Modeling the networks

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Outline

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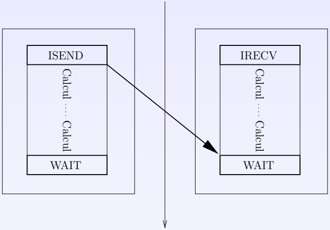
In an ideal world

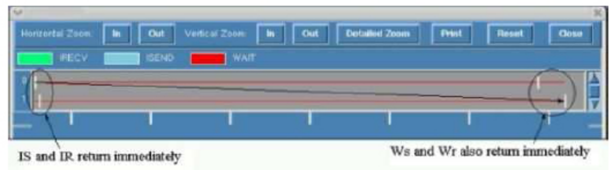
- While sending and receiving messages, one can compute.
- The computational power is not at all affected by the communications.
- Communications are realized using non blocking primitives.

Experimental scheme



- Two processes (on two different processors): the first one sends a message to the second one.
- The two processes execute “at the same time” their asynchronous send and receive commands.
- After a (long) computational time, the two processes put themselves in wait of the termination of the communications.

Actual behavior: 1 kilobyte message



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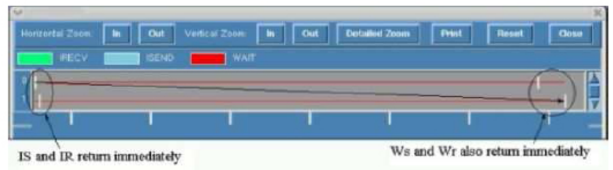
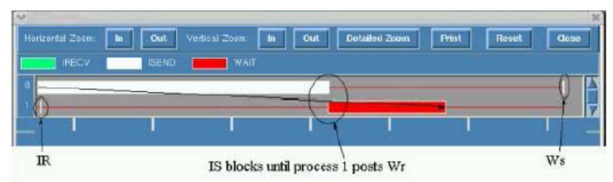


Expected behavior: the apparent behavior of the primitives is non-blocking.

Actual behavior: 60 kilobyte message (1)



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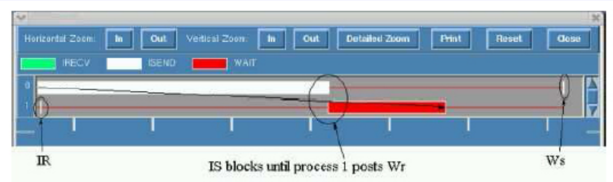
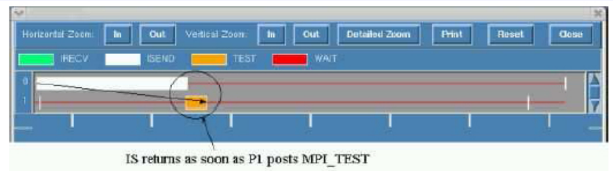


The sending, which was supposed to be non-blocking, is in fact blocking.

Actual behavior: 60 kilobyte message (2)



60 kilobyte message with a non-blocking test of communication completion in the middle of the computational phase of the receiving process.

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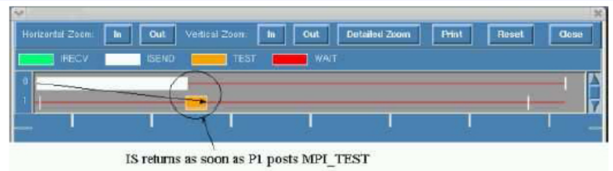

60 kilobyte message with a non-blocking test of communication completion in the middle of the computational phase of the receiving process.

The sending is blocked until the receiver posts the termination test.

Actual behavior: other results

- The symptoms are the same, when one is sending one 60 kilobyte message or three 20 kilobyte messages.
- When one is enlarging the size of the TCP/IP socket buffer to 64 kilobytes, the sending has a non-blocking behavior.

Interpretation

- Once we have reached a certain (accumulated) size of messages, the system buffers do not allow anymore to copy locally the messages. MPI moves then to a rendezvous protocol which needs a synchronization of the two processes which are supposed to communicate.
- This behavior is not a bug. It is inherently linked to the limits of the architectures used.

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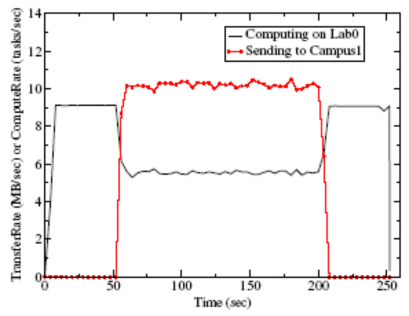
What happens in practice ?

Are the existing influences significant ?

Experimental setup

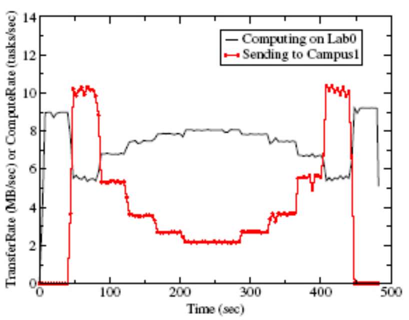
- Program written in Java using native threads.
- Processor architecture: Intel.
- Operating systems: FreeBSD, Linux 2.4 (Debian, RedHat), and Solaris (SunOS).
- Computers in the laboratory (Grail/UCSD), on the campus (UCSD), at UCSB, and on remote sites (Tennessee, Brésil, France).

Simultaneous computations and sends (1)



A processor simultaneously sends messages and computes (constant throughput).

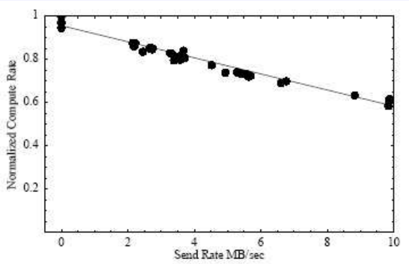
Simultaneous computations and sends (2)



A processor simultaneously sends messages and computes (varying throughput).

(the throughput is defined by introducing some contention on the receiver)

Simultaneous computations and sends (3)



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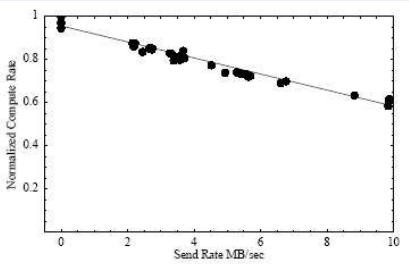

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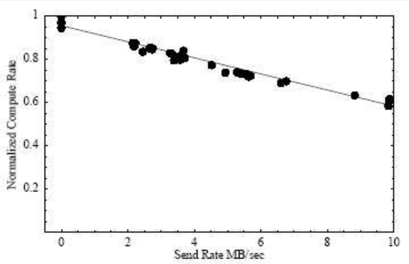

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IR $=$ 0.037 is the Interference Rate

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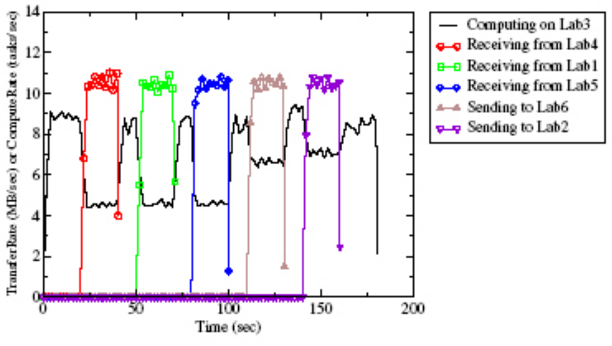
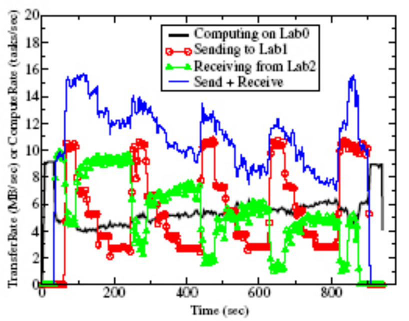
Receptions have more influence than sendings



Simultaneous sends and receives



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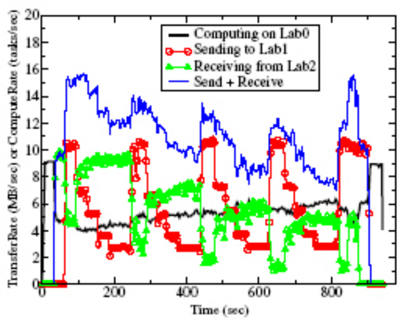

The estimate: $1 - \mathrm{IR}_s \times \mathrm{TR}_s - \mathrm{IR}_r \times \mathrm{TR}_r$

gives (s denoting the sends and r the receives):

$\mathrm{IR}_r = 0.0427$ instead of 0.0502, $\mathrm{IR}_s = 0.0265$ instead of 0.0327

There is a synergistic influence between sends and receives.

Other observations

- The less the memory used by the application, the less important the interferences: the interference rates are application dependent !
- The size of messages has no influence.
- Remote processors have significantly higher interference rates (more than 0.070 for receives), but the achievable bandwidths are far less important.

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- To compute and communicate simultaneously can make one "lose" more than half of the computational power.
- Once the interference rates are measured, for a given application, between each pair of processors, one can deduce a good approximation of the available computational powers.
- What about other platforms ? languages ? (same thing for the C language)

Does this change anything in practice ?

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- The root processor decides which task it executes itself and which tasks it delegates to its sons. Each internal node do the same.
- A node sends work to one of its sons only when the son requests some. When there are simultaneous requests, a scheduling policy solves the conflicts.

Known results

- Hypothesis : 1) no interferences bewteen computations and communications or 2) communications and computations are mutually exclusive.
- Bandwidth centric solution: if there is enough available bandwidth, all the sons work; otherwise, the tasks are sent to the sons which have sufficiently fast communications, priority being given to the son with the fastest communications.

The interference model to be instantiated

- Normalized computation time:

$$1 - \sum_i \mathsf{IR}_s(i) \times \mathsf{TR}_s(i) - \sum_i \mathsf{IR}_r(i) \times \mathsf{TR}_r(i)$$

- To obtain a good approximation of the different interference rate is complicated and costly: it requires the measure of computational capabilities for different throughput of sends and receives.

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$$\mathsf{IR}_s(i) = \frac{\left(1 - \frac{\mathsf{RR}(i)}{\mathsf{MR}^n}\frac{C^n - C_r^n}{C^n} - \frac{C_{sr}^n(i)}{C^n}\right)}{\mathsf{SR}(i)}$$

A more simply instantiated interference model (2)

- One measure for node n and one for each of its sons, instead of a potentially exponential number of measures with a set of different bandwidth values.
- The global precision is of a lesser quality, but a better local precision (inside the convex hull of the measured points).

Using the interference model

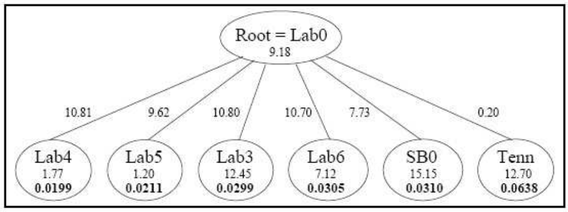
- Multi-port sends: the throughput of each node is maximized when priority is given to the sons with the smaller interference rates.
  One should not give a task to a son for which $\mathsf{IR}_s^n(i) Z C^n \geq 1$, where Z is the size of a task (to send a task to a son is more expensive in computational time than what it saves).
- One-port sends: the throughput of each node is maximized when priority is given to the sons with the highest value for: $B_r^i(1 - \mathsf{IR}_s^n Z C^n)$.

Experimental verification

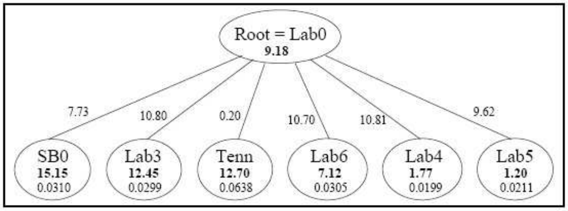
Scheduling policy:

- IA:single : *interference aware, single port*, ordering by increasing IR_s values.
- IA:multi : *interference aware, multiple ports*, ordering by increasing IR_s values.
- FCFS : first come, first serve.
- CompRate : by decreasing computational power.
- BWC : by decreasing bandwidth.
- RootComputes : the root computes everything.

Impact of the policies on the order (1)

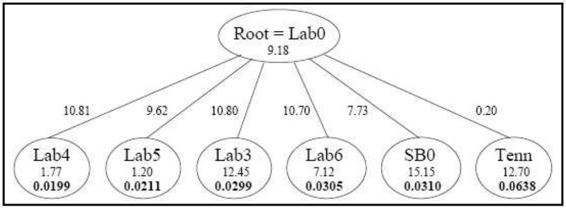


Order : interference rates.



Order : computational power.

Impact of the policies on the order (2)



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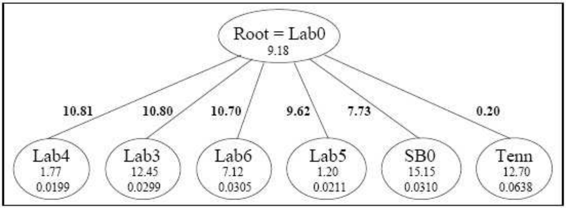
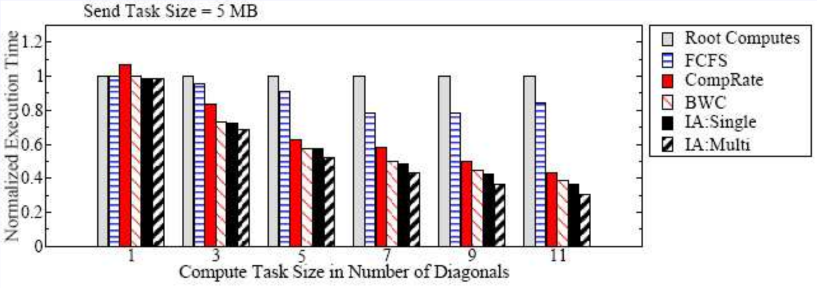


Order : bandwidth.

Results


Send Task Size = 5 MB
Normalized Execution Time
1.2
1
0.8
0.6
0.4
0.2
0
1
3
5
7
9
11
Compute Task Size in Number of Diagonals
Root Computes
FCFS
CompRate
BWC
IA:Single
IA:Multi

Outline

The problem

How can we model the network in order to predict the time needed for messages to be sent ?

Network latencies (1)

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  Justification: to ease or enable problem solving.
- Problem: the solution obtained may be unrealistic has the sending of an infinitely small message is not penalized.
  (Ex.: distributing an infinite number of infinitely small rounds.)

Network latencies (2)

Are latencies actually negligible ?

An example: TeraGrid between SDSC and NCSA (USA)

- Latency: $\approx$ 100ms
- Bandwidth: 40 Gb/s.
- Sending a 1 gigabyte message: more than a third of the time needed to send a message is due to the latency.

Conclusion ?

Computation latencies

- Programs may have a non negligible initialization cost.
- Launching a process may be long: on Globus Toolkit 2.0, launching a task doing no work (*no-op job*) may take 25 seconds (authentication, resources acquisition, process creation, etc.).

It may be necessary to take into account computation latencies even for programs whose execution time is linear in the size of input data.

Classical model of networks

Classical model

- Processors are interconnected through point to point connections (no routers, no switch, etc.).
- Multi port model: a processor can simultaneously send messages to several other processors.
- One port model: a processor can, at one time, send at most one message to one other processor.
- Only one message can travel along a given link at a given time.
- A complete graph can model a switch, but not an Ethernet network.

First problem of the classical model of network

Different logical communication links can share physical communication links.

Simultaneous communications from A to B and from C to D. Logically: no interferences; In practice: one needs to know the network topology to be able to predict what is going to happen.

Bad predictions of contentions.

Second problem of the classical model of network

It may be an advantage to share the use of the communication links.

Example (a caricature)

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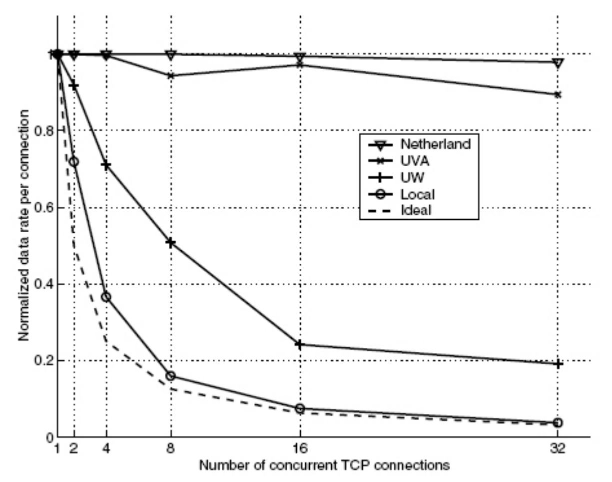
- FIFO policy: $\frac{1}{2}(\frac{20}{20} + \frac{25}{20}) = 1.125$.
- Second task first: $\frac{1}{2}(\frac{26}{20} + \frac{20}{20}) = 1.15$.
- Fair bandwidth sharing: $\frac{1}{2}(\frac{21}{20} + \frac{21}{20}) = 1.05$.

Bandwidth sharing: classical model

- We have supposed that when x communications share a same communication link of bandwidth B, each was receiving a bandwidth of $\frac{B}{x}$.
- This is the classical model.
- This is (usually) true on a local network (LAN).

Bandwidth sharing: in practice (1)



Bandwidth sharing: in practice (2)

- The classical model is invalid for long distance networks (WAN).
- With very long distances, all connections receive the same bandwidth than the first opened connection !

  Through a *backbone* travel a very very large number of communications: one more or one less communication does not significantly change the bandwidth allocated to each of the communications.

  Whatever the bandwidth allocated on a backbone, the amount a sender can effectively use is limited by its TCP congestion window.
- Generalizing the classical model: $\dfrac{B}{\alpha + x\beta}$.

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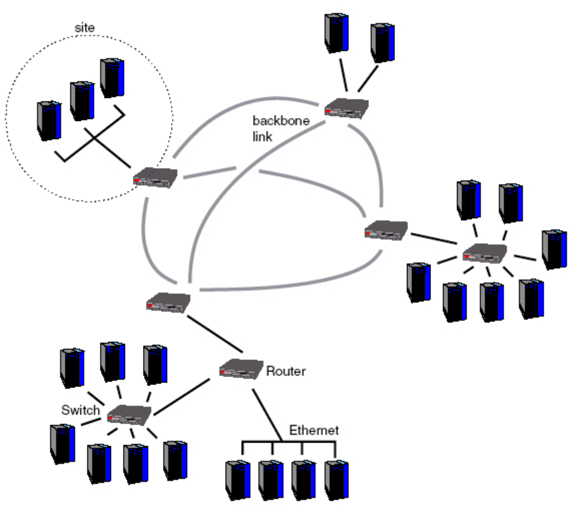
- The limiting factor may be the machine network card or the local network.
- It is necessary to consider the local topology if several machines from a same site may communicate simultaneously.

Network model

- Machines are not linked by backbones: the communication uses a certain number of local links before reaching the backbone(s) used for the long distance communication.
- The bandwidth of a communication using several links is defined by the "slowest" link.
- The limiting factor may be the machine network card or the local network.
- It is necessary to consider the local topology if several machines from a same site may communicate simultaneously.
- TCP behavior: on a congested link, the different communications receive bandwidths which are inversely proportional to their round-trip-times.

Network model



Outline

The problem

- A graph $G = (S, A)$.
  The edge $a \in A$ has a maximal bandwidth $b(a)$.

- A set $\mathcal{R}$ of paths in the graph G.
  Our problem is to allocate a bandwidth λ_r to each path $r \in \mathcal{R}$

- Respecting the available bandwidths:

$$\forall a \in A, \sum_{r \in \mathcal{R}, a \in r} \lambda_r \leq b(a),$$

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How should we allocate the bandwidths ?

Maximizing the total throughput

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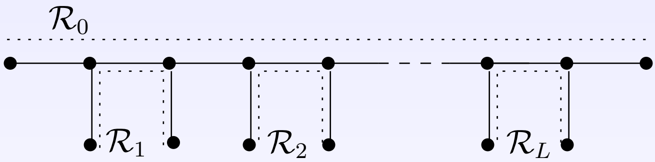
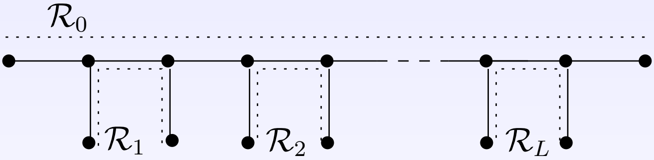


Maximizing the total throughput



In an optimal solution: $\lambda_1 = \lambda_2 = ... = \lambda_L = 1 - \lambda_0$

The total throughput is equal to: $L - (L-1)\lambda_0$ and is maximal when the route $\mathcal{R}_0$ is allocated a null bandwidth !

Max-min fairness: definitions and properties

- Definition: the smallest allocated bandwidth is maximal.

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- Let e be a saturated edge of r. If there exists a path r', $e \in r'$, of bandwidth $\lambda_{r'} > \lambda_r$, then one can give some of r''s bandwidth to r while having $\lambda'_{r'} > \lambda'_r > \lambda_r$.

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  Therefore, there exists a path r such that $\lambda_r = \lambda_{\min}$ and such that there exists an edge $a \in r$ satisfying: $\forall r', a \in r' \Rightarrow \lambda_{r'} = \lambda_{\min}$.

Max-min fairness: allocation

- Minimal allocated bandwidth:

$$\lambda_{\min} = \min_{a \in A} \frac{b(a)}{|\{r \in \mathcal{R} \mid a \in r\}|}.$$

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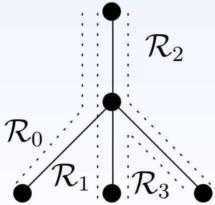
  Obviously not:

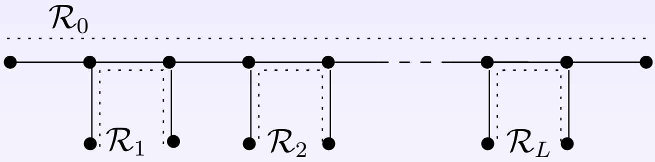


  If all edges are supposed to have a bandwidth of 1, $\lambda_0 = \lambda_1 = \lambda_2 = \frac{1}{3}$ but λ_3 can take any value in the interval $[\frac{1}{3}; \frac{2}{3}]$.

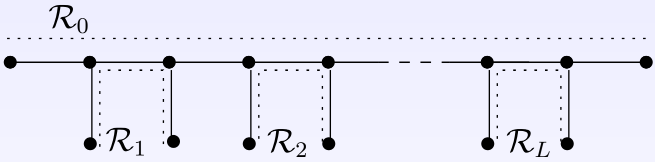
Max-min fairness: the algorithm

Algorithm: we recursively apply the max-min fairness principle: we determine the edges which define the minimal allocated bandwidth, we allocate the corresponding bandwidth to all routes using these edges and one call recursively the algorithm on the remaining paths using the updated available bandwidths.

Back to the example



Back to the example



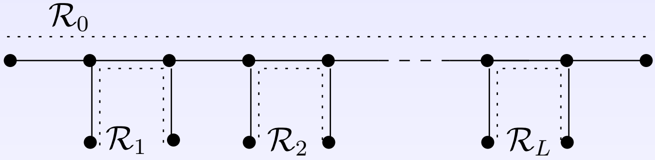
In an optimal solution: $\lambda_1 = \lambda_2 = ... = \lambda_L = \lambda_0 = \frac{1}{2}$

The total throughput is equal to: $\frac{L+1}{2}$ (which is far below the optimal value of L).

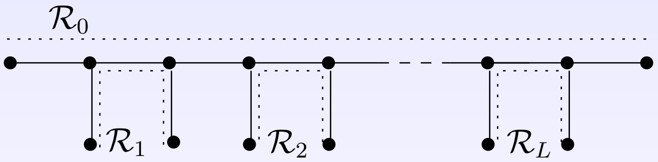
Proportional fairness

- Definition: one want to maximize $\sum_{\mathcal{R}} \log \lambda_{\mathcal{R}}$.
- Closer to TCP behavior.

Back on the example



Back on the example



In an optimal solution: $\lambda_1 = \lambda_2 = ... = \lambda_L = 1 - \lambda_0$.
One want to maximize: $L \log(1 - \lambda_0) + \log(\lambda_0)$.
One thus want to maximize: $(1 - \lambda_0)^L \lambda_0$.
The maximum is reached when $\lambda_0 = \frac{1}{L+1}$.

The total throughput is then equal to: $L - \frac{L-1}{L+1}$ (which is far closer to the optimum L).
The proportional fairness penalizes the long distance route for the benefit of the total throughput.

Conclusion

- A complicated reality.
- Need to have a model which enables to predict the behavior of applications.
- Need to take into account the topology and, more generally, the behavior of networks.
- The solution needs to be adapted to the targeted platforms and applications.