Image deblurring

Loïc Denis
a joint work with Eric Thiébaut, Ferréol Soulez
and the other members of ANR MiTiV

Workshop SIERRA: Signal-Image en Région Rhône-Alpes

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Before starting... 

Short presentation of our research group:

ANR MiTiV: Méthodes Inverses pour le Traitement en Imagerie du Vivant

Multidisciplinary group lead by Eric Thiébaut (Observatoire de Lyon), gathering people from:

- astronomy (Observatoire de Lyon + Institut d’Astrophysique de Paris),
- image processing (Laboratoire Hubert Curien à St Etienne, CPE Lyon, TIMC-IMAG),
- bio-medical imaging (Centre Commun de Quantimétrie de Lyon 1, Hôpital de la Croix Rousse),
- software development (Shaktiware).
Before starting...

Illustration of the problems considered:

- *gated* compensation (2 “scans”)
- compensation (2 “scans”)

Rit et al.

thèse de Fabien Momey (1 seul “scan”)

1 / 66
Topics of my talk:

1. An introduction to image restoration
2. Models and inversion of shift-variant blur
3. Blind deconvolution in 3D microscopy
Why digitally restore images?

→ to reduce noise
→ to improve resolution of blurred images
→ to improve the contrast of blurred structures

→ to fake a higher resolution:

from “Example-based super-resolution”, Freeman et al., 2002.
Denoising vs Deblurring

**Denoising:** when the most significant part of the degradation comes from noise (little blur)
  → some smoothing is unavoidable

**Deblurring:** when blur is stronger and the signal-to-noise ratio is high enough to improve the resolution
  → special care must be paid to prevent from noise amplification

Denoising and deblurring methodologies have long been very close: Wiener filter, Markov random fields, total variation minimization, wavelets methods. . . .

During last decade, denoising approaches have significantly differed: role played by image patches (selection-based estimation, redundancy of image patches, patch dictionaries). The methodological gap with deblurring methods is still to be filled. . .
The 3 ingredients of a deblurring method

1. The degradation model 1/2: blur

Blur is generally modeled as a **linear** transformation applied on the original crisp image.

Within the anisoplanatism domain, this transformation is a convolution:

\[
\mathcal{M}[f](r) \equiv \int h(r - s)f(s) \, ds
\]
The 3 ingredients of a deblurring method

1. The degradation model 2/2: noise

Noise is considered as a stochastic perturbation of the image (it may have a deterministic origin, e.g., quantization noise).

The simplest (and most used) model is white Gaussian noise. The probability density function is then:

\[- \log p \left( g(r) \mid M[f](r) \right) \propto \frac{(g(r) - M[f](r))^2}{\sigma^2} + \text{cste}\]

Under low/moderate light flux, it is more accurate to vary the variance of noise $\sigma^2$ with the signal amplitude $M[f](r)$.

At very low levels of light (photon counting), or under coherent lighting non-Gaussian pdf are more accurate (e.g., mix of Poisson and Gaussian, gamma distributions for speckle).

Noise at different pixels may show some correlations. This can be modeled through a non-diagonal covariance matrix $\Sigma = W^{-1}$:

\[- \log p \left( g \mid M[f] \right) \propto (g - M[f])^T W(g - M[f]) + \text{cste}\]
2. The model for the class of images considered

It is essential to constrain the recovered image to be in a restricted class of images to prevent from overfitting the noise.

The constraints take the form of regularization terms, or prior models under a Bayesian setting.

A typical example widely used in image deblurring is total variation (or relaxed versions that prevent the staircasing effect):

$$-\log p(f) \propto \sum_i \|\nabla f_i\|_2$$
The 3 ingredients of a deblurring method

3. The estimator

Given the model of the degradation and of the images under consideration, an estimator can be built.

The most widely used estimator for image deblurring is the maximum a posteriori estimator:

$$\hat{f}^{(\text{MAP})} = \arg \min_f \left[ -\log p \left( g \mid M[f] \right) + \mu \log p \left( f \right) \right]$$

Other estimators such as the posterior mean can be computed. They generally require solving more challenging computational problems (e.g., sampling high-dimensional distributions).
Handling image borders

When performing discrete convolutions of images, we need to decide how to handle the borders. What really matters is to reconstruct more than the observed field in order to describe correctly what is seen.

Illustration in the case of a very large PSF: reconstruction in digital holography:

![Data and reconstruction images](image_url)
“Hot” topics

Some current research topics:

– learning rich statistical priors (e.g., dictionary-based: see Remi Gribonval’s talk),

– designing fast minimization methods to compute the MAP estimator (e.g., using majorant approximations: see Jérôme Idier’s talk),

– designing accurate degradation models (e.g., to account for blur variations: more on that next section),

– designing blind methods, i.e., methods that jointly estimate the original image and the PSF (more on that in last section),

– unsupervised setting of all parameters of the method.
1. An introduction to image restoration

2. Models and inversion of shift-variant blur

3. Blind deconvolution in 3D microscopy
Blur changes in wide-field imaging

Observations of wide-fields suffer from shift-variant blurring:

– in astronomy (**surveys**, post adaptive optics),
– in microscopy (widening of the PSF with depth),
– in wide-angle optical systems.

→ mostly independent of the scene.

Blur variations can also occur because of scene/camera relative motion or depth-of-focus

→ depends on the 3D geometry of the scene.

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Need for a fast and accurate model

If blur changes in the field of view are independent of the scene, the image formation model is still linear:

\[
g(r) = \int h(r, s) f(s) \, ds \quad \rightarrow \quad g = H \cdot f
\]

Matrix \( H \) (i.e., the direct operator) is structured in the case of shift-invariant blur (BTTB).

It can take any form in the general case of shift-variant blur. There are potentially \( N^2 \) degrees of freedom for matrix \( H \) pour \( N \)-pixels images (i.e., \( 10^{12} \) elements for mega-pixel images).

\( \sim \) intractable for storage, application and estimation of \( H \)!

We need to enforce a structure to \( H \) that achieves a trade-off between approximation quality (i.e., flexibility) and speed.
Approximating shift-variant blur: continuous point-of-view

The linear formation models write:

\[ g(r) = \int h(r, s) f(s) \, ds \quad \text{and} \quad g(r) = \int h(r - s) f(s) \, ds \]

- **shift-variant case**
- **shift-invariant case (convolution)**

Replacing the PSF with a separable approximation:

\[ h(r, s) \approx \sum_{i=1}^{k} m_i(r - s) w_i(s) \]

leads to an efficient approximation [Gilad & Hardenberg, 2006]:

\[ g(r) = \int h(r, s) f(s) \, ds \approx \int \sum_{i=1}^{k} m_i(r - s) w_i(s) f(s) \, ds \]

\[ \quad \| \quad \sum_{i=1}^{k} \int m_i(r - s) [w_i(s)f(s)] \, ds \]
Approximating shift-variant blur: continuous point-of-view

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and

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shift-variant case

shift-invariant case (convolution)

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\[ g(r) = \int h(r, s) f(s) \, ds \approx \int \sum_{i=1}^{k} m_i(r - s) w_i(s)f(s) \, ds \]

\[ k \, \text{convolutions} \quad \longleftrightarrow \quad \sum_{i=1}^{k} \int m_i(r - s) [w_i(s)f(s)] \, ds \]
Approximating shift-variant blur: discrete point-of-view

Let's consider the structure of the matrix $H$:

\[
\begin{bmatrix}
\ldots
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
\ldots
\end{bmatrix}
\]

shift-variant case

shift-invariant case (convolution)

We need to model changes of the PSF with the position:

\[
\begin{bmatrix}
\ldots
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
\ldots
\end{bmatrix}
\]

shift-variant case

shift-invariant case (convolution)
Approximating shift-variant blur: discrete point-of-view

Possible approximations:

– piecewise-constant:

\[
\begin{pmatrix}
\ldots \\
\ldots \\
\ldots \\
\end{pmatrix} \approx 
\begin{pmatrix}
\ldots \\
\ldots \\
\ldots \\
\end{pmatrix} = 
\begin{pmatrix}
\ldots \\
\ldots \\
\ldots \\
\end{pmatrix} \cdot \begin{pmatrix}
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\ldots \\
\ldots \\
\end{pmatrix} + 
\begin{pmatrix}
\ldots \\
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\ldots \\
\end{pmatrix} \cdot \begin{pmatrix}
\ldots \\
\ldots \\
\ldots \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
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\ldots \\
\ldots \\
\end{pmatrix} \approx 
\begin{pmatrix}
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\ldots \\
\end{pmatrix} = 
\begin{pmatrix}
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\end{pmatrix} \cdot \begin{pmatrix}
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\ldots \\
\ldots \\
\end{pmatrix} + 
\begin{pmatrix}
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\ldots \\
\end{pmatrix} \cdot \begin{pmatrix}
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\end{pmatrix}
\]

\[
\begin{pmatrix}
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\end{pmatrix} \approx 
\begin{pmatrix}
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\ldots \\
\ldots \\
\end{pmatrix} = 
\begin{pmatrix}
\ldots \\
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\ldots \\
\end{pmatrix} \cdot \begin{pmatrix}
\ldots \\
\ldots \\
\ldots \\
\end{pmatrix} + 
\begin{pmatrix}
\ldots \\
\ldots \\
\ldots \\
\end{pmatrix} \cdot \begin{pmatrix}
\ldots \\
\ldots \\
\ldots \\
\end{pmatrix}
\]

conv\(k_1\) diag\(\imath_1\) conv\(k_2\) diag\(\imath_2\)
Approximating shift-variant blur: discrete point-of-view

Possible approximations:

– interpolation of PSF

\[
H \approx \begin{pmatrix}
\text{\ldots} & \text{\ldots} & \text{\ldots} \\
\text{\ldots} & \text{\ldots} & \text{\ldots} \\
\end{pmatrix}
= \begin{pmatrix}
\text{\ldots} \\
\text{\ldots} \\
\end{pmatrix} \cdot (\ldots) + \begin{pmatrix}
\text{\ldots} \\
\text{\ldots} \\
\end{pmatrix} \cdot (\ldots) + \begin{pmatrix}
\text{\ldots} \\
\text{\ldots} \\
\end{pmatrix} \cdot (\ldots)
\]

\[
H \approx \begin{pmatrix}
\text{conv} (k_1) & \text{diag} (\varphi_1) & \text{conv} (k_2) & \text{diag} (\varphi_2) \\
\end{pmatrix}
= \begin{pmatrix}
\text{\ldots} \\
\text{\ldots} \\
\text{\ldots} \\
\text{\ldots} \\
\end{pmatrix} \cdot \begin{pmatrix}
\text{\ldots} \\
\text{\ldots} \\
\text{\ldots} \\
\end{pmatrix} + \begin{pmatrix}
\text{\ldots} \\
\text{\ldots} \\
\text{\ldots} \\
\text{\ldots} \\
\end{pmatrix} \cdot \begin{pmatrix}
\text{\ldots} \\
\text{\ldots} \\
\text{\ldots} \\
\end{pmatrix} + \begin{pmatrix}
\text{\ldots} \\
\text{\ldots} \\
\text{\ldots} \\
\text{\ldots} \\
\end{pmatrix} \cdot \begin{pmatrix}
\text{\ldots} \\
\text{\ldots} \\
\text{\ldots} \\
\text{\ldots} \\
\end{pmatrix}
\]
Approximating shift-variant blur: discrete point-of-view

Possible approximations:

– low-rank approximation: decomposition on PSF modes

\[
\begin{pmatrix}
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{pmatrix}
\approx
\begin{pmatrix}
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{pmatrix}
= \begin{pmatrix}
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{pmatrix}
\cdot \begin{pmatrix}
\cdots \\
\cdots \\
\cdots \\
\cdots \\
\cdots \\
\end{pmatrix}
+ \begin{pmatrix}
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{pmatrix}
\cdot \begin{pmatrix}
\cdots \\
\cdots \\
\cdots \\
\cdots \\
\cdots \\
\end{pmatrix}
+ \cdots
\]

\[
H
\approx
\begin{pmatrix}
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{pmatrix}
= \begin{pmatrix}
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{pmatrix}
\cdot \begin{pmatrix}
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{pmatrix}
+ \begin{pmatrix}
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{pmatrix}
\cdot \begin{pmatrix}
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{pmatrix}
+ \cdots
\]

\[
\text{conv}(u_1) \cdot \text{diag}(\sigma_1 v_1) + \text{conv}(u_2) \cdot \text{diag}(\sigma_2 v_2)
\]
Best localized approximation

Possible approximations:

– optimal approximation with localized weights

[Denis, Thiébaut & Soulez 2011]

\[
\begin{bmatrix}
\ldots
\end{bmatrix}
\approx
\begin{bmatrix}
\ldots
\end{bmatrix}
= \left( \begin{bmatrix}
\ldots
\end{bmatrix} \cdot \left( \begin{bmatrix}
\ldots
\end{bmatrix} \right) + \cdots + \left( \begin{bmatrix}
\ldots
\end{bmatrix} \cdot \left( \begin{bmatrix}
\ldots
\end{bmatrix} \right) \right.
\right.
\]

\[
\begin{bmatrix}
\ldots
\end{bmatrix}
\approx
\begin{bmatrix}
\ldots
\end{bmatrix}
= \left( \begin{bmatrix}
\ldots
\end{bmatrix} \cdot \left( \begin{bmatrix}
\ldots
\end{bmatrix} \right) + \cdots \right.
\]

\[
H \approx \begin{bmatrix}
\ldots
\end{bmatrix}
= \text{conv}(c_1^*) \cdot \text{diag}(w_1^*)
\]

\[
+ \begin{bmatrix}
\ldots
\end{bmatrix}
= \text{conv}(c_4^*) \cdot \text{diag}(w_4^*)
\]
Implementation

Shift-variant blurring can be computed efficiently:

\[ Hf \approx \sum_{i=1}^{k} \text{conv}(m_i)\text{diag}(w_i) f \]
Deblurring illustrations

Simulation of an optical system with aberrations:

(a)  
(b)  
(c)
Deblurring illustrations

Simulation of a blurred image:

original

blurred
Deblurring illustrations

Deblurring results:

5 × 5 interp. PSF (PSNR=25dB)  
1 PSF (PSNR=12dB)
Deblurring illustrations

Deblurring results:

21 × 21 interp. PSF (PSNR=38dB)

1 PSF (PSNR=12dB)
Application to spatio-spectral deblurring

Context: Hyper-spectral imaging with MUSE instrument (integral field spectrometer for deep field astronomical imaging).

The field of view is sliced into rows. Each row of the image is then dispersed to produce a spectro-spatial 2D image. When put together, all spectrum measured form a 3D hyperspectral cube with 300x300 pixels and 3463 bins spectrum.
Application to spatio-spectral deblurring

**Problem:** To improve the quality of the images, it is necessary to deblur the data.

The PSF varies smoothly with the wavelength.

It can be approximated by interpolating few PSF:

\[
h_\lambda(\Delta \theta, \Delta \lambda) \approx \sum_{i=1}^{k} h_i(\Delta \theta, \Delta \lambda) w_i(\lambda)
\]

- spatial profile of the PSF
- spectral profile of the PSF
Application to spatio-spectral deblurring

Result: Deblurring with spatial sparsity constraint (i.e., structured $\ell_1$ norm minimization).

“Restoration of Hyperspectral Astronomical Data with Spectrally Varying Blur”, F. Soulez et al., 2013
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1. An introduction to image restoration

2. Models and inversion of shift-variant blur

3. Blind deconvolution in 3D microscopy
Wide field fluorescence microscopy

- Illumination of the whole sample,
- Observation at the emission wavelength,
- 3D image obtained by scanning the focal plane.

Very coarse depth resolution

How to improve the resolution

- Improve the PSF (confocal, multiphoton, . . . ),
- Use structured illumination (SIM),
- Deconvolution

d’après [Griffa, 2010]
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d’après [Griffa, 2010]
Blind deconvolution

Blur modeled by a convolution: Flou modélisé par la convolution :

\[ g = Hf + n = h * f + n \]

Deconvolution:
Estimate the *crisp image* \( f \) from the blurry and noisy observations \( g \), for a given *PSF* \( h \) and noise statistics.

In practice... The PSF is unknown

- use the theoretical PSF (i.e., diffraction-limited) lacks some flexibility
- experimentally measure the PSF (beads) hard, noisy
- direct estimation of the PSF from the data blind deconvolution
Blind deconvolution

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Blind deconvolution

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3. Blind deconvolution in 3D microscopy

**Blind deconvolution**

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- use the theoretical PSF (i.e., diffraction-limited) (lacks some flexibility)
- experimentally measure the PSF (beads) (hard, noisy)
- direct estimation of the PSF from the data (blind deconvolution)
Blind deconvolution

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  hard, noisy
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Blind deconvolution

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hard, noisy  
blind deconvolution
### Blind deconvolution

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- direct estimation of the PSF from the data

lacks some flexibility
hard, noisy
blind deconvolution
Degradation model

- **Gaussian noise**: 
  
  data fitting term: 
  \[
  (g - H \cdot f)^T \cdot \Sigma^{-1} \cdot (g - H \cdot f)
  \]

- **Unstationary white noise**: 
  
  data fitting term: 
  \[
  \sum_{\text{pixels } j} \frac{1}{\sigma_j^2} [(H \cdot f)_j - g_j]^2
  \]

  Missing pixels \(\rightarrow \sigma_j^2 = \infty\).

- **Photon noise** \(\approx\) unstationary Gaussian noise

  \[
  \sigma_j = \gamma (H \cdot f)_j + \sigma_a^2 \approx \gamma \max(g_j, 0) + \sigma_a^2
  \]

  \(\gamma\): quantization gain

  \(\sigma_a^2\): additive noise sources (e.g. reading noise...).
Degradation model

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- **Unstationary white noise**: 
  
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  \(\gamma\): quantization gain
  \(\sigma_a^2\): additive noise sources (e.g. reading noise...).
Degradation model: PSF parameterization

The PSF $h$ is defined by the pupil function [Markham,1999; Hanser,2004]

$$h(r_j, z) = \left| \sum_k F_{j,k} a_k(z) \right|^2,$$

with $r_j$ the location of pixel $j$, $F$ the discrete Fourier transform, $a_k(z)$ the pupil function at frequency $k$ and at depth $z$.

$$a_k(z) = \rho_k \exp(i 2\pi (\phi_k + z \psi_k)),$$

$$\rho_k = \sum_n \beta_n Z^n_k,$$

$$\phi_k = \sum_n \alpha_n Z^n_k,$$

$$\psi_k = \sqrt{k_0^2 - (\kappa_x - \delta_x)^2 - (\kappa_y - \delta_y)^2}.$$

$Z^n_k$: $n$-th Zernike polynomial, $k_0 = (n_i/\lambda)$ with $n_i$: optical index and $(\delta_x, \delta_y)$: the location of the optical axis.

PSF parameterized by $\{k_0, \alpha, \beta, \delta_x, \delta_y\}$
Degradation model: PSF parameterization

The PSF $h$ is defined by the pupil function [Markham,1999; Hanser,2004]

$$ h(r_j, z) = \left| \sum_k F_{j,k} a_k(z) \right|^2, $$

with $r_j$ the location of pixel $j$, $F$ the discrete Fourier transform, $a_k(z)$ the pupil function at frequency $k$ and at depth $z$.

$$ a_k(z) = \rho_k \exp(i 2\pi (\phi_k + z \psi_k)), $$

$$ \rho_k = \sum_n \beta_n Z_k^n, $$

$$ \phi_k = \sum_n \alpha_n Z_k^n, $$

$$ \psi_k = \sqrt{k_i^2 - (\kappa_x - \delta_x)^2 - (\kappa_y - \delta_y)^2} $$

$Z_k^n$: $n$-th Zernike polynomial, $k_0 = (n_i/\lambda)$ with $n_i$: optical index and $(\delta_x, \delta_y)$: the location of the optical axis.

PSF parameterized by $\{k_0, \alpha, \beta, \delta_x, \delta_y\}$
Degradation model: PSF parameterization

The PSF $h$ is defined by the pupil function [Markham, 1999; Hanser, 2004]

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**PSF parameterized by** \( \{k_0, \alpha, \beta, \delta_x, \delta_y\} \)
3. Blind deconvolution in 3D microscopy

Interest of this parameterization

- model derived from optics (realistic PSF, positive . . . ),
- requires only the knowledge of the wavelength \( \lambda \) and the numerical aperture \( NA \),
- few parameters (a few tenths),
- no other prior: no regularization, no hyperparameter to tune,
- possible simplifications (e.g. use of only radial modes to obtain a symmetrical PSF).
Suppression of degeneracies

Degeneracies of blind deconvolution:

- permutation \( x \ast h = h \ast x \)
- scaling \( (\alpha x) \ast h = x \ast \left( \frac{1}{\alpha} h \right) \)
- shift \( x \ast h = (\delta_{-s} \ast x) \ast (\delta_s \ast h) \)
- identity \( \delta \ast y = y \)
- reducibility \( (g \ast x) \ast h = x \ast (g \ast h) \)
- inversion \( x \ast h = (s \ast x) \ast (s^{-1} \ast h) \)

object \( \neq \) diffraction pattern
normalized PSF: \( \sum_n \beta_n^2 = 1 \)
centered PSF \( \alpha_1 = \alpha_2 = 0 \) (tip-tilt)
limited bandwidth: \( h \neq \delta \)
empirical evidence
empirical evidence

The proposed parameterization solves classical blind deconvolution degeneracies.
Suppression of degeneracies

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object ≠ diffraction pattern

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centered PSF \(\alpha_1 = \alpha_2 = 0\) (tip-tilt)

limited bandwidth: \(h ≠ \delta\)

empirical evidence

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3. Blind deconvolution in 3D microscopy

Suppression of degeneracies

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The proposed parameterization solves classical blind deconvolution degeneracies.
Blind deconvolution algorithm

\[
\{f^+, n_i^+, \alpha^+, \beta^+, \delta^+\} = \arg \min_{f, n_i, \alpha, \beta} \left\{ J_{\text{data}}(f, h(n_i, \alpha, \beta, \delta; g)) + \mu J_{\text{prior}}(f) \right\}
\]

Non-convex and ill-conditioned problem solved by alternated minimization

- \( t = 0 \), start with the diffraction-limited PSF \( h^{(0)} (\alpha = \beta = 0) \),
- \( t = t + 1 \), estimation of the optimal non-negative object \( f^{(t)} \) with given PSF \( h^{(t-1)} \):
  \[
f^{(t)} \approx \arg \min_{f \geq 0} \left(J_{\text{data}}(f, h^{(t-1)}) + \mu J_{\text{prior}}(f)\right),
\]
- Estimation of defocus parameters:
  \[
  \{k_0^{(t)}, \delta^{(t)}\} = \arg \min_{\{k, \delta\}} J_{\text{data}} \left( f^{(t)}, h(\alpha^{(t-1)}, \beta^{(t-1)}, k_0, \delta) \right).
\]
- Estimation of phase parameters:
  \[
  \alpha^{(t)} = \arg \min_\alpha J_{\text{data}} \left( f^{(t)}, h(\alpha, \beta^{(t-1)}, k_0^{(t)}, \delta^{(t)}) \right).
\]
- Estimation of module parameters:
  \[
  \beta^{(t)} = \arg \min_{\sum \beta^2 = 1} J_{\text{data}} \left( f^{(t)}, h(\alpha^{(t)}, \beta, k_0^{(t)}, \delta^{(t)}) \right).
\]
- until convergence.
Blind deconvolution algorithm

\[
\{f^+, n_i^+, \alpha^+, \beta^+, \delta^+\} = \arg \min_{f, n_i, \alpha, \beta} \left\{ \mathcal{J}_\text{data}(f, h(n_i, \alpha, \beta, \delta; g)) + \mu \mathcal{J}_\text{prior}(f) \right\}
\]

Non-convex and ill-conditionned problem solved by alternated minimization

1. \(t = 0\), start with the diffraction-limited PSF \(h^{(0)}\) (\(\alpha = \beta = 0\)),
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   \[
   \{k_0^{(t)}, \delta^{(t)}\} = \arg \min_{\{k_0, \delta\}} \mathcal{J}_\text{data} \left( f^{(t)}, h(\alpha^{(t-1)}, \beta^{(t-1)}, k_0, \delta) \right).
   \]
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   \[
   \alpha^{(t)} = \arg \min_{\alpha} \mathcal{J}_\text{data} \left( f^{(t)}, h(\alpha, \beta^{(t-1)}, k_0^{(t)}, \delta^{(t)}) \right).
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   \beta^{(t)} = \arg \min_{\sum \beta^2 = 1} \mathcal{J}_\text{data} \left( f^{(t)}, h(\alpha^{(t)}, \beta, k_0^{(t)}, \delta^{(t)}) \right),
   \]
6. until convergence.
Simulations

Comparison of several methods from [Kenig,2010]:

MSE =
(a) ground truth 35.81 25.47 19.30 19.10 9.59 10.55
(b) data (c) AIDA (d) KPCA (e) Keuper (f) dec. (g) bl. dec.

AIDA: [Hom,2007], KPCA: [Kenig,2010], Keuper: [Keuper,2013]
Experimental results: calibration beads

— Diameter: 2.5µm, NA = 1.4
— 256³ pixels 64.5 × 64.5 × 160nm³
data from [Griffa,2010]
Deconvolution with the theoretical PSF

— Diameter: 2.5 µm, NA = 1.4
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Blind deconvolution

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data from [Griffa,2010]
3D radial profile of the bead

Performance of 3 deconvolution methods from [Griffa, 2010], compared with our method.

Hyugens et AutoDeblur are commercial software Deconvolution Lab is a plugin for imageJ.
3. Blind deconvolution in 3D microscopy

Calibration beads: PSF

Theoretical PSF and estimated PSF in XY slice, XZ slice, and YZ slice.
3. Blind deconvolution in 3D microscopy

Experimental data

Blind deconvolution of S2 cell compared to wide field and structured illumination microscopy. 

(1024 × 1024 × 44) voxels of 40 × 40 × 110 nm³. NA = 1.4.
Experimental data

xy slice and xz slice of a filament:

Resolution close to that of a structured illumination microscope (SIM) with a simple wide field microscope!
Experimental data: C. Elegans

Embryon de C. Elegans

- microscope lens $\times 63$, 1.4 NA,
- $672 \times 712 \times 104$ voxels,
- voxels $64.5 \times 64.5 \times 200 \text{ nm}^3$

data from [Griffa,2010]