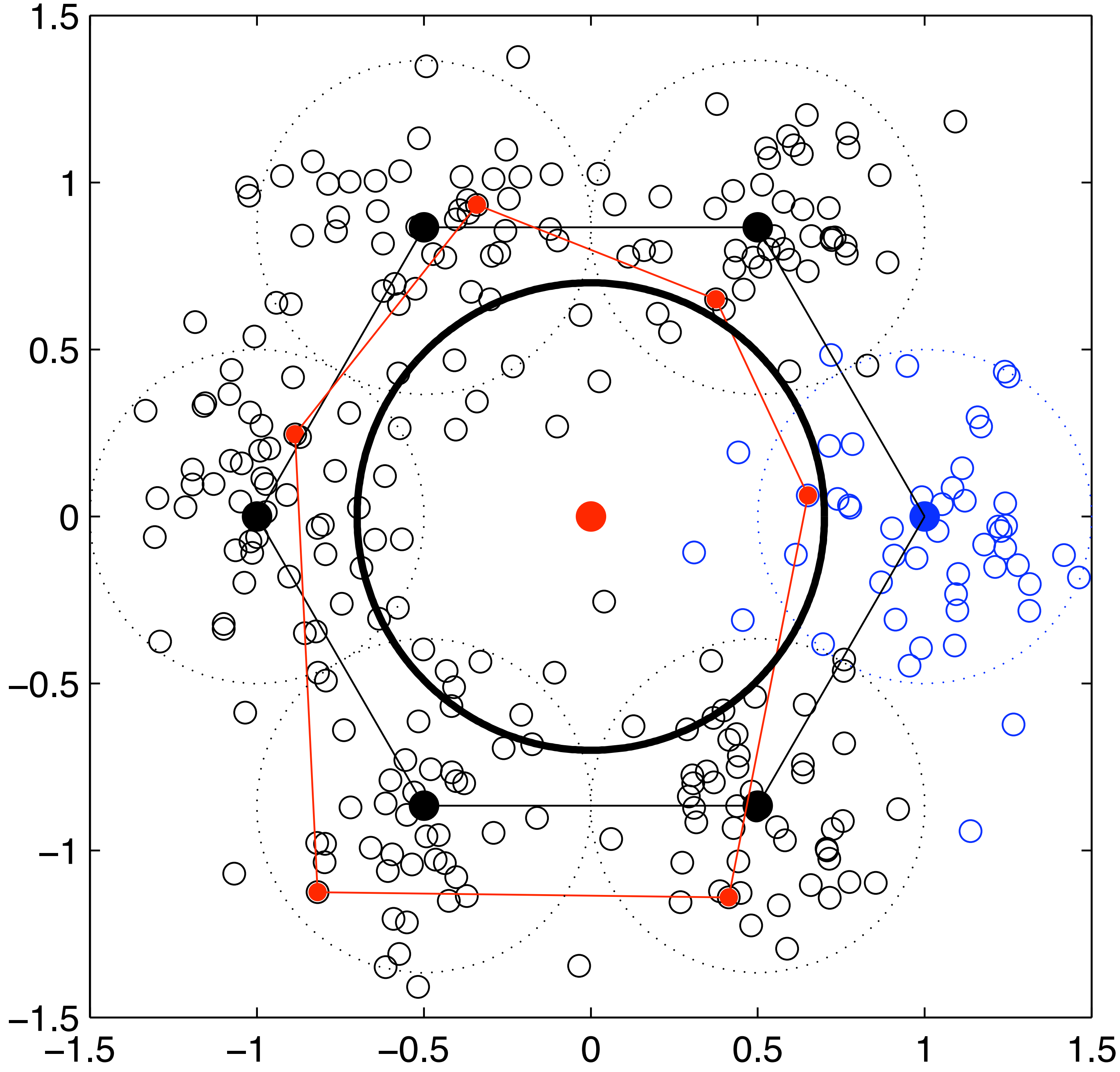
« Complete Spatial Randomness »: distribution of nearest-neighbour distance such that $\Pr\{D \leq d\} = 1 - \exp\{-\lambda\pi d^2\}$



Spectrogram extrema are not Poisson



A randomized lattice model for local maxima



Constrained spatial randomness

$$\mathrm{Prob}(D \leq d) = 1 - \left(1 - \int_0^d F(r; m, 2\sigma^2) dr\right)^6$$

with

$$F(r; m, \sigma^2) = \frac{r}{\sigma^2} \exp\left\{-\frac{1}{2\sigma^2}\left(r^2 + m^2\right)\right\} I_0\left(\frac{rm}{\sigma^2}\right)$$

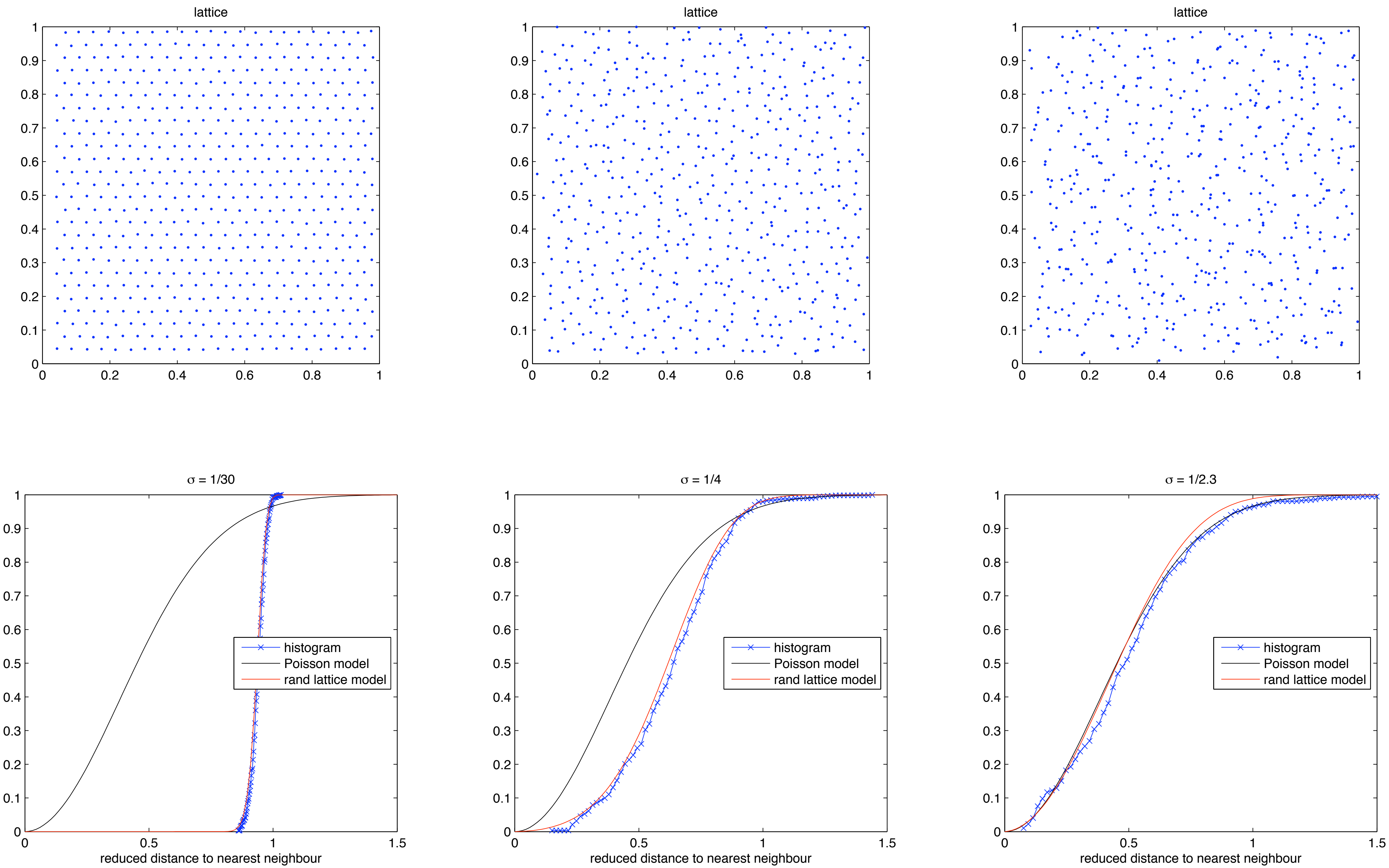
and

$$I_0(x) = \frac{1}{2\pi} \int_0^{2\pi} \exp\{x \cos\theta\} d\theta$$

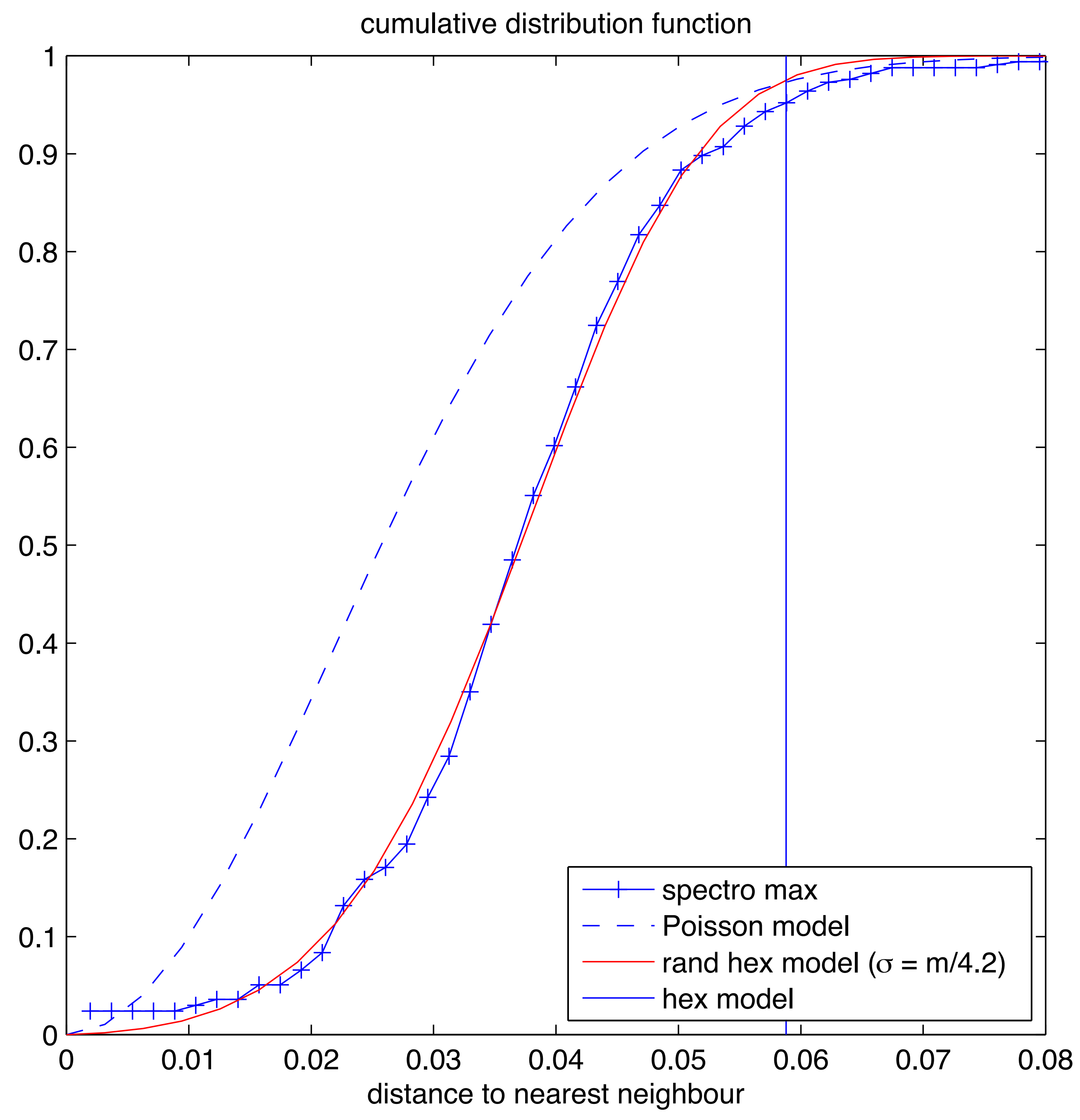
modified Bessel function of 1st kind

(adapted from [Stirling Churchman *et al.*, '08])

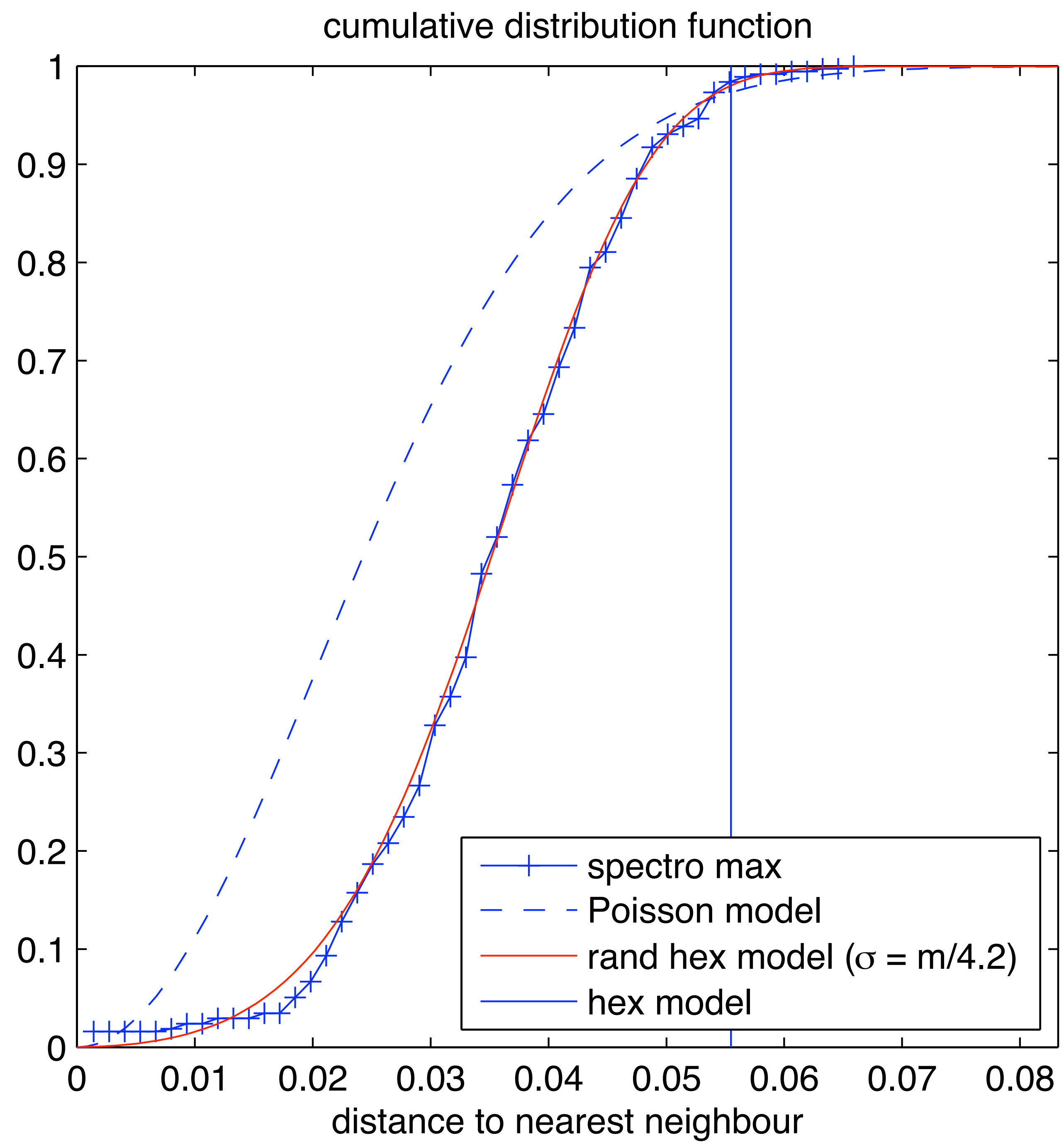
From order to disorder within the model



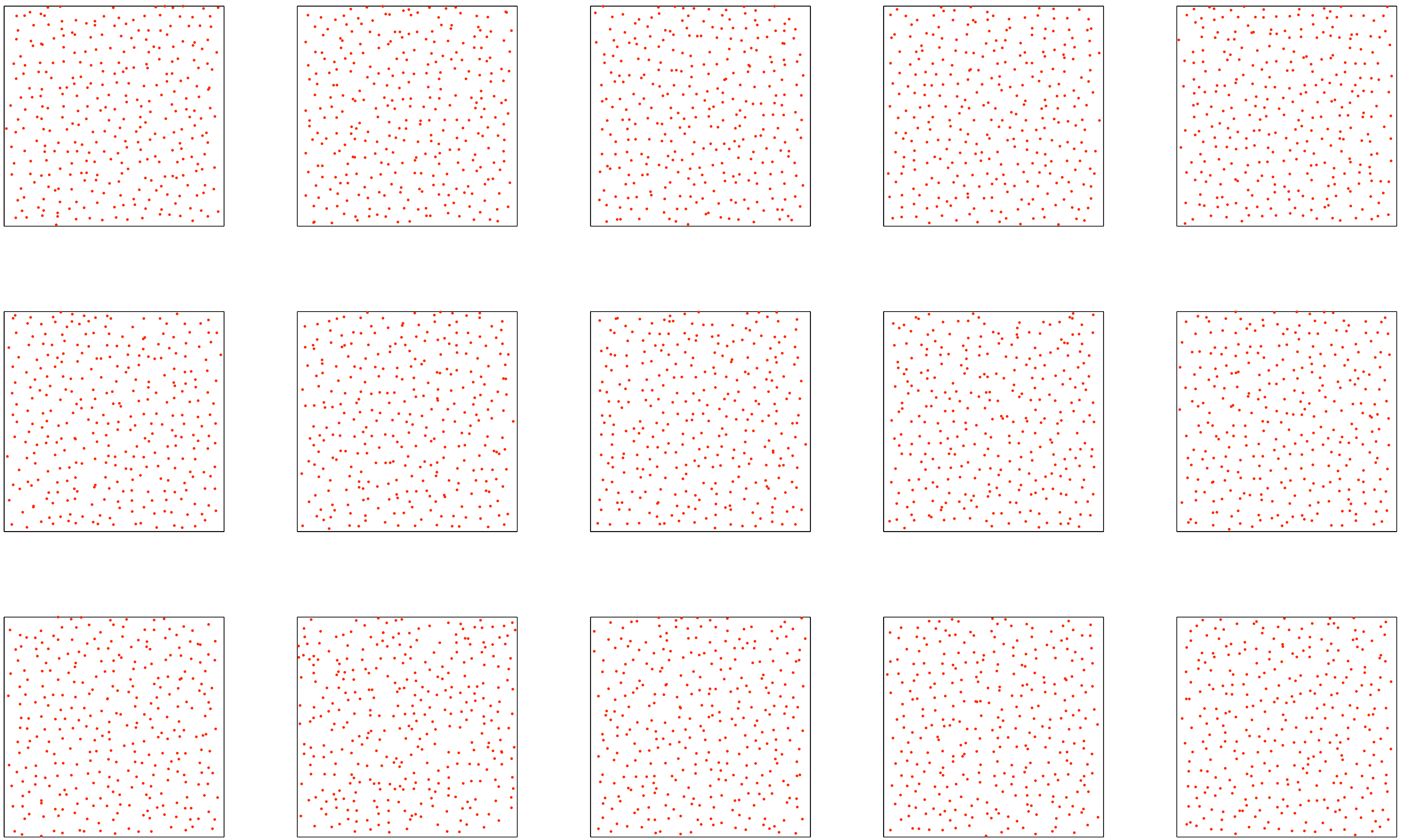
Actual spectrogram extrema vs. Poisson & model



Synthetic spectrogram extrema vs. Poisson & model

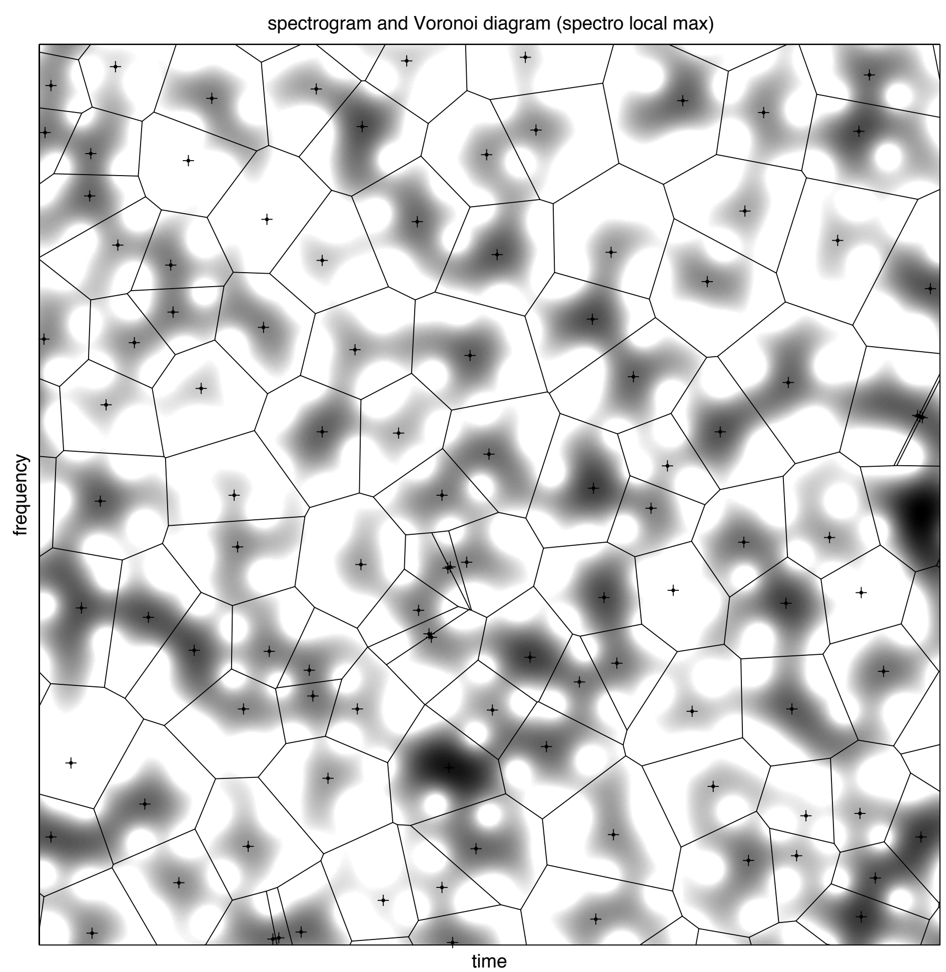


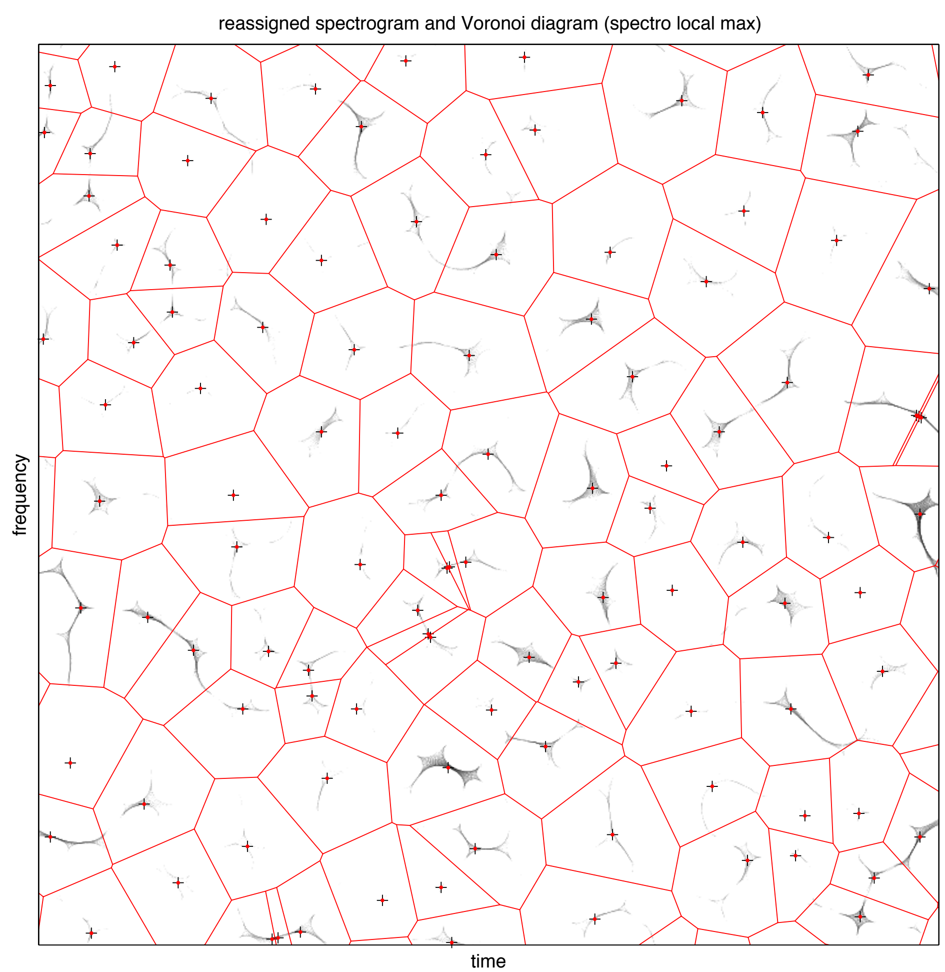
Spectrogram extrema vs. model



Voronoi diagram

Paving of the plane by means of disjoint, adjacent cells attached to local maxima and made of all points closer to a given maximum than to any other one [Okabe *et al.*, ’00]





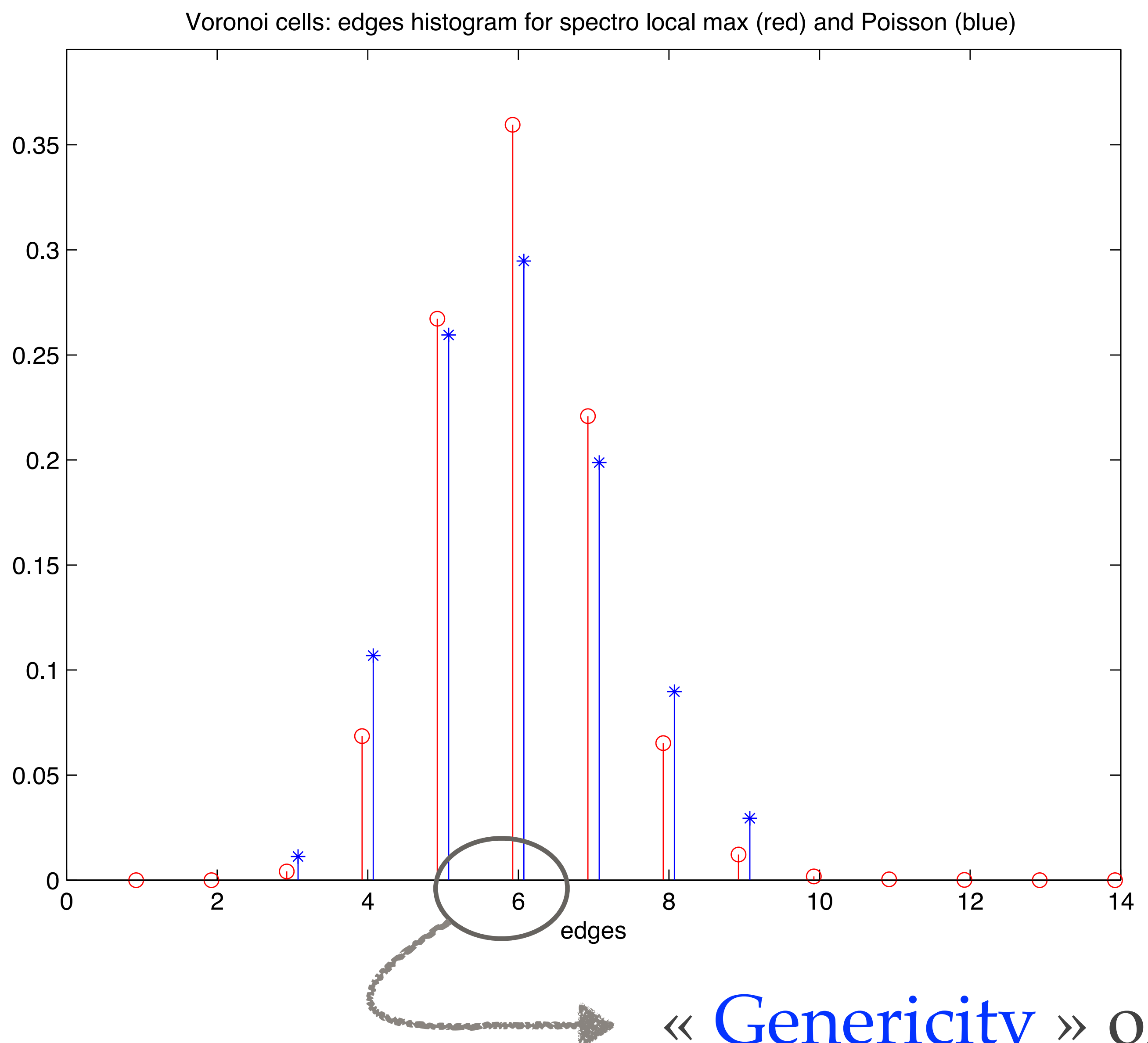
Voronoi diagram

- Simplified, polygonal representation of local energy patches
- Cells as basins of attraction for reassignment vector field [Chassande-Mottin *et al.*, ’97]:

$$\mathbf{r}_x(t,\omega) = \frac{1}{2}\nabla \log S_x^{(g)}(t,\omega)$$

- Several known results for statistics of Voronoi cells features in the Poisson case ([Calka, ’03][Ference & Néda, ’07][Frey & Schmidt, ’98][Lucarini, ’08][Stirling Churchman *et al.*, ’06][Stoyan *et al.*, ’95])

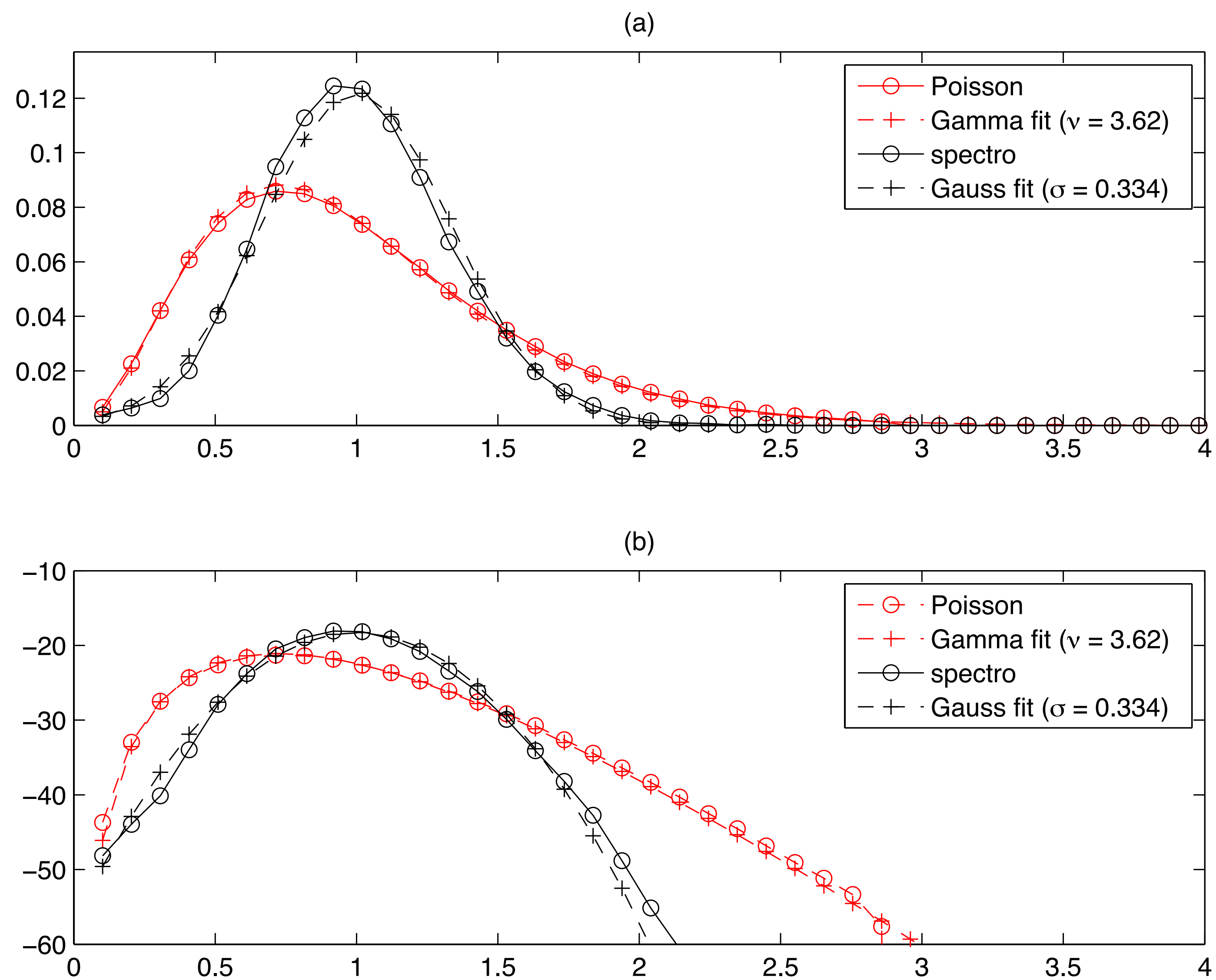
Cell edges



« Genericity » of hexagons [Lucarini, ’08]

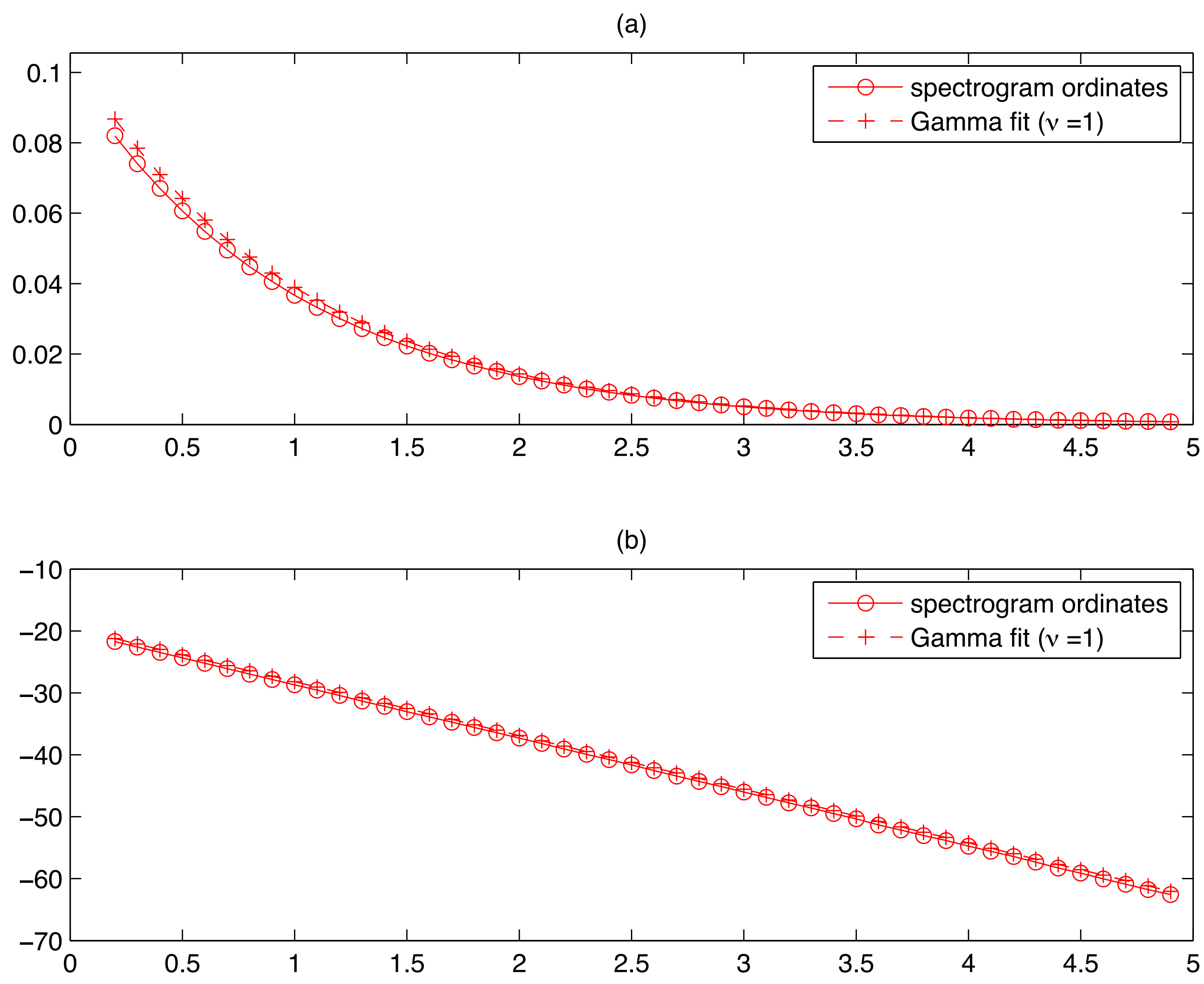
Cell area

No proof, but ample empirical evidence for Gamma in the CSR case (see, e.g., [Ference & Néda, '07] or [Senthil Kumar & Kumaran, '05])



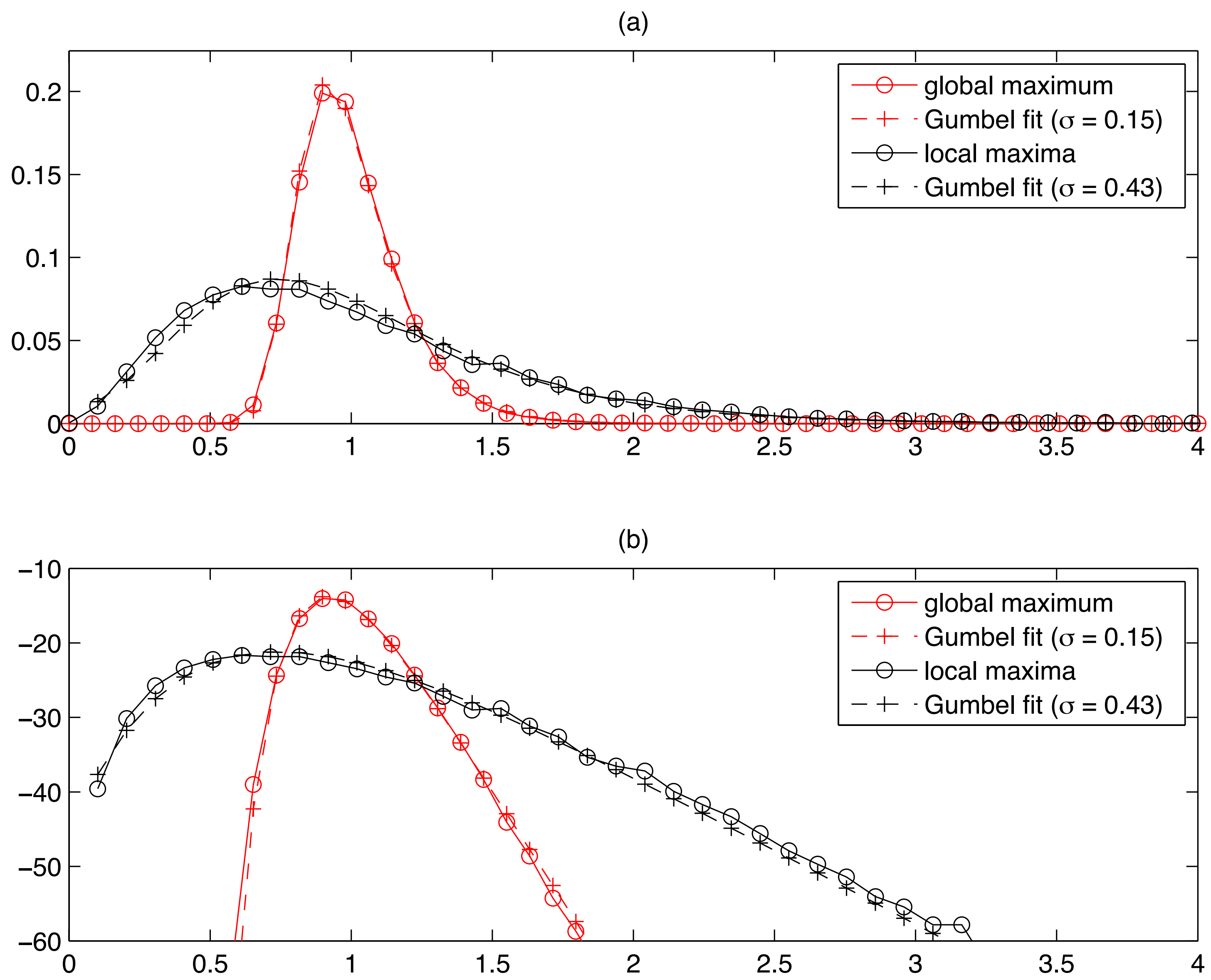
Spectrogram ordinates

Exponential distribution (see, e.g., [Durrani & Nightingale, ’73] or [Huillery *et al.*, ’08])

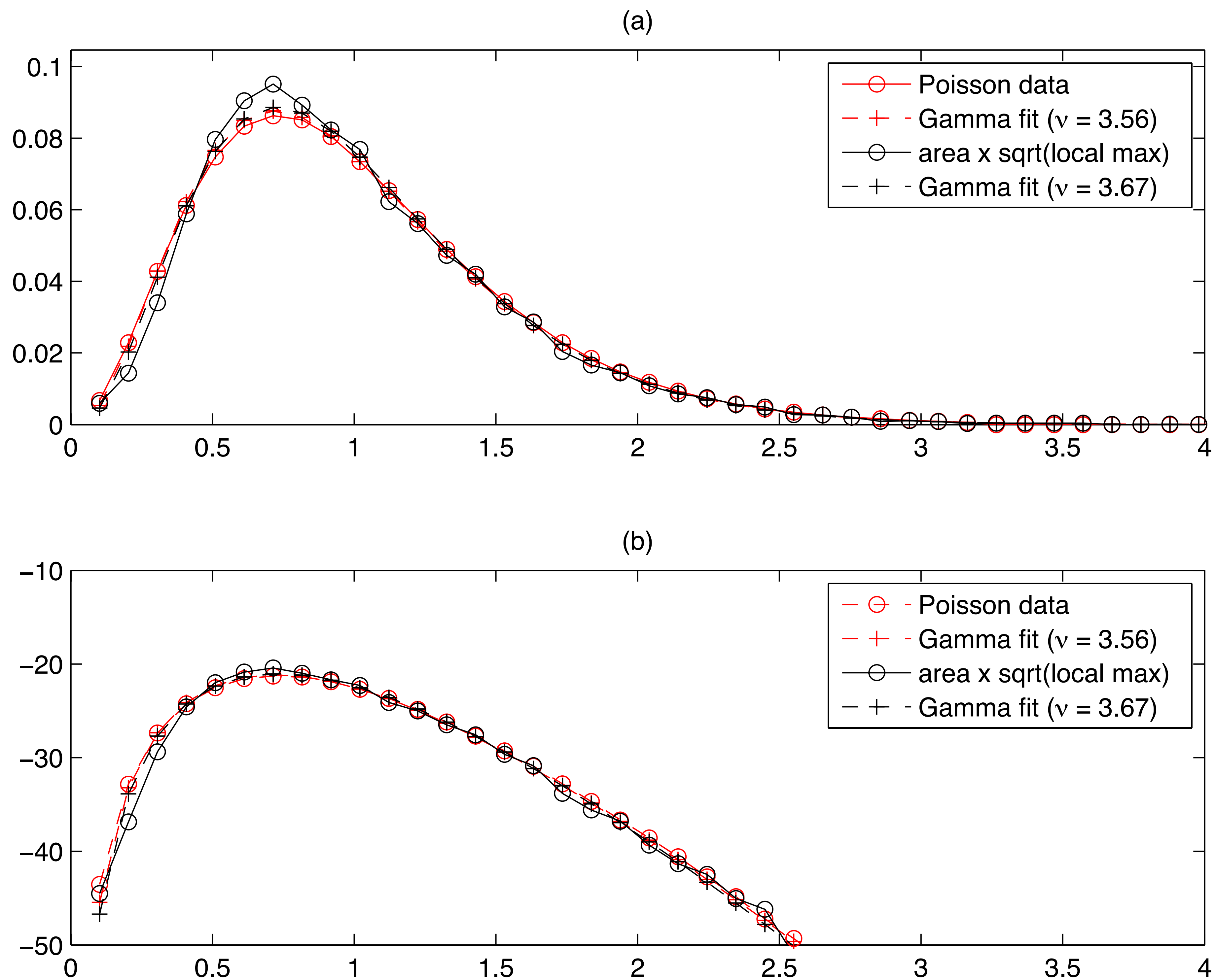


Global & local maxima

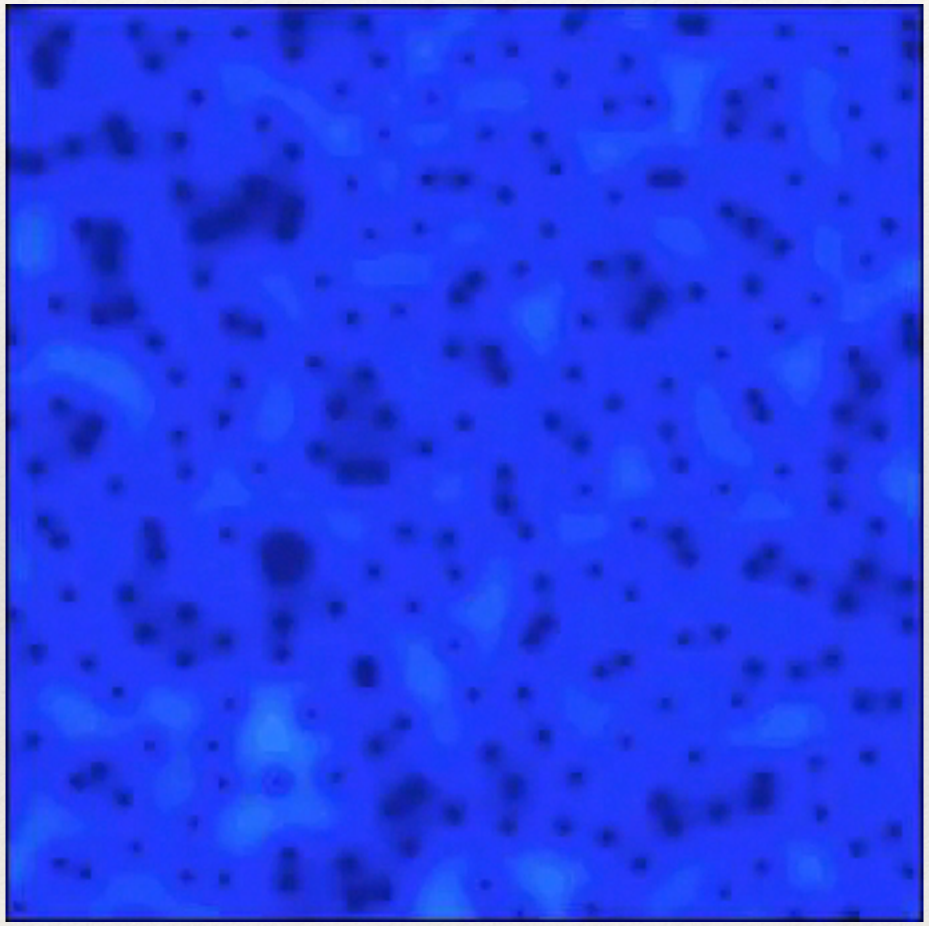
Gumbel distribution for global extremes of exponentially distributed ordinates [Embrechts et al., ’97]: what about local maxima?

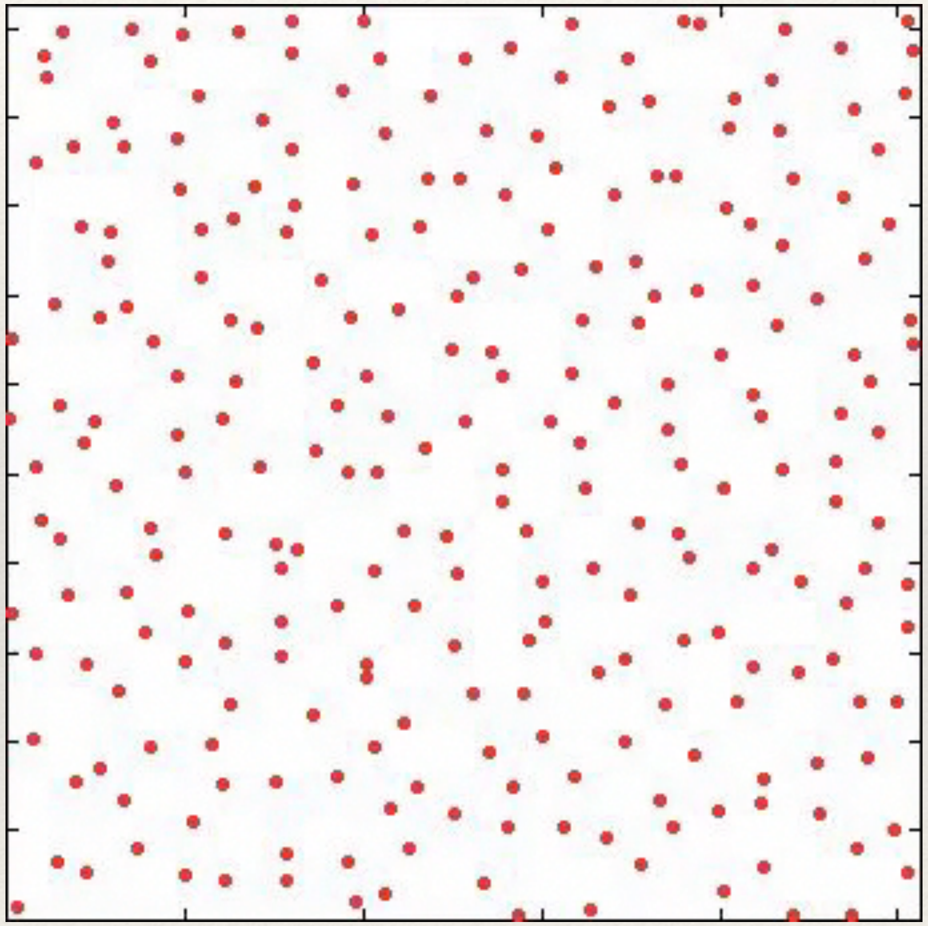


Cell « volume » vs. Poisson (?!)

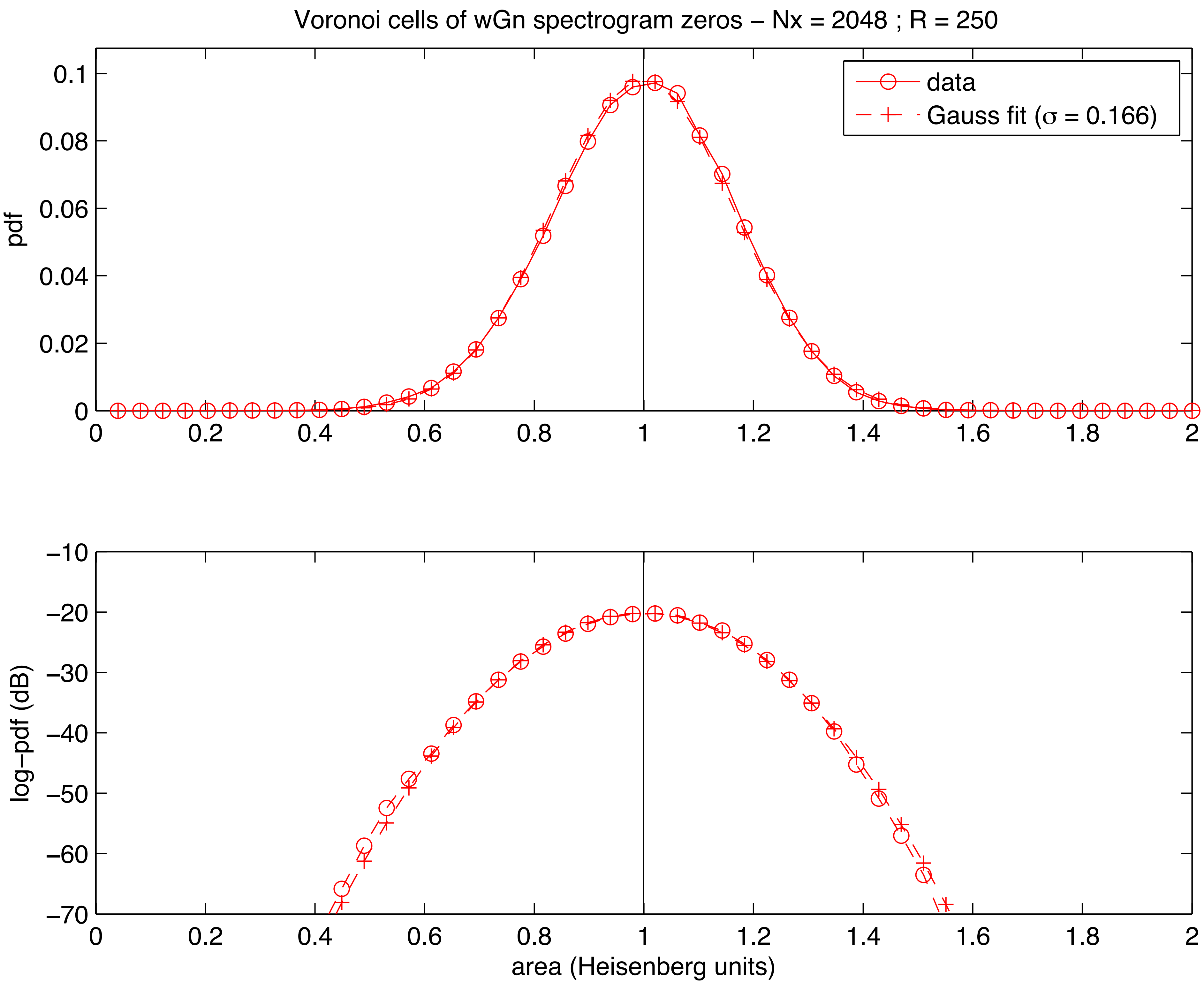


Zeros





Voronoi cells attached to zeros



Using zeros?

- Complete description (factorization theorem)
- Distinctive geometrical features for signal and noise regions:
  - *regular alignment on component contours*
  - *random distribution in noise-only regions*
  - *noise = 1 zero per Heisenberg cell (on average)*

Zeros-based « filtering »