Competitive Self-Stabilizing *k*-Clustering

Yvan Rivierre

Joint work with
Ajoy K. Datta, Stéphane Devismes,
Karel Heurtefeux, and Lawrence L. Larmore

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Verimag

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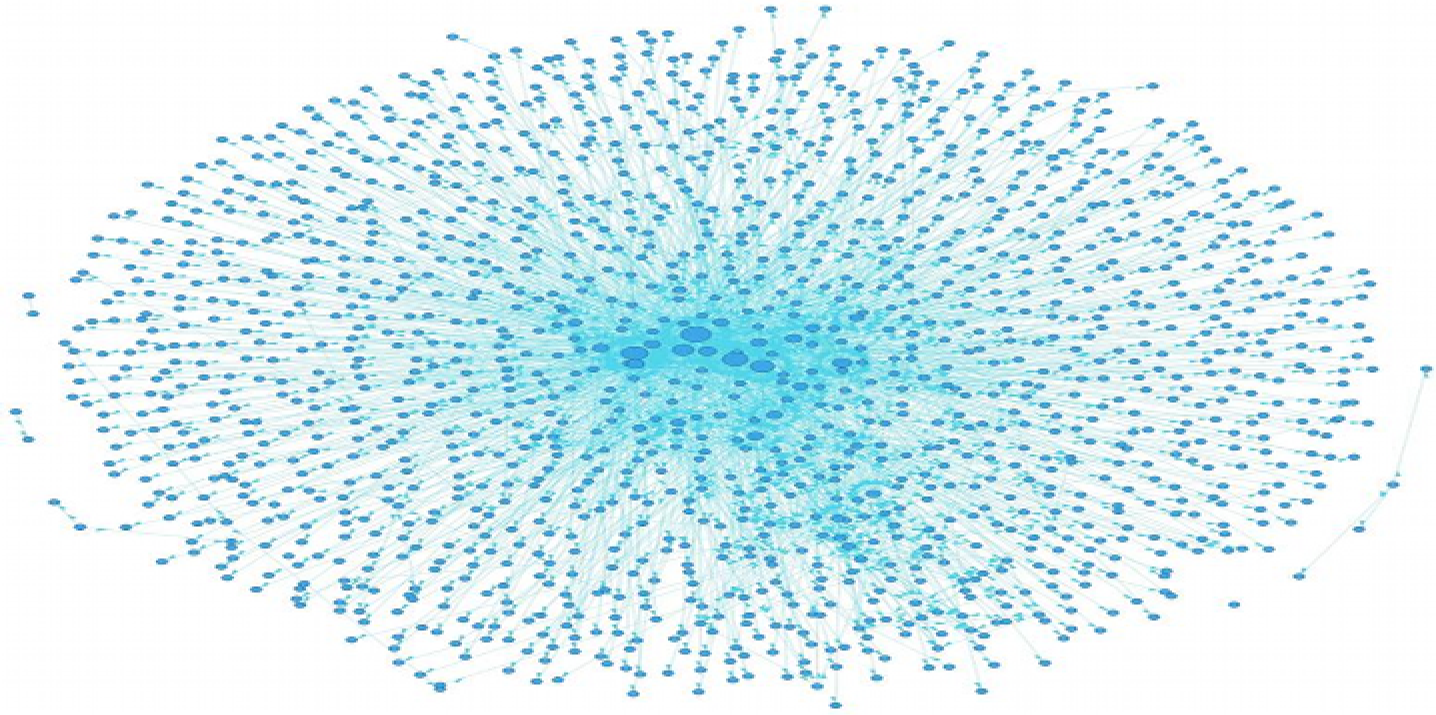
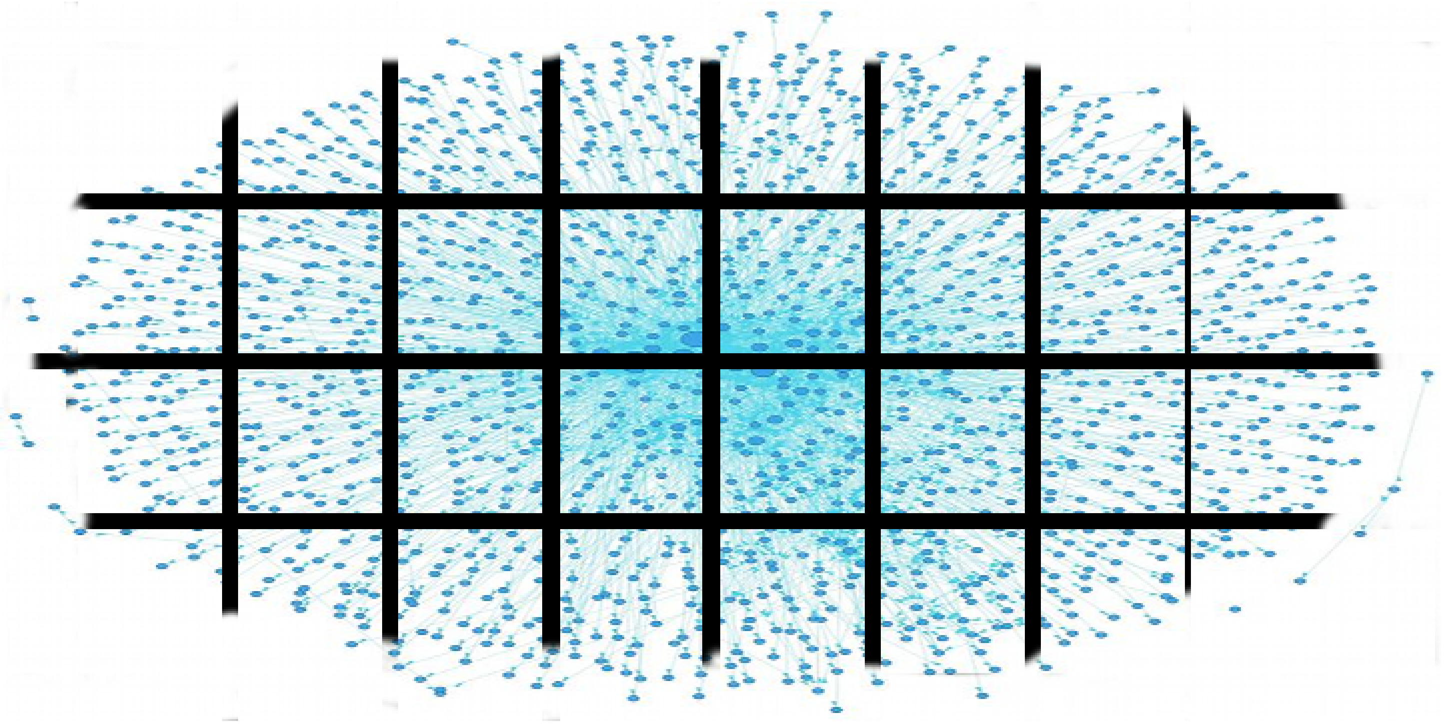
Introduction to *k*-Clustering



Hierarchical organization of processes

Introduction to *k*-Clustering



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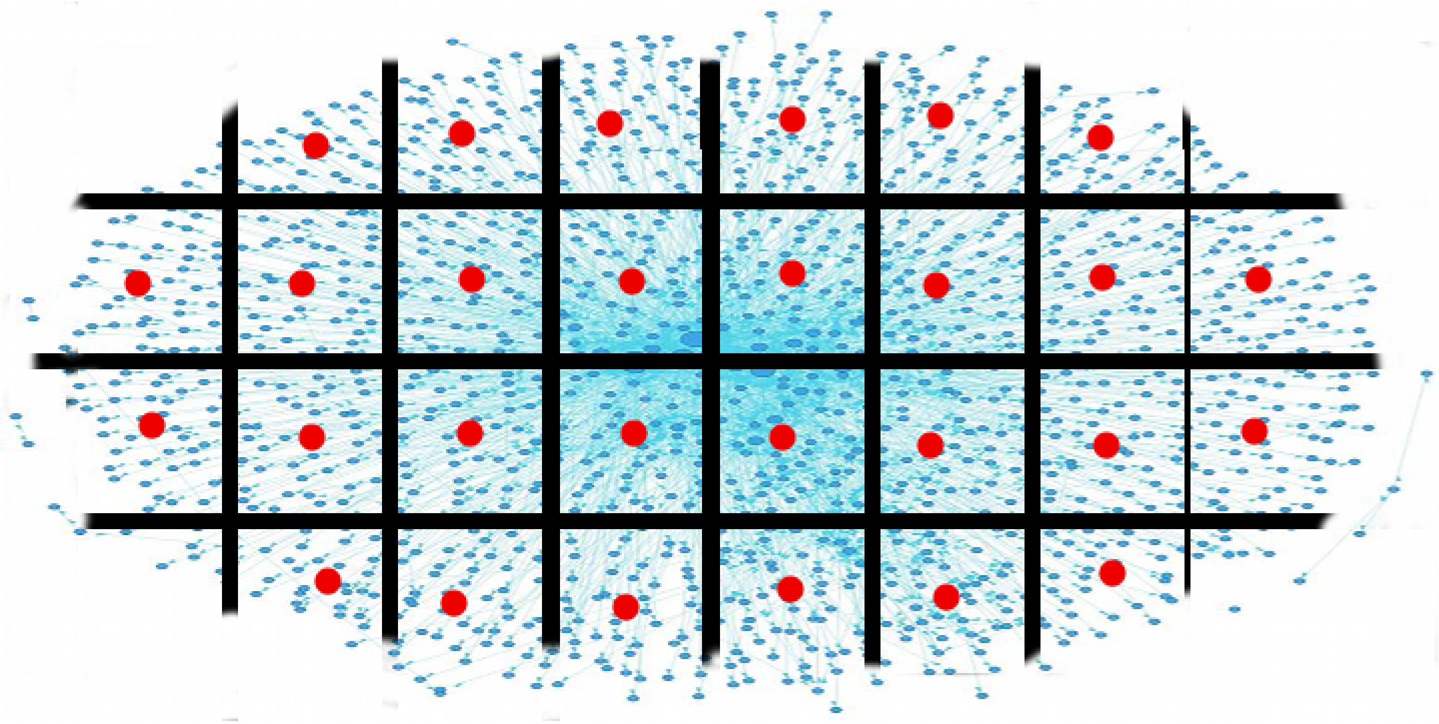

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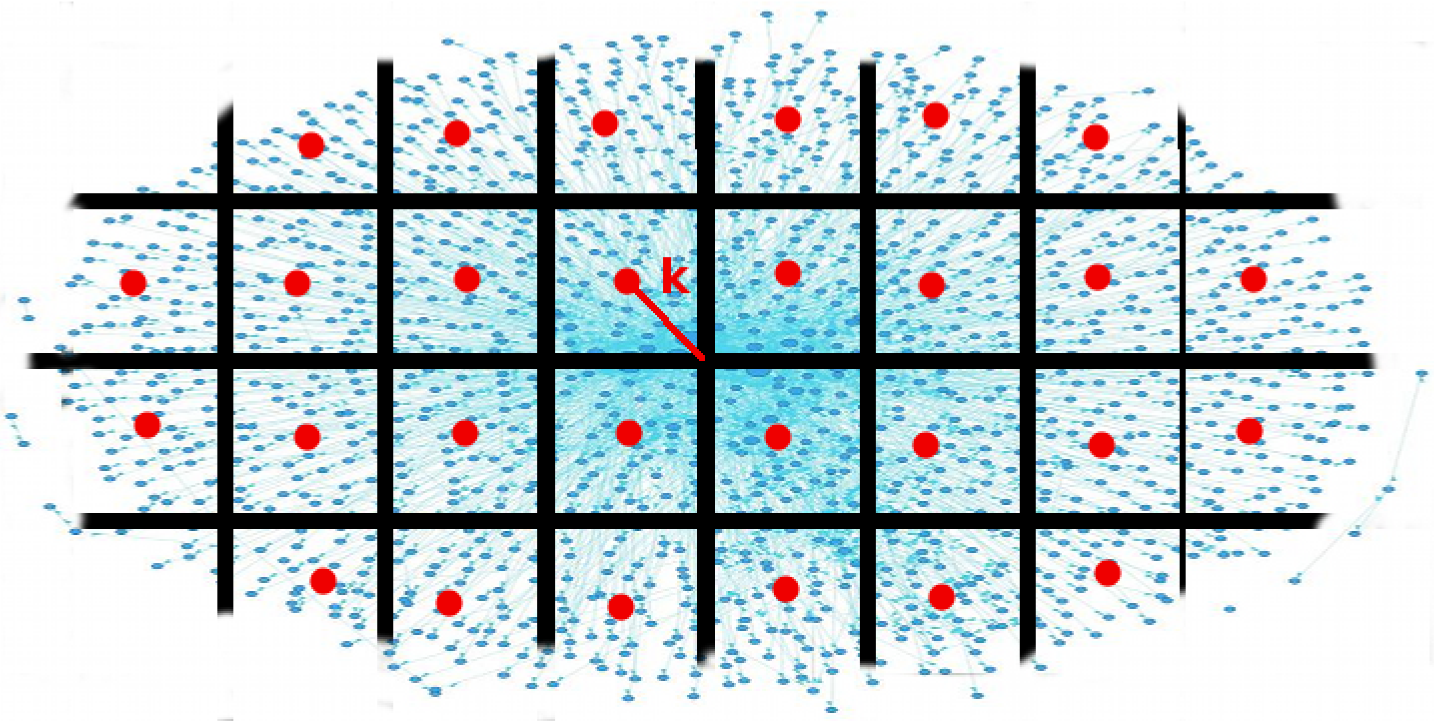
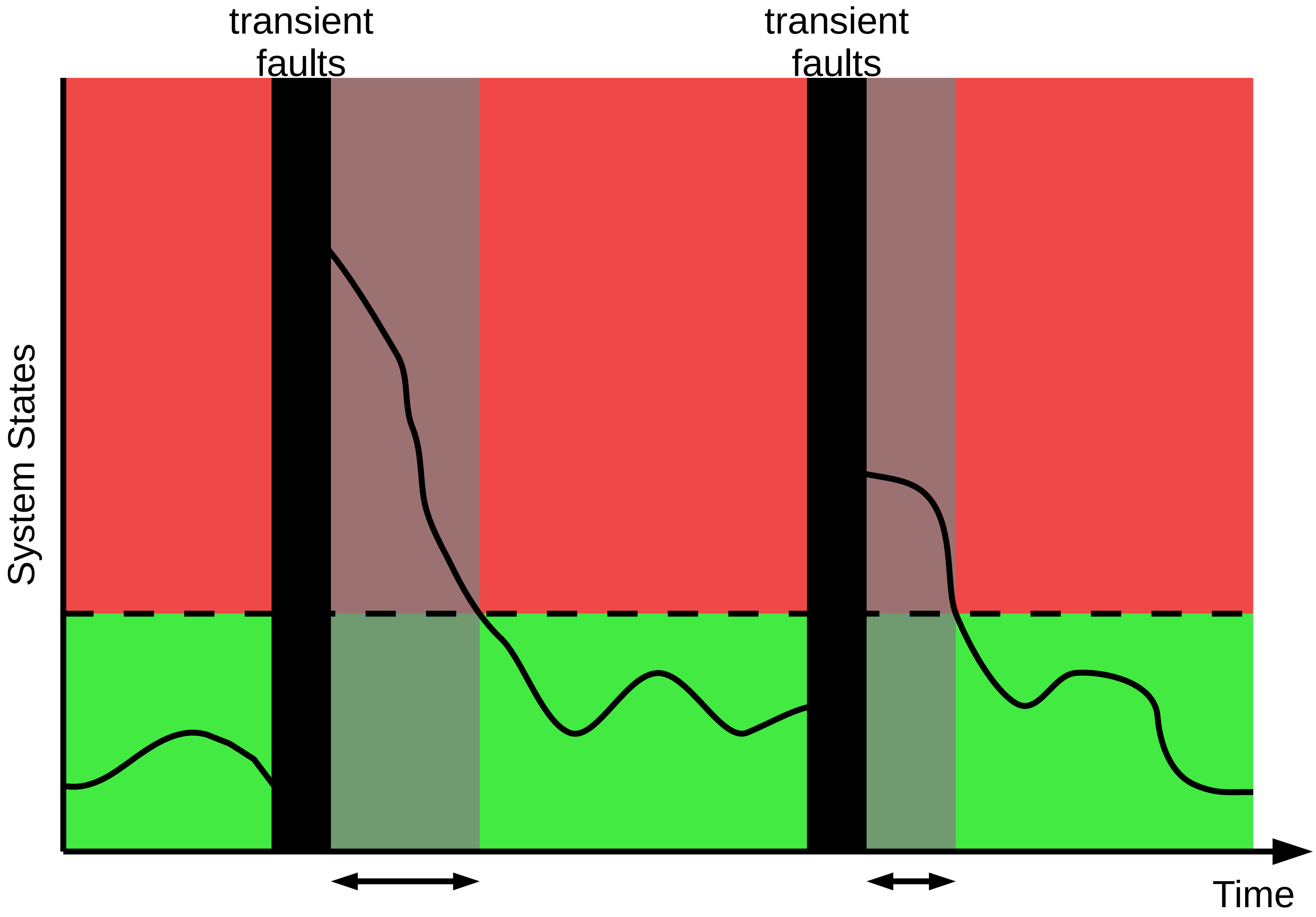


Hierarchical organization of processes

Self-Stabilization

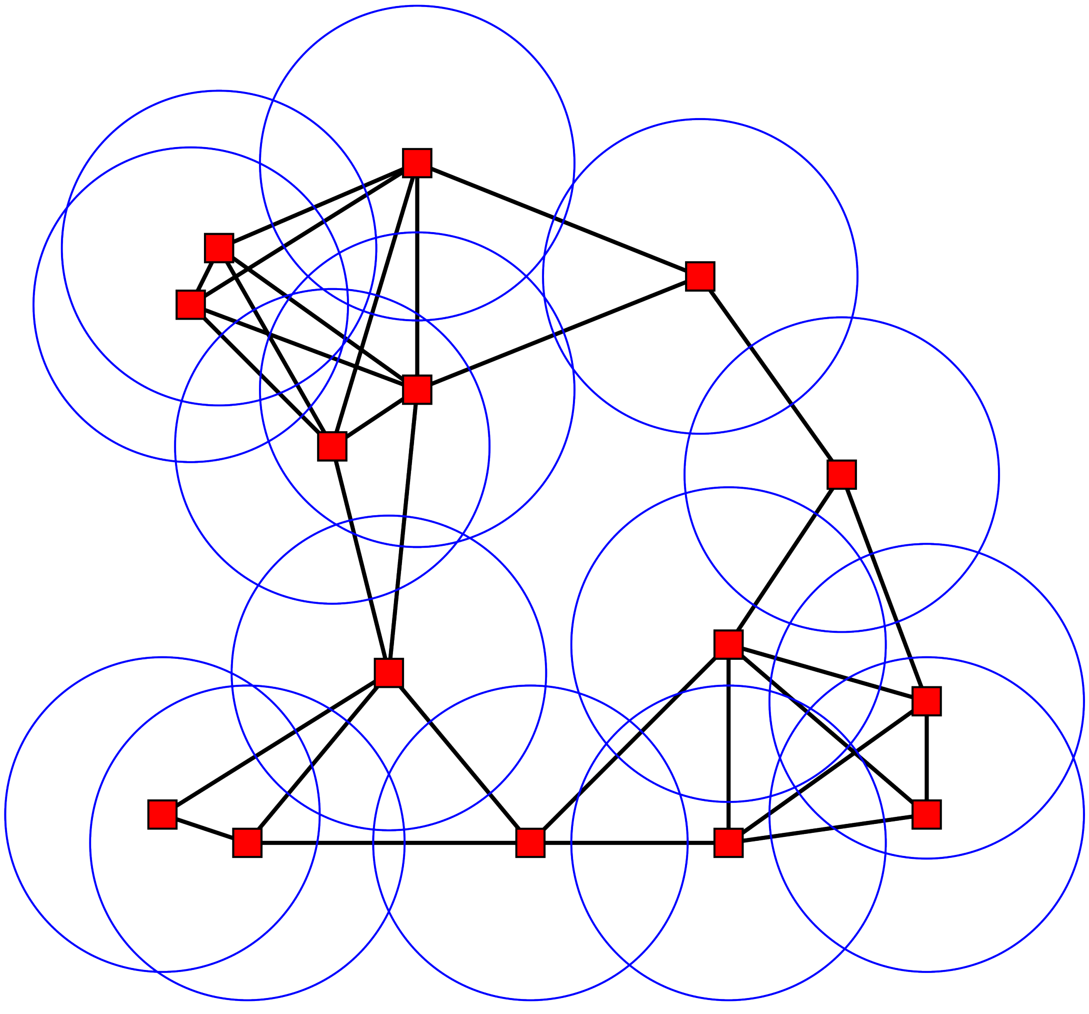


Competitiveness

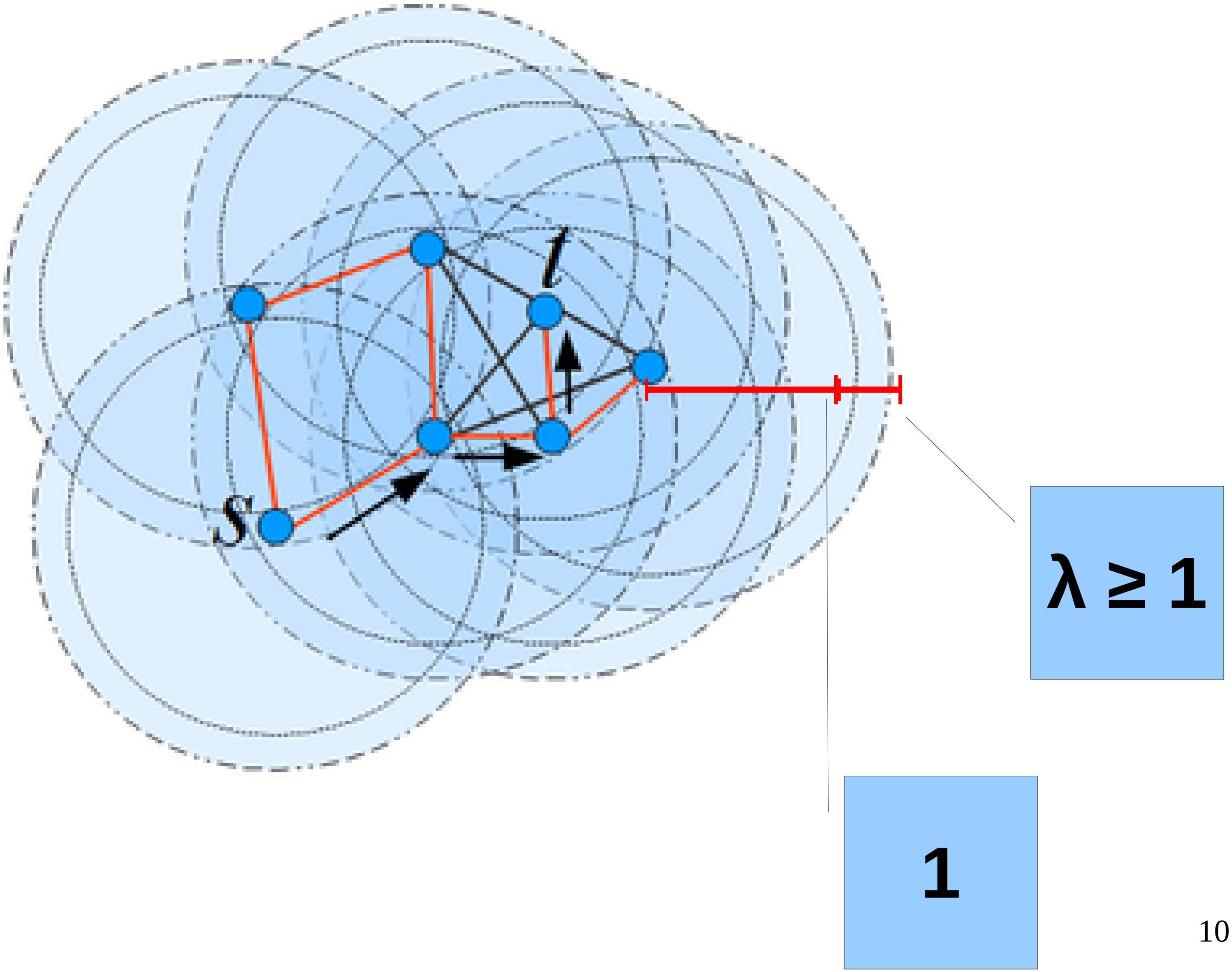
Finding a minimum k-clustering set is *NP*-hard.

An algorithm is *X-competitive* if it builds a k-clustering of size at most X times the smallest possible number of k-clusters.

Unit Disk Graphs



Quasi Unit Disk Graphs



Contribution

A *k*-clustering algorithm that is

- self-stabilizing,
- for identified networks,
- in the shared memory model,
- building at most $O(n/k)$ *k*-clusters,
- $7.2552k+O(1)$-competitive in UDGs,
- and $7.2552\lambda^2k+O(\lambda)$-competitive in QUDGs.

Related Work

- Self-stabilizing *k*-Clustering algorithms
  (Datta & al., 2009) (Caron & al., 2010)
- *8k+O(1)*-competitive *k*-Clustering algorithm
  (Fernandess & Malkhi, 2002)

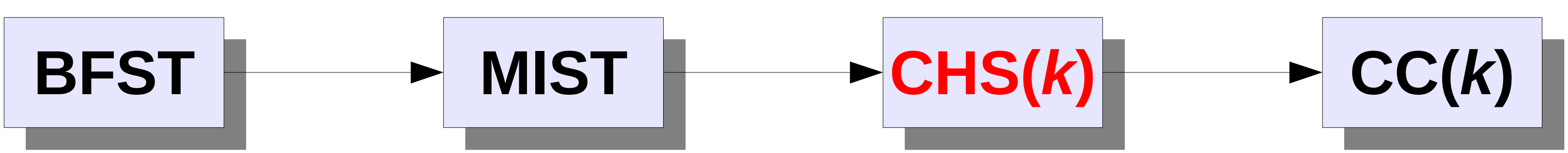
Algorithm: General Overview

Combination of self-stabilizing algorithms:

- Spanning Tree Construction (Huang & Chen, 1992)
- MIS Tree Build (imp. Fernandess & Malkhi, 2002)

  $\rightarrow$ *for the sole purpose of competitiveness*
- *k*-Clusterheads Selection
- *k*-Clustering Construction

Parallel execution of both algorithms



k-Clusterheads Selection: *α*

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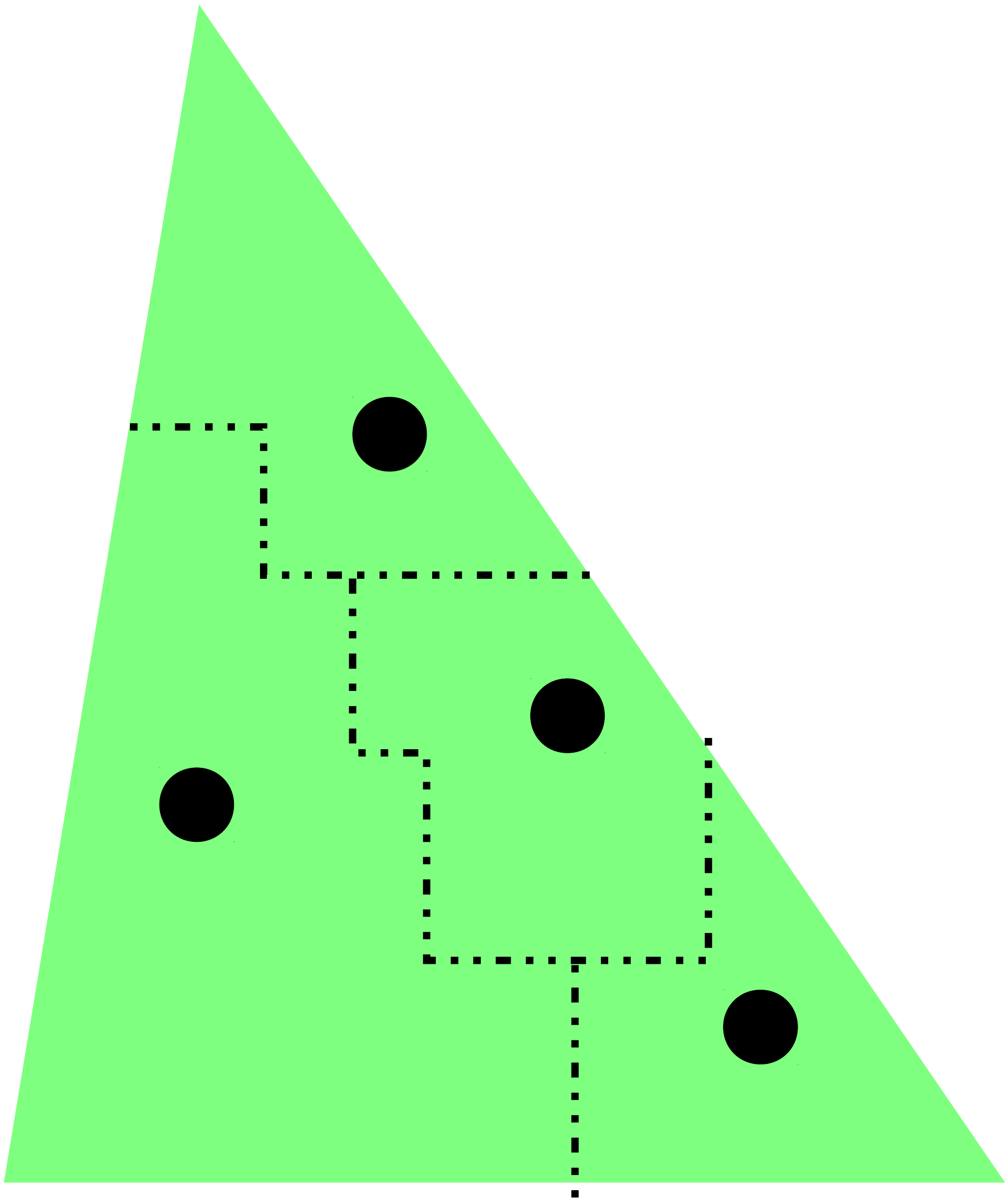

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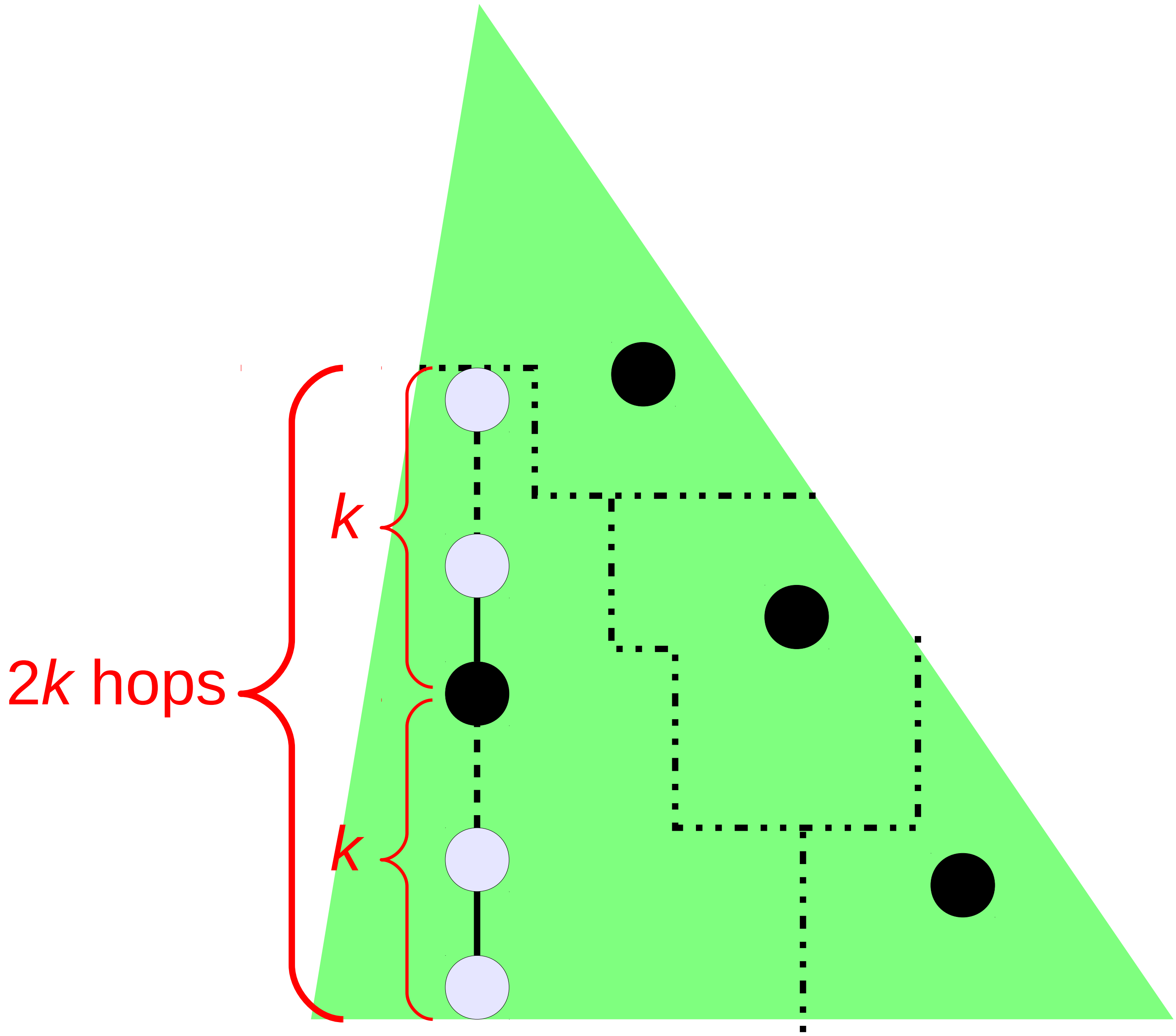

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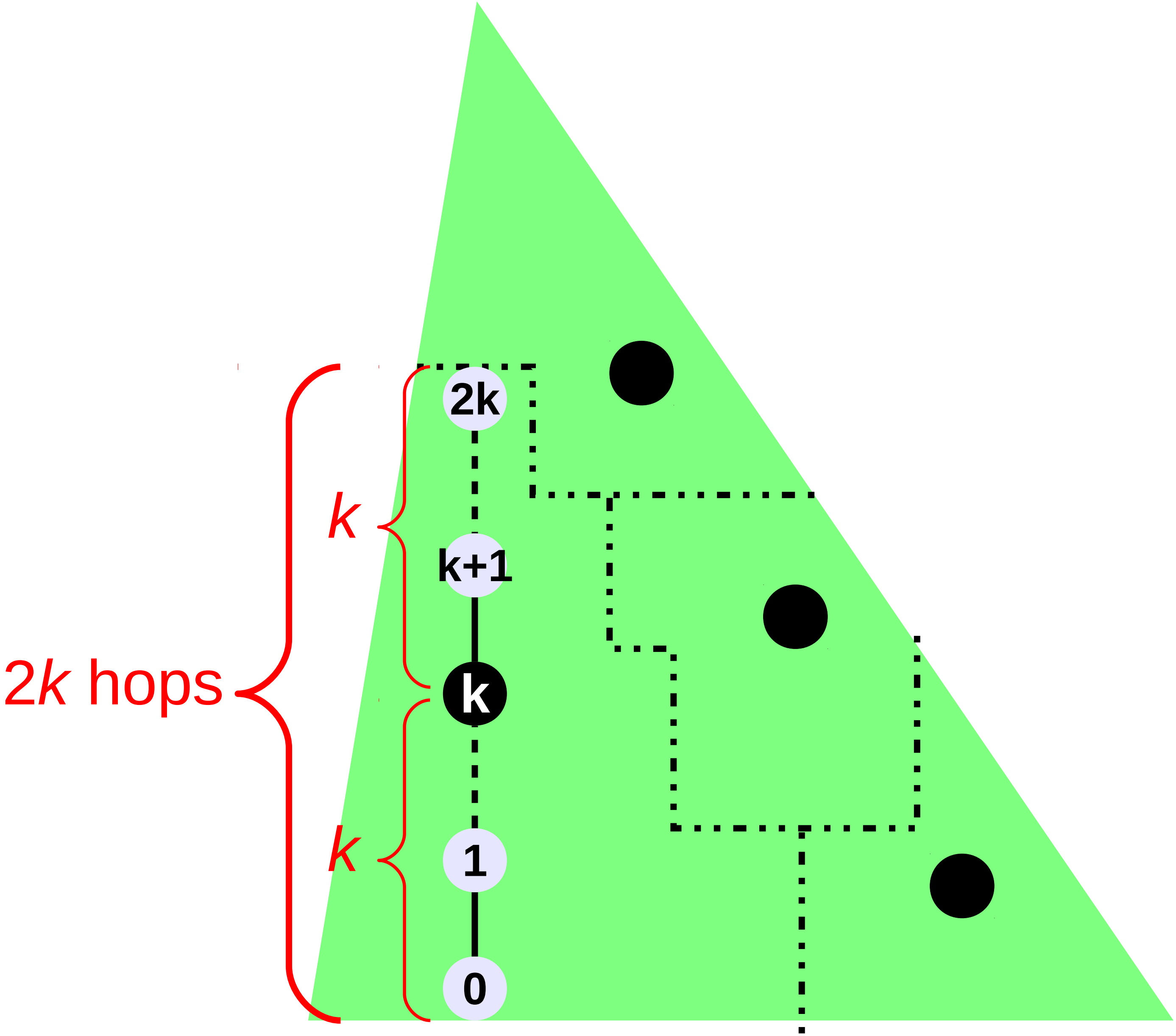

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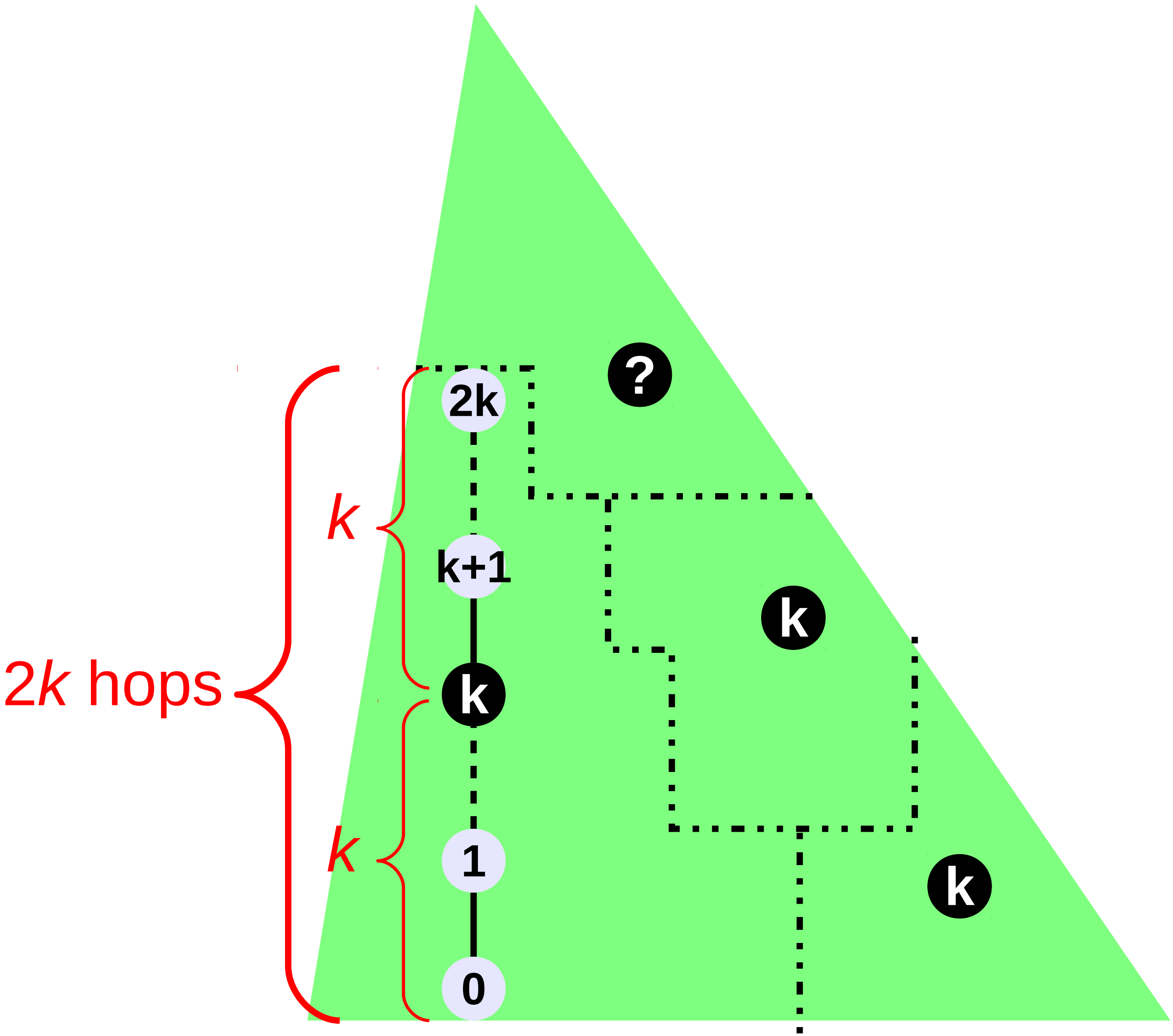

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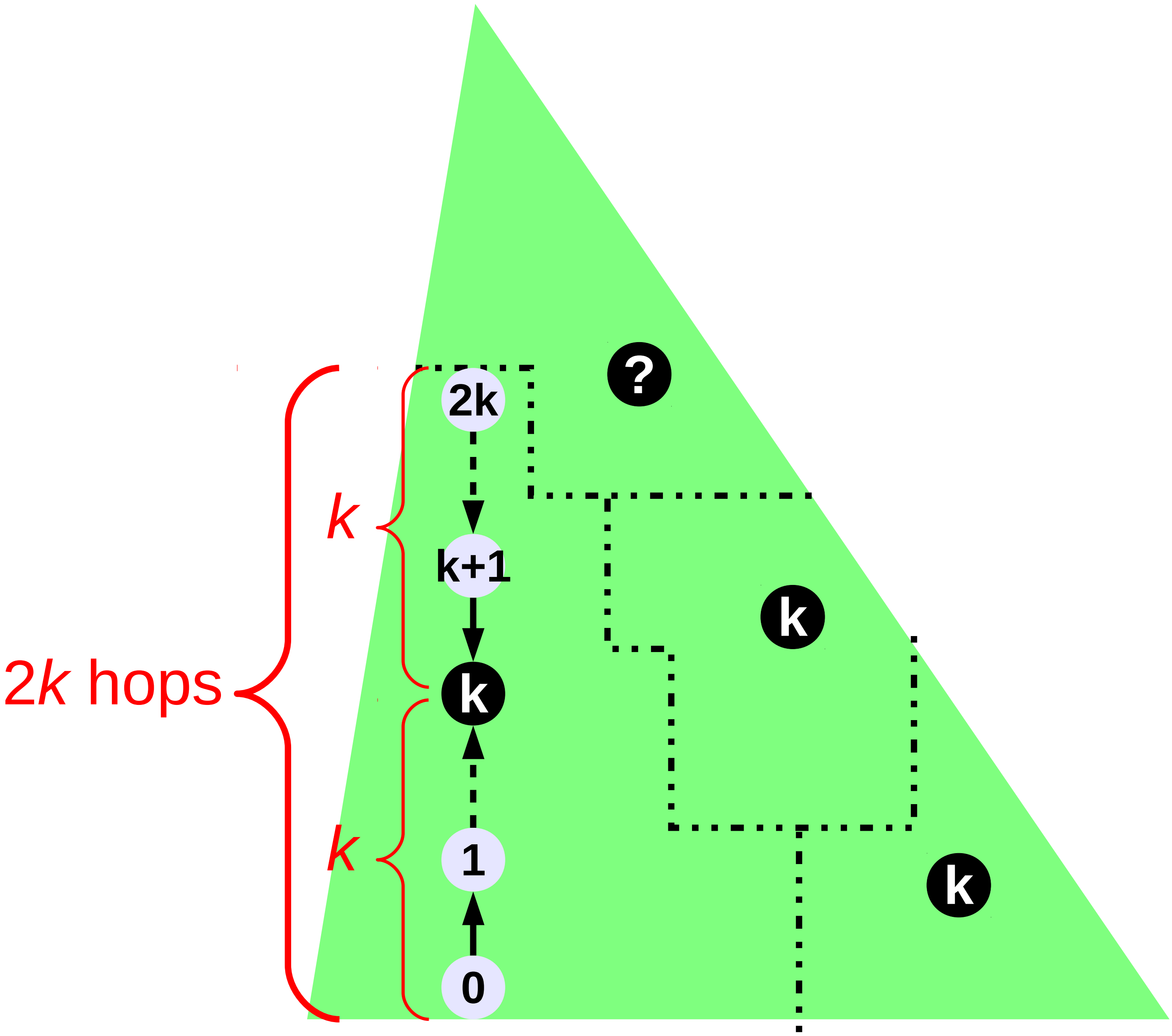

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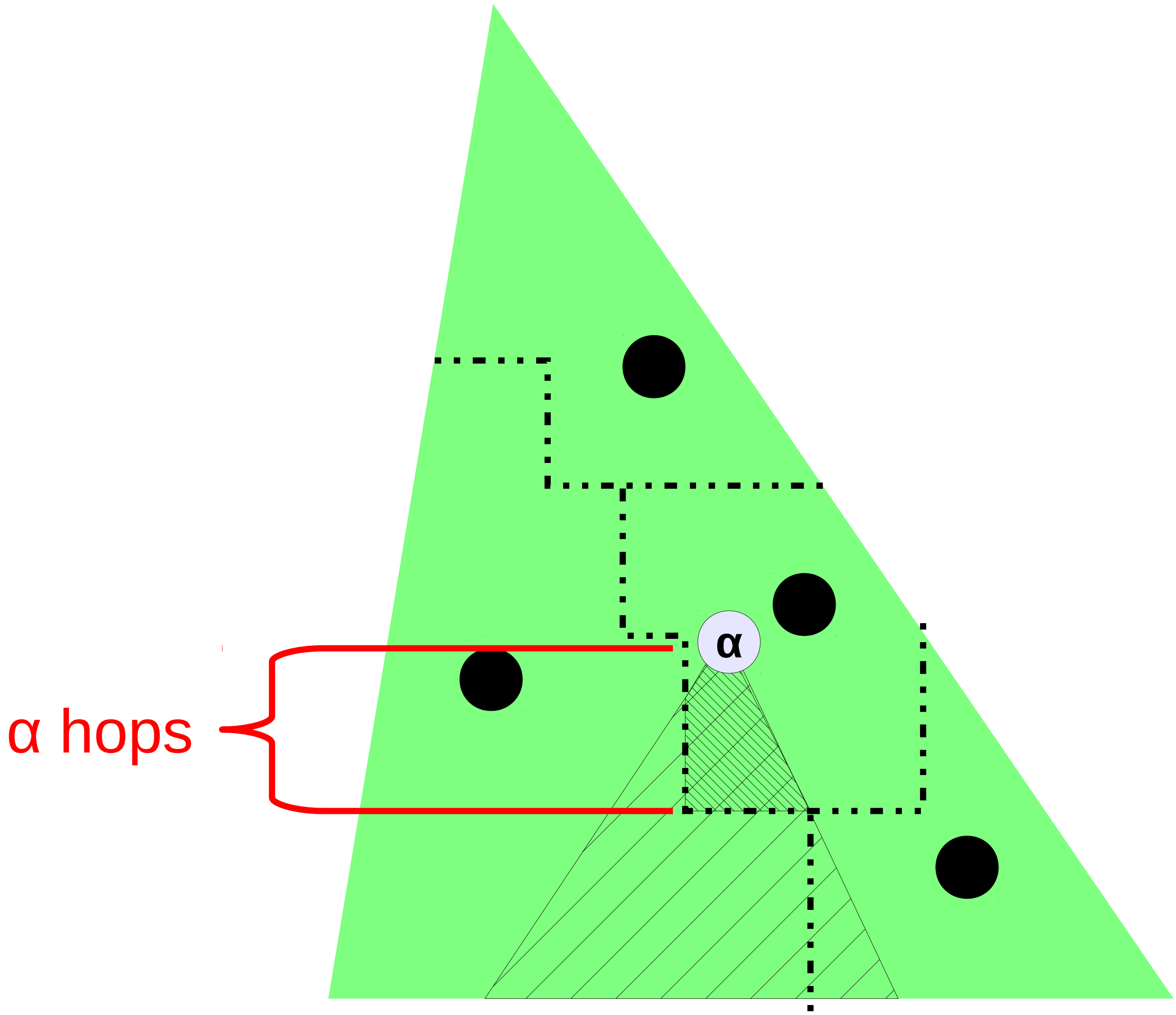
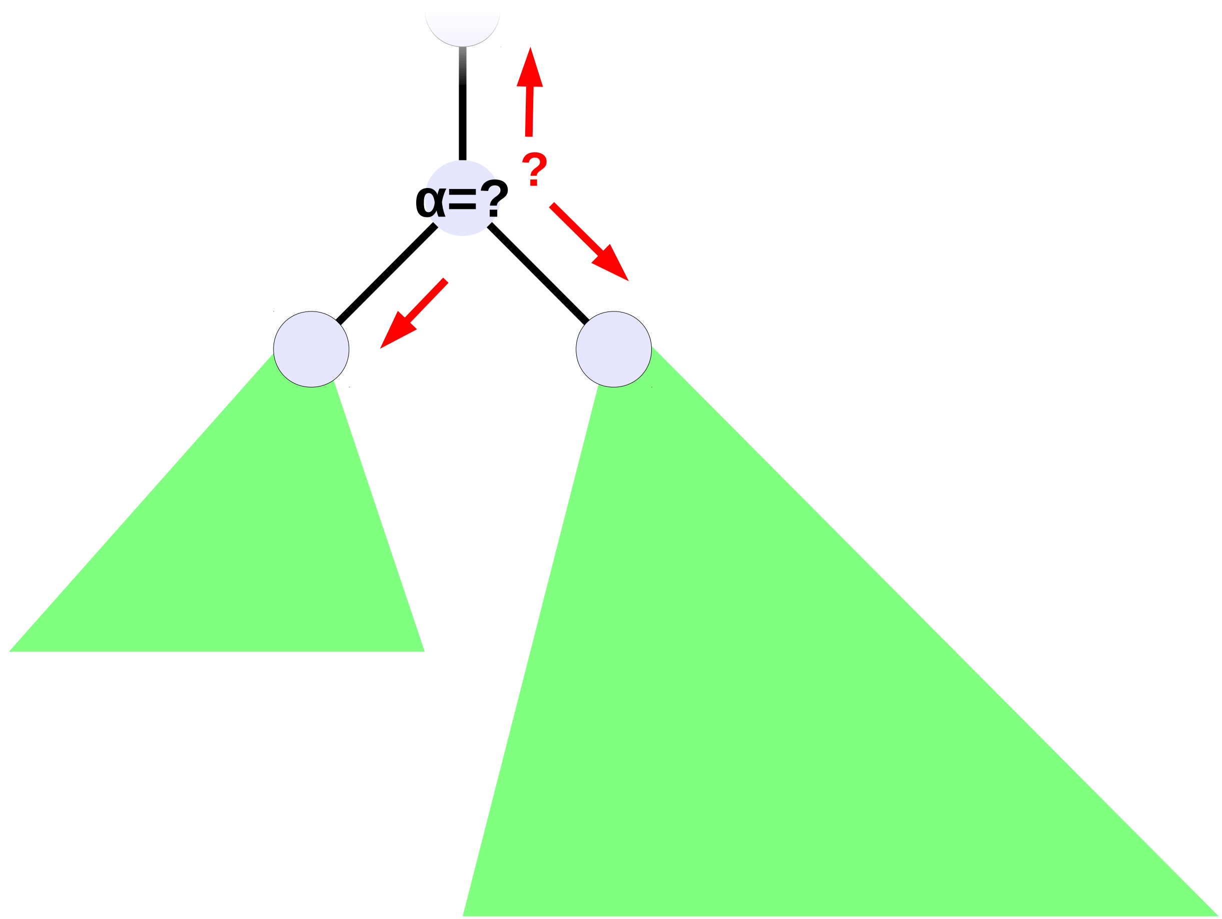


Inductive computation of α



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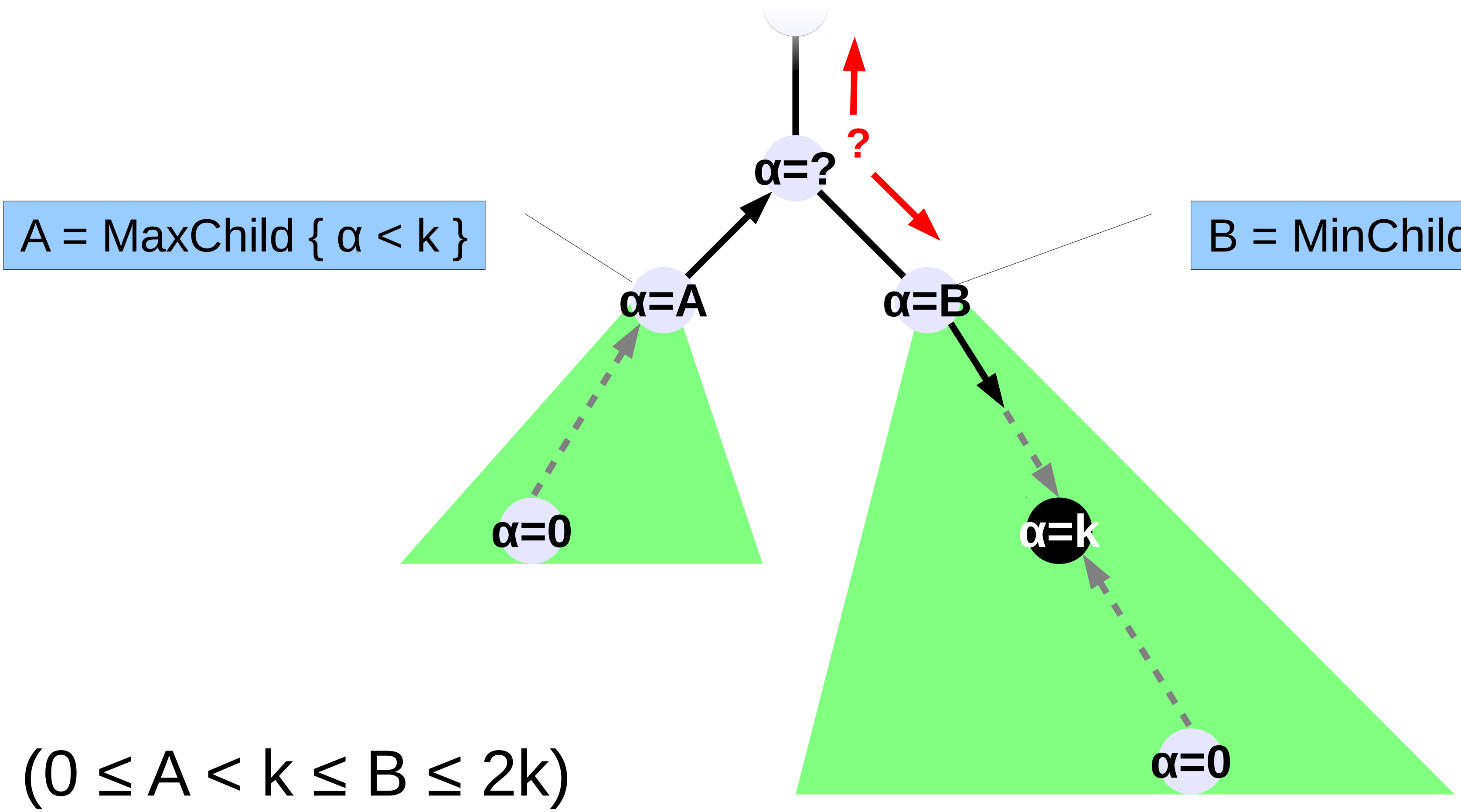
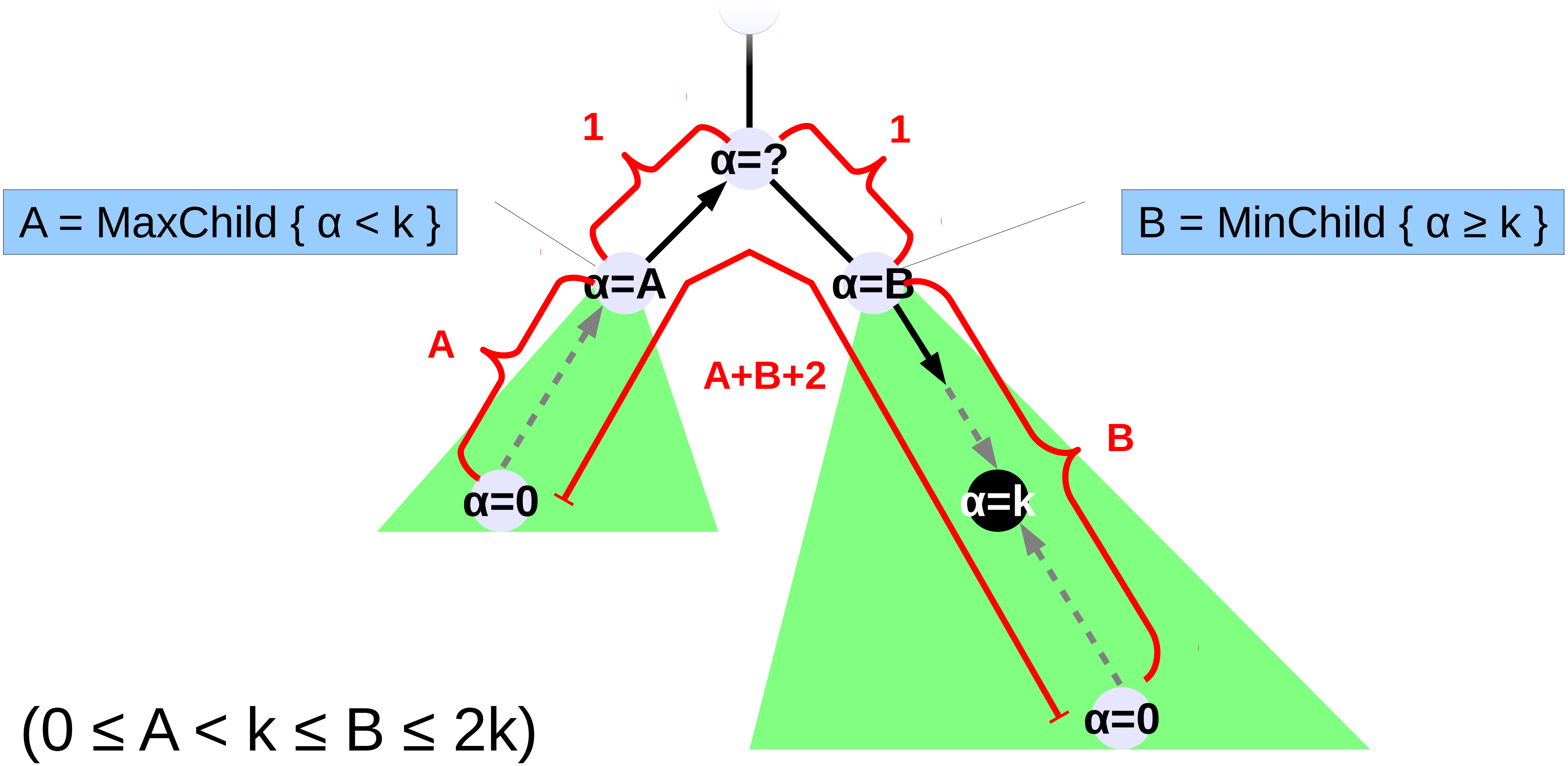


$(0 \leq A < k \leq B \leq 2k)$

Inductive computation of α



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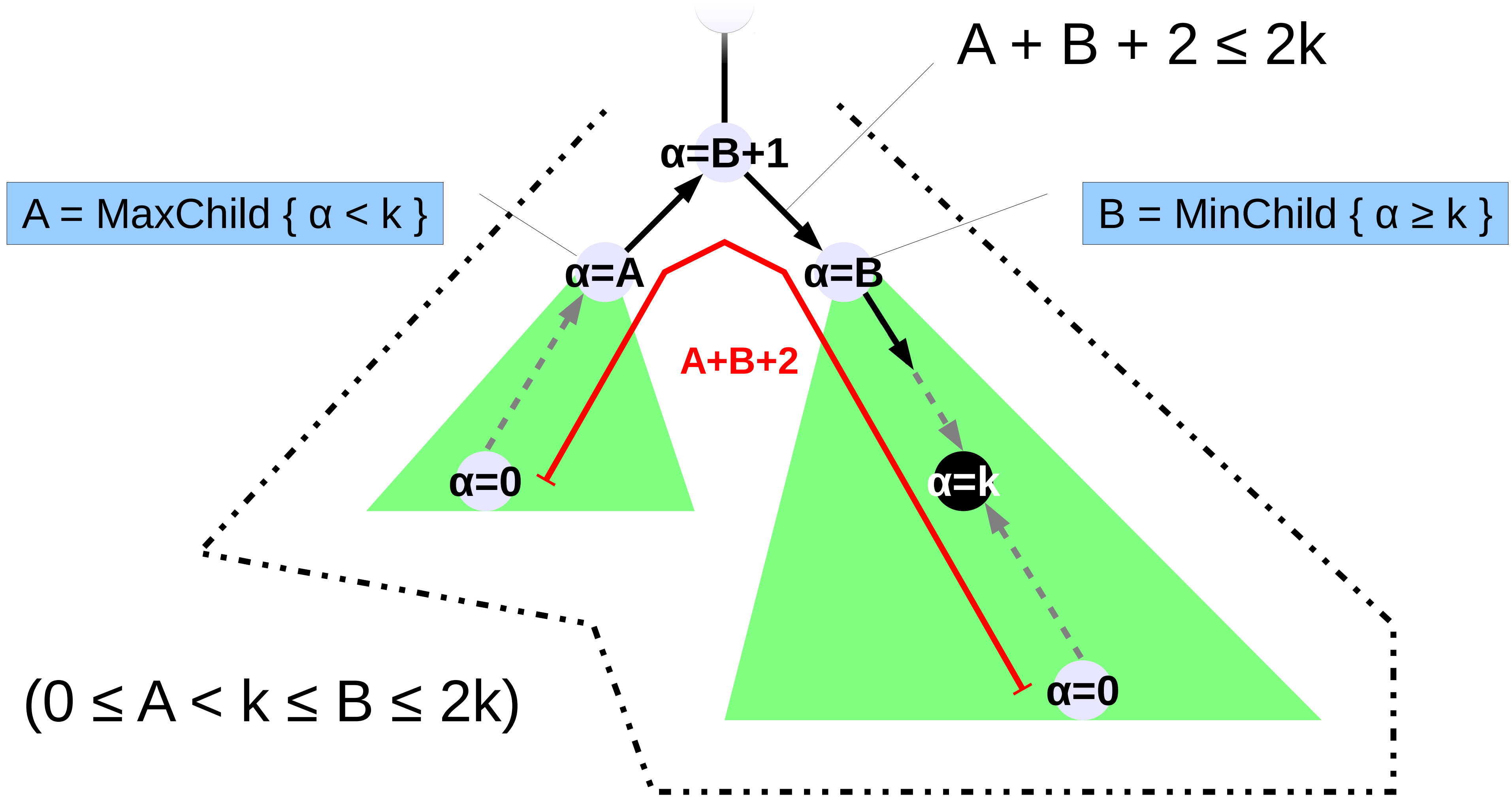
Inductive computation of *α*



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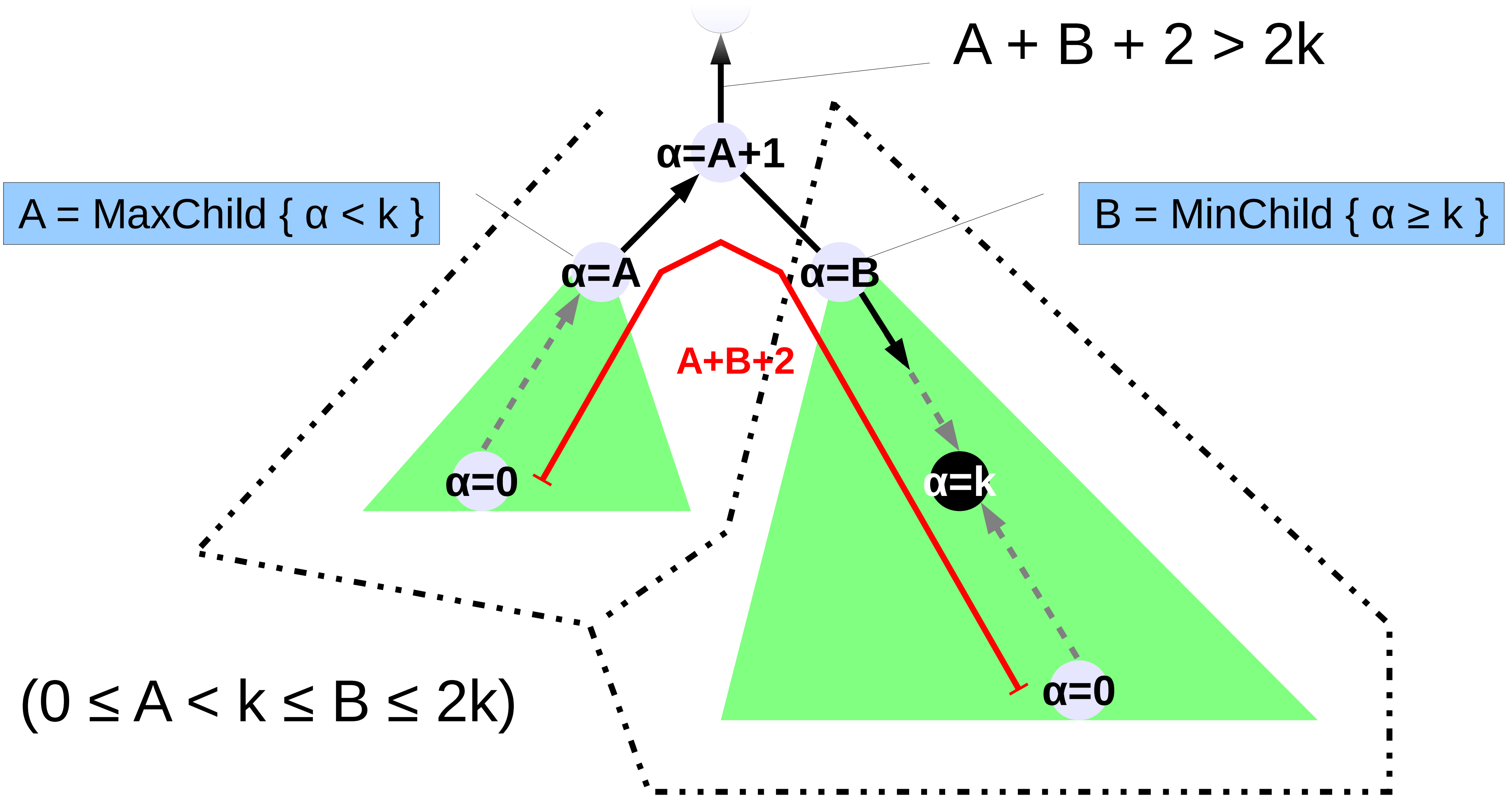
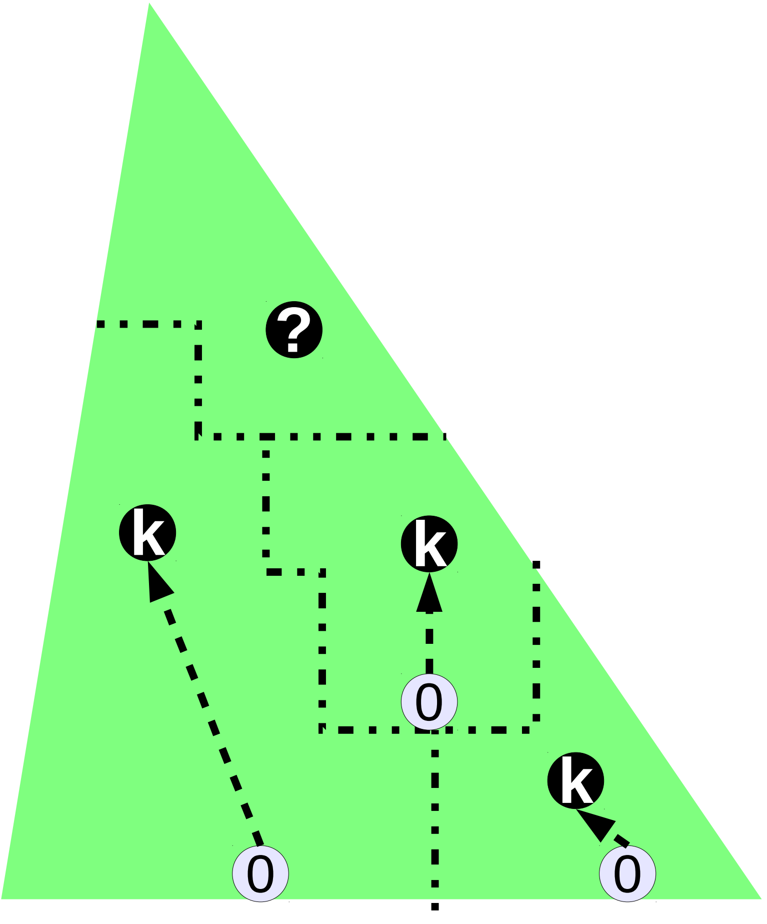


7.2552*k*+0(1)-competitive in UDGs



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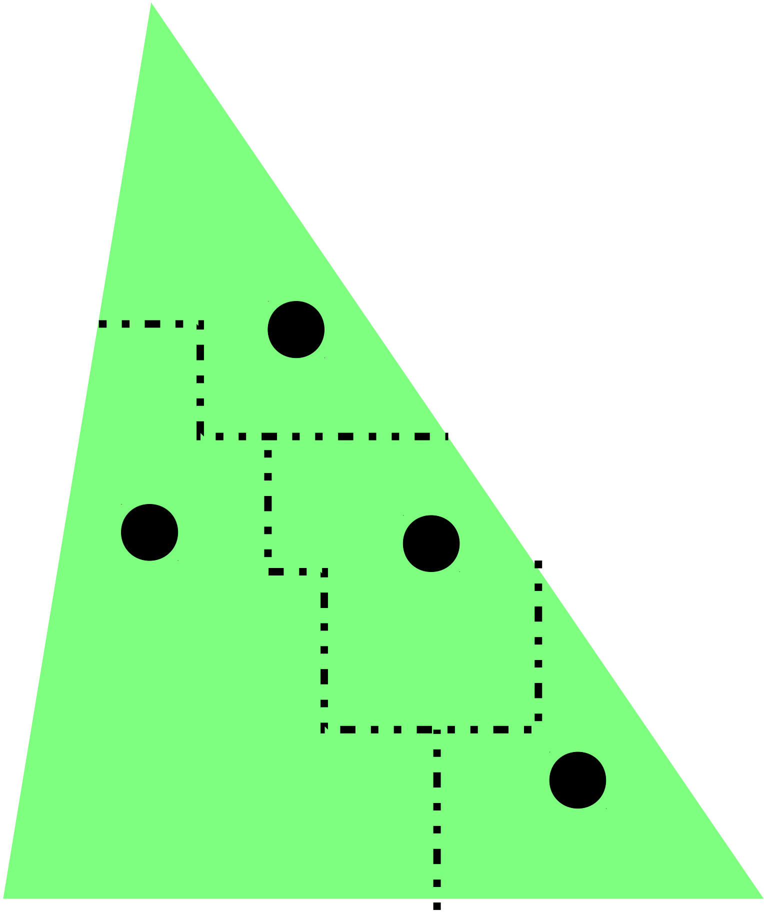

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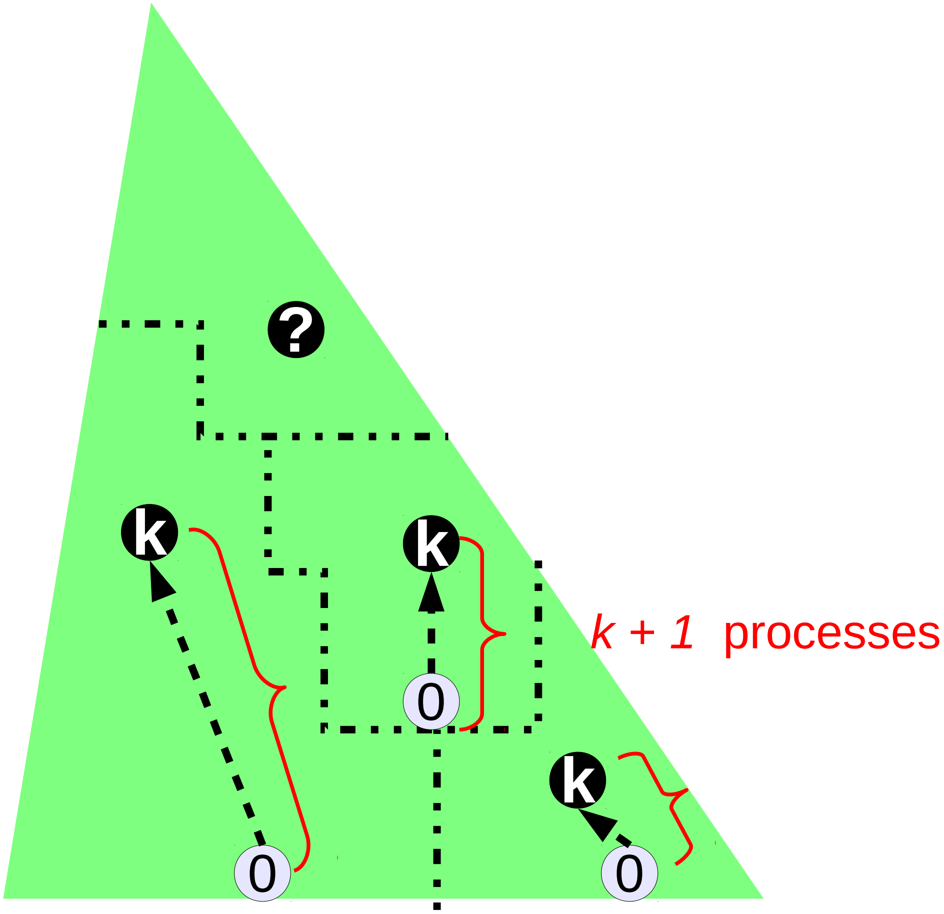

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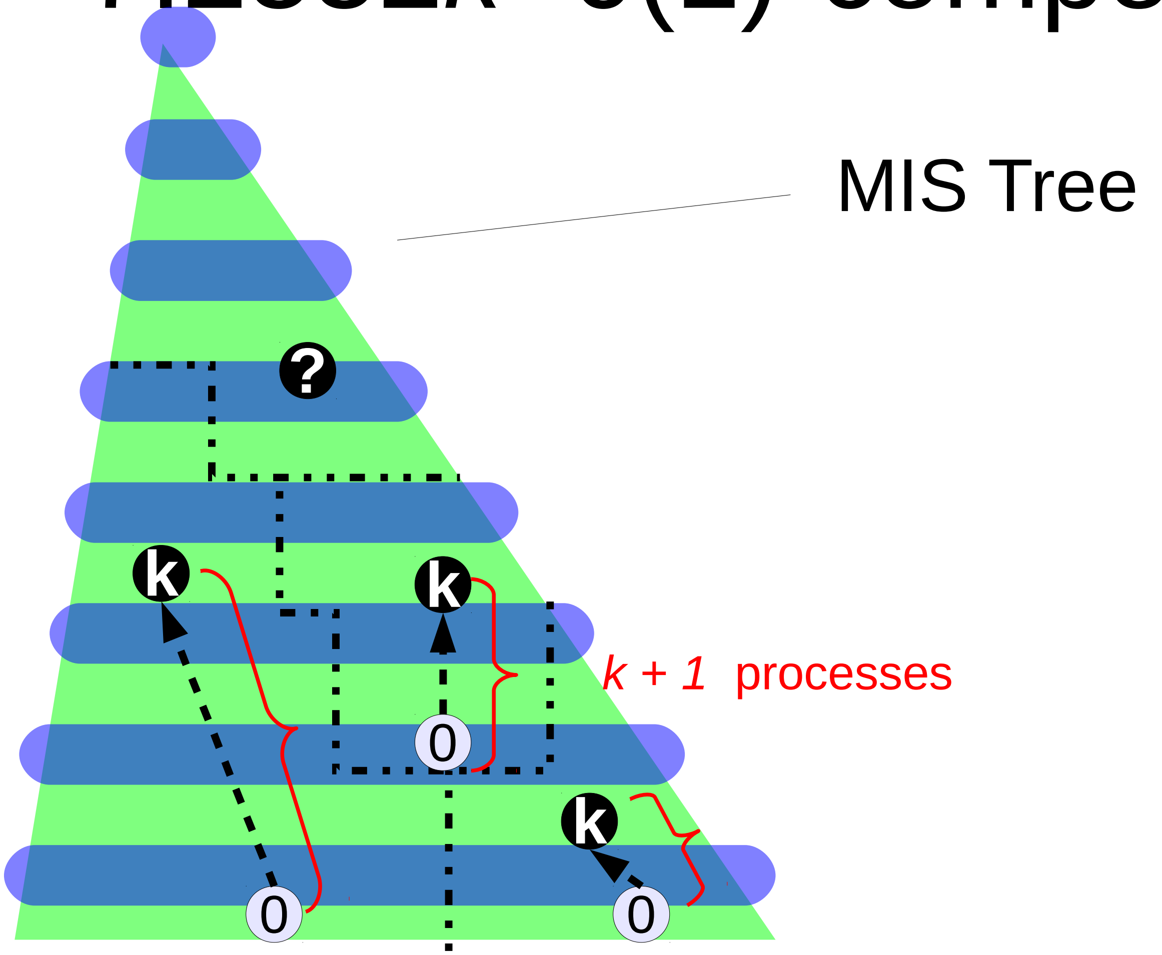

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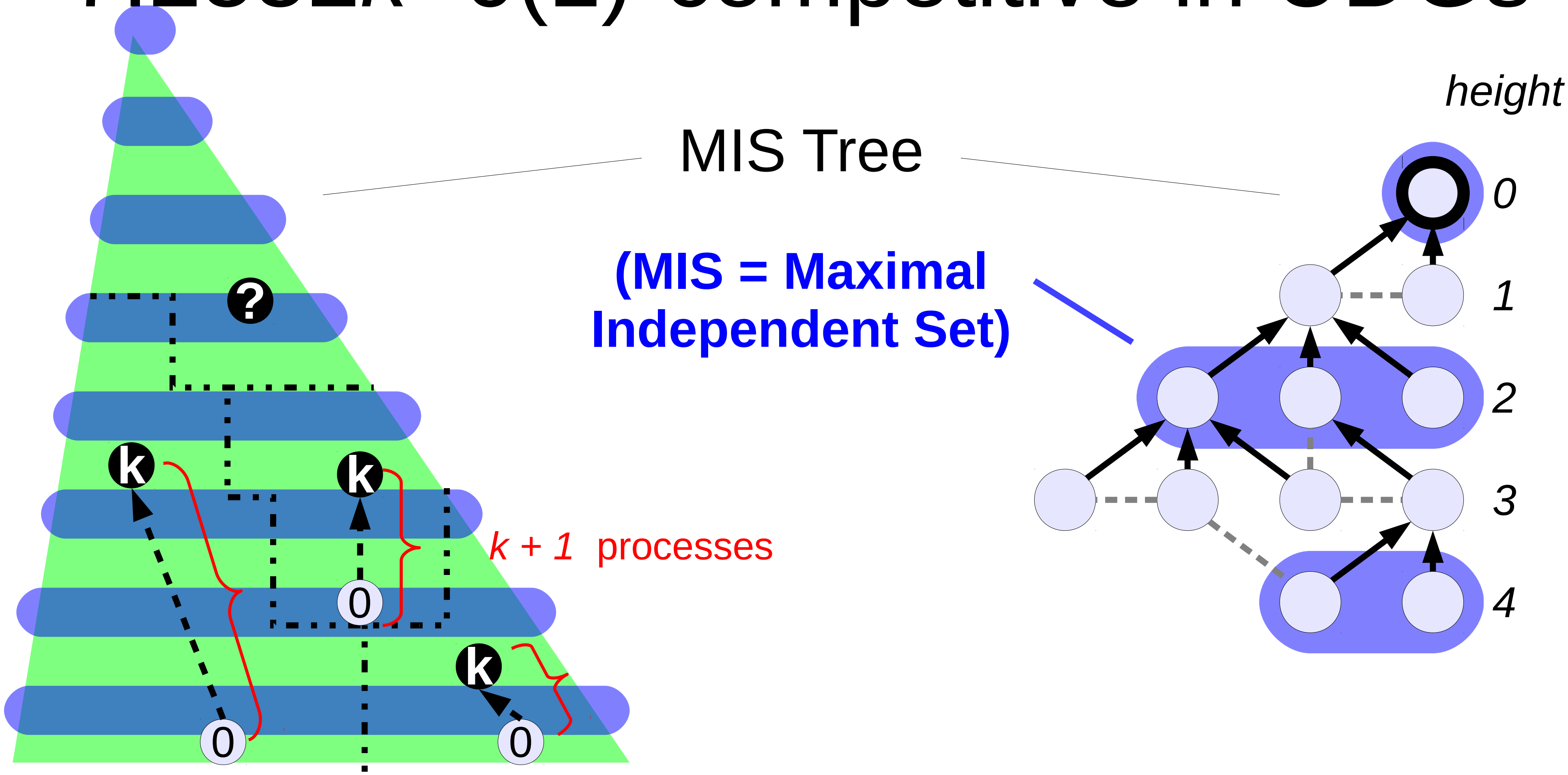

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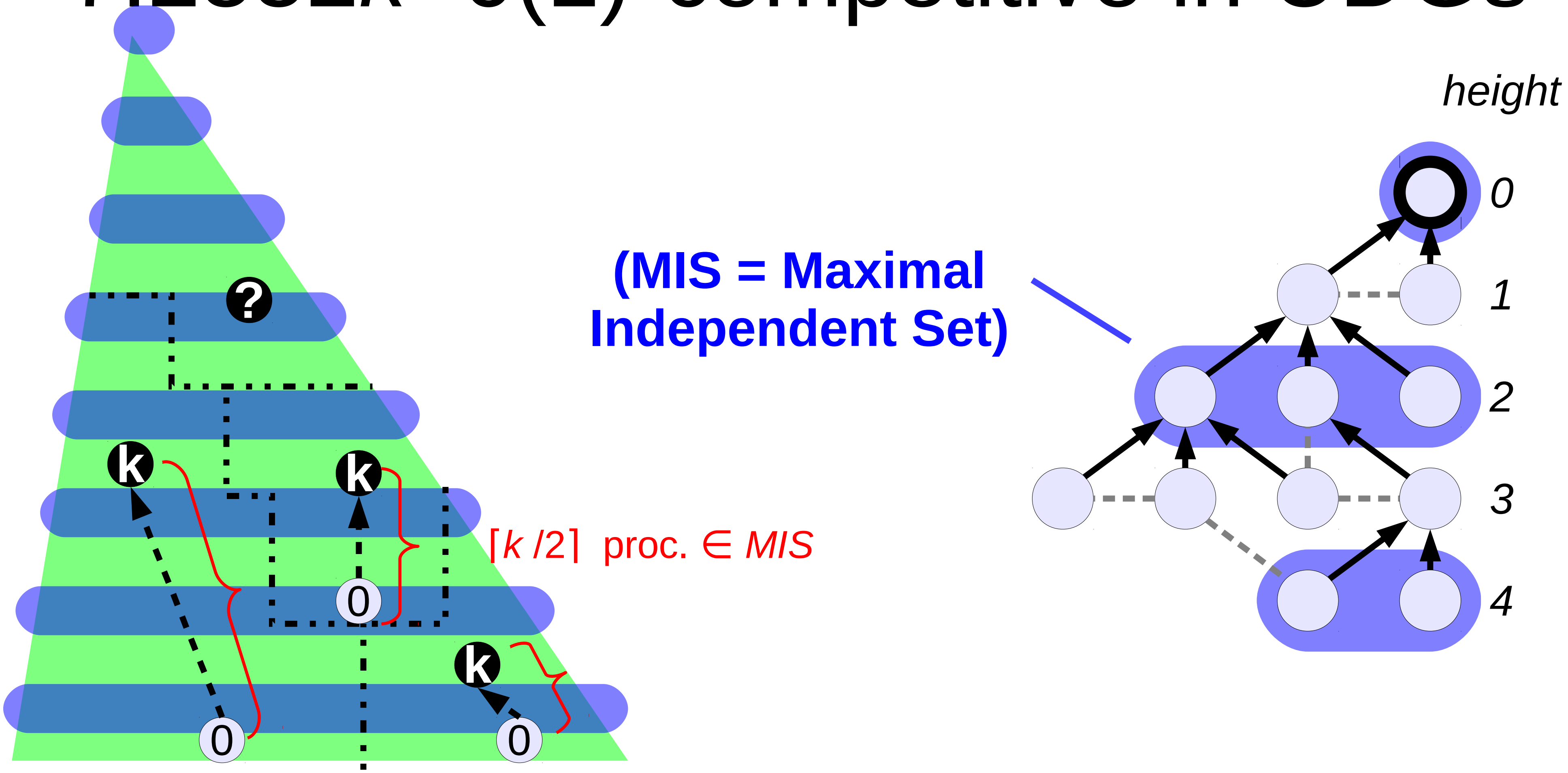

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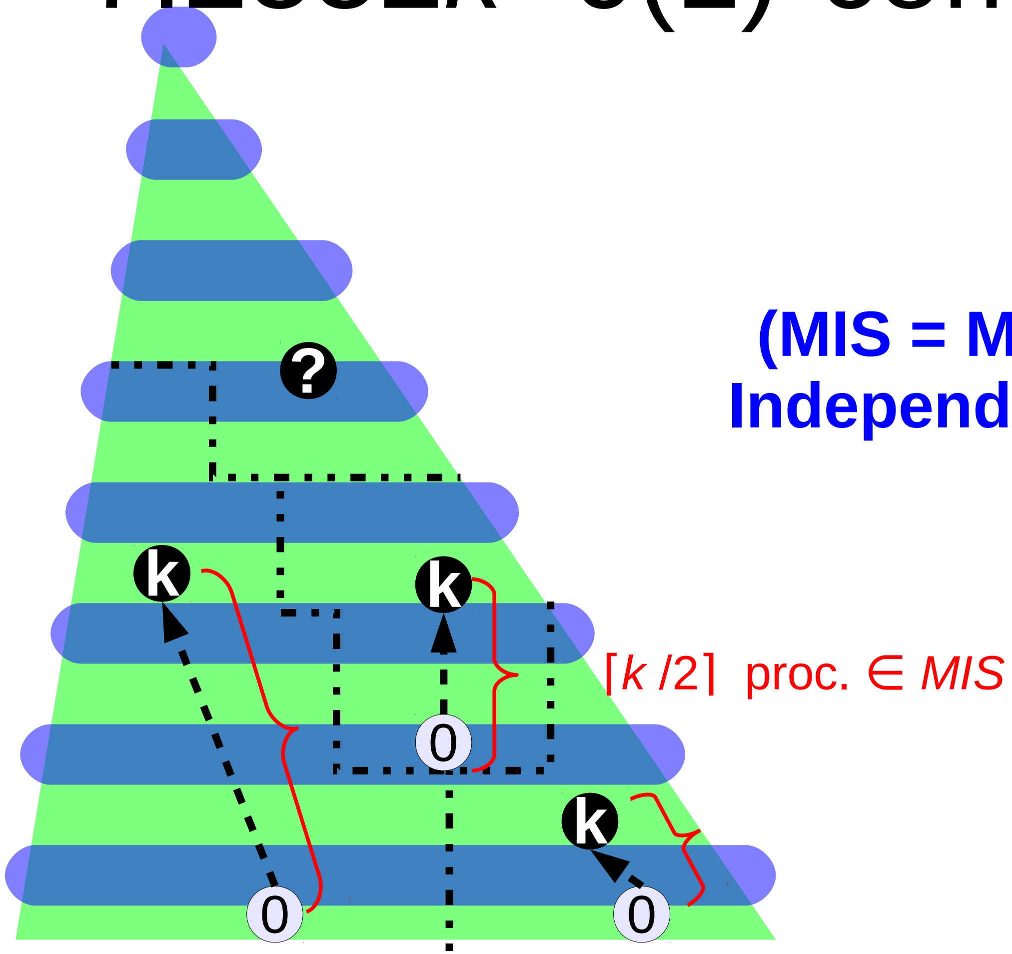
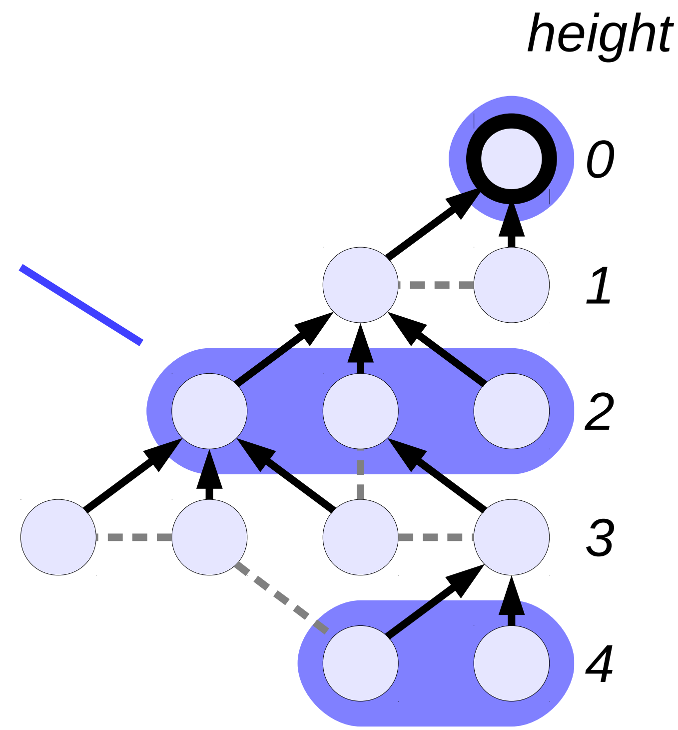


(MIS = Maximal Independent Set)



$(|Clr| - 1) \lceil k/2 \rceil \leq |MIS| - 1$

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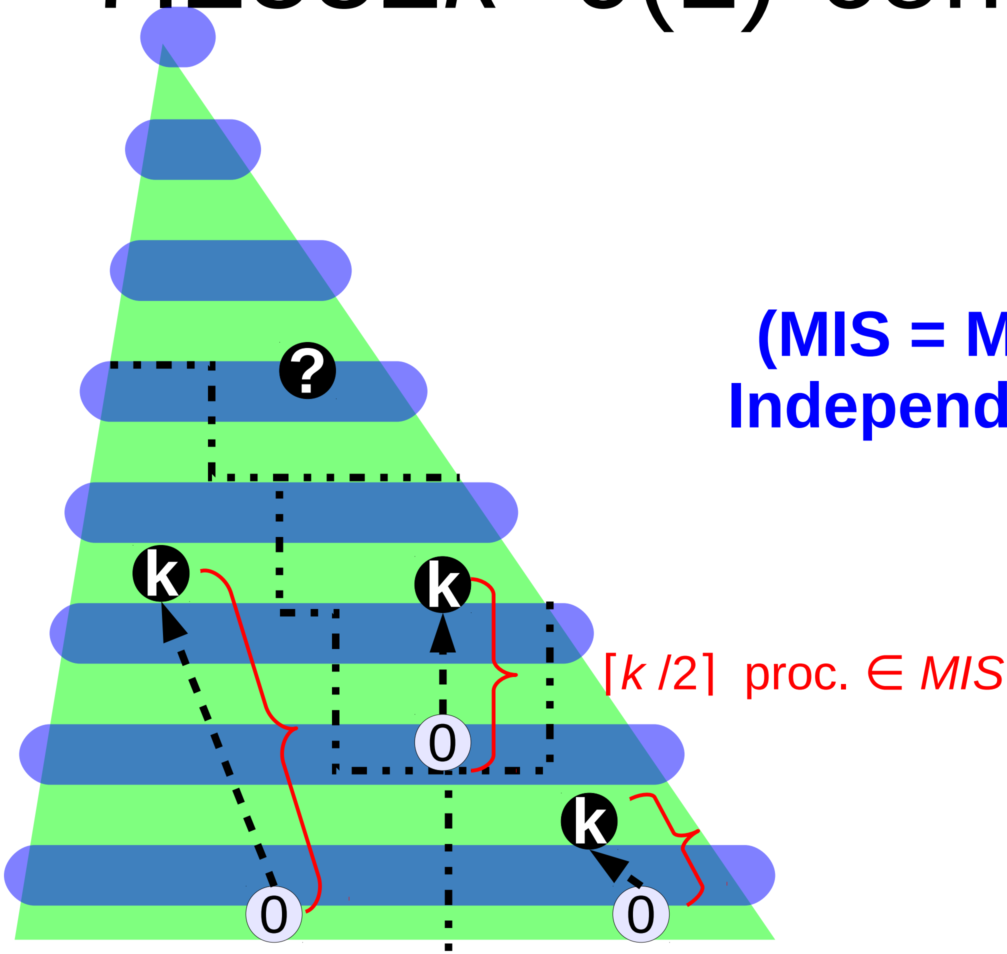
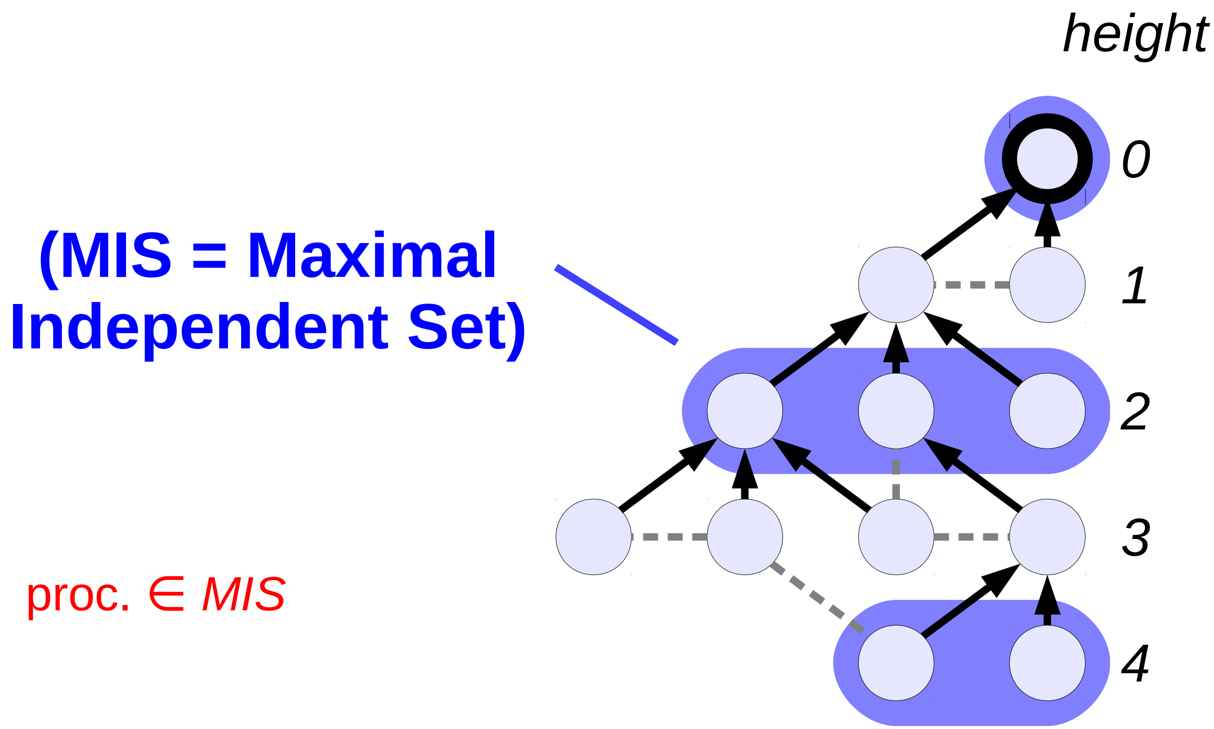
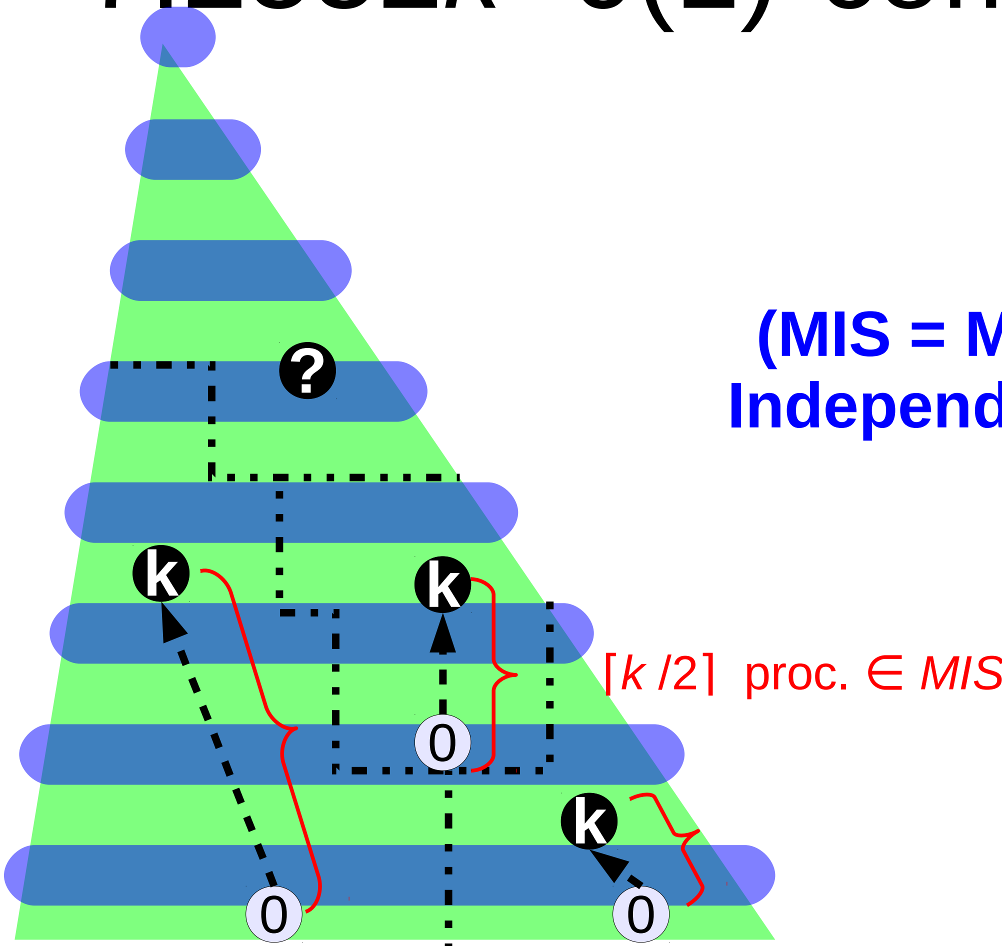




$(|Clr| - 1) \lceil k/2 \rceil \leq |MIS| - 1$ $|IS| \leq ((2\pi/\sqrt{3})k^2 + \pi k + 1)\ |Opt|$

7.2552k+0(1)-competitive in UDGs



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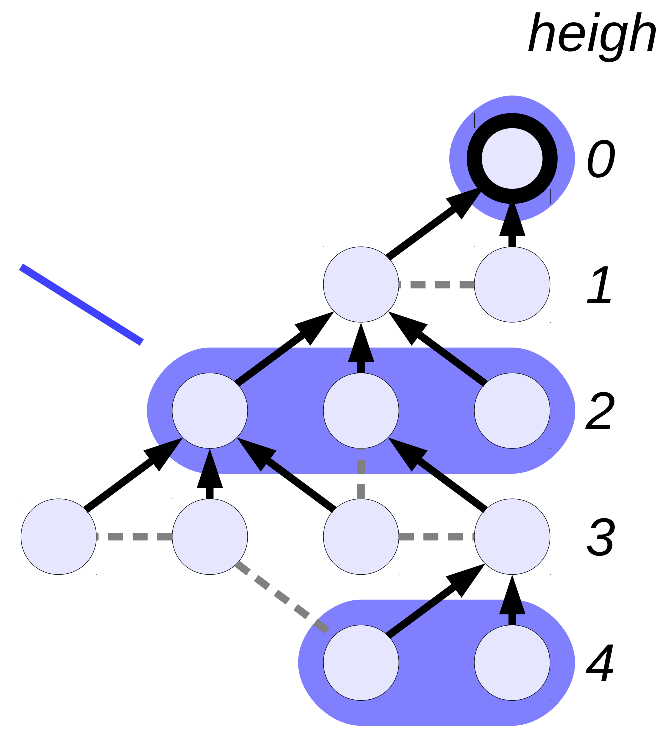

$(|Clr| - 1)\lceil k/2 \rceil \le |MIS| - 1$

$|IS| \le ((2\pi/\sqrt{3})k^2 + \pi k + 1)\,|Opt|$

$|Clr| \le 1 + ((4\pi/\sqrt{3})k + 2\pi)\,|Opt| \approx 7.2552k+0(1)\,|Opt|$

Conclusion

Self-stabilizing k-clustering algorithm which

- stabilizes in $O(n)$ rounds,
- uses O(log n) space per process,
- builds at most $O(n/k)$ k-clusters,
- is competitive in UDGs and QUDGs.

Thanks for your attention.