Games

A game board is a finite directed graph. The vertices are called *cells*, and each cell may be in a finite number of states. Hence there are finitely many different configurations for the board. The rules of the game state how one may move legally from a configuration to the next one. There is one initial configuration. The set of final configurations is divided between *losing* and *winning* configurations.

We will consider only games between two players, called Alice and Bob. Alice plays first and then the two players alternate. Alice has a winning strategy if she can win no matter what Bob plays. The GAME is then the set of initial configurations for which Alice has a winning strategy.

Example: the game of geography. Alice names a fixed initial city, for instance Lyon. As this city ends with the letter N, Bob must reply with a city that begins with the same letter, for instance Nice. Alice may then play Edinburgh, or any other city that begins with an E. A city may be chosen only once, and a player who cannot cite any city loses.

1. For the game of geography define carefully the board, the states, the initial and final configurations, and the rules of the game.

2. One can generalize this game by working with an arbitrary finite set of "city names" over an arbitrary finite alphabet. Can any directed graph be obtained as the board of such a game?

3. GEO is the following problem.

   INPUT: a directed graph G and a vertex s of G (the "initial city").

   QUESTION: is there a winning strategy for Alice beginning with city s ?

   Give an example of an instance that is in GEO, and of an instance that is outside GEO.

4. Explain how TQBF can be viewed as a game.

5. Show that GEO is in PSPACE.

6. Show that GEO is PSPACE-complete.