Strategies for Replica Placement in Tree Networks

December 11, 2006

Introduction and motivation

- Replica placement in tree networks
- Set of clients (tree leaves): flows of requests with QoS constraints, known in advance
- Internal nodes may be provided with a replica;
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Total replica cost?

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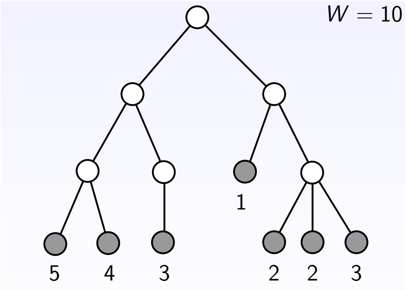

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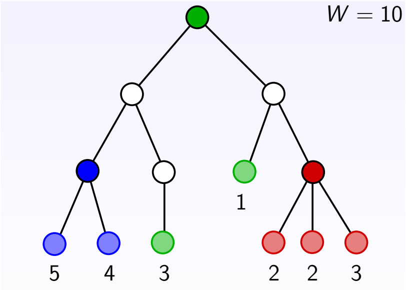

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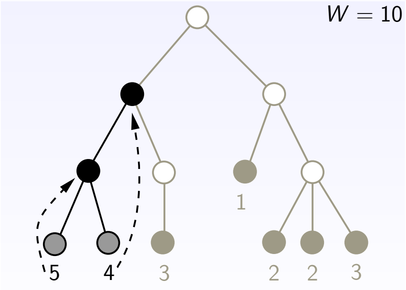

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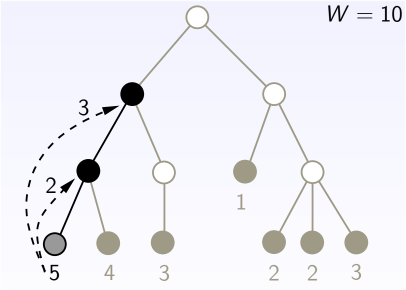

Outline

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- Distribution tree $\mathcal{T}$, clients $\mathcal{C}$ (leaf nodes), internal nodes $\mathcal{N}$

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- **Tree edge:** $l \in \mathcal{L}$ (communication link between nodes)
  - Communication time $comm_l$
  - Bandwidth limit BW_l

Tree notations

- r: tree root
- children(j): set of children of node $j \in \mathcal{N}$
- parent(k): parent in the tree of node $k \in \mathcal{N} \cup \mathcal{C}$
- link $l : k \to \text{parent}(k) = k'$. Then succ($l$) is the link $k' \to \text{parent}(k')$ (when it exists)
- Ancestors(k): set of ancestors of node k
- If $k' \in \text{Ancestors}(k)$, then path[$k \to k'$]: set of links in the path from k to k'
- subtree(k): subtree rooted in k, including k.

Problem instances

- Goal: place replicas to process client requests
- Client $i \in \mathcal{C}$: Servers(i) $\subseteq \mathcal{N}$ set of servers responsible for processing its requests
- $r_{i,s}$: number of requests from client i processed by server s ($\sum_{s \in \text{Servers}(i)} r_{i,s} = r_i$)
- $R = \{s \in \mathcal{N} | \ \exists i \in C \ , \ s \in \text{Servers}(i)\}$: set of replicas

Constraints

- **Server capacity**

$$\forall s \in R, \sum_{i \in \mathcal{C} | s \in \mathrm{Servers}(i)} r_{i,s} \leq \mathrm{W}_s$$

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- **QoS**

$$\forall i \in \mathcal{C}, \forall s \in \mathrm{Servers}(i), \sum_{l \in \mathrm{path}[i \rightarrow s]} \mathrm{comm}_l \leq \mathrm{q}_i.$$

Objective function

- Min $\sum_{s \in R} sc_s$

Objective function

- Min $\sum_{s \in R} \mathsf{sc}_s$
- Restrict to case where $\mathsf{sc}_s = \mathsf{W}_s$
- Replica Cost problem: no QoS nor bandwidth constraints; heterogeneous servers
- Replica Counting problem: idem, but homogeneous platforms

Outline

Single server vs. Multiple servers

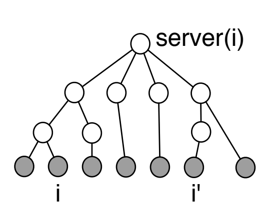
Single server – Each client i is assigned a single server server(i), that is responsible for processing all its requests.

Multiple servers – A client i may be assigned several servers in a set Servers(i). Each server $s \in \text{Servers}(i)$ will handle a fraction $r_{i,s}$ of the requests.

In the literature: single server policy with additional constraint.

Closest policy

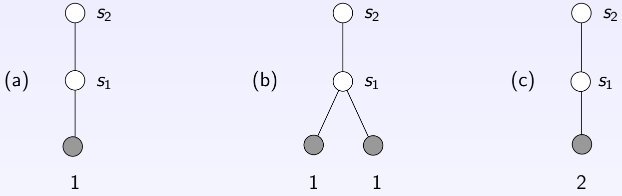
- *Closest*: single server policy
- Server of client i is constrained to be first server found on the path that goes from i upwards to the tree root
- Consider a client i and its server server(i):
  $\forall i' \in \text{subtree}(\text{server}(i)), \quad \text{server}(i') \in \text{subtree}(\text{server}(i))$
- Requests from i' cannot "traverse" server(i) and be served higher



Upwards and *Multiple* policy

- New policies not studied in the literature
- *Upwards*: *Closest* constraint is relaxed
- *Multiple*: relax single server restriction
- Expect more solutions with new policies, at a lower cost
- QoS constraints may lower difference between policies

Example: existence of a solution



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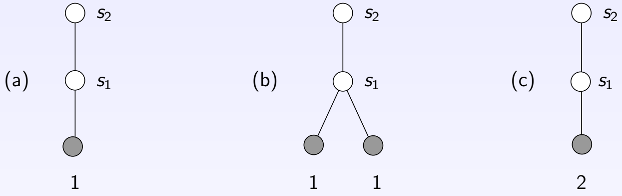

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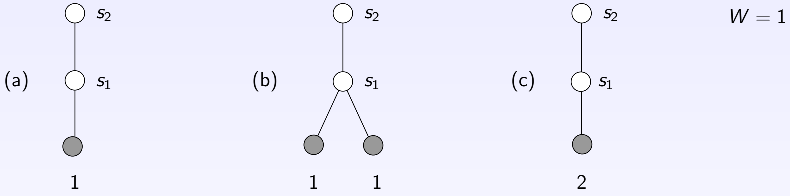

- (a): solution for all policies
- (b): no solution with *Closest*

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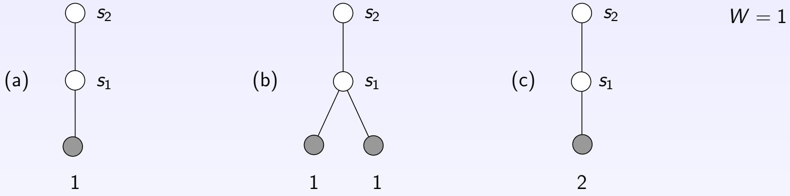
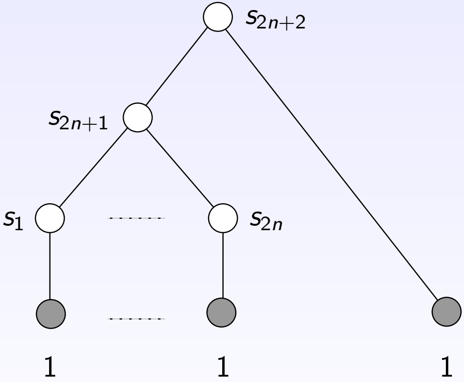


- (a): solution for all policies
- (b): no solution with *Closest*
- (c): no solution with *Closest* nor *Upwards*

Upwards versus *Closest*



$W = n$

- *Upwards*: 3 replicas in s_{2n}, s_{2n+1} and s_{2n+2}
- *Closest*: at least $n + 2$ replicas (replica in s_{2n+1} or not)

Multiple versus *Upwards*

- Replica Counting: *Multiple* twice better than *Upwards*.
- Performance ratio: open problem.

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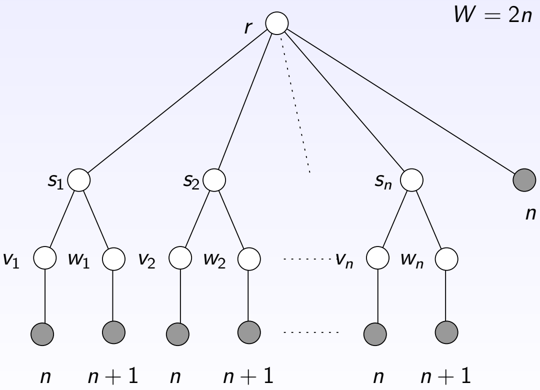
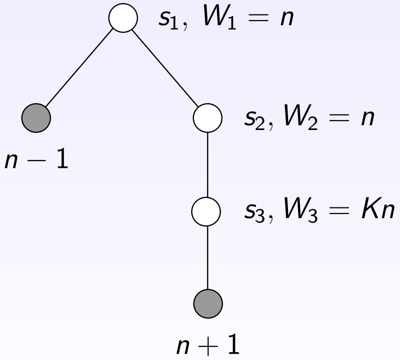


Multiple: $n+1$ replicas / *Upwards*: $2n$ replicas

Multiple versus *Upwards*

- Replica Cost: *Multiple* arbitrarily better than *Upwards*



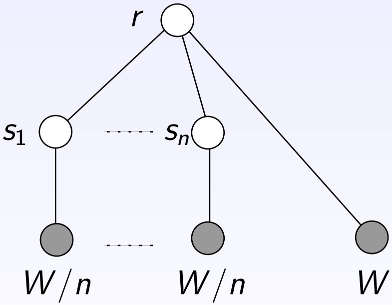
Multiple: cost $2n$ / *Upwards*: cost $(K + 1)n$

Lower bound for the Replica Counting problem

Obvious lower bound: $\left\lceil \frac{\sum_{i \in \mathcal{C}} r_i}{W} \right\rceil$

Lower bound for the REPLICA COUNTING problem

Obvious lower bound: $\left\lceil \frac{\sum_{i \in \mathcal{C}} r_i}{W} \right\rceil = 2$



All policies require $n+1$ replica (one at each node).

Outline

Complexity results - Basic problem

	REPLICA COUNTING **Homogeneous**	REPLICA COST **Heterogeneous**
Closest **Upwards** **Multiple**	polynomial [Cidon02,Liu06]	

Table: Complexity results for the different instances of the problem

- *Closest*/Homogeneous: only known result (Cidon et al. 2002, Liu et al. 2006)

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Upwards	NP-complete	NP-complete
Multiple	polynomial algorithm	NP-complete

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- All instances for the Heterogeneous case are NP-complete

Complexity results - QoS and Bandwidth

- *Closest*/Homogeneous + QoS: Polynomial (Liu et al.)

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Complexity results - QoS and Bandwidth

- *Closest*/Homogeneous + QoS: Polynomial (Liu et al.)
- *Closest*/Homogeneous + Bandwidth: ??
- *Multiple*/Homogeneous + QoS: NP-complete (reduction to 2-PARTITION)
- *Multiple*/Homogeneous + Bandwidth: Polynomial? Algorithm quite similar to the case without BW, but proof still to be checked.

Multiple/Homogeneous: greedy algorithm

3-pass algorithm:

- Select nodes which can handle W requests
- Select some extra servers to fulfill remaining requests
- Decide which requests are processed where

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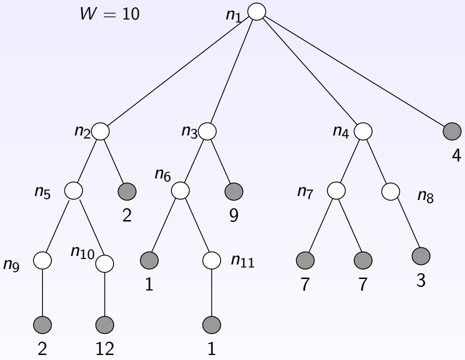
Example to illustrate algorithm (informally)

Proof of optimality: any optimal solution can be transformed into a solution similar to the one of the algorithm (moving requests from one server to another)

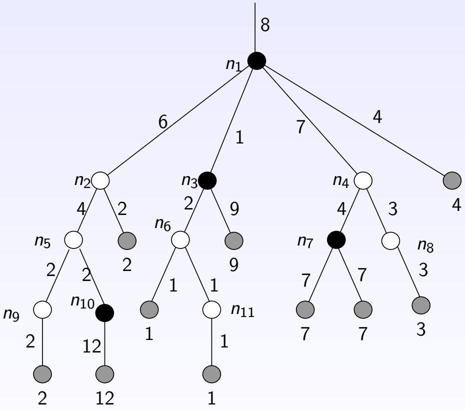
Multiple/Homogeneous: example



Initial network

The example network

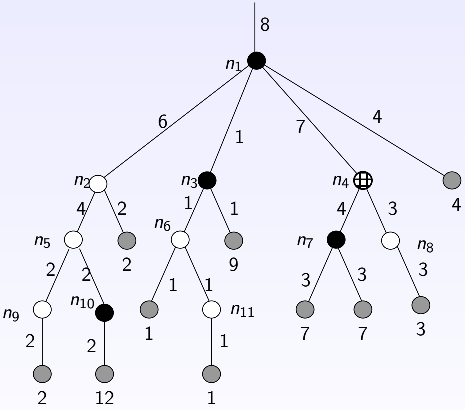
Multiple/Homogeneous: example



Pass 1

Placing *saturated* replicas

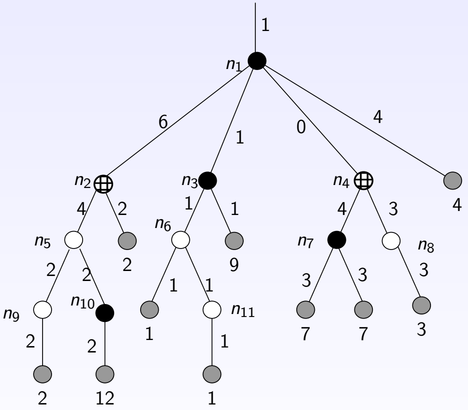
Multiple/Homogeneous: example



Pass 2

Placing extra replicas: n_4 has maximum useful flow

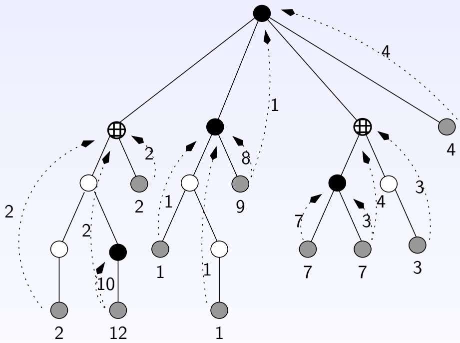
Multiple/Homogeneous: example



Pass 2

Placing extra replicas: n_2 is of maximum useful flow 1

Multiple/Homogeneous: example



Pass 3

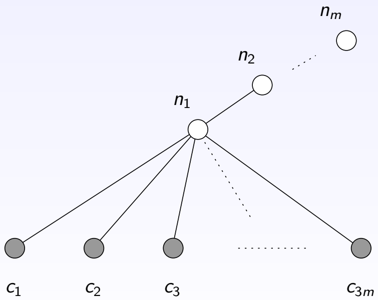
Deciding where requests are processed

Upwards/Homogeneous

- The Replica Counting problem with the *Upwards* strategy is NP-complete in the strong sense

Upwards/Homogeneous

- The REPLICA COUNTING problem with the *Upwards* strategy is NP-complete in the strong sense
- Reduction from 3-PARTITION



$W = B$

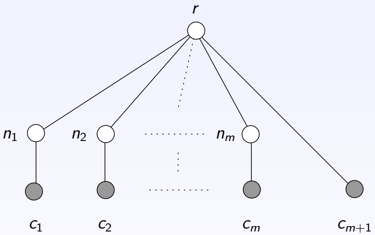
$\sum_{i=1}^{3m} c_i = mB$

Heterogeneous network: Replica Cost problem

- All three instances of the Replica Cost problem with heterogeneous nodes are NP-complete

Heterogeneous network: REPLICA COST problem

- All three instances of the REPLICA COST problem with heterogeneous nodes are NP-complete
- Reduction from 2-PARTITION



$$\sum_{i=1}^{m} c_i = S,\ c_{m+1} = 1,\ W_j = c_j,\ W_r = S/2 + 1$$

Heterogeneous network: REPLICA COST problem

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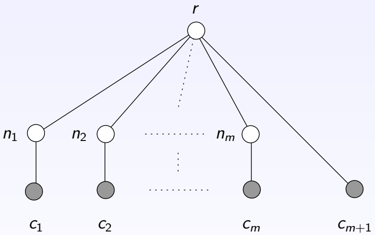

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Solution with total storage cost $S + 1$?

Outline

Linear programming

- General instance of the problem
  - Heterogeneous tree
  - QoS and bandwidth constraints
  - *Closest*, *Upwards* and *Multiple* policies
- Integer linear program: no efficient algorithm
- Absolute lower bound if program solved over the rationals (using the GLPK software)
- *Closest*/*Upwards* LP formulation

Linear program: variables

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Objective function: $\sum_{j \in \mathcal{N}} \text{sc}_j x_j$

Linear program: constraints

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- Conservation: $\forall i \in \mathcal{C}, \forall l : j \to j' = \text{parent}(j) \in \text{path}[i \to r]$,
$$z_{i,\text{succ}(l)} = z_{i,l} - y_{i,j'}$$

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- Server capacity: $\forall j \in \mathcal{N}, \sum_{i \in \mathcal{C}} r_i y_{i,j} \leq \mathsf{W}_j x_j$
- Bandwidth limit: $\forall l \in \mathcal{L}, \sum_{i \in \mathcal{C}} r_i z_{i,l} \leq \mathsf{BW}_l$

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- QoS constraint: $\forall i \in \mathcal{C}, \forall j \in \text{Ancestors}(i), \text{dist}(i,j) y_{i,j} \le \text{q}_i$

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- Bandwidth limit: $\forall l \in \mathcal{L}, \sum_{i \in \mathcal{C}} r_i z_{i,l} \leq \text{BW}_l$
- QoS constraint: $\forall i \in \mathcal{C}, \forall j \in \text{Ancestors}(i), \text{dist}(i,j) y_{i,j} \leq \text{q}_i$
- *Closest* constraint: $\forall i \in \mathcal{C}, \forall j \in \text{Ancestors}(i) \setminus \{r\}$,
  $$\forall i' \in \mathcal{C} \cap \text{subtree}(j), y_{i,j} + z_{i',j \to \text{parent}(j)} \leq 1$$

Multiple formulation

Multiple

- Similar formulation, with
  - $y_{i,j}$: integer variable = nb requests from client i processed by node j
  - $z_{i,l}$: integer variable = nb requests flowing through link l
- Constraints are slightly modified

An ILP-based lower bound

- Solving over the rationals: solution for all practical values of the problem size
  - Not very precise bound
  - *Upwards*/*Closest* equivalent to *Multiple* when solved over the rationals

An ILP-based lower bound

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  - *Upwards*/*Closest* equivalent to *Multiple* when solved over the rationals
- Integer solving: limitation to $s \leq 50$ nodes and clients
- Mixed bound obtained by solving the *Upwards* formulation over the rational and imposing only the x_j being integers
  - Resolution for problem sizes $s \leq 400$
  - Improved bound: if a server is used only at 50% of its capacity, the cost of placing a replica at this node is not halved as it would be with $x_j = 0.5$.

Outline

Heuristics

- Polynomial heuristics for the REPLICA COST problem
  - Heterogeneous platforms
  - No QoS nor bandwidth constraints

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  - Heterogeneous platforms
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- Experimental assessment of the relative performance of the three policies
- Traversals of the tree, bottom-up or top-down
- Worst case complexity $O(s^2)$,
  where $s = |\mathcal{C}| + |\mathcal{N}|$ is problem size

Heuristics for *Closest*

Closest Top Down All **CTDA**

- Breadth-first traversal of the tree
- When a node can process the requests of all the clients in its subtree, node chosen as a server and exploration of the subtree stopped
- Procedure called until no more servers are added

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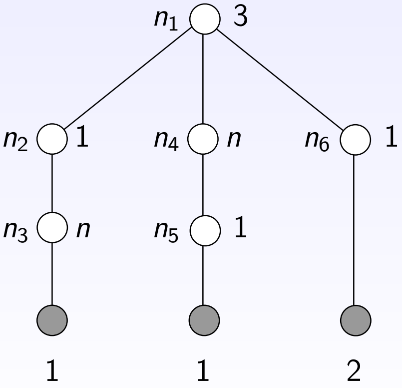
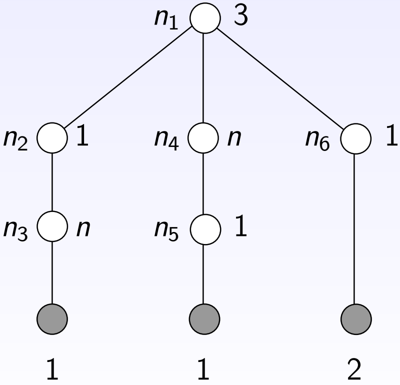


Heuristics for *Closest*

Closest Top Down Largest First **CTDLF**

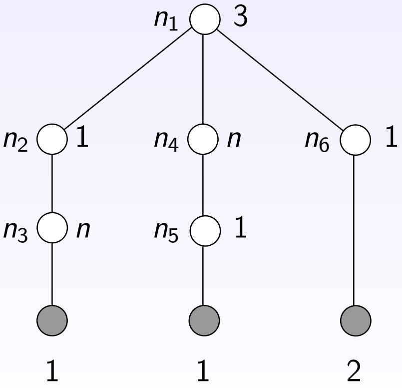
- Traversal of the tree, treating subtrees that contains most requests first
- When a node can process the requests of all the clients in its subtree, node chosen as a server and traversal stopped
- Procedure called until no more servers are added
- Choosing n_2 and then n_1



Heuristics for *Closest*

Closest Bottom Up **CBU**

- Bottom-up traversal of the tree
- When a node can process the requests of all the clients in its subtree, node chosen as a server
- Choosing n_3, n_5, n_1



Heuristics for *Upwards*

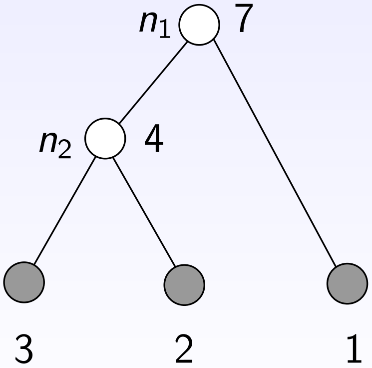
Upwards Top Down **UTD**

- 2-pass algorithm
- Select first saturating nodes, then extra nodes

Heuristics for *Upwards*

Upwards Top Down **UTD**

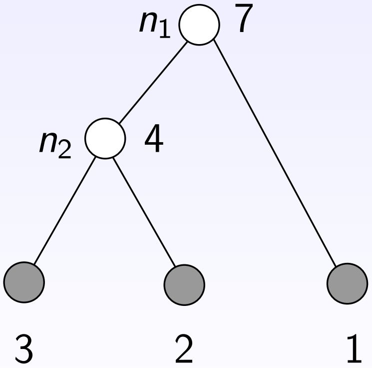
- 2-pass algorithm
- Select first saturating nodes, then extra nodes
- Choosing n_2 (for c_1) and in second pass n_1 (for c_2, c_3)



Heuristics for *Upwards*

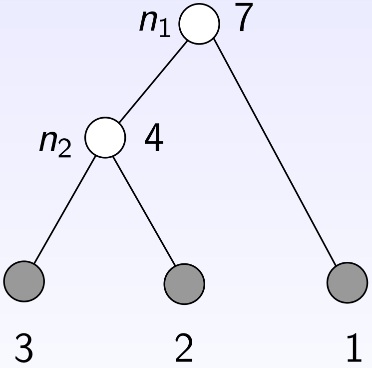
Upwards Big Client First **UBCF**

- Sorting clients by decreasing request numbers, and finding the server of minimal available capacity to process its requests.
- Choosing n_2 for c_1, n_1 for c_2 and n_1 for c_3



Heuristics for *Multiple*

A greedy heuristic **MG**, similar to Pass 3 of the polynomial algorithm for *Multiple*/Homogeneous: fill all servers as much as possible in a bottom-up fashion



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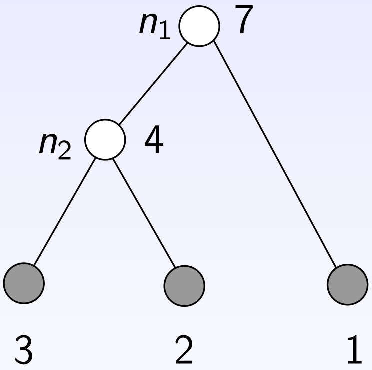

- MG affects 4 requests to n_2, and then the remaining 2 requests to n_1
- CTDLF better on this example: selects n_1 only

Heuristics for *Multiple*

- A top-down and a bottom-up heuristic in 2-passes (**MTD**, **MBU**)
- Heuristic MixedBest **MB** which picks up best result over all heuristics: solution for the *Multiple* policy

Plan of experiments

- Assess impact of the different access policies
- Assess performance of the polynomial heuristics

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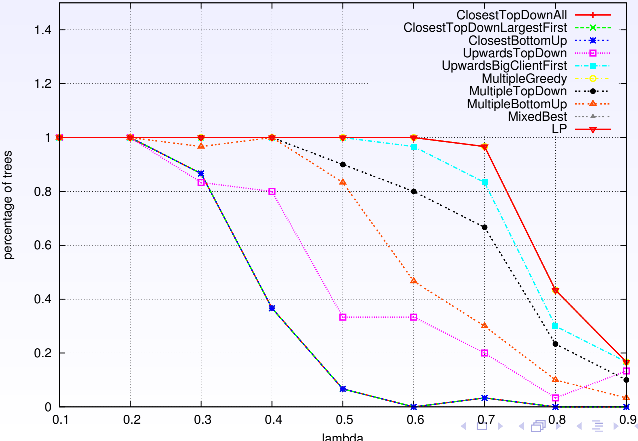
- Assess impact of the different access policies
- Assess performance of the polynomial heuristics
- Important parameter:

$$\lambda = \frac{\sum_{i \in \mathcal{C}} r_i}{\sum_{j \in \mathcal{N}} \mathsf{W}_i}$$

- 30 trees for each $\lambda = 0.1, 0.2, ..., 0.9$
- Problem size $s = |\mathcal{C}| + |\mathcal{N}|$ such that $15 \leq s \leq 400$
- Computation of the LP lower bound for each tree

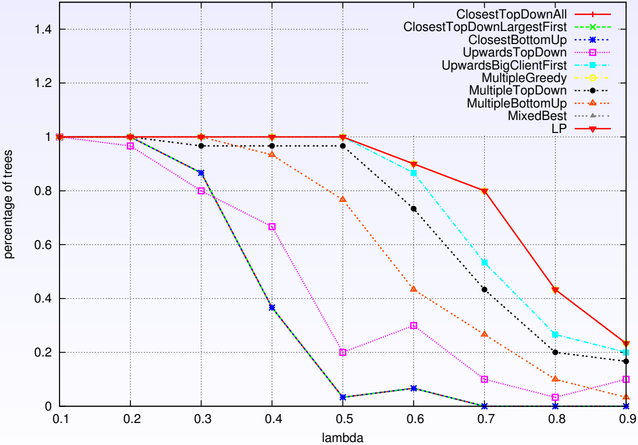
Results - Percentage of success

- Number of solutions for each lambda and each heuristic
- No LP solution → No solution for any heuristic
- Homogeneous case



Results - Percentage of success

- Heterogeneous trees: similar results



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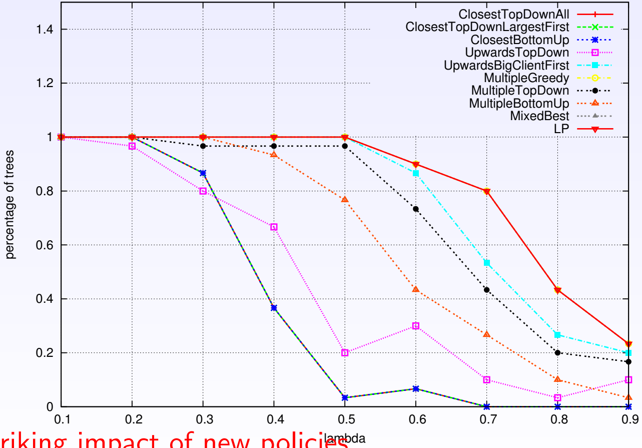

- Striking impact of new policies

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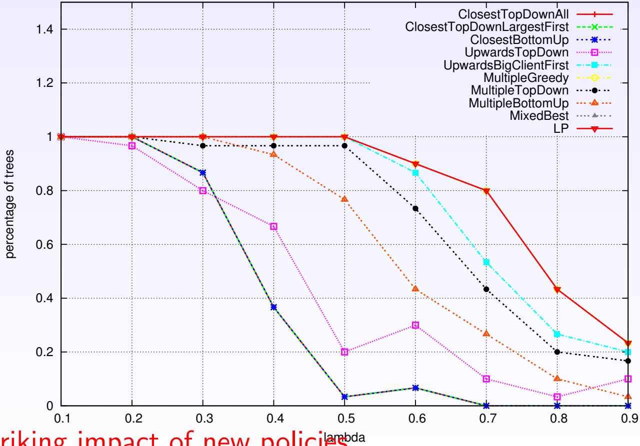

- Striking impact of new policies
- MG and MB always find the solution

Results - Solution cost

- Distance of the result (in terms of replica cost) of the heuristic to the lower bound

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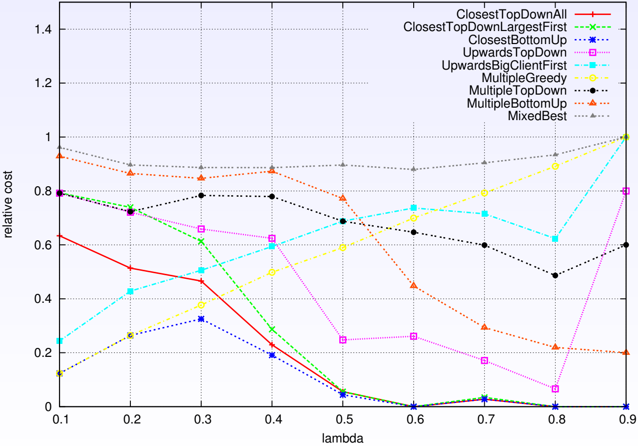
- Distance of the result (in terms of replica cost) of the heuristic to the lower bound
- T_λ: subset of trees with a solution
- Relative cost:

$$rcost = \frac{1}{|T_\lambda|} \sum_{t \in T_\lambda} \frac{cost_{LP}(t)}{cost_h(t)}$$

- $cost_{LP(t)}$: lower bound cost on tree t
- $cost_h(t)$: heuristic cost on tree t; $cost_h(t) = +\infty$ if h did not find any solution

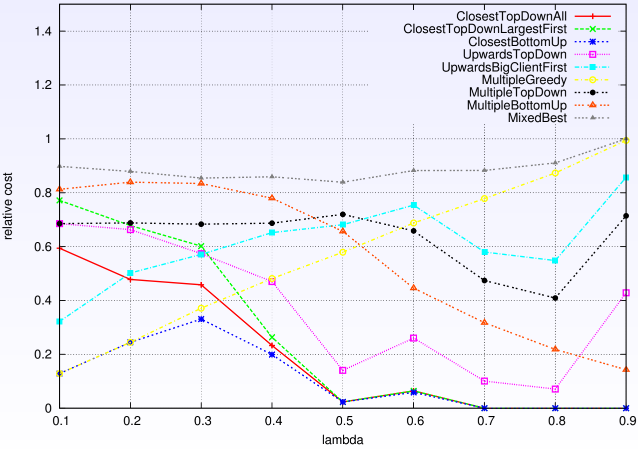
Results - Solution cost

- Homogeneous results

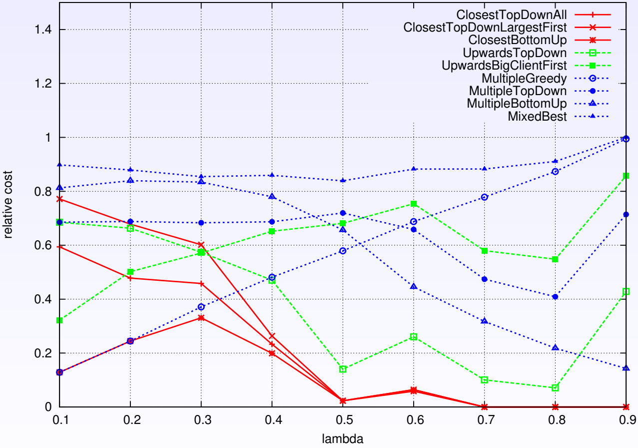


Results - Solution cost

- Heterogeneous results - similar to the homogeneous case



Results - Hierarchy



Summary

- Striking effect of new policies: many more solutions to the REPLICA PLACEMENT problem
- *Multiple* $\geq$ *Upwards* $\geq$ *Closest*: hierarchy observed within our heuristics
- Best *Multiple* heuristic (MB) always at 85% of the lower bound: satisfactory result

Outline

Related work

- Several papers on replica placement, but...

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- REPLICA PLACEMENT in a general graph is NP-complete
- Wolfson and Milo: impact of the *write* cost, use of a minimum spanning tree for updates. Tree networks: polynomial solution
- Cidon et al (multiple objects) and Liu et al (QoS constraints): polynomial algorithms for homogeneous networks.
- Kalpakis et al: NP-completeness of a variant with bidirectional links (requests served by any node in the tree)
- Karlsson et al: comparison of different objective functions and several heuristics. No QoS, but several other constraints.
- Tang et al: real QoS constraints
- Rodolakis et al: *Multiple* policy but in a very different context

Conclusion

- Introduction of two new policies for the REPLICA PLACEMENT problem
- *Upwards* and *Multiple*: natural variants of the standard *Closest* approach $\longrightarrow$ surprising they have not already been considered

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- Introduction of two new policies for the Replica Placement problem
- *Upwards* and *Multiple*: natural variants of the standard *Closest* approach → surprising they have not already been considered

Theoretical side – Complexity of each policy, for homogeneous and heterogeneous platforms

Practical side

- Design of several heuristics for each policy
- Comparison of their performance
- Striking impact of the policy on the result
- Use of a LP-based lower bound to assess the absolute performance, which turns out to be quite good.