Modèles de calcul, fiche de TD numéro 5

Friday, October 29, 2004

1. **Spira's theorem.** We shall work with terms in the language $\{\vee, \wedge, \neg, 0, 1\}$ (such terms are also often called *boolean formulas*). We recall that terms over a set of variables $\{x_1, \ldots, x_n\}$ are defined inductively as follows:

   (i) the constants 0 and 1 and the variables x_i are terms.

   (ii) if t_1 and t_2 are terms then $t_1 \vee t_2$, $t_1 \wedge t_2$ and $\neg t_1$ are terms.

   In other words, a term is a circuit where all gates (except the output gate) have fanout 1. Note that a given variable may label several input gates.

   Show that any term Θ of size t is equivalent to a term of depth at most $4 \log t$. Hint: the idea of the proof is reminiscent of the "carry-look-ahead" method for addition. More precisely, note that if T is a subterm of Θ, one may consider in parallel the two cases $T = 0$ and $T = 1$, and then select the correct result.

2. Show that families of boolean circuits of logarithmic depth, families of terms of logarithmic depth and families of terms of polynomial size all recognize the same languages.