

# Corrigé du TD de logique n°6

## $\lambda$ -calcul – types simples

### Exercice 1 : Types simples

1°)

$$\frac{\overline{x : \alpha \vdash x : \alpha} \text{ (var)}}{\vdash : \alpha \rightarrow \alpha} \text{ (abs)}$$

2°)

$$\frac{\overline{x : \alpha, y : \beta \vdash x : \alpha} \text{ (var)}}{x : \alpha \vdash \lambda y. x : \beta \rightarrow \alpha} \text{ (abs)}$$

$$\vdash K : \alpha \rightarrow \beta \rightarrow \alpha \text{ (abs)}$$

$$\frac{\frac{\overline{\Gamma \vdash x : \alpha \rightarrow \beta \rightarrow \gamma} \text{ (var)} \quad \overline{\Gamma \vdash z : \alpha} \text{ (var)} \quad \overline{\Gamma \vdash x : \alpha \rightarrow \beta \rightarrow \gamma} \text{ (var)} \quad \overline{\Gamma \vdash y : \alpha \rightarrow \beta} \text{ (var)}}{\Gamma \vdash xz : \beta \rightarrow \gamma} \text{ (app)} \quad \overline{\Gamma \vdash yz : \beta} \text{ (app)}}{\Gamma = x : \alpha \rightarrow \beta \rightarrow \gamma, y : \alpha \rightarrow \beta, z : \alpha \vdash xz(yz) : \gamma} \text{ (app)}$$

$$\vdash S : (\alpha \rightarrow \beta \rightarrow \gamma) \rightarrow (\alpha \rightarrow \beta) \rightarrow \alpha \rightarrow \gamma \text{ (abs)}^3$$

[cf. S]

[cf. K]

$$\frac{\overline{\vdash S : (\alpha \rightarrow (\alpha \rightarrow \alpha) \rightarrow \alpha) \rightarrow (\alpha \rightarrow \alpha \rightarrow \alpha) \rightarrow \alpha \rightarrow \alpha} \quad \overline{\vdash K : \alpha \rightarrow (\alpha \rightarrow \alpha) \rightarrow \alpha} \text{ (app)} \quad \overline{\vdash K : \alpha \rightarrow \alpha \rightarrow \alpha} \text{ (app)}}{\vdash SK : (\alpha \rightarrow \alpha \rightarrow \alpha) \rightarrow \alpha \rightarrow \alpha} \text{ (app)}$$

$$\vdash SKK : \alpha \rightarrow \alpha$$

3°)  $SKK = (\lambda x. \lambda y. \lambda z. xz(yz))KK \xrightarrow{\beta} \lambda z. Kz(Kz) \xrightarrow{\beta} \lambda z. z.$ 4°)  $\lambda x. \lambda y. \lambda z. x(yz)$ 

5°)

$$\frac{\overline{\Gamma \vdash x : \alpha \rightarrow \beta \rightarrow \gamma \rightarrow \delta} \text{ (Var)} \quad \overline{\Gamma \vdash y : \beta \rightarrow \alpha} \text{ (Var)} \quad \overline{\Gamma \vdash w : \beta} \text{ (Var)}}{\Gamma \vdash x(yw) : \beta \rightarrow \gamma \rightarrow \delta} \text{ (App)} \quad \overline{\Gamma \vdash w : \beta} \text{ (Var)} \quad \overline{\Gamma \vdash z : \beta \rightarrow \gamma} \text{ (Var)} \quad \overline{\Gamma \vdash w : \beta} \text{ (Var)}}{\Gamma \vdash x(yw)w : \gamma \rightarrow \delta} \text{ (App)} \quad \overline{\Gamma \vdash zw : \gamma} \text{ (App)}$$

$$\Gamma = x : \alpha \rightarrow \beta \rightarrow \gamma \rightarrow \delta, y : \beta \rightarrow \alpha, z : \beta \rightarrow \gamma, w : \beta \vdash x(yw)w(zw) : \delta \text{ (App)}$$

$$\vdash \lambda x. \lambda y. \lambda z. \lambda w. x(yw)w(zw) : (\alpha \rightarrow \beta \rightarrow \gamma \rightarrow \delta) \rightarrow (\beta \rightarrow \alpha) \rightarrow (\beta \rightarrow \gamma) \rightarrow \beta \rightarrow \delta \text{ (abs*4)}$$