How do many-particle systems evolve in time? Our daily experience is that when we perturb a many-particle system (for instance, we stir a cup of coffee with a spoon) it will rapidly relax to an equilibrium state, unless the perturbation is persistently applied to it — namely unless the system is persistently driven out of equilibrium.

Does quantum mechanics add anything to this picture? If you agitate a system at very low temperature / entropy so that its degrees of freedom exhibit some degree of quantum coherence, is the dynamics any different?

The answer is “yes!” in at least three different ways:

1) The coffee you stir is in contact with the cup that contains it, as well as with the outer air. Therefore in general what you are really probing is not the response of the coffee alone. On the other hand, engineering many-particle systems using assemblies of very few (real or artificial) atoms (from some tens up to some hundred thousands) allows to realize highly controlled systems— so called quantum simulators, based on ultracold atoms [1], superconducting circuits [2], etc. — which can be considered as decoupled from any form of environment over long times, and this happens naturally in a regime in which such systems are quantum coherent;

2) The very same setups for quantum simulation allow one to put the environment back into the picture, but this time in an engineered manner, and in particular with a coupling to the system of interest that can also be engineered [3];

3) Finally, because of quantum coherent effects, quantum dynamics can behave in a radically different way with respect to classical systems. One dramatic quantum phenomenon is that of localization of the dynamics in the presence of strong disorder: for sufficiently strong disorder, quantum systems — unlike classical ones — may completely fail to relax back to thermal equilibrium after being disturbed, realizing exotic non-equilibrium stationary states with dramatic disorder-dependent features, including singularities at a (many-body) localization transition.

In the face of this richness and fundamental interest, the quantitative study of non-equilibrium quantum dynamics still represents a great challenge to theoretical methods, because of the absence of scalable, numerically exact methods for quantum many-body dynamics which can predict arbitrary stages of the time evolution of the system.

The goal of this thesis project is multiple. On the one hand, we will focus on efficient semi-analytical as well as numerical techniques to address the study of non-equilibrium quantum many-body systems, either isolated from any environment as well as coupled to an engineered environment. The general strategy we propose to find an approximate solution to the time-dependent many-body Schrödinger equation is a variational one, based on the choice of an explicit, compact form for the time-dependent many-body wavefunction; such a strategy
underpins some of the most popular approaches, as *e.g.* those based on (weakly entangled) matrix product states. At variance with other approaches, our variational choice will be based on strongly entangled quantum states, either of semiclassical character — in the form of Gaussian Ansätze for both bosons or fermions, which allow for a semi-analytical solution of the quantum dynamics [4]; or fully quantum, based on recent generalizations of the entangled-plaquette state Ansatz [5]. These approaches have the unique advantage of accommodating the fast buildup of a very strong entanglement along the time evolution; and they can be straightforwardly adapted to the variational study of a system coupled to a bath — namely to the search of an approximate solution to the stochastic Schrödinger equation which describes the evolution of the system perturbed by the bath [6].

Armed with this analytical and numerical background, we can attack a broad range of different questions which are animating the current research on quantum many-body dynamics [7], namely:

a) in disordered quantum systems, what is the critical behavior at the transition between the weak-disorder regime — in which the system relaxes to thermal equilibrium — and the strong-disorder regime — in which the system is many-body localized away from equilibrium?

b) what is the effect of dimensionality / long-range interactions on many-body localization?

c) in open quantum systems, can one have a strong competition between a stationary state stabilized by the Hamiltonian evolution (for instance, a thermal equilibrium state, or a many-body-localized state) and a different state stabilized by the coupling with a (properly engineered) bath? And what is the nature of this transition?

The theoretical work is framed within a close collaboration with Fabio Mezzacapo (ENS de Lyon) for the development of variational Ansätze in strongly correlated systems. Moreover many of these questions are tightly related to experiments in cold atoms, and will be addressed in the framework of two funded projects (a French one and a European one) involving a network of experimental and theory teams which include: the Laboratoire de Physique des Lasers, Paris XIII; the Center for Ultracold Atoms and Quantum Gases, University of Innsbruck (Austria); the 5th Institute of Physics of the University of Stuttgart (Germany); the CNR-INO in Pisa (Italy); the Institute for Photonic Sciences (Spain); and the Institute of Physics of the Polish Academy of Sciences (Poland).