Functional programming languages

Part V: functional intermediate representations

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Intermediate representations in a compiler

Between high-level languages and machine code, compilers generally go through one or several intermediate representations where, in particular:

- Expressions are decomposed in a sequence of processor-level instructions.

```
        x = (y + z) * (a - b)
-->
        t1 = y + z; t2 = a - b; x = t1 * t2;
```

- Temporary variables (`t1`, `t2`) are introduced to hold intermediate results.
- These temporaries, along with program variables, can later be placed in concrete locations: processor registers or stack slots.

Outline

1. A conventional IR: RTL-CFG
2. CPS as a functional IR
3. Another functional IR: A-normal forms

A conventional intermediate representation: RTL-CFG

(Register Transfer Language with Control-Flow Graph.)

A function = a set of processor-level instructions operating over variables and temporaries, e.g.

```
x = y + z
t = load(x + 8)
if (t == 0)
```

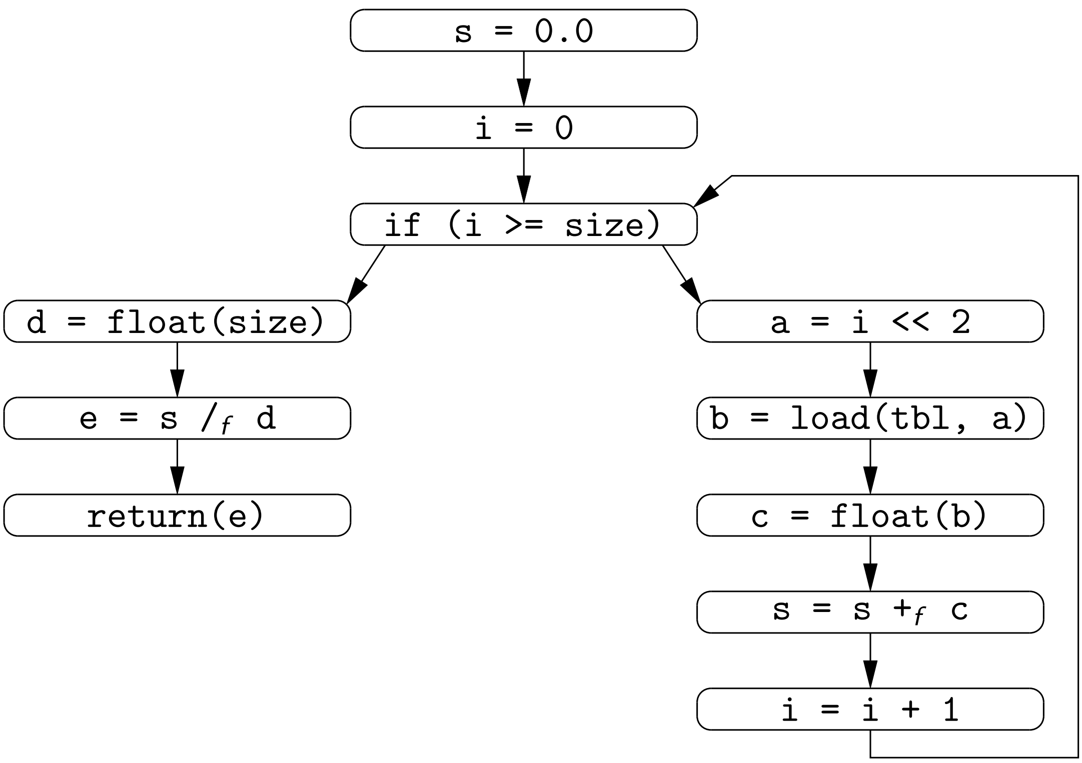
Organized in a control-flow graph:

- Nodes = instructions.
- Edge from I to J $=$ J can execute just after I.

Example: some source code

```
double average(int * tbl, int size)
{
    double s = 0.0;
    int i;
    for (i = 0; i < size; i++)
        s = s + tbl[i];
    return s;
}
```


Example: the corresponding RTL graph



Classic optimizations over RTL

Many classic optimizations can be performed on the RTL form.

- Constant propagation

```
a = 1                    a = 1
b = 2          -->       b = 2
c = a + b                c = 3
d = x - a                d = x + (-1)
```

- Dead code elimination

```
a = 1                    nop
b = 2          -->       b = 2
c = 3                    c = 3
```

(if a unused later)

- Common subexpression elimination

```
c = a                  c = a
d = a + b      -->     d = a + b
e = c + b              e = d
```

- Hoisting of loop-invariant computations

```
L: c = a + b                 c = a + b
   ...              -->   L: ...
   ... -> L                  ... -> L
```

- Induction variable elimination

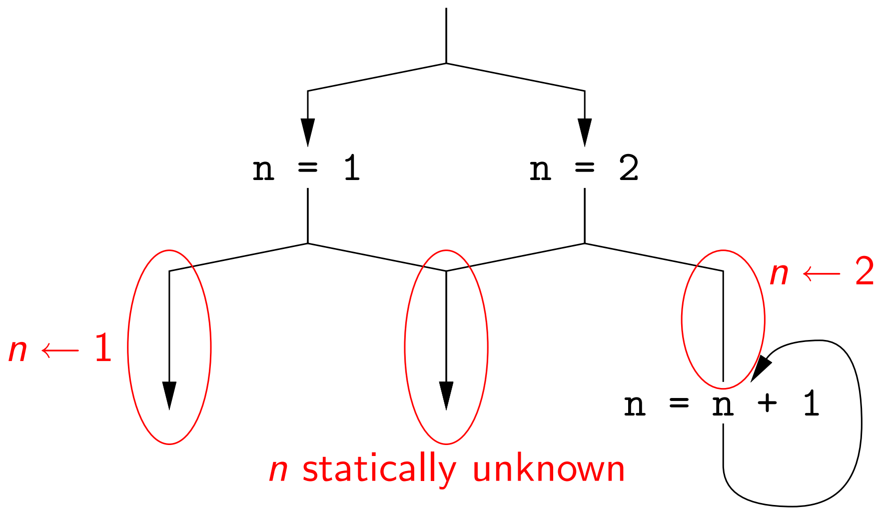
```
   i = 0                     i = 0
L: a = i * 4       -->       b = p
   b = p + a              L: ...
   ...                       b = b + 4
   i = i + 1 -> L            i = i + 1 -> L
```

- ... and much more. (See e.g. Steven Muchnick, *Advanced Compiler Design and Implementation*, Morgan Kaufmann Publishers.)

RTL optimizations and dataflow analysis

Problem: it is not obvious to see where these optimizations apply, because

1. A given variable or temporary can be defined several times. (Unavoidable if the source language is imperative.)
2. The CFG is not a structured representation of control.

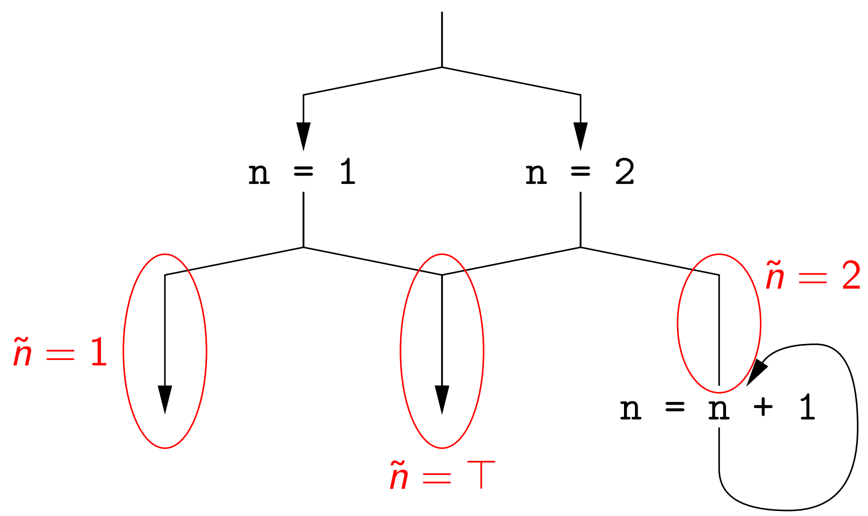


RTL optimizations and dataflow analysis

Solution: use static analyses to determine opportunities for optimization, e.g. dataflow analyses (a simple case of abstract interpretation).

Example: for constant propagation, use the abstract lattice

$$a := \top \mid \bot \mid N$$



Outline

CPS as a functional IR

CPS terms share many features of intermediate representations. In particular, expressions are decomposed in individual operations and intermediate results are named.

Example: source term `let` $x = (y + z) * (a - b)$ `in ...`.

CPS	RTL

`(y + z) $ (λt.`
`(a - b) $ (λu.`
`(t * u) $ (λx.`
`...)))`
	`t = y + z;`
	`u = a - b;`
	`x = t * u;`
	`...`

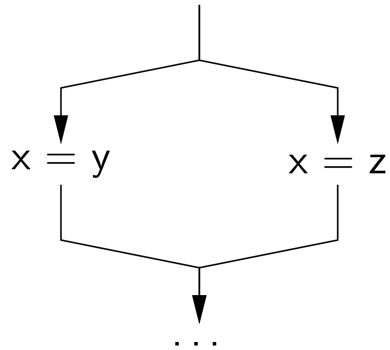
(We write \$ for reverse function application: $a \, \$ \, b = b\ a$.)

CPS as a functional IR

Likewise, `let`-bound continuations correspond to join points in a control-flow graph.

Example: source term `let` $x =$ (`if` c `then` y `else` z) `in` ...

```
let k = λx ... in
if c
then k y
else k z
```



Optimizations on CPS terms

When expressed over CPS terms, many classic optimizations boil down to β or arithmetic reductions, or variants thereof.

Example: constant propagation $\approx \beta$, arithmetic reduction.

$$\begin{array}{ll} & 1 \ \$ \ (\lambda x. \ \ldots \ x+1 \ \ldots \ x+y \ \ldots) \\ \rightarrow & \ldots \ 2 \ \ldots \ 1+y \ \ldots \end{array}$$

Example: common subexpression elimination $\approx$ inverse β

```
(a + b) $ (λx.               (a + b) $ (λx.
...                  --->    ...
(a + b) $ (λy.               x $ (λy.
...                          ...
```

Back to direct style

To support stack-allocation of activation records, several functional compilers perform an inverse CPS transformation after CPS optimization, to recover direct-style function calls.

$$\begin{tikzcd}
\text{DS} & \text{Source} \arrow[d, "\text{CPS transf.}"'] & & {} \\
\text{CPS} & {} \arrow[r, "\text{administrative}", "\text{reductions}"'] & {} \arrow[r, "\text{optimizing}", "\text{reductions}"'] & {} \arrow[u, "\text{inverse CPS transf.}"']
\end{tikzcd}$$

The origin of ANF

In 1993, Flanagan, Sabry and Felleisen showed that this detour through CPS can be avoided, and indeed is unnecessary in the following formal sense:

$$\begin{tikzcd}
\text{DS} & \text{Source} \arrow[r, "\text{ANF}", "\text{conversion}"'] \arrow[d, "\text{CPS transf.}"'] & {} \arrow[r, "\text{optimizing}", "\text{reductions}"'] & {} \\
\text{CPS} & {} \arrow[r, "\text{administrative}", "\text{reductions}"'] & {} \arrow[u, "\text{inverse CPS transf.}"'] \arrow[r, "\text{optimizing}", "\text{reductions}"'] & {} \arrow[u, "\text{inverse CPS transf.}"']
\end{tikzcd}$$

ANF stands for "administrative normal form", and is the direct-style sub-language that is the target of inv-CPS-transf $\circ$ adm-red $\circ$ CPS-transf.

(C. Flanagan, A. Sabry, M. Felleisen, *The essence of compiling with continuations*, PLDI 1993.)

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Syntax of ANF

Atom:

$$a ::= x \mid N \mid \lambda \vec{x}.b$$

Computation:

$c ::= a_1 \; op \; a_2$	arithmetic
$\mid a(\vec{a})$	function application
$\mid C(\vec{a})$	datatype constructor
$\mid \texttt{closure}(a, \vec{a})$	closure constructor

Body:

$b ::= c$	tail computation
$\mid \texttt{let } x = c \texttt{ in } b$	sequencing
$\mid \texttt{if } a \texttt{ then } b_1 \texttt{ else } b_2$	conditional
$\mid \texttt{match } a \texttt{ with } \ldots p_i \to b_i \ldots$	pattern-matching

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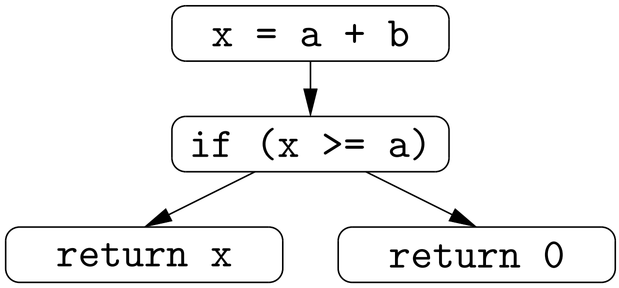
ANF as a CFG

```
let x = a + b in
if (x >= a)
then x
else 0
```



Conversion to ANF

Step 1: perform monadic conversion.

Example 1

Source term: $1 + ($`if` $x >= 0$ `then` $f(x)$ `else` $0)$

Monadic conversion:

```
bind (if x >= 0 then f(x) else ret 0)
     (λt. 1 + t)
```

Conversion to ANF

Step 2: interpret the result in the Identity monad:

$$\begin{array}{rcl} \texttt{ret}\ a & \mapsto & a \\ \texttt{bind}\ a\ (\lambda x.b) & \mapsto & \texttt{let}\ x = a\ \texttt{in}\ b \end{array}$$

Example 2

Source term: $1 + (\texttt{if}\ x >= 0\ \texttt{then}\ f(x)\ \texttt{else}\ 0)$

Monadic conversion + identity monad:

```
let t = if x >= 0 then f(x) else ret 0
in 1 + t
```


Conversion to ANF

Step 3: "flatten" the nesting of `let`, `if` and `match`.

$$\begin{array}{l} \texttt{let}\ x = (\texttt{let}\ y = a\ \texttt{in}\ b)\ \texttt{in}\ c \\ \quad \to \quad \texttt{let}\ y = a\ \texttt{in}\ \texttt{let}\ x = b\ \texttt{in}\ c \quad (\text{if } y \text{ not free in } c) \\ \texttt{let}\ x = (\texttt{match}\ a\ \texttt{with}\ \ldots p_i \to b_i \ldots)\ \texttt{in}\ c \\ \quad \to \quad \texttt{match}\ a\ \texttt{with}\ \ldots p_i \to \texttt{let}\ x = b_i\ \texttt{in}\ c \ldots \\ \texttt{match}\ (\texttt{match}\ a\ \texttt{with}\ \ldots p_i \to b_i \ldots)\ \texttt{with}\ \ldots q_j \to c_j \ldots \\ \quad \to \quad \texttt{match}\ a\ \texttt{with}\ \ldots p_i \to (\texttt{match}\ b_i\ \texttt{with}\ \ldots q_j \to c_j \ldots) \ldots \end{array}$$

Example 3

```
if x >= 0
then let t = f(x) in 1 + t
else let t = 0 in 1 + t
```

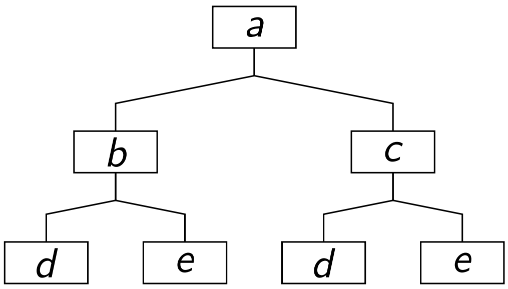
Tail duplication, and how to avoid it

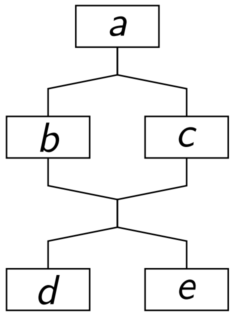
Note that possibly large terms can be duplicated:

$$\begin{array}{l} \texttt{if}\ (\texttt{if}\ a\ \texttt{then}\ b\ \texttt{else}\ c)\ \texttt{then}\ d\ \texttt{else}\ e \\ \quad \to \quad \texttt{if}\ a\ \texttt{then}\ (\texttt{if}\ b\ \texttt{then}\ {\color{red}d}\ \texttt{else}\ {\color{red}e})\ \texttt{else}\ (\texttt{if}\ c\ \texttt{then}\ {\color{red}d}\ \texttt{else}\ {\color{red}e}) \end{array}$$

This can be avoided by using auxiliary functions:

$$\begin{array}{l} \texttt{if}\ (\texttt{if}\ a\ \texttt{then}\ b\ \texttt{else}\ c)\ \texttt{then}\ d\ \texttt{else}\ e \\ \quad \to \quad \texttt{let}\ f(x) = \texttt{if}\ x\ \texttt{then}\ d\ \texttt{else}\ e\ \texttt{in}\ \texttt{if}\ a\ \texttt{then}\ f(b)\ \texttt{else}\ f(c) \end{array}$$





Optimizations on ANF terms

As in the case of CPS, classic optimizations boil down to β or arithmetic reductions over ANF terms.

Example: constant propagation $\approx \beta$, arithmetic reduction.

$$\begin{array}{ll} & \texttt{let}\ x = 1\ \texttt{in}\ \ldots\ x+1\ \ldots\ x+y\ \ldots \\ \to & \ldots\ {\color{red}2}\ \ldots\ {\color{red}1+y}\ \ldots \end{array}$$

Example: common subexpression elimination $\approx$ inverse β

```
let x = a + b in                 let x = a + b in
...                    -->       ...
let y = a + b in                 let y = x in
...                              ...
```

Register allocation

The register allocation problem: place every variable in hardware registers or stack locations, maximizing the use of hardware registers.

Naive approach:
Assign the N hardware registers to the N most used variables; assign stack slots to the other variables.

Finer approach:
Notice that the same hardware register can be assigned to several distinct variables, provided they are never used simultaneously.

Example 4

```
if ... then (let x = ... in ...) else (let y = ... in ...)
```

`x` and `y` can share a register.

Register allocation on ANF

On functional intermediate representations like ANF, register allocation boils down to α-conversion:

The register allocation problem, revisited: rename variables, using hardware registers or stack locations as new names, in such a way that

- (Correctness) the renamed term is α-equivalent to the original;
- (Efficiency) hardware registers are used as much as possible.

Example 5

```
if ... then (let x = ... in ...) else (let y = ... in ...)
```

can be α-converted to

```
if ... then (let R1 = ... in ...) else (let R1 = ... in ...)
```

The interference graph

An undirected graph,

- Nodes: names of variables
- Edges: between any two variables that cannot be renamed to the same location, as this would violate α-equivalence.

Constructing the interference graph: at each point where a variable x is bound, add edges with all other variables that occur free in the continuation of this binding.

$\texttt{let } x = c \texttt{ in } b$

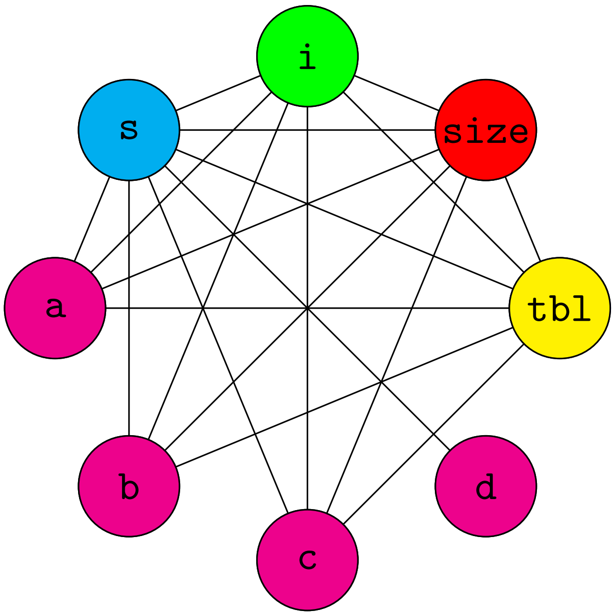
$\quad \rightarrow$ add edges between x and all $y \in FV(b) \setminus \{x\}$

$\texttt{match } a \texttt{ with } \ldots C(x_1, \ldots, x_n) \rightarrow b \ldots$

$\quad \rightarrow$ add edges between x_i and all $y \in FV(b) \setminus \{x_i\}$.

Example of an interference graph

```
let s = 0.0 in
let i = 0 in
let rec f(s,i) =
  if (ri < size) then
    let a = i*4 in
    let b = load(tbl+a) in
    let c = float(b) in
    let s = s +f c in
    let i = i + 1 in
    f(s,i)
  else
    let d = float(size) in
    s /f d
in f(s,i)
```



Register allocation by graph coloring

Correct register allocations correspond to colorings of the interference graph: each node should be assigned a color (= a register or stack location) so that adjacent nodes have different colors.

If the interference graph can be colored with at most N colors (where N is the number of hardware register), we obtain a perfect register allocation.

Otherwise, the coloring is a good starting point to determine which variables go into registers.

A. Appel, *Modern compiler implementation in ML*, Cambridge U. Press, esp. chapter 11.
F. Pereira and J. Palsberg, *Register allocation via coloring of chordal graphs*, APLAS 2005.