Strategies for Replica Placement
in Tree Networks

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Introduction and motivation

- Replica placement in tree networks
- Set of clients (tree leaves): requests with QoS constraints, known in advance
- Internal nodes may be provided with a replica; in this case they become servers and process requests (up to their capacity limit)

How many replicas required?
Which locations?
Total replica cost?

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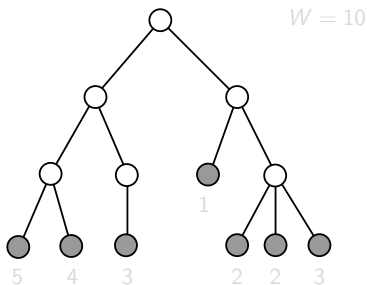
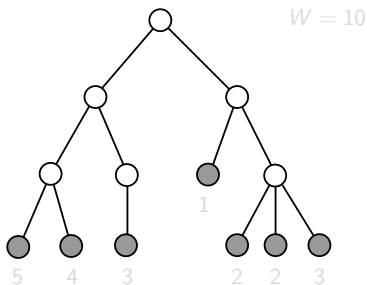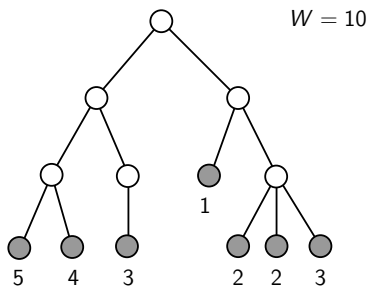How many replicas required?
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Rule of the game

- Handle all client requests, and minimize cost of replicas
- → REPLICA PLACEMENT problem
- Several policies to assign replicas

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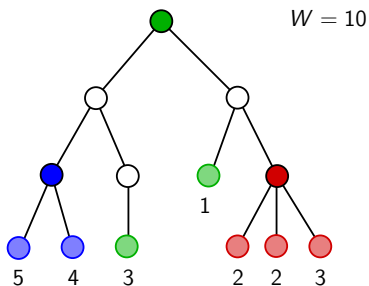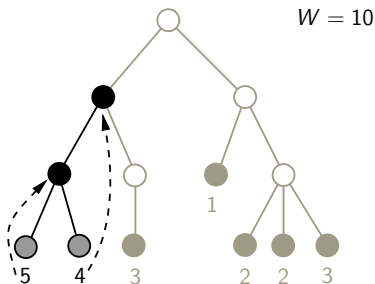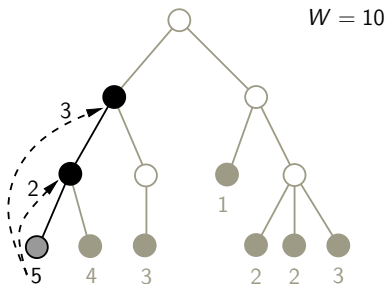- → REPLICA PLACEMENT problem
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Major contributions

Theory New access policies
Problem complexity
LP-based lower bound to cost of REPLICA
PLACEMENT

Practice Heuristics for each policy
Experiments to assess impact of new policies

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Outline

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1. **Framework**

2. Access policies

3. Complexity results

4. Linear programming formulation

5. Heuristics for REPLICA COST problem

6. Experiments

7. Extensions

8. Conclusion

Definitions and notations

- Distribution tree $\mathcal{T}$, clients $\mathcal{C}$ (leaf nodes), internal nodes $\mathcal{N}$
- **Client** $i \in \mathcal{C}$:
    - Sends r_i requests per time unit (number of accesses to a single object database)
    - Quality of service q_i (response time)
- **Node** $j \in \mathcal{N}$:
    - Can contain the object database replica (server) or not
    - Processing capacity W_j
    - Storage cost sc_j
- **Tree edge:** $l \in \mathcal{L}$ (communication link between nodes)
    - Communication time $comm_l$
    - Bandwidth limit BW_l

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Tree notations

- r: tree root
- children(j): set of children of node $j \in \mathcal{N}$
- parent(k): parent in the tree of node $k \in \mathcal{N} \cup \mathcal{C}$
- link $l : k \rightarrow \text{parent}(k) = k'$. Then succ($l$) is the link $k' \rightarrow \text{parent}(k')$ (when it exists)
- Ancestors(k): set of ancestors of node k
- If $k' \in$ Ancestors(k), then path$[k \rightarrow k']$: set of links in the path from k to k'
- subtree(k): subtree rooted in k, including k.

Problem instances

- Goal: place replicas to process client requests
- Client $i \in \mathcal{C}$: Servers$(i) \subseteq \mathcal{N}$ set of servers responsible for processing its requests
- $r_{i,s}$: number of requests from client i processed by server s ($\sum_{s \in \text{Servers}(i)} r_{i,s} = r_i$)
- $R = \{s \in \mathcal{N} \mid \exists i \in C , s \in \text{Servers}(i)\}$: set of replicas

Constraints

- **Server capacity** $- \forall s \in R, \sum_{i \in \mathcal{C} | s \in \text{Servers}(i)} r_{i,s} \leq W_s$

- **QoS** $- \forall i \in \mathcal{C}, \forall s \in \text{Servers}(i), \sum_{l \in \text{path}[i \rightarrow s]} \text{comm}_l \leq q_i.$

- **Link capacity** $- \forall l \in \mathcal{L} \sum_{i \in \mathcal{C}, s \in \text{Servers}(i) | l \in \text{path}[i \rightarrow s]} r_{i,s} \leq BW_l$

Objective function

- Min $\sum_{s \in R} sc_s$

- Restrict to case where $sc_s = W_s$

- REPLICA COST problem: no QoS nor bandwidth constraints; heterogeneous servers

- REPLICA COUNTING problem: idem, but homogeneous platforms

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Single server vs Multiple servers

Single server – Each client i is assigned a single server $server(i)$, that is responsible for processing all its requests.

Multiple servers – A client i may be assigned several servers in a set $Servers(i)$. Each server $s \in Servers(i)$ will handle a fraction $r_{i,s}$ of the requests.

In the literature: single server policy with additional constraint.
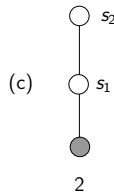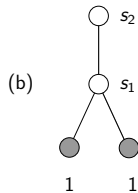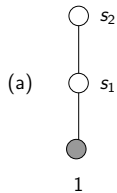
Closest policy

- *Closest*: single server policy
- Server of client i is constrained to be first server found on the path that goes from i upwards to the tree root
- Consider a client i and its server server(i):
  $\forall i' \in$ subtree(server(i)),   server(i') $\in$ subtree(server(i))
- Requests from i' cannot "traverse" server(i) and be served higher
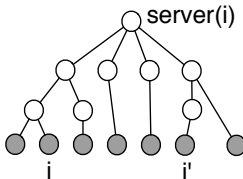
Upwards and *Multiple* policy

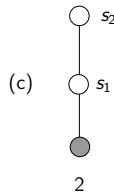- New policies not studied in the literature

- *Upwards*: *Closest* constraint is relaxed
- *Multiple*: relax single server restriction

- Expect more solutions with new policies, at a lower cost
- QoS constraints may lower difference between policies

Example: existence of a solution

- (a): solution for all policies

- (b): no solution with *Closest*

- (c): no solution with *Closest* nor *Upwards*

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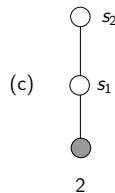
Upwards versus *Closest*
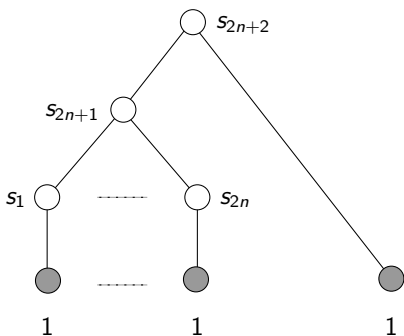
$W = n$

- *Upwards*: 3 replicas in s_{2n}, s_{2n+1} and s_{2n+2}
- *Closest*: at least $n + 2$ replicas (replica in s_{2n+1} or not)

Intro Framework **Policies** Complexity LP Heuristics Experiments Extensions Conclusion
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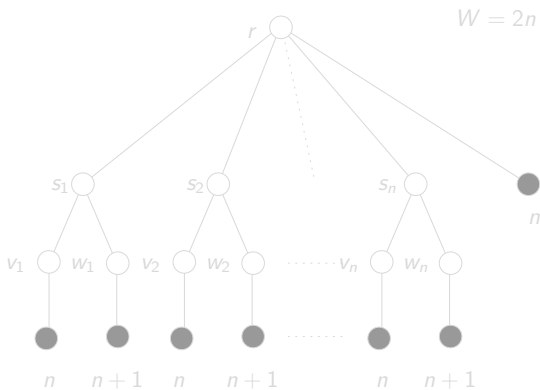
Multiple versus *Upwards*

- REPLICA COUNTING: *Multiple* twice better than *Upwards*.
- Performance ratio: open problem.

Multiple: $n + 1$ replicas / *Upwards*: $2n$ replicas

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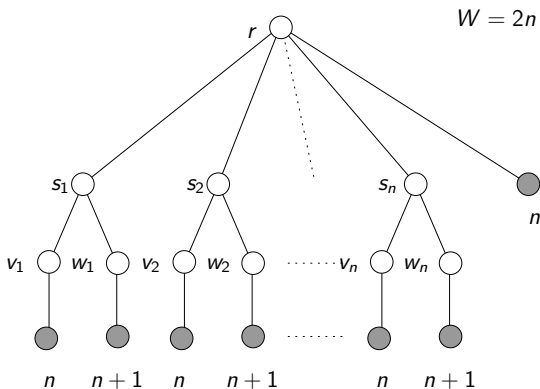
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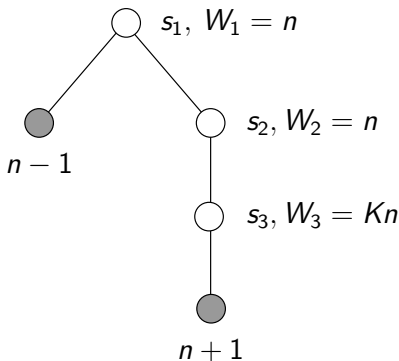
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Lower bound for the REPLICA COUNTING problem

Obvious lower bound: $\left\lceil \frac{\sum_{i \in \mathcal{C}} r_i}{W} \right\rceil = 2$
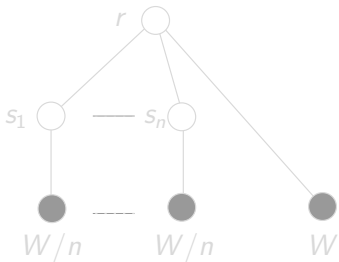
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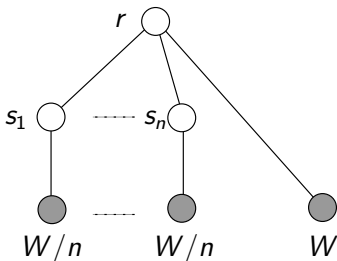
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3. **Complexity results**

4. **Linear programming formulation**

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Complexity results - Basic problem

| | REPLICA COUNTING | REPLICA COST |
	Homogeneous	**Heterogeneous**
Closest	polynomial [Cidon02,Liu06]	
Upwards		
Multiple		

Table: Complexity results for the different instances of the problem

- *Closest*/Homogeneous: only known result (Cidon et al. 2002, Liu et al. 2006)

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Upwards	NP-complete	NP-complete
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- All instances for the Heterogeneous case are NP-complete

Complexity results - QoS and Bandwidth

- *Closest*/Homogeneous + QoS: Polynomial (Liu et al.)

- *Closest*/Homogeneous + Bandwidth: Polynomial - algorithm quite similar to the case with QoS

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Multiple/Homogeneous: greedy algorithm

3-pass algorithm:

- Select nodes which can handle W requests
- Select some extra servers to fulfill remaining requests
- Decide which requests are processed where

Example to illustrate algorithm (informally)

Proof of optimality: any optimal solution can be transformed into
a solution similar to the one of the algorithm (moving requests
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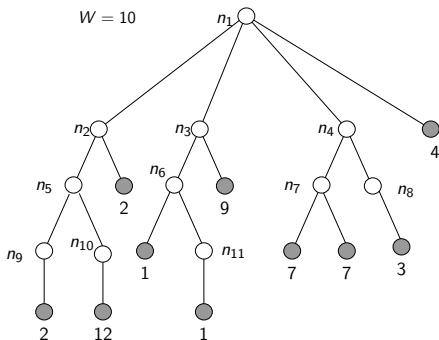
Initial network

The example network

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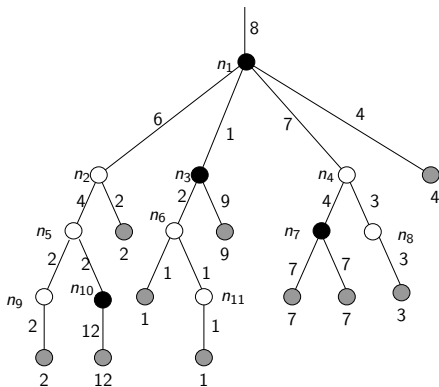
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Placing *saturated* replicas

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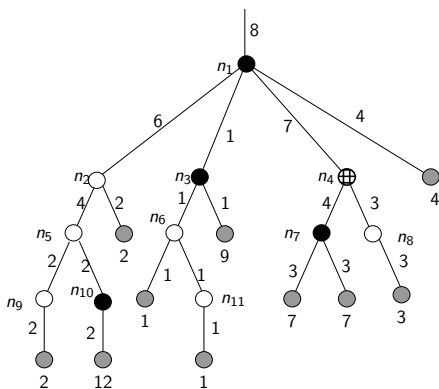
Pass 2

Placing extra replicas: n_4 has maximum useful flow

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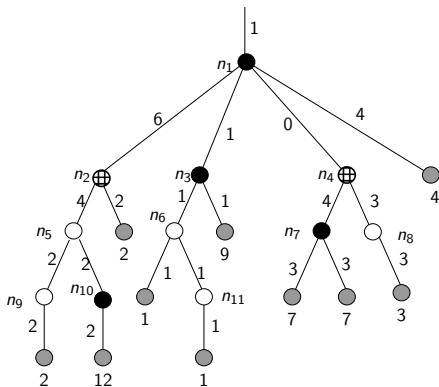
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Placing extra replicas: n_2 is of maximum useful flow 1

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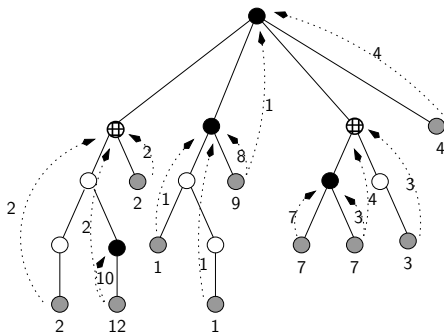
Pass 3

Deciding where requests are processed

Upwards/Homogeneous

- The REPLICA COUNTING problem with the *Upwards* strategy is NP-complete in the strong sense
- Reduction from 3-PARTITION

n_m

$W = B$

n_2

n_1

$\sum_{i=1}^{3m} a_i = mB$

c_1 c_2 c_3 c_{3m}
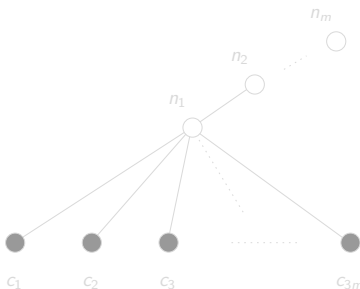
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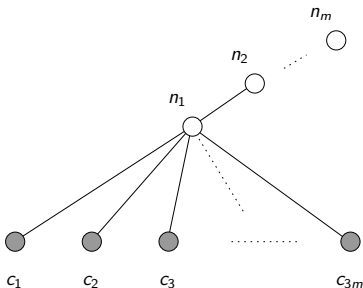
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Heterogeneous network: REPLICA COST problem

- All three instances of the REPLICA COST problem with heterogeneous nodes are NP-complete
- Reduction from 2-PARTITION

$\sum_{i=1}^{m} a_i = S$, $a_{m+1} = 1$, $W_j = a_i$, $W_r = S/2 + 1$

Solution with total storage cost $S + 1$?

Heterogeneous network: $\textsc{Replica Cost}$ problem

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Linear programming

- General instance of the problem
    - Heterogeneous tree
    - QoS and bandwidth constraints
    - *Closest*, *Upwards* and *Multiple* policies
- Integer linear program: no efficient algorithm
- Absolute lower bound if program solved over the rationals (using the GLPK software)
- *Closest/Upwards* LP formulation

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Linear program: variables

- x_j: boolean variable equal to 1 if j is a server (for one or several clients)
- $y_{i,j}$: boolean variable equal to 1 if $j = \text{server}(i)$
    - If $j \notin \text{Ancests}(i)$, $y_{i,j} = 0$
- $z_{i,l}$: boolean variable equal to 1 if link $l \in \text{path}[i \to r]$ used when i accesses $\text{server}(i)$
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Objective function: $\sum_{j \in \mathcal{N}} \text{sc}_j x_j$

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Linear program: constraints

- **Servers**: $\forall i \in \mathcal{C}, \sum_{j \in \text{Ancestors}(i)} y_{i,j} = 1$

- Links: $\forall i \in \mathcal{C}, z_{i,i \rightarrow \text{parent}(i)} = 1$

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Multiple formulation

Multiple

- Similar formulation, with
  - $y_{i,j}$: integer variable = nb requests from client i processed by node j
  - $z_{i,l}$: integer variable = nb requests flowing through link l

- Constraints are slightly modified

A mixed integer LP-based optimal solution

- Solving over the rationals: solution for all practical values of the problem size
  - Not very precise bound
  - *Upwards*/*Closest* equivalent to *Multiple* when solved over the rationals

- Integer solving: limitation to $s \leq 50$ nodes and clients

- Mixed bound obtained by solving the *Upwards* formulation over the rational and imposing only the x_j being integers
  - Resolution for problem sizes $s \leq 400$
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Outline

| Intro | Framework | Policies | Complexity | LP | **Heuristics** | Experiments | Extensions | Conclusion |
| 0000 | 00000 | 00000000 | 000000 | 00000 | ●0000 | 0000000000 | 000 | 000 |

Heuristics

- Polynomial heuristics for REPLICA COST problem
    - Heterogeneous platforms
    - No bandwidth constraints
    - Heuristics with and without QoS

- Experimental assessment of relative performance of the three policies

- Impact of QoS

- Traversals of the tree, bottom-up or top-down

- Worst case complexity $O(s^2)$,
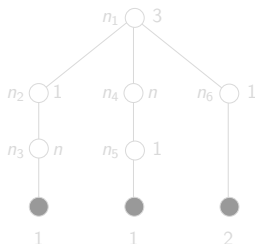 where $s = |\mathcal{C}| + |\mathcal{N}|$ is problem size

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Heuristics for *Closest*

- Closest Top Down All **CTDA**
    - Breadth-first traversal of the tree
    - When a node can process the requests of all the clients in its subtree, node chosen as a server and exploration of the subtree stopped
    - Procedure called until no more servers are added
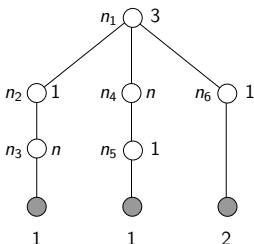   - Choosing n_2, n_4 and then n_1

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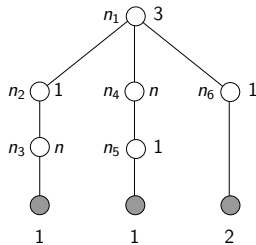   - Choosing n_2, n_4 and then n_1

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    - Choosing n_3, n_5, n_1
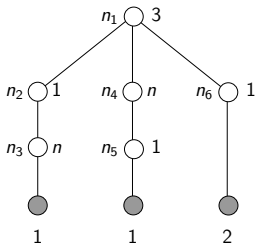
Heuristics for *Upwards*

- Upwards Top Down **UTD**
  - 2-pass algorithm
  - Select first saturating nodes, then extra nodes
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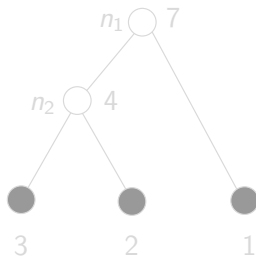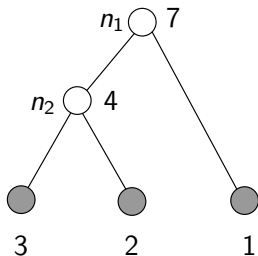
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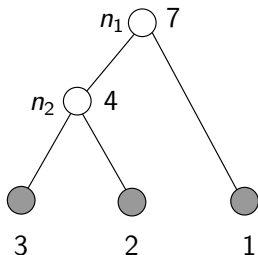
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  - Sorting clients by decreasing request numbers, and finding the server of minimal available capacity to process its requests.
  - Choosing n_2 for c_1, n_1 for c_2 and n_1 for c_3

Heuristics for *Multiple*

- A top-down and a bottom-up heuristic in 2-passes (**MTD**, **MBU**)

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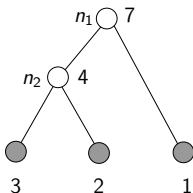
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- CTDLF better on this example: selects n_1 only
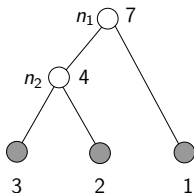
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- Heuristic MixedBest **MB** which picks up best result over all heuristics: solution for the *Multiple* policy

Heuristics with QoS

- Bunch of similar polynomial heuristics with QoS constraints
- Tradeoff between big and QoS-critic clients
- Identifying indispensable servers: clients with QoS=1 need be served by their parent, and so on
- MixedBest heuristic which picks up the best result

Outline

1 Framework

2 Access policies

3 Complexity results

4 Linear programming formulation

5 Heuristics for REPLICA COST problem

6 Experiments

7 Extensions

8 Conclusion

Plan of experiments

- Assess impact of the different access policies
- Assess performance of the polynomial heuristics
- Assess impact of QoS

- Important parameter:

$$\lambda = \frac{\sum_{i \in \mathcal{C}} r_i}{\sum_{j \in \mathcal{N}} W_i}$$

- 30 trees for each $\lambda = 0.1, 0.2, ..., 0.9$
- Problem size $s = |\mathcal{C}| + |\mathcal{N}|$ such that $15 \le s \le 400$
- Computation of the LP optimal solution for each tree

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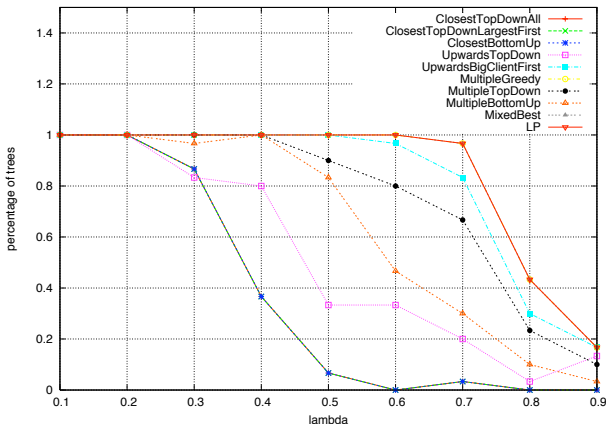
- Number of solutions for each lambda and each heuristic
- No LP solution $\rightarrow$ No solution for any heuristic
- Homogeneous case

Results - Percentage of success

- Heterogeneous trees: similar results

- Striking impact of new policies
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Results - Solution cost

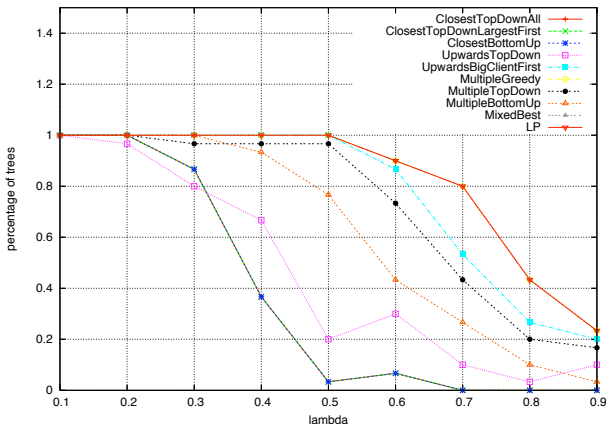- Distance of the result (in terms of replica cost) of the heuristic to the optimal
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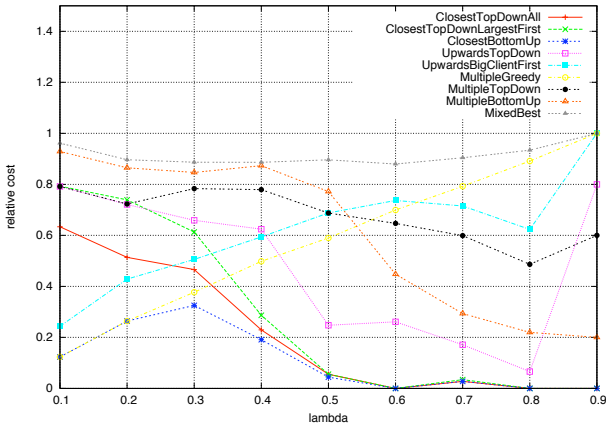
Results - Solution cost

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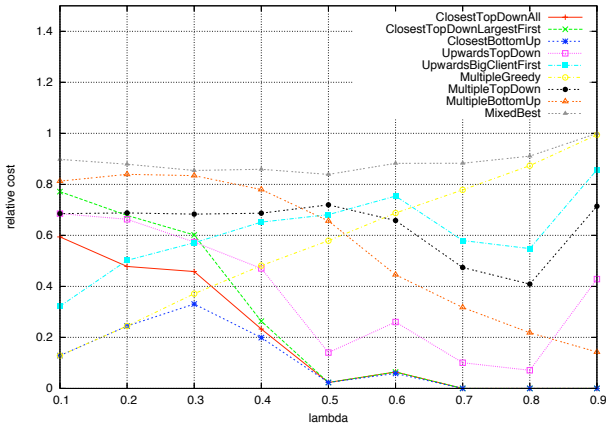
Summary

- Striking effect of new policies: many more solutions to the REPLICA PLACEMENT problem
- *Multiple* $\geq$ *Upwards* $\geq$ *Closest*: hierarchy observed within our heuristics
- Best *Multiple* heuristic (MB) always at 85% of the optimal: satisfactory result

Results - QoS impact

Big trees, $average(QoS) = height/2$
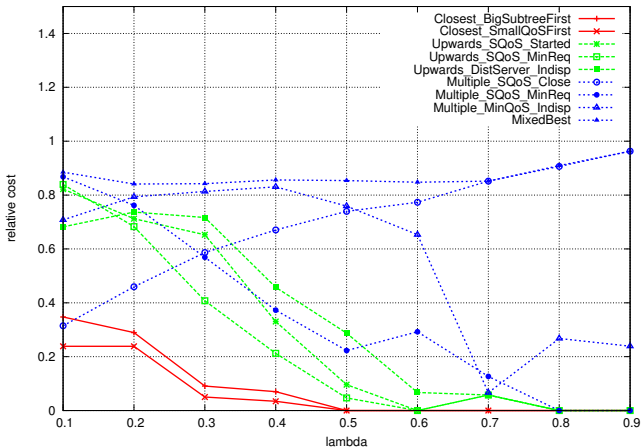
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Big trees, $QoS \in \{1, 2\}$ - very constrained
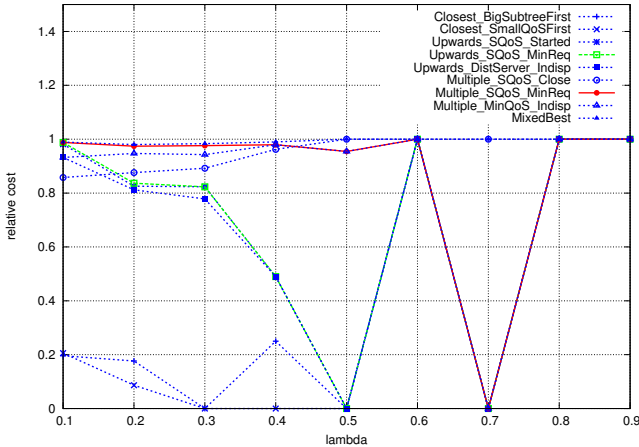
Summary

- *Multiple* ≥ *Upwards* ≥ *Closest*: hierarchy also under QoS-constraints
- Best *Multiple* heuristic (MB) at 80% of optimal solution with average QoS constraints: $average(qos) = height/2$
- Better results with big trees (height between 16 and 21) than smaller trees

$$QoS \in \{1,2\}: \quad 95\% \text{ (vs 90\% with an exception)}$$
$$average(QoS) = height/2: \quad 85\% \text{ (vs 80\%)}$$
$$\text{no QoS}: \quad 85\% \text{ (vs 70\%)}$$

- Good performance of heuristics with QoS

Outline

Extensions

- Simplified problem instance for this work

- Possible generalizations:
  - Several objects
  - More complex objective function

Extensions - Several objects

- We considered a single object: all replicas are identical
- Different types of objects need to be accessed: clients have requests of different types

- New parameters:
  - Requests per object r_i^k, and $q_i^{(k)}$
  - Size of the object, computation time involved, storage cost, ...

- Constraints and objective function slightly modified
- Constraints/Objective function add up linearly for different objects: LP-formulation easily extended.

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Cost of replica – What we considered in this work

Communication cost – This cost is the *read* cost

Update cost – The *write* cost is the extra cost due to an update of the replicas

Linear combination – A quite general objective function can be obtained by a linear combination of the three different costs

$$\alpha \sum_{\text{servers, objects}} \text{replica cost} + \beta \sum_{\text{requests}} \text{read cost} + \gamma \sum_{\text{updates}} \text{write cost}$$

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Update cost – The *write* cost is the extra cost due to an update of the replicas

Linear combination – A quite general objective function can be obtained by a linear combination of the three different costs

$$\alpha \sum_{\text{servers, objects}} \textit{replica cost} + \beta \sum_{\text{requests}} \textit{read cost} + \gamma \sum_{\text{updates}} \textit{write cost}$$

Efficient heuristics in this case: challenging problem

Outline

1. Framework

2. Access policies

3. Complexity results

4. Linear programming formulation

5. Heuristics for REPLICA COST problem

6. Experiments

7. Extensions

8. Conclusion

Related work

- Several papers on replica placement, but...

- ...all consider only the *Closest* policy

- REPLICA PLACEMENT in a general graph is NP-complete

- Wolfson and Milo: impact of the *write* cost, use of a minimum spanning tree for updates. Tree networks: polynomial solution

- Cidon et al (multiple objects) and Liu et al (QoS constraints): polynomial algorithms for homogeneous networks.

- Kalpakis et al: NP-completeness of a variant with bidirectional links (requests served by any node in the tree)

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Intro   Framework   Policies   Complexity   LP   Heuristics   Experiments   Extensions   **Conclusion**
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Conclusion

Introduction of two new policies for the REPLICA PLACEMENT problem, *Upwards* and *Multiple*: natural variants of the standard *Closest* approach → surprising they have not already been considered

Theoretical side – Complexity of each policy, for homogeneous and heterogeneous platforms

Practical side

- Design of several heuristics for each policy
- Comparison of their performance
- Striking impact of the policy on the result
- QoS is not changing the hierarchy of policies
- Use of a LP-based optimal solution to assess the absolute performance, which turns out to be quite good.

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Future work

Short term

- More simulations for the REPLICA COST problem: shape of the trees, distribution law of the requests, degree of heterogeneity of the platforms
- Designing heuristics for more general instances of the REPLICA PLACEMENT problem (bandwidth constraints): these constraints may lower the difference between policies

Longer term

- Consider the problem with several object types
- Extension with more complex objective functions

Still a lot of challenging algorithmic problems ☺

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