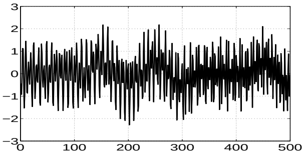
2D Hilbert-Huang Transform

Nelly Pustelnik
(joint work with Jérémy Schmitt, Pierre Borgnat and Patrick Flandrin)
Laboratoire de Physique – ENS Lyon
CNRS UMR 5672

ANR ASTRES
20 octobre 2014

Motivations (in signal processing)

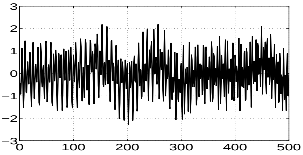
- Goal : Extract instantaneous amplitude and frequencies [Ville, 1948]



$$\longrightarrow \qquad (\alpha,\, \xi)\,?$$

Motivations (in signal processing)

- Goal : Extract instantaneous amplitude and frequencies [Ville, 1948]



$\longrightarrow$ (α, ξ) ?

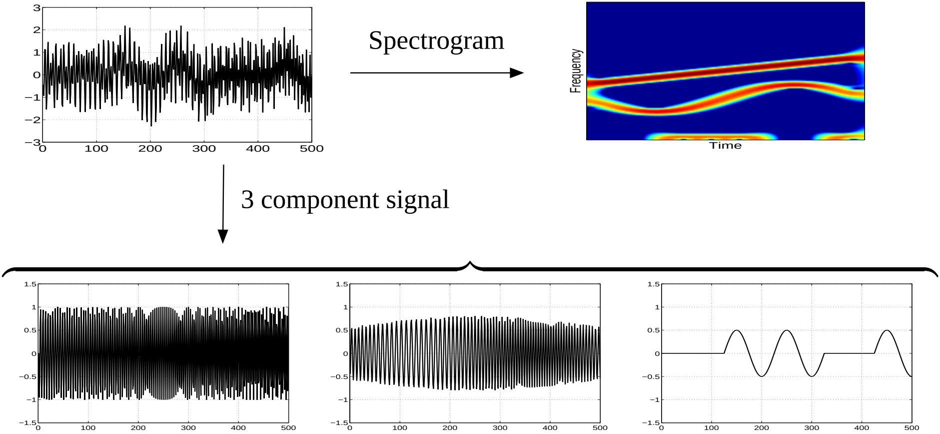
- ∘ x : real-valued 1D signal,
- ∘ $\mathcal{H}$: convolution whose transfer function is $H(\omega) = -j\text{sign}(\omega)$.
- ∘ $\mathcal{H}(x)$: Hilbert transform of x,
- ∘ x_a : analytic signal such that

$$\begin{aligned} x_a &= x + j\mathcal{H}(x) \\ &= \alpha e^{j\xi} \end{aligned}$$

$\rightarrow$ require a signal oscillating around zero

Motivations (in signal processing)

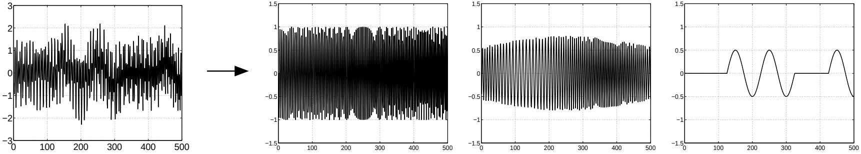
- Goal : Extract instantaneous amplitude and frequencies [Huang et al., 1998][Daubechies et al., 2010]



Motivations (in signal processing)

- Goal : Extract instantaneous amplitudes and frequencies [Huang et al., 1998][Daubechies et al., 2010]

1. Extract the components oscillating around zero (IMFs) + the tend



Signal x → IMF d_1 IMF d_2 Trend a_2

2. Compute the instantaneous amplitudes ans frequencies from each IMF.

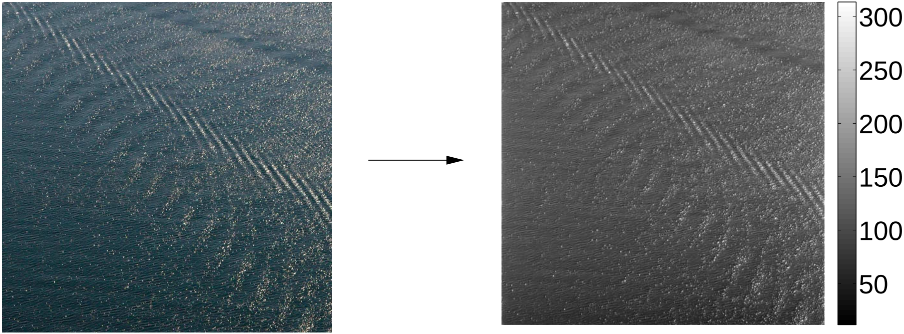
$\rightarrow$ Analytic signal for d_1 leads to (α_1, ξ_1)
$\rightarrow$ Analytic signal for d_2 leads to (α_2, ξ_2)

Motivations (in image processing)

- Objective : spectral analysis of a nonstationary image.
  $\Rightarrow$ extract local amplitude $\boldsymbol{\alpha}$, phase $\boldsymbol{\xi}$ and orientation $\boldsymbol{\theta}$.

Motivations (in image processing)

- Objective : spectral analysis of a nonstationary image.
  $\Rightarrow$ extract local amplitude α, phase ξ and orientation θ.

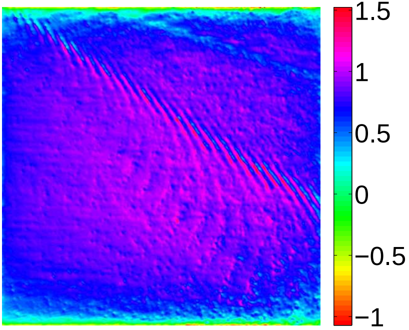


Motivations (in image processing)

- Objective : spectral analysis of a nonstationary image.
  $\Rightarrow$ extract local amplitude α, phase ξ and orientation θ.

Motivations (in image processing)

- Objective : spectral analysis of a nonstationary image.
  $\Rightarrow$ extract local amplitude α, phase ξ and orientation θ.



$\Rightarrow$ Superposition of several components (waves, noise, illumination...)

$\Rightarrow$ Poor performance of the spectral estimation

Introduction

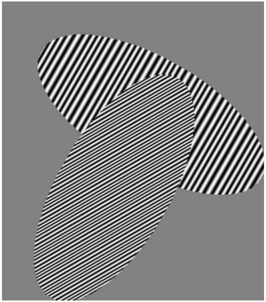
- Proposed solution : a two-step procedure
  1. Decomposition step $\Rightarrow \mathbf{x} = \sum_{k=1}^{K} \underbrace{\mathbf{d}_k}_{\text{IMF}} + \underbrace{\mathbf{a}_K}_{\text{Trend}}$

$\mathbf{x}$

Introduction

- Proposed solution : a two-step procedure
  1. Decomposition step $\Rightarrow \mathbf{x} = \sum_{k=1}^{K} \underbrace{\mathbf{d}_k}_{\text{IMF}} + \underbrace{\mathbf{a}_K}_{\text{Trend}}$

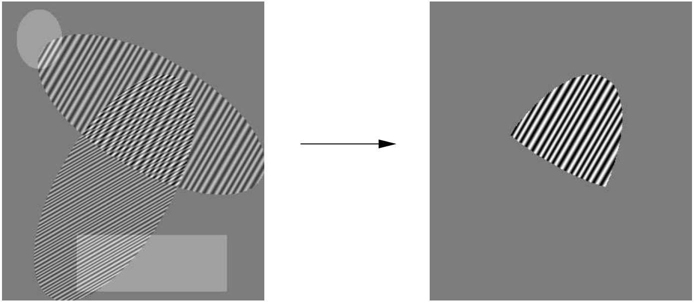
$\mathbf{x}$



$\mathbf{d}_1$

Introduction

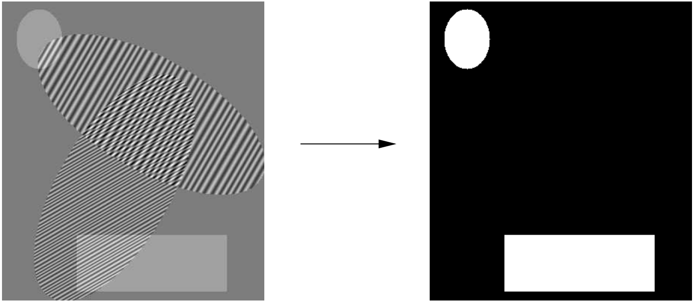
- Proposed solution : a two-step procedure
  1. Decomposition step $\Rightarrow \mathbf{x} = \sum_{k=1}^{K} \underbrace{\mathbf{d}_k}_{\text{IMF}} + \underbrace{\mathbf{a}_K}_{\text{Trend}}$



$\mathbf{x}$ $\mathbf{d}_2$

Introduction

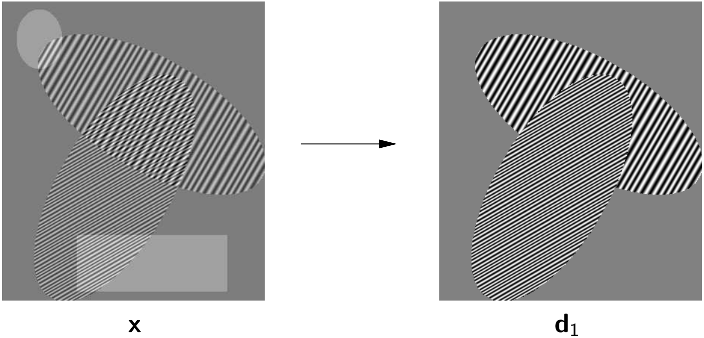
- Proposed solution : a two-step procedure
    1. Decomposition step $\Rightarrow \mathbf{x} = \sum_{k=1}^{K} \underbrace{\mathbf{d}_k}_{\text{IMF}} + \underbrace{\mathbf{a}_K}_{\text{Trend}}$



$\mathbf{x}$ $\mathbf{a}_2$

Introduction

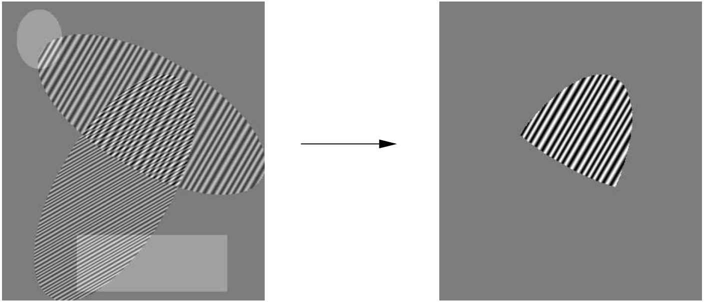
- Proposed solution : a two-step procedure
  1. Decomposition step ⇒ $\mathbf{x} = \sum_{k=1}^{K} \underbrace{\mathbf{d}_k}_{\text{IMF}} + \underbrace{\mathbf{a}_K}_{\text{Trend}}$



  2. Monogenic analysis on $\mathbf{d}_k$ ⇒ amplitude α_k, phase ξ_k, orientation θ_k .

Introduction

- Proposed solution : a two-step procedure
  1. Decomposition step $\Rightarrow \mathbf{x} = \sum_{k=1}^{K} \underbrace{\mathbf{d}_k}_{\text{IMF}} + \underbrace{\mathbf{a}_K}_{\text{Trend}}$



$\mathbf{x}$ $\mathbf{d}_2$

  2. Monogenic analysis on $\mathbf{d}_k \Rightarrow$ amplitude α_k, phase ξ_k, orientation θ_k .

State-of-the-art

- Riesz-Laplace transform [Unser et. al., 2009] :
    - Combination of a 2D wavelet transform with monogenic analysis.
    - Lack of adaptivity.

State-of-the-art

- Riesz-Laplace transform [Unser et. al., 2009] :
  - Combination of a 2D wavelet transform with monogenic analysis.
  - Lack of adaptivity.
- Image empirical mode decomposition (IEMD) + monogenic analysis [Jager, 2010] :
  - Good adaptivity.
  - Lack of robustness and of convergence guarantees.

State-of-the-art

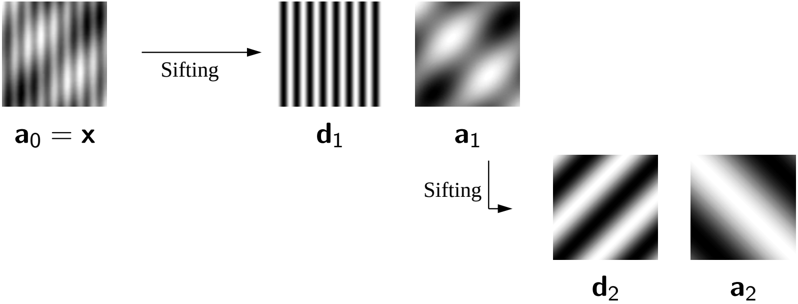
- Riesz-Laplace transform [Unser et. al., 2009] :
  - Combination of a 2D wavelet transform with monogenic analysis.
  - Lack of adaptivity.
- Image empirical mode decomposition (IEMD) + monogenic analysis [Jager, 2010] :
  - Good adaptivity.
  - Lack of robustness and of convergence guarantees.
- Texture-geometry decomposition + monogenic analysis [Aujol, 2008, Gilles, Osher, 2011] :
  - Good convergence guarantees.
  - Lack of adaptivity.

State-of-the-art

- Riesz-Laplace transform [Unser et. al., 2009] :
  - Combination of a 2D wavelet transform with monogenic analysis.
  - Lack of adaptivity.
- Image empirical mode decomposition (IEMD) + monogenic analysis [Jager, 2010] :
  - Good adaptivity.
  - Lack of robustness and of convergence guarantees.
- Texture-geometry decomposition + monogenic analysis [Aujol, 2008, Gilles, Osher, 2011] :
  - Good convergence guarantees.
  - Lack of adaptivity.
- Proposed solution : combine IEMD + texture-geometry + monogenic analysis

Principle of IEMD [Linderhed, 2009]

- Decomposition of $\mathbf{a}_{k-1}$ into $(\mathbf{a}_k, \mathbf{d}_k)$ based on sifting process.

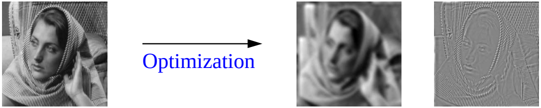


- Sifting process : research of local extrema and computation of a mean envelope until getting a fluctuation $\mathbf{d}_k$ oscillating symmetrically around zero : intrinsic mode function (IMF).

$\Rightarrow$ Problem : lack of convergence guarantees

Texture-geometry decomposition [Aujol, 2008]

- Decomposition of $\mathbf{x}$ into $(\mathbf{a}, \mathbf{d})$ based on optimization procedure.



$\mathbf{y}$ $\mathbf{a}$ $\mathbf{d}$

- Optimization procedure :

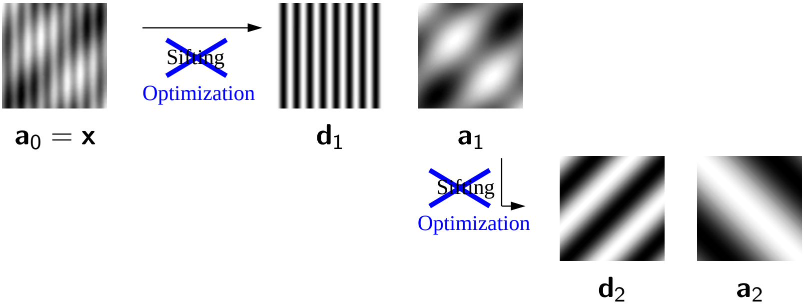
$$\text{Find} \quad (\hat{\mathbf{a}}, \hat{\mathbf{d}}) \in \underset{(\mathbf{a},\mathbf{d})}{\text{Argmin}} \|\mathbf{x} - \mathbf{a} - \mathbf{d}\|_2^2 + \phi(\mathbf{a}) + \psi(\mathbf{d})$$

  - ϕ imposes the trend behavior to $\mathbf{a}$ (smoothness) : Total Variation.
  - ψ imposes the fluctuation behavior to $\mathbf{d}$ (oscillatory behavior) :
    - $\psi = 0 \rightarrow$ TV [Aujol, 2008]
    - $\psi = \| \cdot \|_G \rightarrow$ TV-G [Gilles, Osher, 2011]
      (G : Banach space of signals with large oscillations)

$\Rightarrow$ Problem : lack of adaptivity

Proposed method

- Decomposition of $\mathbf{a}_{k-1}$ into $(\mathbf{a}_k, \mathbf{d}_k)$ based on optimization procedure.



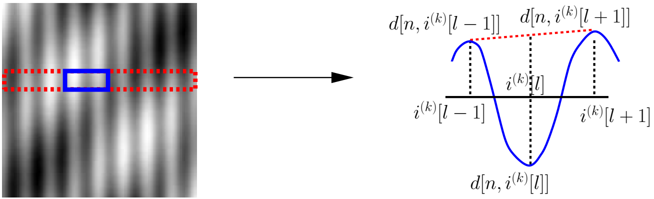
- Optimization procedure for every $k = \{1, \cdots, K\}$:

$$\text{Find} \quad (\hat{\mathbf{a}}_k, \hat{\mathbf{d}}_k) \in \underset{(\mathbf{a},\mathbf{d})}{\text{Argmin}} \|\mathbf{a}_{k-1} - \mathbf{a} - \mathbf{d}\|_2^2 + \phi_k(\mathbf{a}) + \psi_k(\mathbf{d})$$

  - ϕ_k imposes the trend behavior to $\mathbf{a}_k$ (smoothness) : Total Variation.
  - ψ_k imposes the IMF behavior to $\mathbf{d}_k$

Choice of ψ_k

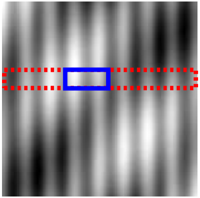
- Constraint applied separately on rows, columns, diagonals and anti-diagonals of $\mathbf{d}$.
- Example of the ℓ-th extremum of the n-th row constraint



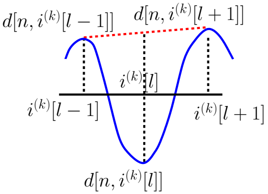
$\mathbf{a}_{k-1}$ $\mathbf{d}[n, \cdot]$

Choice of ψ_k

- Constraint applied separately on rows, columns, diagonals and anti-diagonals of **d**.
- Example of the ℓ-th extremum of the n-th row constraint



$\mathbf{a}_{k-1}$

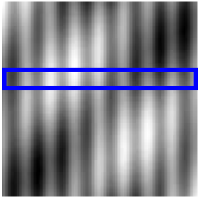


$\mathbf{d}[n, \cdot]$

$$\left| \mathbf{d}[n, i^{(k)}[\ell]] + \frac{\alpha_1^{(k)}[\ell]\mathbf{d}[n, i^{(k)}[\ell-1]] + \alpha_2^{(k)}[\ell]\mathbf{d}[n, i^{(k)}[\ell+1]]}{\alpha_1^{(k)}[\ell] + \alpha_2^{(k)}[\ell]} \right|$$

Choice of ψ_k

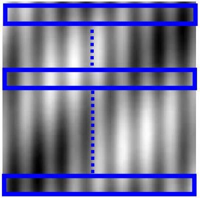
- Constraint applied separately on rows, columns, diagonals and anti-diagonals of $\mathbf{d}$.
- Example of the n-th row constraint



$$\sum_\ell \left| \mathbf{d}[n, i^{(k)}[\ell]] + \frac{\alpha_1^{(k)}[\ell]\mathbf{d}[n, i^{(k)}[\ell-1]] + \alpha_2^{(k)}[\ell]\mathbf{d}[n, i^{(k)}[\ell+1]]}{\alpha_1^{(k)}[\ell] + \alpha_2^{(k)}[\ell]} \right|$$

Choice of ψ_k

- Constraint applied separately on rows, columns, diagonals and anti-diagonals of **d**.
- Example of the constraint for every rows



$$\sum_n \sum_\ell \left| \mathbf{d}[n, i^{(k)}[\ell]] + \frac{\alpha_1^{(k)}[\ell]\mathbf{d}[n, i^{(k)}[\ell-1]] + \alpha_2^{(k)}[\ell]\mathbf{d}[n, i^{(k)}[\ell+1]]}{\alpha_1^{(k)}[\ell] + \alpha_2^{(k)}[\ell]} \right|$$

Choice of ψ_k

- Constraint applied separately on rows :

$$\sum_n \sum_\ell \left| \mathbf{d}[n, i^{(k)}[\ell]] + \frac{\alpha_1^{(k)}[\ell]\mathbf{d}[n, i^{(k)}[\ell-1]] + \alpha_2^{(k)}[\ell]\mathbf{d}[n, i^{(k)}[\ell+1]]}{\alpha_1^{(k)}[\ell] + \alpha_2^{(k)}[\ell]} \right|$$

$$\Rightarrow \quad \boxed{\|M_1^{(k)} d\|_1}$$

- Constraints on rows, columns, diagonals and antidiagonals :

$$(\forall i \in \{1, \ldots, 4\}) \quad \|M_i^{(k)} d\|_1$$

- Resulting penalization term :

$$\psi_k(\mathbf{d}) = \sum_{i=1}^{4} \lambda_i^{(k)} \|M_i^{(k)} d\|_1 \quad \text{with} \quad \lambda_i^{(k)} > 0 \quad \Rightarrow \quad \text{convex function}$$

Criterion

- For every $k = 1, \cdots K$:

$$(\hat{\mathbf{a}}_k, \hat{\mathbf{d}}_k) \in \underset{(\mathbf{a},\mathbf{d})}{\mathrm{Argmin}} \|\mathbf{a}_{k-1} - \mathbf{a} - \mathbf{d}\|_2^2 + \phi_k(\mathbf{a}) + \psi_k(\mathbf{d})$$

Criterion

- For every $k = 1, \cdots K$:

$$(\hat{\mathbf{a}}_k, \hat{\mathbf{d}}_k) \in \underset{(\mathbf{a},\mathbf{d})}{\mathrm{Argmin}} \|\mathbf{a}_{k-1} - \mathbf{a} - \mathbf{d}\|_2^2 + \phi_k(\mathbf{a}) + \psi_k(\mathbf{d})$$

  - $\phi_k(\mathbf{a}) = \eta^{(k)} \|L\,\mathbf{a}\|_{2,1}$ with $L = [H^* V^*]^*$ and $\eta^{(k)} > 0$
    H, V : linear operators horizontal/vertical finite differences
    $\|\cdot\|_{2,1}$: non-smooth proximable convex function

Criterion

- For every $k = 1, \cdots K$:

$$(\hat{\mathbf{a}}_k, \hat{\mathbf{d}}_k) \in \underset{(\mathbf{a},\mathbf{d})}{\operatorname{Argmin}} \|\mathbf{a}_{k-1} - \mathbf{a} - \mathbf{d}\|_2^2 + \phi_k(\mathbf{a}) + \psi_k(\mathbf{d})$$

  - $\phi_k(\mathbf{a}) = \eta^{(k)} \|L\ \mathbf{a}\|_{2,1}$ with $L = [H^* V^*]^*$ and $\eta^{(k)} > 0$
    H, V : linear operators horizontal/vertical finite differences
    $\| \cdot \|_{2,1}$: non-smooth proximable convex function
  - $\psi_k(\mathbf{d}) = \sum_{i=1}^{4} \lambda_i^{(k)} \|M_i^{(k)} d\|_1$ with $\lambda_i^{(k)} > 0$
    $M_i^{(k)}$: linear operators
    $\| \cdot \|_1$: non-smooth proximable convex function

Criterion

- For every $k = 1, \cdots K$:

$$(\hat{\mathbf{a}}_k, \hat{\mathbf{d}}_k) \in \underset{(\mathbf{a},\mathbf{d})}{\operatorname{Argmin}} \|\mathbf{a}_{k-1} - \mathbf{a} - \mathbf{d}\|_2^2 + \phi_k(\mathbf{a}) + \psi_k(\mathbf{d})$$

  - $\phi_k(\mathbf{a}) = \eta^{(k)} \|L\ \mathbf{a}\|_{2,1}$ with $L = [H^* V^*]^*$ and $\eta^{(k)} > 0$
    H, V : linear operators horizontal/vertical finite differences
    $\|\cdot\|_{2,1}$: non-smooth proximable convex function
  - $\psi_k(\mathbf{d}) = \sum_{i=1}^{4} \lambda_i^{(k)} \|M_i^{(k)} d\|_1$ with $\lambda_i^{(k)} > 0$
    $M_i^{(k)}$: linear operators
    $\|\cdot\|_1$: non-smooth proximable convex function

- Non-smooth convex optimization problem.

Criterion

- For every $k = 1, \cdots K$:

$$(\hat{\mathbf{a}}_k, \hat{\mathbf{d}}_k) \in \underset{(\mathbf{a},\mathbf{d})}{\mathrm{Argmin}} \|\mathbf{a}_{k-1} - \mathbf{a} - \mathbf{d}\|_2^2 + \phi_k(\mathbf{a}) + \psi_k(\mathbf{d})$$

  - $\phi_k(\mathbf{a}) = \eta^{(k)} \|L\ \mathbf{a}\|_{2,1}$ with $L = [H^* V^*]^*$ and $\eta^{(k)} > 0$
    H, V : linear operators horizontal/vertical finite differences
    $\|\cdot\|_{2,1}$: non-smooth proximable convex function
  - $\psi_k(\mathbf{d}) = \sum_{i=1}^{4} \lambda_i^{(k)} \|M_i^{(k)} d\|_1$ with $\lambda_i^{(k)} > 0$
    $M_i^{(k)}$: linear operators
    $\|\cdot\|_1$: non-smooth proximable convex function

- Non-smooth convex optimization problem.
- Solved with Condat-Vũ primal-dual splitting algorithm [Condat, 2013, Vũ, 2013].

2D–EMD Algorithm

STEP 1 – **Initialization**

Set $\mathbf{a}_0 = \mathbf{x}$,
Choose the number of IMFs K to be extracted,
Set $k = 1$.

STEP 2 – **2D proximal mode decomposition** : extract $\mathbf{a}_k$ and $\mathbf{d}_k$ from $\mathbf{a}_{k-1}$.

Compute $(M_i^{(k)})_{1 \leq i \leq 4}$ from $\mathbf{a}_{k-1}$,
Compute $\mathbf{a}_k$ and $\mathbf{d}_k$ by minimizing the convex criterion :

$$(\hat{\mathbf{a}}_k, \hat{\mathbf{d}}_k) = \underset{(\mathbf{a},\mathbf{d})}{\mathrm{Argmin}} \|\mathbf{a}_{k-1} - \mathbf{a} - \mathbf{d}\|_2^2 + \eta^{(k)} \|L\ \mathbf{a}\|_{2,1} + \sum_{i=1}^4 \lambda_i^{(k)} \|M_i^{(k)} d\|_1$$

While $k < K$, set $k \leftarrow k + 1$ and return to STEP 2.

Experiments

- IEMD [Linderhed, 2009] :

x

Experiments

- IEMD [Linderhed, 2009] :

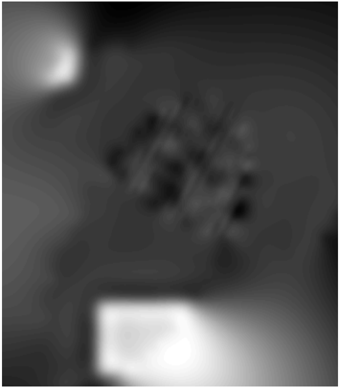
$\rightarrow$

$\mathbf{x}$ $\mathbf{d}_1$

Experiments

- IEMD [Linderhed, 2009] :

→



x $\mathbf{d}_2$

Experiments

- IEMD [Linderhed, 2009] :

$\rightarrow$

$\mathbf{x}$ $\mathbf{a}_2$

$\Rightarrow$ The components are not separated

Experiments

- TV-G Decomposition [Gilles, Osher, 2011] :

x

Experiments

- TV-G Decomposition [Gilles, Osher, 2011] :

$\mathbf{x}$

$\rightarrow$

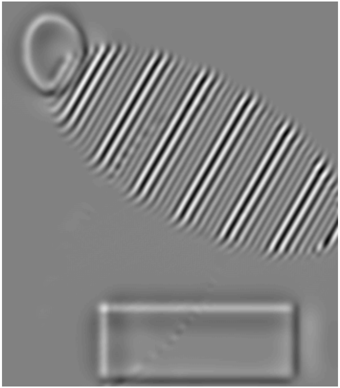
$\mathbf{d}_1$

Experiments

- TV-G Decomposition [Gilles, Osher, 2011] :

$\mathbf{x}$

$\rightarrow$

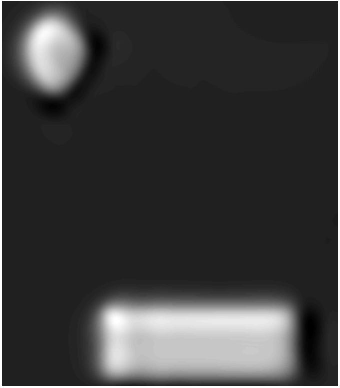


$\mathbf{d}_2$

Experiments

- TV-G Decomposition [Gilles, Osher, 2011] :

→



$\mathbf{x}$ $\mathbf{a}_2$

⇒ Scale mixing

Experiments

- Proposed method :

x

Experiments

- Proposed method :

$\mathbf{x}$

$\rightarrow$

$\mathbf{d}_1$

Experiments

- Proposed method :

$\rightarrow$

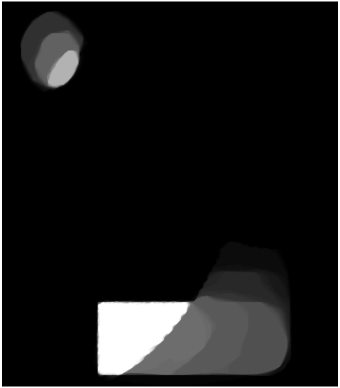
$\mathbf{x}$ $\mathbf{d}_2$

Experiments

- Proposed method :

$\mathbf{x}$

$\rightarrow$



$\mathbf{a}_2$

$\Rightarrow$ EMD behavior : extract fastest oscillations

Spectral analysis

1D Hilbert Transform	2D Riesz Transform
$x_h = h * x$	$\mathbf{x}_r = (h^{(1)} * \mathbf{x}, h^{(2)} * \mathbf{x}) = (\mathbf{x}^{(1)}, \mathbf{x}^{(2)})$
$H(\omega) = -j\omega/\lvert\omega\rvert$	$H^{(i)}(\underline{\omega}) = -j\omega_i/\lVert\underline{\omega}\rVert$ for $i = \{1, 2\}$ $\underline{\omega} = (\omega_1, \omega_2)$

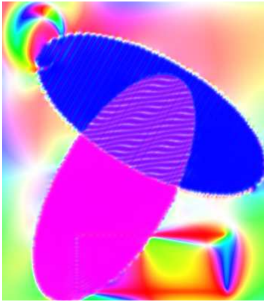
Analytic signal :	Monogenic signal :
$x_a = x + jx_h = \alpha e^{j\xi}$	$\mathbf{x}_m = (\mathbf{x}, \mathbf{x}^{(1)}, \mathbf{x}^{(2)})$
$\alpha = \lvert x_a\rvert = \sqrt{(x)^2 + (x_h)^2}$	$\boldsymbol{\alpha} = \sqrt{(\mathbf{x})^2 + (\mathbf{x}^{(1)})^2 + (\mathbf{x}^{(2)})^2}$
$\xi = \arg(x_a) = \arctan\left(\frac{x_h}{x}\right)$	$\boldsymbol{\xi} = \arctan\left(\frac{\sqrt{(\mathbf{x}^{(1)})^2 + (\mathbf{x}^{(2)})^2}}{\mathbf{x}}\right)$
	$\boldsymbol{\theta} = \arctan(\mathbf{x}^{(2)}/\mathbf{x}^{(1)})$

Experiments

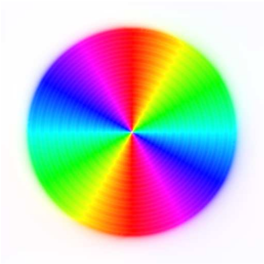
- IEMD + Monogenic analysis ($\boldsymbol{\theta}_k$)

$\mathbf{d}_1$

$\boldsymbol{\theta}_1$



Colormap of $\boldsymbol{\theta}_1$

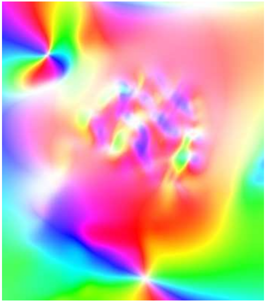


Experiments

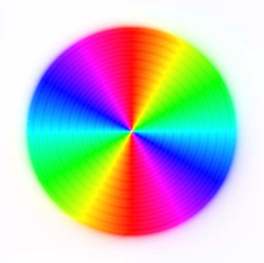
- IEMD + Monogenic analysis ($\boldsymbol{\theta}_k$)

$\mathbf{d}_2$

$\boldsymbol{\theta}_2$



Colormap of $\boldsymbol{\theta}_2$



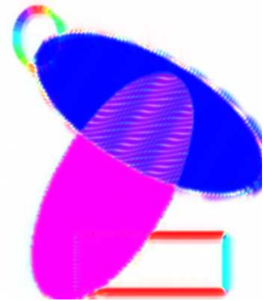
$\Rightarrow$ Poor results due to bad separation

Experiments

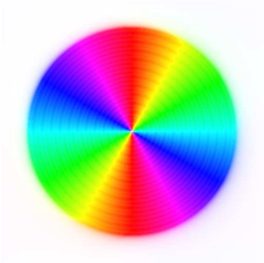
- TV-G decomposition + Monogenic analysis ($\boldsymbol{\theta}_k$)

$\mathbf{d}_1$

$\boldsymbol{\theta}_1$



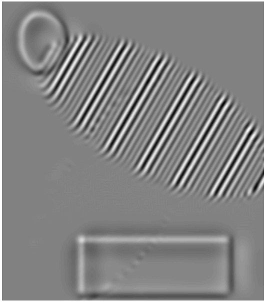
Colormap of $\boldsymbol{\theta}_1$



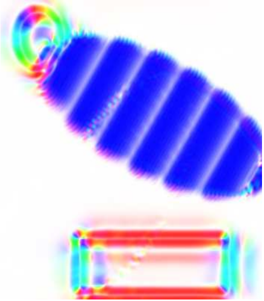
Experiments

- TV-G decomposition + Monogenic analysis ($\boldsymbol{\theta}_k$)

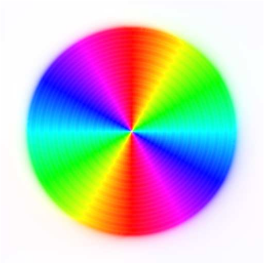
$\mathbf{d}_2$



$\boldsymbol{\theta}_2$



Colormap of $\boldsymbol{\theta}_2$



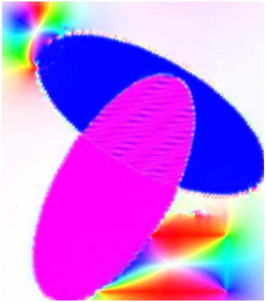
⇒ Poor results due to scale mixing

Experiments

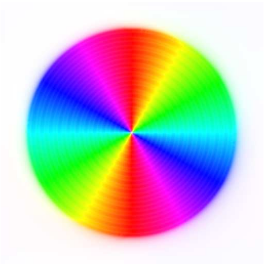
- Proposed method + Monogenic analysis ($\boldsymbol{\theta}_k$)

$\mathbf{d}_1$

$\boldsymbol{\theta}_1$



Colormap of $\boldsymbol{\theta}_1$



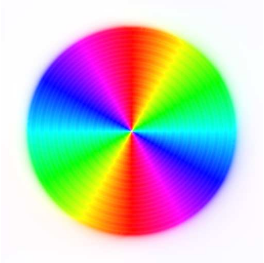
Experiments

- Proposed method + Monogenic analysis ($\boldsymbol{\theta}_k$)

$\mathbf{d}_2$

$\boldsymbol{\theta}_2$

Colormap of $\boldsymbol{\theta}_2$



$\Rightarrow$ Good results

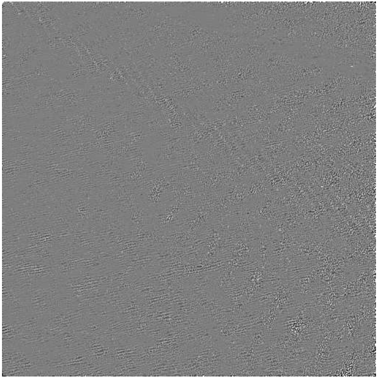
Proposed method on real data : decomposition

x

Proposed method on real data : decomposition

$\mathbf{x}$

$\rightarrow$

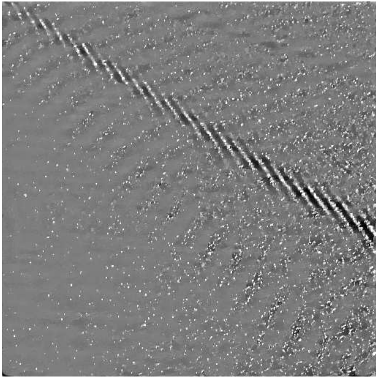


$\mathbf{d}_1$

Proposed method on real data : decomposition

$\mathbf{x}$

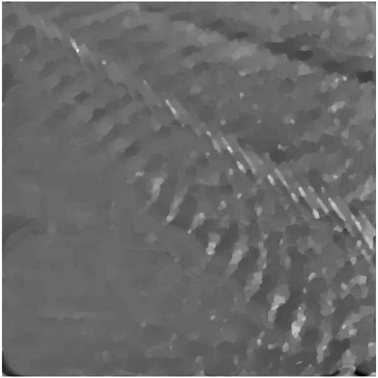
$\rightarrow$



$\mathbf{d}_2$

Proposed method on real data : decomposition

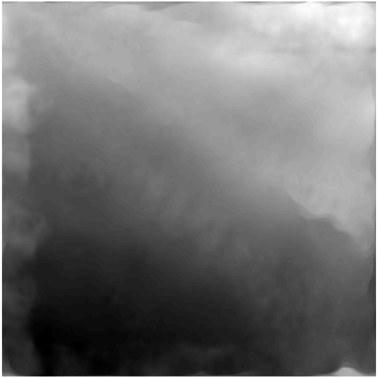
$\rightarrow$



$\mathbf{x}$ $\mathbf{d}_3$

Proposed method on real data : decomposition

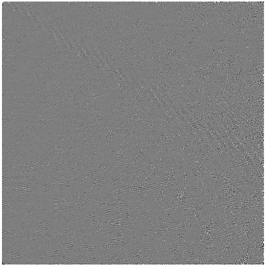
$\rightarrow$



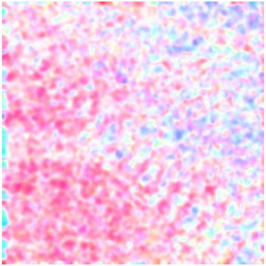
$\mathbf{x}$ $\mathbf{a}_3$

Proposed method on real data : monogenic analysis

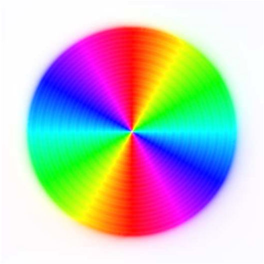
$\mathbf{d}_1$



$\boldsymbol{\theta}_1$



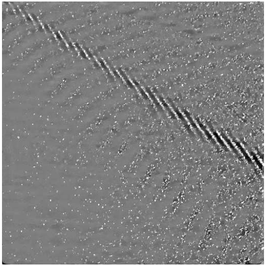
Colormap of $\boldsymbol{\theta}_1$



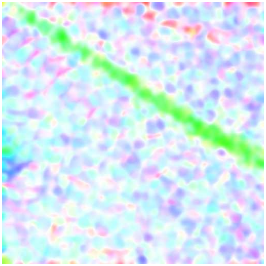
$\Rightarrow$ Extract smallest waves.

Proposed method on real data : monogenic analysis

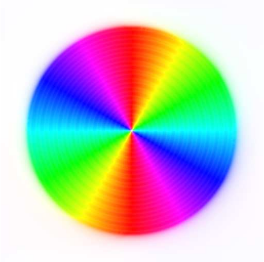
$\mathbf{d}_2$



$\boldsymbol{\theta}_2$



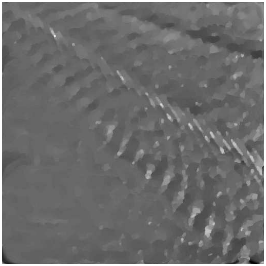
Colormap of $\boldsymbol{\theta}_2$



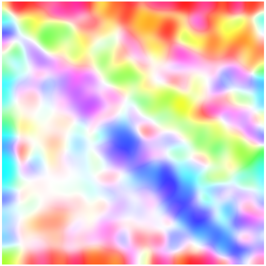
$\Rightarrow$ Extract a secondary wave.

Proposed method on real data : monogenic analysis

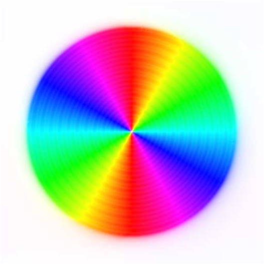
$\mathbf{d}_3$



$\boldsymbol{\theta}_3$



Colormap of $\boldsymbol{\theta}_3$



$\Rightarrow$ Extract the 2 longest waves.

Conclusions

- Convergence guarantees.
- Good numerical results.
- Flexible approach due to various possible choices for ϕ_k, ϕ_k, and φ_k.
- Robustness to sampling effects and allows to deal with non-smooth trend.

References

- N. Pustelnik, P. Borgnat, and P. Flandrin, "*A multicomponent proximal algorithm for Empirical Mode Decomposition*," EUSIPCO, Bucharest, Romania, 27-31 August, 2012.
- N. Pustelnik, P. Borgnat, and P. Flandrin, "*Empirical Mode Decomposition revisited by multicomponent non smooth convex optimization*," Signal Processing, vol. 102, pp. 313-331, Sept. 2014.
- J. Schmitt, N. Pustelnik, P. Borgnat, and P. Flandrin, *2D Hilbert-Huang Transform*, IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP), Florence, Italy, May 4-9, 2014
- J. Schmitt, N. Pustelnik, P. Borgnat, P. Flandrin, L. Condat, *2-D Prony–Huang Transform : A New Tool for 2-D Spectral Analysis*, IEEE Trans. Image Proc., 2014.