Modèles de calcul, fiche de TD numéro 4

Friday, October 22, 2004

1. Show that any logspace reduction runs in in polynomial time.

2. Show that for every integer $k \geq 1$ there is a logspace Turing machine which on input 1^n outputs the number n^k written in binary.

3. Show that any polynomial-time problem can be recognized by a family of boolean circuits of polynomial size.

4. **Circuit Value.**

   Instance: a boolean circuit C with n inputs, and a word $a \in \{0, 1\}^n$.

   Question: $C(a) = 1$?

   Show that Circuit Value is a P-complete problem.

5. A circuit is said to be *monotone* if it is made only of AND and OR gates. Show that Monotone Circuit Value is a P-complete problem.

6. **Linear Programming.**

   Instance: a $n \times n$ rational matrix A, and a vector $b \in \mathbb{Q}^n$.

   Question: does there exist $x \in \mathbb{Q}^n$ such that $Ax \geq b$?

   Integer Linear Programming (ILP) is defined in the same way, except that the condition $x \in \mathbb{Q}^n$ is replaced by $x \in \mathbb{Z}^n$.

   Show that ILP is NP-hard (one can in fact show that ILP is NP-complete).

7. One can show that Linear Programming is in P (a deep result). Show that this problem is P-complete.