L and NL

1. We recall that REACHABILITY is the following problem: given a directed graph G and two nodes s and t of G, decide whether there is a path from s to t in G.

   Give a polynomial time algorithm for this problem.

2. What do you know about the (deterministic) space complexity of this problem ?

3. L denotes the class of languages recognized by logarithmic space Turing machines, and NL the class of languages recognized by nondeterministic logspace Turing machine.

   Obviously, L $\subseteq$ NL. The "$L = NL$?" problem is one of the basic open problems of complexity theory.

   Show that REACHABILITY is in NL.

4. What can you say about the "NL = coNL ?" problem ?

5. You must already have seen a polynomial time algorithm for 2SAT. Recall that algorithm, and show that 2SAT is in NL.

6. Show that NL $\subseteq$ P.

7. Show that REACHABILITY is NL-complete (for logspace reductions).

8. Show that 2SAT is NL-complete.

9. Show that the reachability problem for acyclic graph remains NL-complete.

   Remark: We have studied here the Reachability problem in directed graphs. Reachability in undirected graphs is also a natural problem. After 25 years of intensive work, it has been announced recently (November 2004) that this problem is in L.