Modèles de calcul, fiche de TD numéro 2

Friday, October 1, 2004

A Random Access Machine (RAM) over the integers has access to an infinite set R_1, R_2,... of registers. Each register may contain an arbitrary nonnegative integer. The program of a RAM is a finite list of instructions which may be of the following type:

$R_i := 0$
$R_i := 1$
$R_i := R_j + R_k$
$R_i := R_j - R_k$ (the result is 0 if $R_j < R_k$)
$R_i := \lfloor R_i/2 \rfloor$
$R_i := R_{R_j}$
$R_{R_i} := R_j$
goto line m
if $R_i > 0$ goto line m
stop.

Question 1

Write a RAM program which computes the product of R_1 and R_2.

Question 2

Write a RAM program which on input n computes $n!$.

Question 3

Write a RAM program which on inputs x and i computes the bit of x of weight 2^i.

Question 4

Write a RAM program which on inputs x et y computes the product $x \times y$ in time proportional to the length of its inputs.

Question 5

Let $n = \min\{j;\ R_j = 0 \text{ and } j \geq 5\}$. Write a RAM program which computes $\sum_{j=5}^{n} R_j$ if n exists.

Question 6

Show that there exist a RAM which, given two integers x and y, computes the quotient of the euclidean division of x by y in time proportional to the length of its inputs.