Probabilités discrètes et continues

1 Simulation of discrete distributions

1.1 Dice

In the early 1600s, Galileo was asked to explain the fact that, although the number of triples of integers from 1 to 6 with sum 9 is the same as the number of such triples with sum 10, when three dice are rolled, a 9 seemed to come up less often than a 10, supposedly in the experience of gamblers.

▷ **Question 1** *Write a program to simulate the roll of three dice a large number of times and keep track of the proportion of times that the sum is 9 and the proportion of times it is 10.*

▷ **Question 2** *Were the gamblers correct ?*

1.2 Roulette

In Las Vegas, a roulette wheel has 38 slots numbered 0, 00, 1, 2, . . . , 36. The 0 and 00 slots are green and half of the remaining 36 slots are red and half are black. A croupier spins the wheel and throws in an ivory ball. If you bet 1 dollar on red, you win 1 dollar if the ball stops in a red slot and otherwise you lose 1 dollar.

▷ **Question 3** *Write a program to find the total winnings for a player who makes 1000 bets on red.*

Another form of bet for roulette is to bet that a specific number (say 17) will turn up. If the ball stops on your number, you get your dollar back plus 35 dollars. If not, you lose your dollar.

▷ **Question 4** *Write a program that will plot your winnings when you make 1000 plays betting each time on the number 17. What differences do you see in the graphs of your winnings on these two occasions ?*

The Labouchere system for roulette is played as follows. Write down a list of numbers, usually 1, 2, 3, 4. Bet the sum of the first and last, $1 + 4 = 5$, on red. If you win, delete the first and last numbers from your list. If you lose, add the amount that you last bet to the end of your list. Then use the new list and bet the sum of the first and last numbers (if there is only one number, bet that amount). Continue until your list becomes empty. Show that, if this happens, you win the sum, $1 + 2 + 3 + 4 = 10$, of your original list.

▷ **Question 5** *Simulate this system and see if you do always stop and, hence, always win. If so, why is this not a foolproof gambling system ?*

2 Discrete probability distributions

Let $\Omega = \{a, b, c\}$ be a sample space. Let $m(a) = 1/2$, $m(b) = 1/3$, and $m(c) = 1/6$.

▷ **Question 6** *Find the probabilities for all eight subsets of* Ω.

Let $\Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

▷ **Question 7** *Describe in words the events specified by the following subset of* Ω. *Give their probability.*

1. $E =$ *HHH,HHT,HTH,HTT.*

2. $E =$ *HHH,TTT.*

3. $E =$ *HHT,HTH,THH.*

4. $E =$ *HHT,HTH,HTT,THH,THT,TTH,TTT.*

Let A and B be events such that $P(A \cap B) = 1/4$, $P(\neg A) = 1/3$, and $P(B) = 1/2$.

▷ **Question 8** *What is $P(A \cup B)$?*

A student must choose exactly two out of three electives : art, French, and mathematics. He chooses art with probability 5/8, French with probability 5/8, and art and French together with probability 1/4.

▷ **Question 9** *What is the probability that he chooses mathematics ? What is the probability that he chooses either art or French ?*

In a horse race, the odds that Romance win are listed as 2 : 3 and that Downhill win are 1 : 2.

▷ **Question 10** *What odds should be given for the event that either Romance or Downhill wins ?*

John and Mary are taking a mathematics course. The course has only three grades : A, B, and C. The probability that John gets a B is .3. The probability that Mary gets a B is .4. The probability that neither gets an A but at least one gets a B is .1.

▷ **Question 11** *What is the probability that at least one gets a B but neither gets a C ?*

A, B, and C are any three events.

▷ **Question 12** *show that $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$*

3 Simulation of continuous probabilities

In the spinner problem, divide the unit circumference into three arcs of length 1/2, 1/3, and 1/6.

▷ **Question 13** *Write a program to simulate the spinner experiment 1000 times*

▷ **Question 14** *Write a program to estimate the area of the circle of radius 1/2 with center at (1/2, 1/2) inside the unit square by choosing 1000 points at random and give an estimation of π*

4 Continuous probability densities

Suppose Mr. and Mrs. Lockhorn agree to meet at the Hanover Inn between 5 :00 and 6 :00 P.M. on Tuesday. Suppose each arrives at a time between 5 :00 and 6 :00 chosen at random with uniform probability.

▷ **Question 15** *What is the distribution function for the length of time that the first to arrive has to wait for the other ? What is the density function ?*

Suppose you choose at random a real number X from the interval [2, 10].

▷ **Question 16** *Find the density function f(x) and the probability of an event E for this experiment, where E is a subinterval [a, b] of [2, 10]. Find the probability that $X > 5$, that $5 < X < 7$, and that $X^2 - 12X + 35 > 0$.*

Suppose you throw a dart at a circular target of radius 10 inches. Assuming that you hit the target and that the coordinates of the outcomes are chosen at random,

▷ **Question 17** *Find the probability that the dart falls :*
(a) within 2 inches of the center.
(b) within 2 inches of the rim.
(c) within the first quadrant of the target.
(d) within the first quadrant and within 2 inches of the rim.