MHD and plasma turbulence in accretion discs

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Talk outline

- Astrophysical introduction to accretion
  - What accretion discs are and where they can be found
  - Various accretion regimes
- Accretion discs dynamics
  - The basics
  - Angular momentum transport and stresses
- Linear stability of differentially rotating fluid flows
  - Differential rotation on a cylinder
  - Linear hydrodynamic stability
  - MHD : the magneto-rotational instability

Talk outline (2)

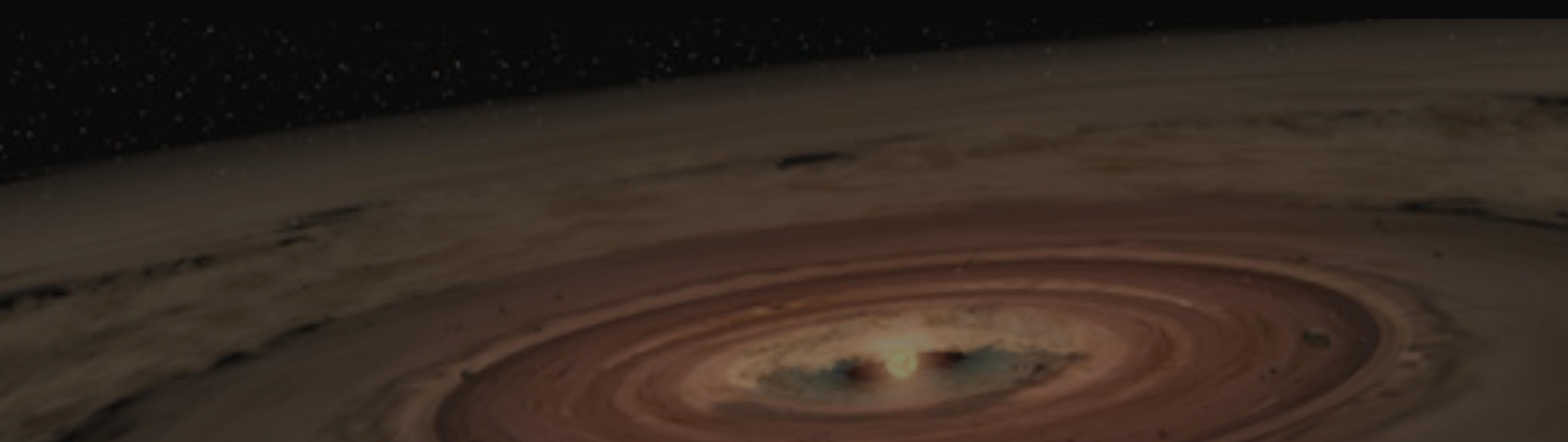
- Local approaches to rotating shear flows
  - The infinite shearing sheet
  - Rotating plane Couette flow
  - The shearing box
- Nonlinear hydrodynamic stability in fluid discs
  - Short review of past theoretical, numerical and experimental work
  - Subcritical shear turbulence and self-sustaining processes
- MHD turbulence in fluid discs
  - Magneto-rotational turbulence phenomenology
  - Turbulent transport and their Pm / Rm dependences
  - Magnetorotational dynamo action

Talk outline (3 !)

- Plasma physics and turbulence in hot accretion flows
  - A major actor : pressure anisotropy
  - MHD, Braginskii MHD, Kinetic MHD and other things
  - Kinetic MRI and magnetoviscous instability
  - The hidden role of pressure anisotropy-driven instabilities
- Conclusions and the future

Yes, it is going to be very long and painful !

Astrophysical introduction to accretion

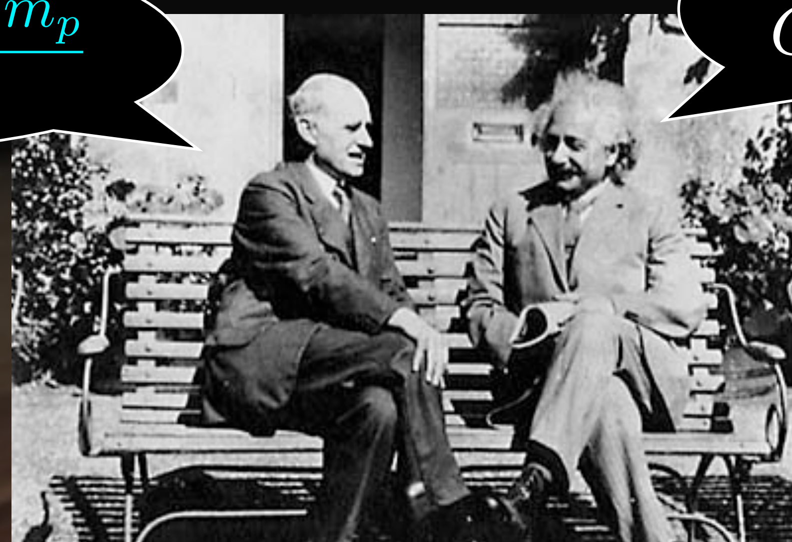


Accretion basics

- Matter falling onto a massive central object
  - Release of gravitational energy leads to radiation
  - Accretion luminosity

$$\mathcal{L} \sim \frac{\mathcal{G} M_* \dot{M}}{R}$$

- Eddington luminosity limit



Accretion discs

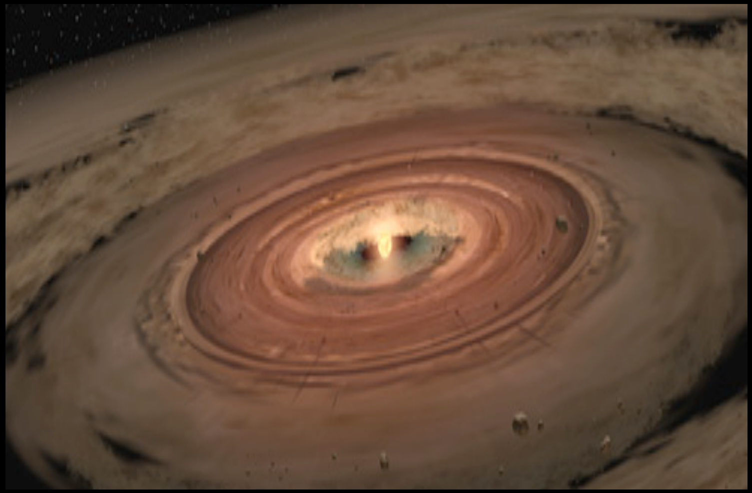
- Formation
  - Almost everything spins at least weakly in the Universe
  - Angular momentum conservation
    - accreted matter does not fall directly onto the central accreting object
  - An accretion disc forms
- Depending on conditions, their composition involves
  - plasma (fluid or collisionless)
  - neutral, non-conducting gas
  - solid dust grains
  - a combination of all these states

Motivation for studying accretion discs

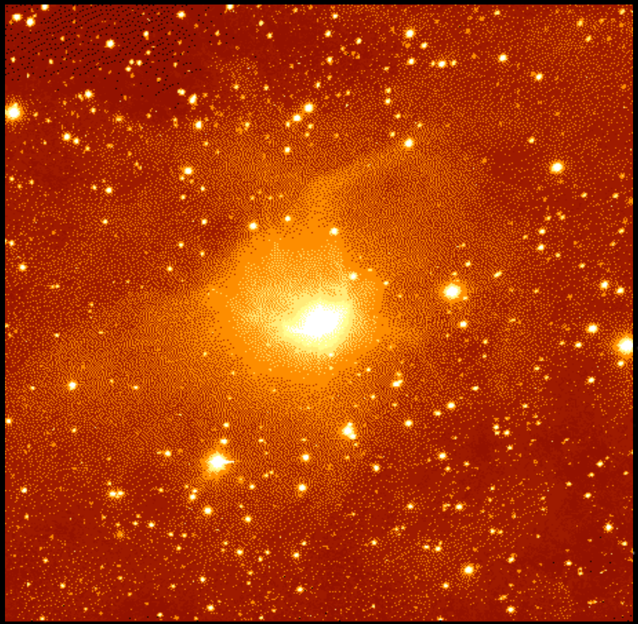
- Very common objects in the Universe
  - Systematic transient state regulating
    - the dynamics and thermal evolution of rotating accreting systems
    - the formation of astrophysical structures
- Discs naturally provide
  - a reservoir of energy, in the form of gravitational energy
  - a reservoir of mass
- Key astrophysics problems linked to accretion
  - stellar and planetary formation
  - the physics of luminous sources in the Universe, such as AGNs
    - the physics of our galactic centre

Extremely diverse manifestations

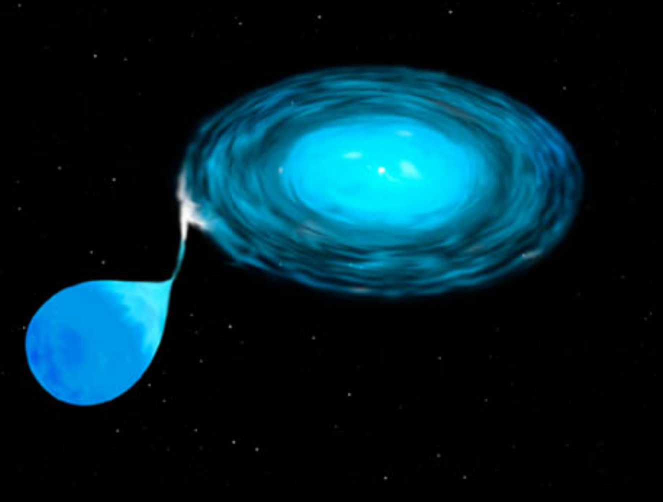
Protoplanetary discs



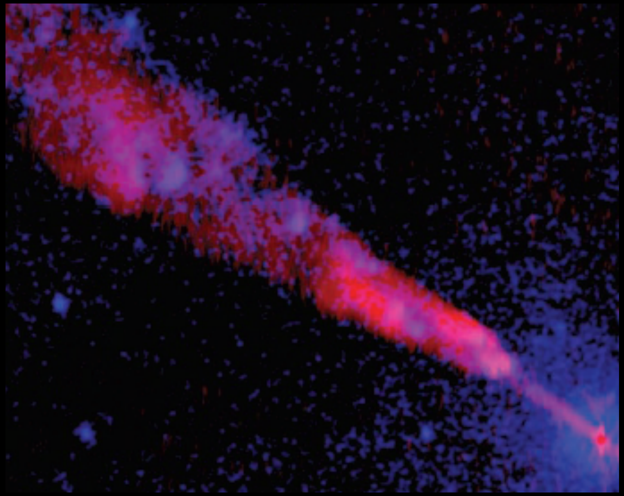
Protostellar discs (FU Orionis)

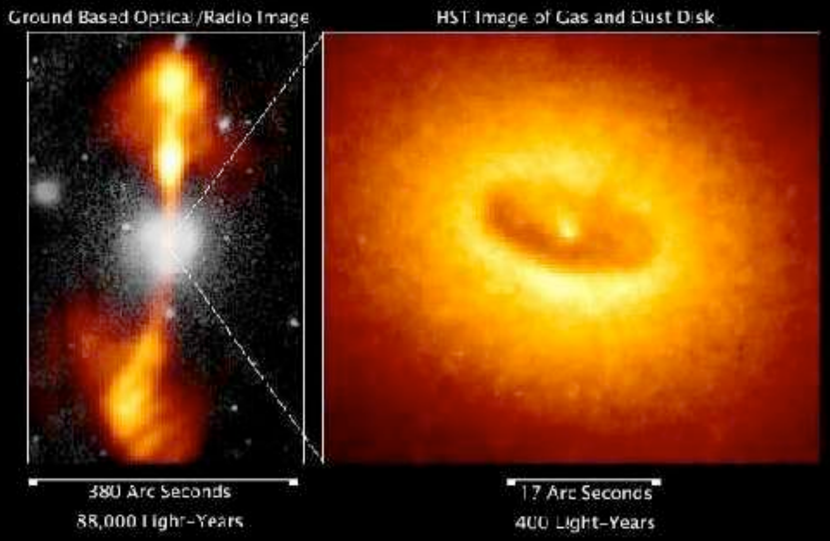


Binary systems
(cataclysmic variables)



Active Galactic Nuclei



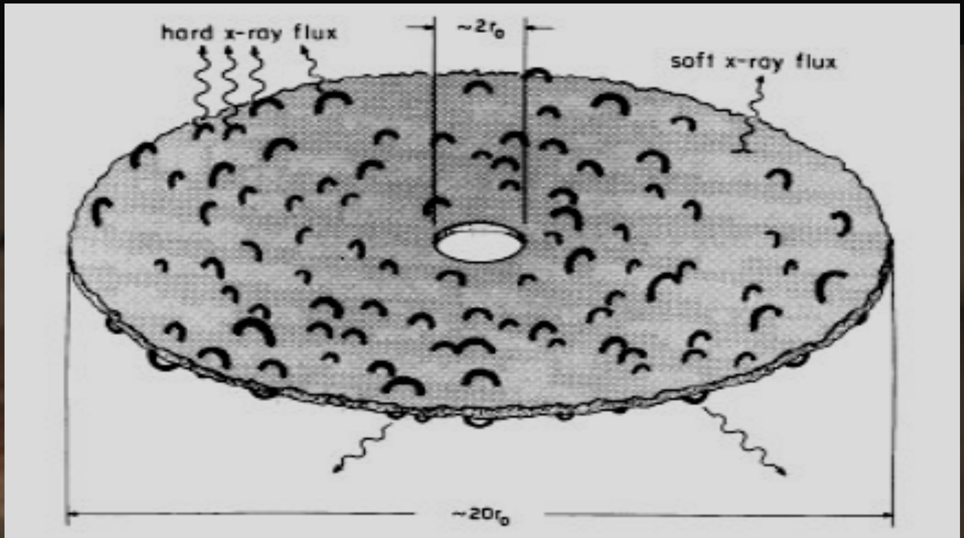


Thin accretion discs

- Geometrically thin, gravitational energy lost by radiation
- Keplerian differential rotation
  - disc supported by centrifugal force

$$\Omega^2(R)R = \frac{\mathcal{G}M_*}{R^2} \quad \Rightarrow \quad \Omega(R) = \frac{\sqrt{\mathcal{G}M_*}}{R^{3/2}}$$

  - disc scale height and sound speed: $H/R \ll 1, \quad c_s = \Omega\, H$
- Largely collisional
- Typical of
  - protoplanetary discs
  - protostellar discs
  - cataclysmic variables



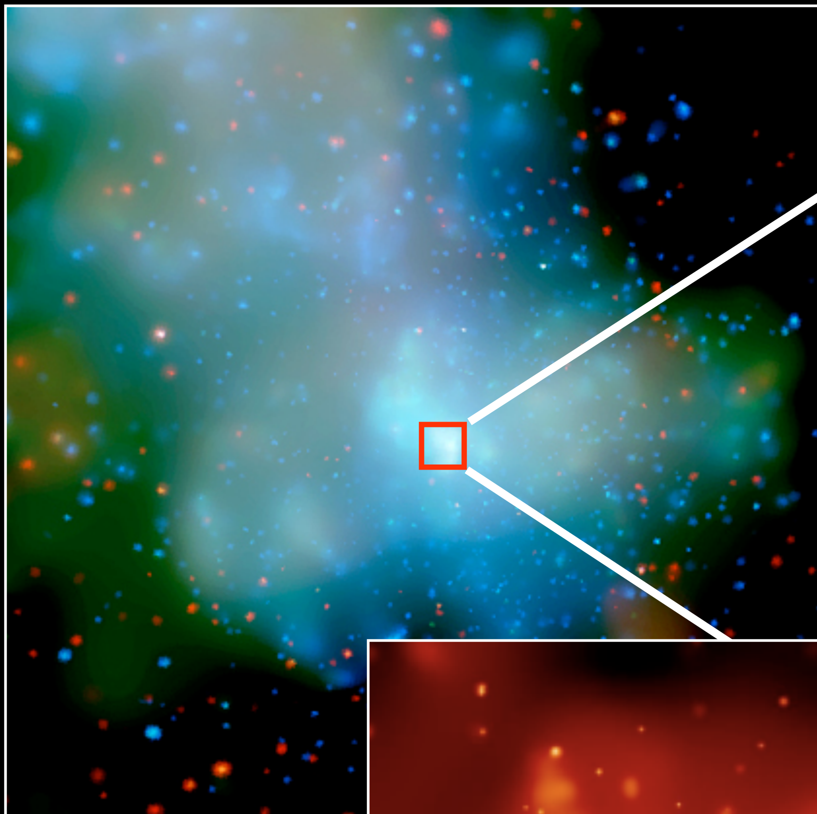
The galactic centre: Sgr A*

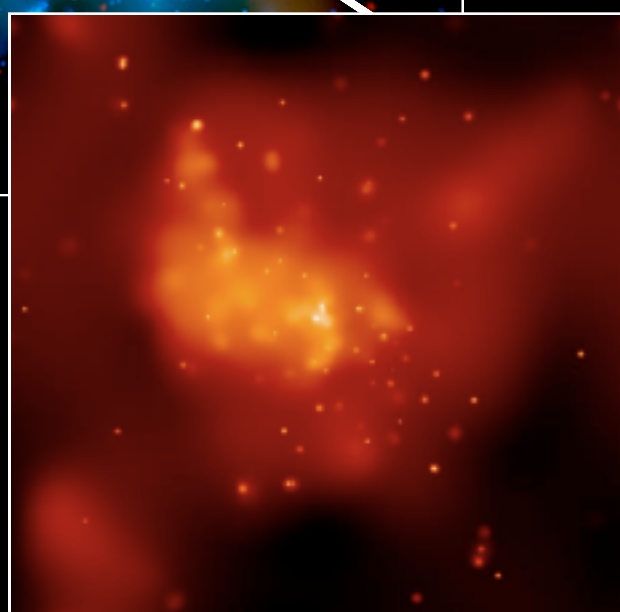
Evidence for a supermassive
$4.4 \times 10^6 M_\odot$ black hole

[Excellent new review by Genzel et al., Rev. Mod. Phys. 82, 3121 (2010)]

Black hole accretion in Sgr A*

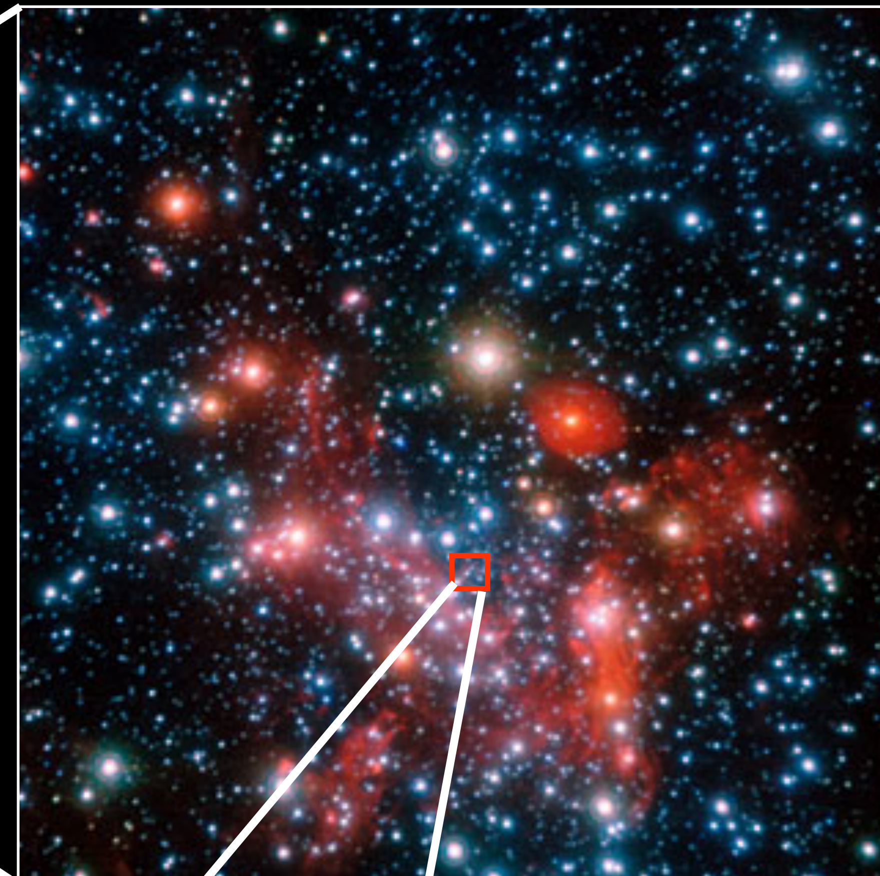
Chandra, X-ray observations





10^8 K plasma,
1-10 light year

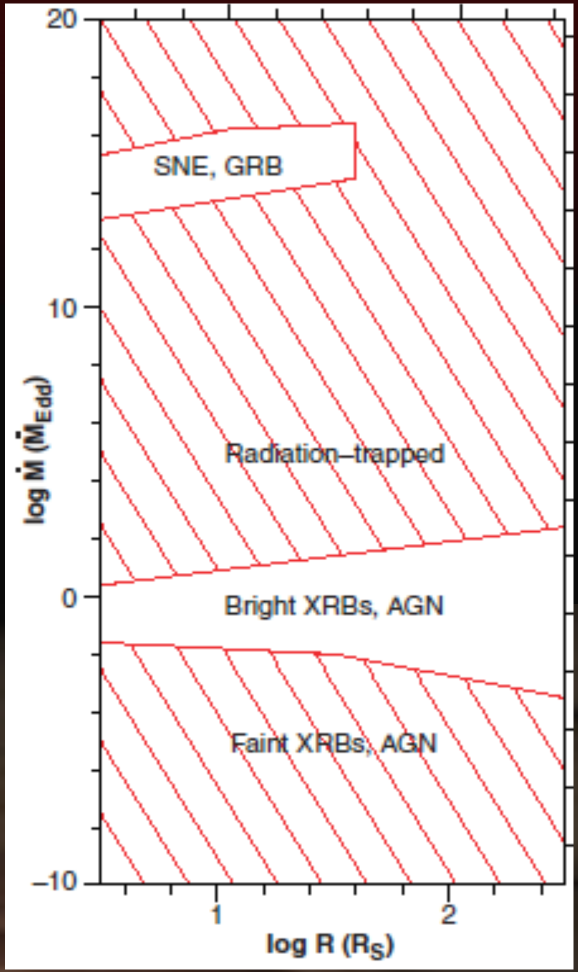
VLT, IR composite, 5 light-years



VLT,
~0.1 light year

AGNs and black hole accretion

- AGNs are luminous sources at the centre of galaxies
  - L up to 10^{13} $L_\odot$
    - Accretion onto supermassive black holes
  - But many AGNs are actually suprisingly faint
    - Sgr A* : $L \sim 10^{2\text{-}3}$ $L_\odot$!
      - Accretion disc not detected
  - Thin disc component cannot always explain spectral emission in X-rays
- A different accretion mode close to BH ?
  - Radiatively inefficient accretion flows (RIAFs)



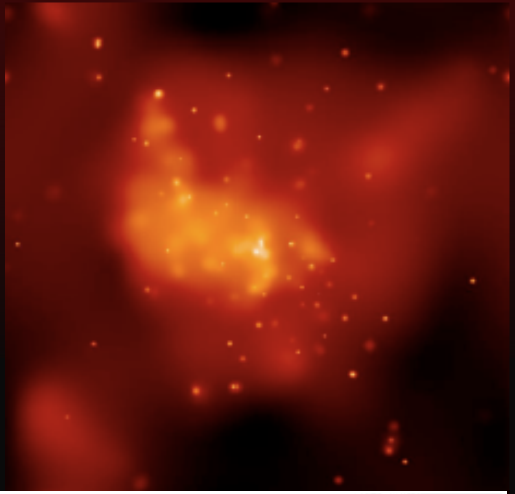
[Narayan & Quataert, Science 307, 77 (2005)]

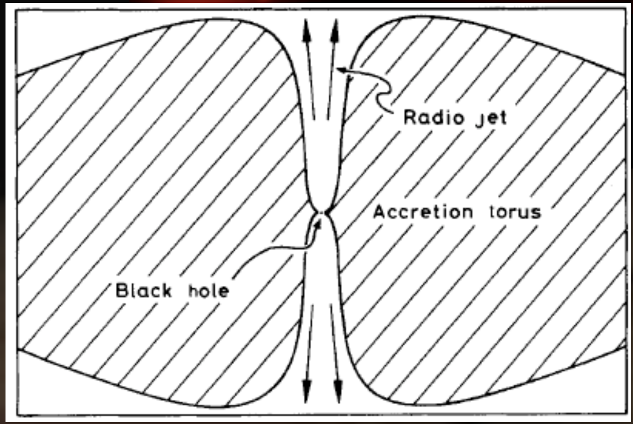
Hot accretion flows theory

- Gravitational energy not efficiently radiated: $\mathcal{L} \ll \mathcal{L}_E$
  - stored as thermal energy

$$kT_i \sim \mathcal{G} M_* m_i / R,\ H/R \sim 1$$

  - pressure supported, puffy structure with a non-Keplerian rotation curve
- Very hot, low density plasma
  - $T_i \sim 10^{11\text{-}12}$ K, $n \sim 10^{12}$ m^{-3}
- Low collisionality
  - 2-temperature plasma: $T_i > T_e$
  - ion m.f.p. ~ 10^{10} km ~ R_{gas}
  - e-i collision time ~ 10^3 y > accretion time ~ 10^2 y

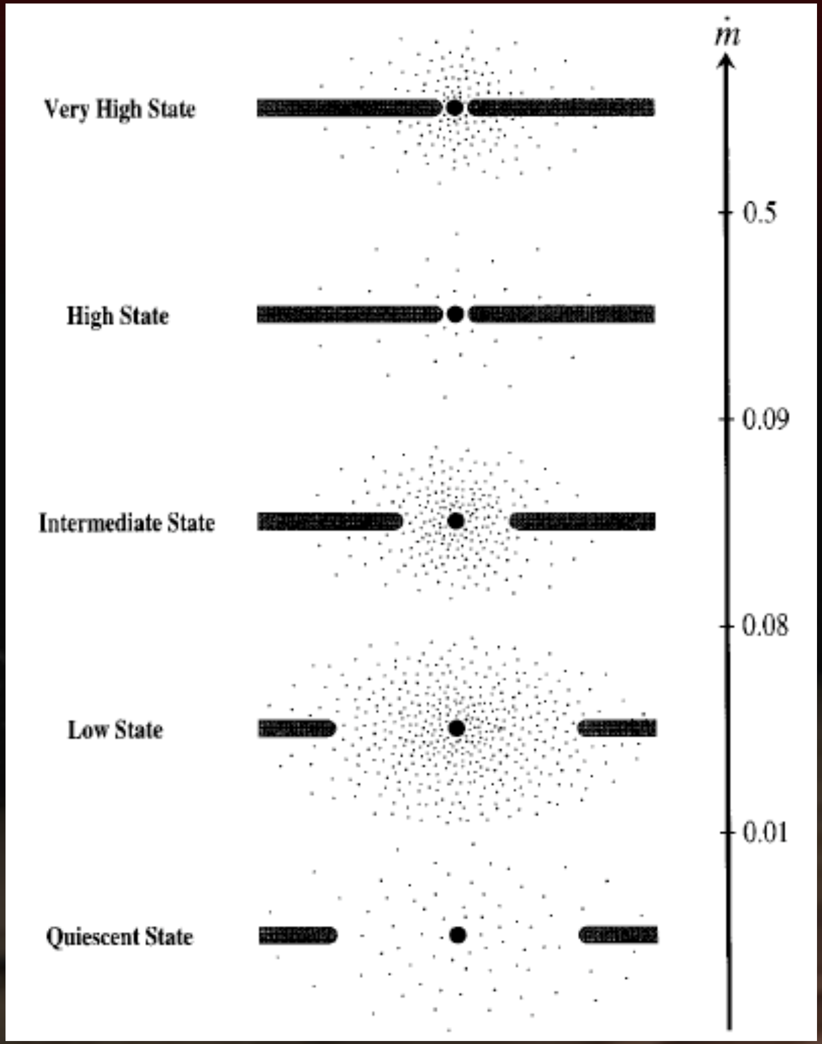


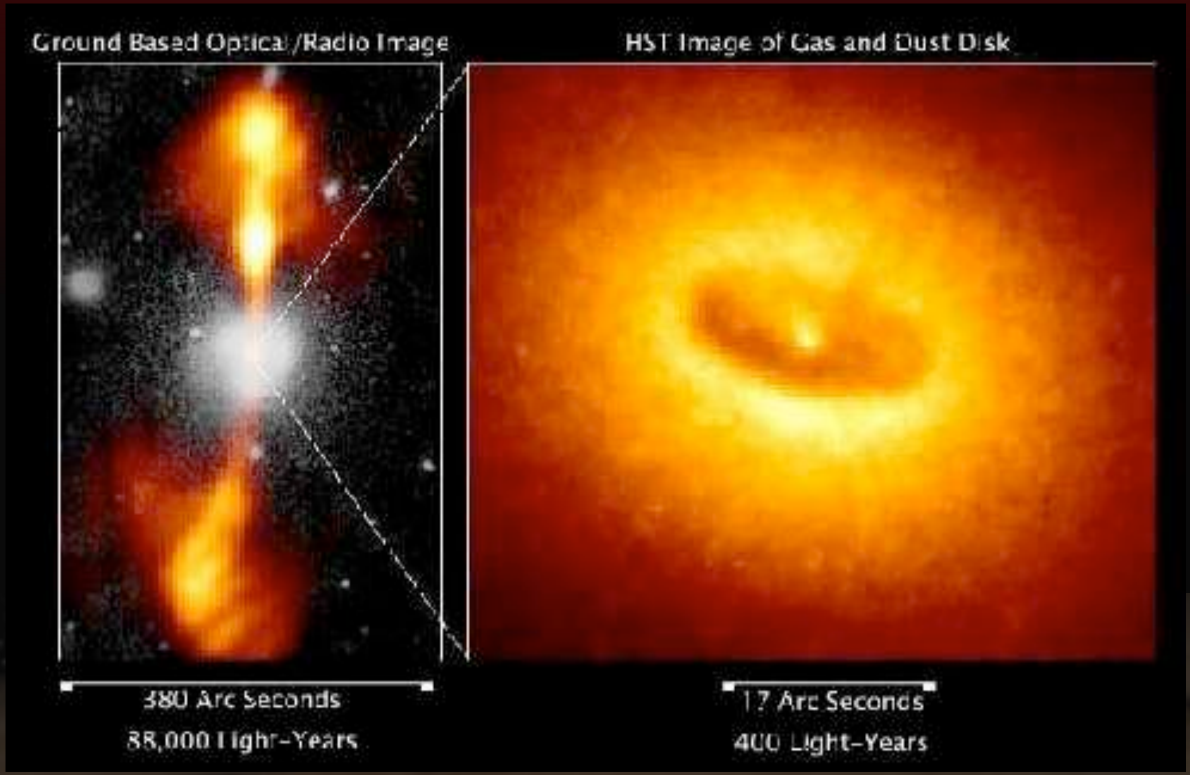


[e. g. Rees et al., Nat. 295, 17 (1982),
Narayan & Quataert, Science 307, 77 (2005)
Quataert, ASPC 224, 711 (2001)]

Hot accretion flows and thin discs

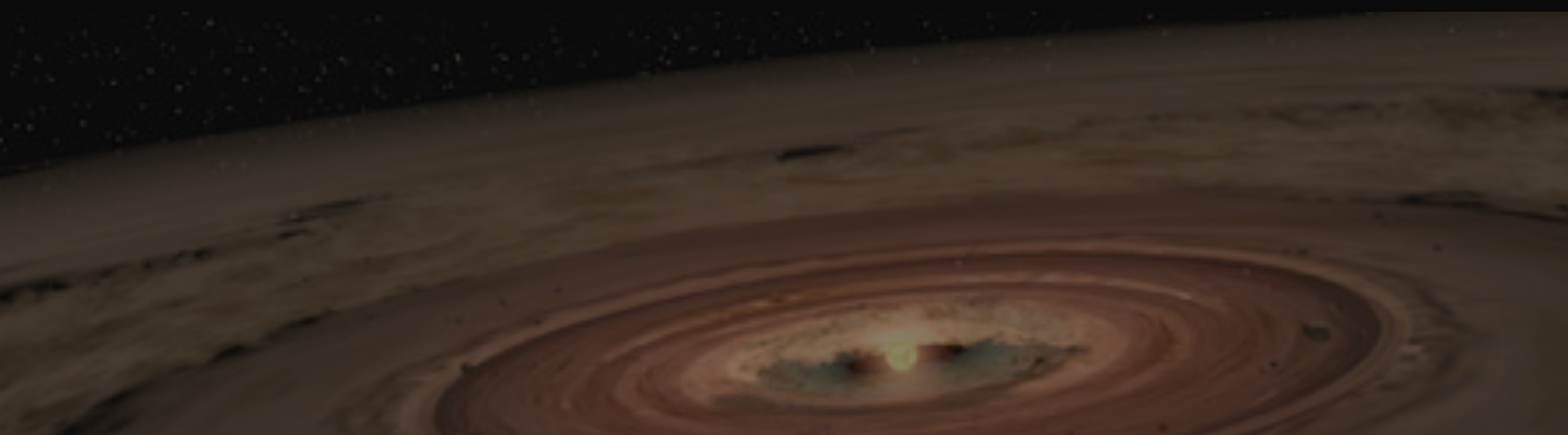
- Transition between different accretion regimes ?





[Esin et al., ApJ 489, 865 (1997), see reviews by Lasota, Phys. Rep. 311, 247 (1999), Dubus, EAS Pub. Series 7, 283 (2003)]

Accretion discs dynamics



The basics

- Consider two closely orbiting particles
  - masses $m_1,\, m_2 \,/\, M = m_1 + m_2$ is conserved
  - distances $R_1,\ R_2$, rotation rates $\Omega_1,\ \Omega_2$
  - angular momenta $L_1 = m_1 \ell_1,\ L_2 = m_2 \ell_2 \,/\, L = L_1 + L_2$ is conserved
  - energies $E_1 = m_1 \varepsilon_1,\ E_2 = m_2 \varepsilon_2$
- Allow for angular momentum and mass exchange
  - How does total energy vary ?

Note: $\frac{d}{dR}(\varepsilon - \ell\Omega) = -\ell \frac{d\,\Omega}{dR}$

$$dE = dE_1 + dE_2 = (\Omega_1 - \Omega_2)\, dL_1 + ([\varepsilon_1 - \ell_1\Omega_1] - [\varepsilon_2 - \ell_2\Omega_2])\, dm_1$$

- For $\frac{d\Omega}{dR} < 0$, energy is lowered if the innermost particle
  - gains mass: inwards mass transport = accretion
  - loses angular momentum: outwards angular momentum transport

[Lynden-Bell & Pringle, MNRAS 168, 603 (1974)]

Collisional transport estimate...

- Viscosity [Spitzer, 1962]

$$\nu = 1.4 \times 10^{11} \frac{T^{5/2}}{n \ln \Lambda_{ii}} \, \mathrm{m}^2 \, \mathrm{s}^{-1}$$

- For a typical Keplerian thin disc

$$T \sim 10^4 \, K, \;\; n \sim 10^{21} \;\; \mathrm{m}^{-3}, \;\; M_* \sim 1 \, M_\odot, \;\; R \sim 10^8 \, \mathrm{m}$$

$$c_s \sim 10 \, \mathrm{km} \, \mathrm{s}^{-1}, \; H/R \sim 10^{-2}$$

- Viscous evolution timescale

$$\tau_\nu = \frac{R^2}{\nu} \sim 10^9 \, \mathrm{yr}$$

- Way, way too long compared to luminosity variations...
  - Something else must be at work !

Angular momentum transport / extraction processes

- Many possibilities a priori...
  - Waves
  - Winds & Jets
  - Hydrodynamic turbulence (shear, convection, etc.)
  - MHD turbulence

[Papaloizou & Lin, Ann. Rev. Astron. Astrophys. 33, 505 (1995) Balbus, Ann. Rev. Astron. Astrophys. 41, 555 (2003)]

A complete talk could be given on any of these...

- Focus on the most likely candidate : turbulent transport
  - Disc Reynolds numbers (using previous estimates)

$$\mathrm{Re} = \frac{c_s H}{\nu} > 10^{10}$$

  - Surely, this must be turbulent...(but wait for a few more minutes)

Turbulent transport

- In practical applications / accretion disc models, the gory details are hidden in a turbulent "α viscosity" prescription

$$\nu_T \sim \alpha c_s H$$

[Shakura & Sunyaev, A&A 24, 337 (1973)]

"The main difficulty in applying the above considerations to galaxies is the calculation of the frictional mechanism. In application to the Galaxy molecular viscosity in the gas in negligibly weak, so any evolution of this type will be due to turbulence eddy viscosity or to some form of magnetic friction. The theory of both these is still in a lamentable state so only the crudest estimates can be made."

[Lynden-Bell & Pringle, MNRAS 168, 603 (1974)]

- So, listen to the pioneers and try to address the problem...
  - How does turbulence originate in discs ? (is α non-zero ?)
  - How efficient is it at transporting angular momentum ? (how big is α)
  - Is an α prescription correct ?

Generic dynamical fluid equations

- Mass conservation equation

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho\,\nabla\cdot\mathbf{v}$$

- Momentum equation

$$\rho\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\nabla\left(p + \frac{B^2}{8\pi}\right) - \rho\,\nabla\Phi + \frac{\mathbf{B}\cdot\nabla\ \mathbf{B}}{4\pi} + \mu\left(\nabla^2\mathbf{v} + \frac{1}{3}\nabla\ (\nabla\cdot\mathbf{v})\right)$$

- Induction equation + solenoidality condition

$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} = \mathbf{B}\cdot\nabla\,\mathbf{v} - \mathbf{B}(\nabla\cdot\mathbf{v}) - \nabla\times(\eta\nabla\times\mathbf{B}) \qquad \nabla\cdot\mathbf{B} = 0$$

- Entropy equation

$$\frac{p}{\gamma-1}\frac{\mathrm{d}\ln p\,\rho^{-\gamma}}{\mathrm{d}t} = Q^+ - Q^-$$

  - Q^+: sources (ohmic and viscous dissipation)
  - Q^-: sinks (e.g. radiative losses)

Angular momentum transport and stress tensors

[full derivation: Balbus & Hawley, Rev. Mod. Phys. 70, 1 (1998)]

- Vertical angular momentum conservation
  - assume axisymmetric gravitational potential
  - neglect microscopic viscosity

$$\frac{\partial(\rho\, R\, v_\phi)}{\partial t} + \nabla \cdot \left[R \left(\rho\, v_\phi \mathbf{v} - \frac{B_\phi\, \mathbf{B}}{4\pi} + \left(P + \frac{B^2}{8\pi} \right) \mathbf{e}_\phi \right) \right] = 0$$

- Now, consider deviations from the background flow

$$\mathbf{u} = \mathbf{v} - R\, \Omega(R)\, \mathbf{e}_\phi$$

  - Radial flux of angular momentum

$$\mathcal{F}_R = R \left(\rho\, u_R \left(R\, \Omega(R) + u_\phi \right) - \frac{B_R\, B_\phi}{4\pi} \right)$$

  - Fluctuating part is proportional to

$$\rho\, u_R\, u_\phi - \frac{B_R\, B_\phi}{4\pi}$$

Stress tensor and accretion in alpha-discs

[full derivation: Balbus & Hawley, Rev. Mod. Phys. 70, 1 (1998)]

- Averaging for basic 1D alpha-disc model

$$\langle X \rangle_\rho = \frac{1}{2\pi\Sigma\Delta R}\int_{-\infty}^{+\infty}\int_0^{2\pi}\int_{\Delta R} \rho\, X\, dr\, d\phi\, dz \qquad \Sigma = \int_{-\infty}^{+\infty} \rho\, dz$$

  - Averaged stress tensor is

$$W_{R\phi} = \langle u_R\, u_\phi - u_{AR}\, u_{A\phi} \rangle \quad \text{with} \quad \mathbf{u}_A = \frac{\mathbf{B}}{\sqrt{4\pi\rho}}$$

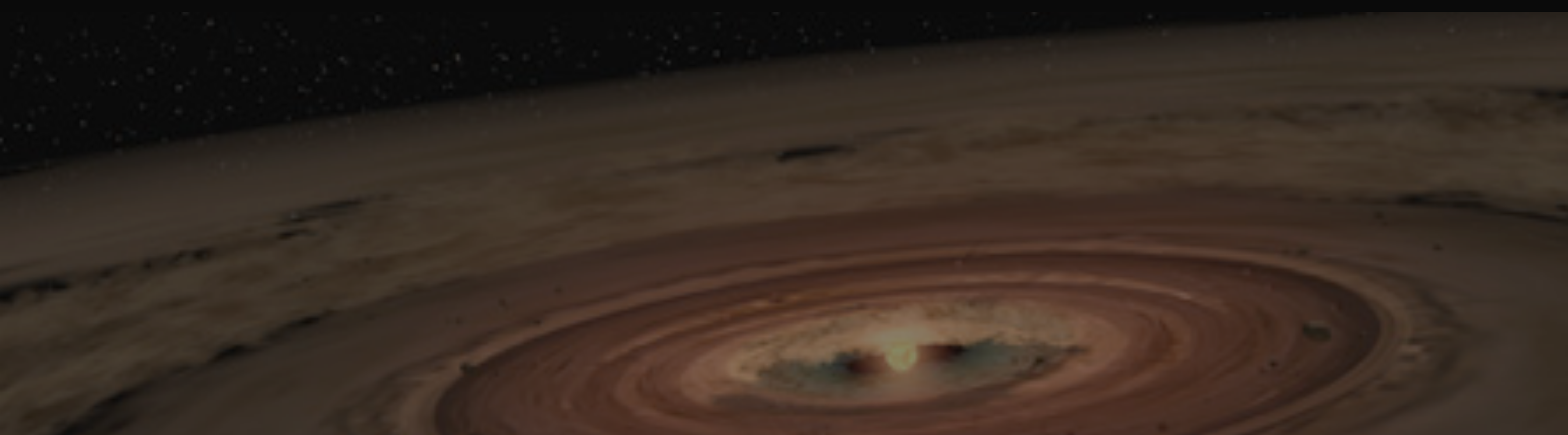
- The alpha ansatz : $\alpha = W_{R\phi}/c_s^2$

  - For accretion to take place for $\frac{d\Omega}{dR} < 0$, this quantity must be positive

In the end, a good part of the game (but not the only part of it) is to understand the behaviour of $W_{R\phi}$ from theory and numerical simulations

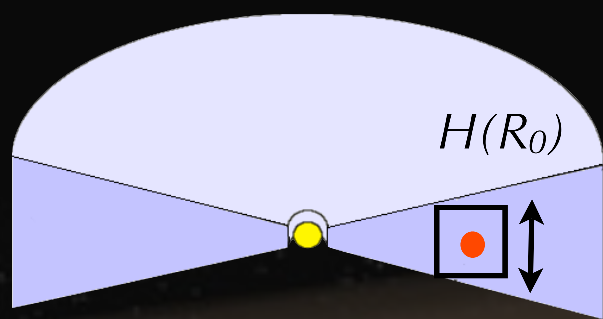
The origins of turbulence…

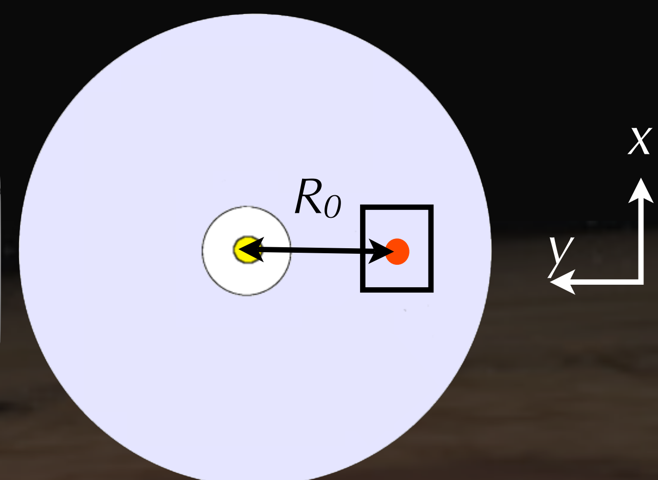
Linear stability of differentially rotating fluid flows



Differential rotation on a cylinder

- Locally, differential rotation on a cylinder combines
  - pure shear (velocity gradient): $S y \, \mathbf{e}_x$
  - pure rotation: $\Omega \, \mathbf{e}_z$
- For a generic differential rotation profile $\Omega(R) \sim R^{-q}$
  - Around $R=R_0$, $\Omega = \Omega(R_0) = C^{st}, \;\; S = q\Omega$





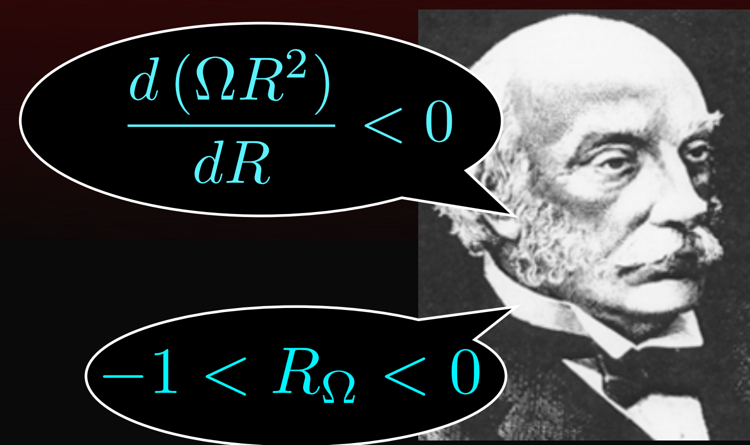
  - Rotation number

$$R_\Omega = -\frac{2\Omega}{S} = -\frac{2}{q}$$

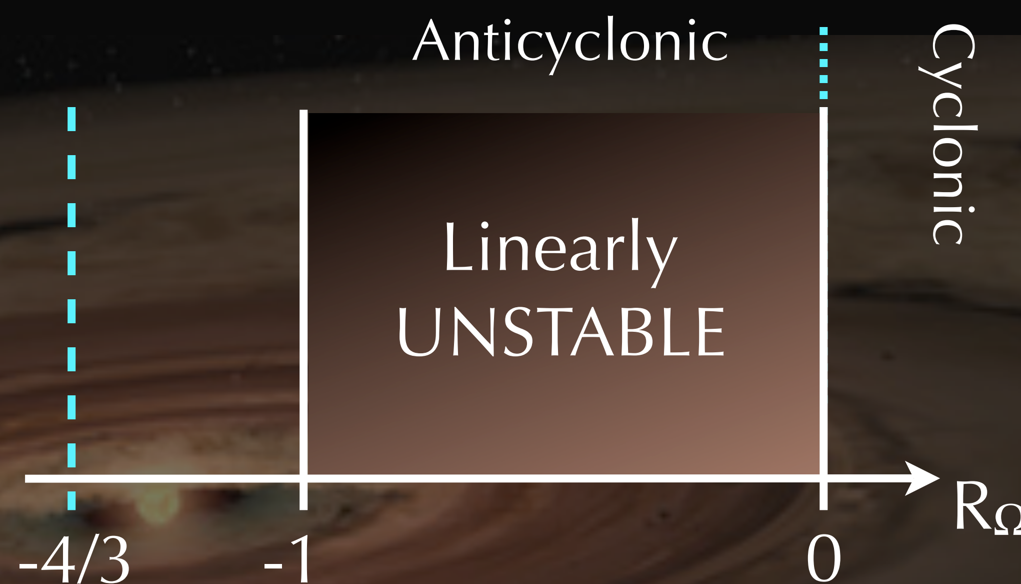
  - For the Keplerian flow, $q=3/2$, $S=3/2\ \mathbf{\Omega}$, $R_\Omega=-4/3$

Linear hydrodynamic stability

- Rayleigh criterion: centrifugal instability (axisymmetric)
  - requires a radially decreasing angular momentum profile
  - Taylor vortices in Taylor-Couette flow

- Important cases
  - No rotation (R_Ω=0, infinite q)
  - C^{st} ang. momentum (R_Ω=-1, q=2)
  - Keplerian flow is stable (R_Ω=-4/3, q=3/2)



MHD: the magneto-rotational instability

- All you need for linear instability is a weak magnetic field !
  - Discovered initially by Chandrasekhar & Velikhov (1953,1959)
  - Revisited for Keplerian discs by Balbus & Hawley [ApJ, 376, 214 (1991]

$$\frac{d\Omega^2}{dR} < 0$$

$$-\infty < R_\Omega < 0$$

- Basic requirements for MRI
  - Radially decreasing angular velocity profile
  - Wave-vector component parallel to the local magnetic field

$$\mathbf{k} \cdot \mathbf{B} \neq 0$$

MRI (ideal MHD, simple vertical field)

- Linear dispersion relation

[once again, see Balbus & Hawley, Rev. Mod. Phys. 70, 1 (1998)]

  - Expand perturbations as plane waves $\propto e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}$

$$\omega^4 - (\kappa^2 + 2\,(\mathbf{k}\cdot\mathbf{u}_A)^2)\,(\mathbf{k}\cdot\mathbf{u}_A)^2 + (\mathbf{k}\cdot\mathbf{u}_A)^2((\mathbf{k}\cdot\mathbf{u}_A)^2 - 2\,\Omega\,S) = 0$$

with $\kappa^2 = 2\,\Omega\,(2\,\Omega - S)$

- Instability condition

$$2\,\Omega\,S > (\mathbf{k}\cdot\mathbf{u}_A)^2 \quad \text{or} \quad -\frac{d\Omega^2}{d\ln R} > (\mathbf{k}\cdot\mathbf{u}_A)^2$$

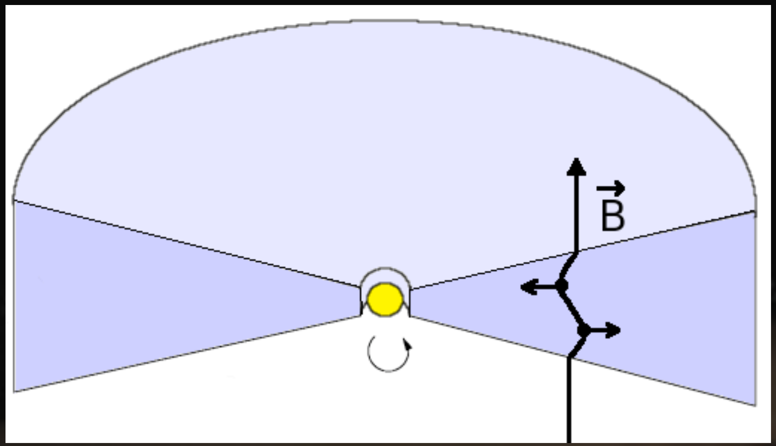
  - k can formally be very small, so instability criterion is just $\frac{d\Omega^2}{d\ln R} < 0$
  - Actually, in a real disc, the minimum is $k \sim \pi/H \sim \pi\Omega/c_s$
    - Instability limited to $\beta = \frac{c_s^2}{u_A^2} > \frac{\pi^2}{2\,q}$: won't disrupt your local tokamak !

- Keplerian discs are MRI unstable

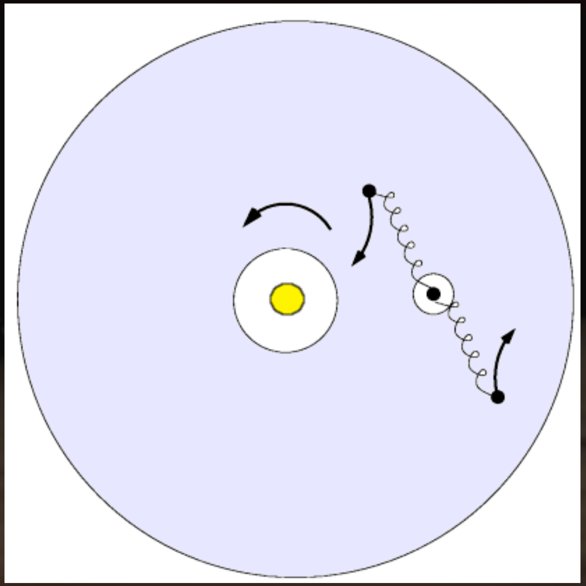
  - Maximum growth rate $\gamma_{\max} = 3/4\,\Omega$ is very large

MRI physics

- Magnetic tension allows angular momentum exchange
  - innermost particle decelerates & falls: rotates faster on a shorter orbit
  - outermost particle accelerates and expands to a longer orbit
  - tension increases: innermost fluid particle decelerates even more...



(This again is a sketch by Geoffroy !)

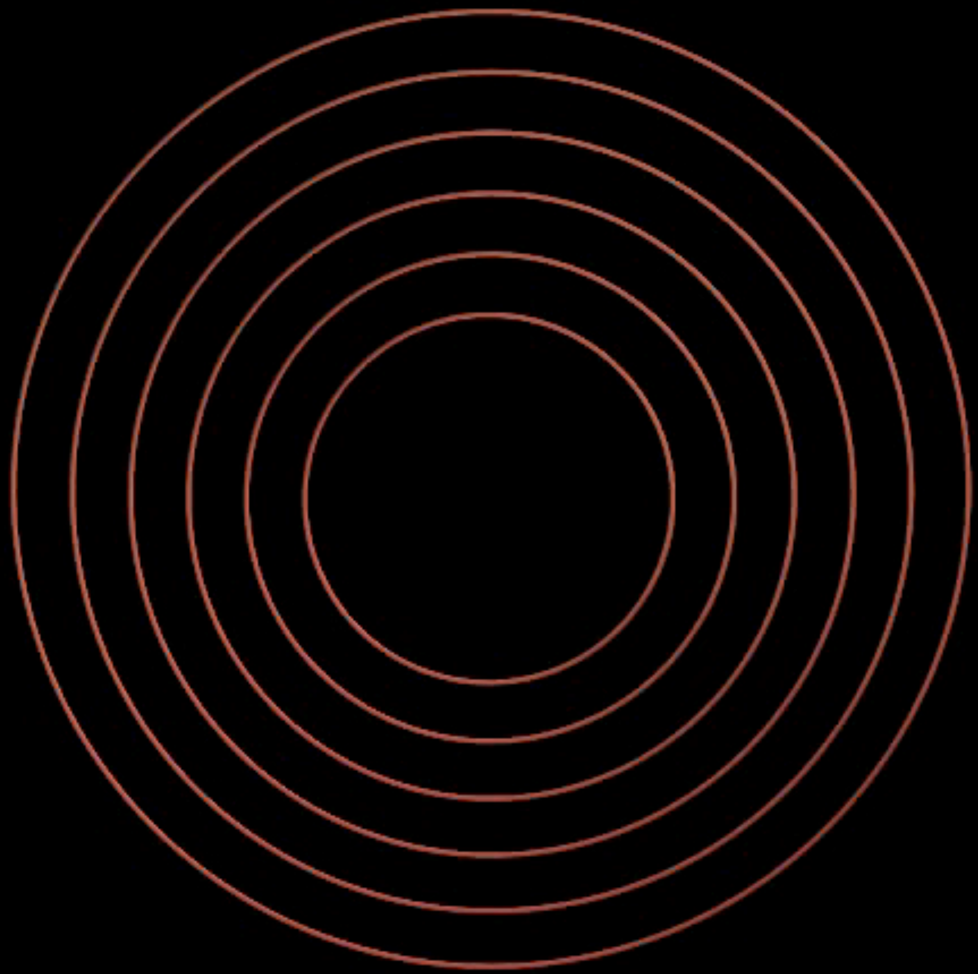


- Ultimately, energy is extracted from the shear
  - magnetic tension only plays a mediating role

Illustration of MRI of a toroidal field

- Toroidal MRI is non-axisymmetric: consider m=1 pertubations

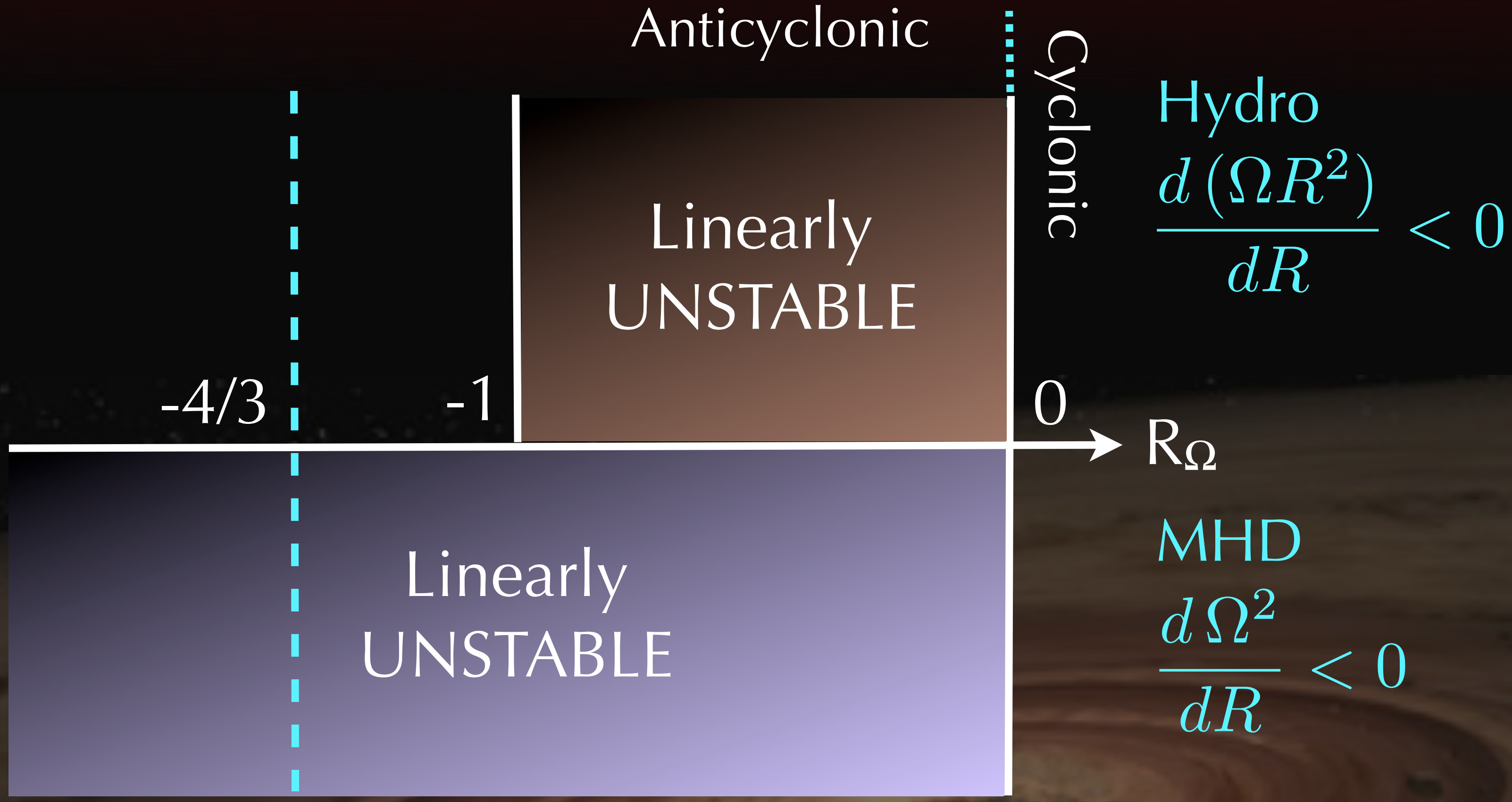
[Ogilvie & Pringle, MNRAS 279, 152 (1996) - Terquem & Papaloizou, MNRAS 279, 767 (1996)]



Note: this is a toy simulation, not a real solution

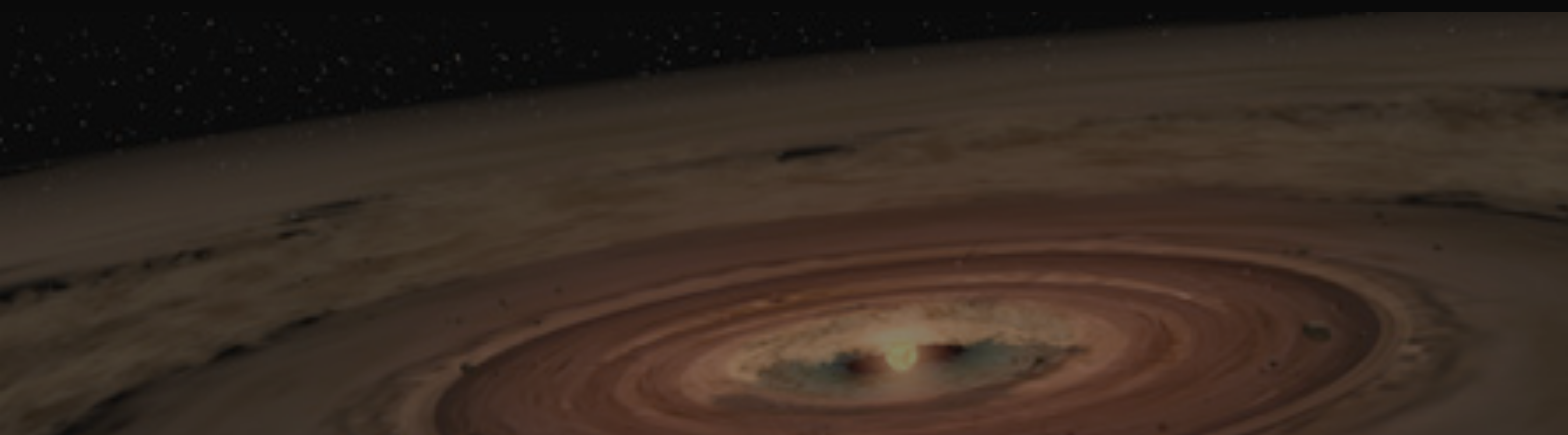
Summary: linear stability of rotating shear flows

- In the Keplerian regime $R_\Omega = -4/3$ ($q=3/2$)
  - No centrifugal instability
  - The MRI is present…if the disc is magnetized !



Before we go nonlinear, we need to think about...

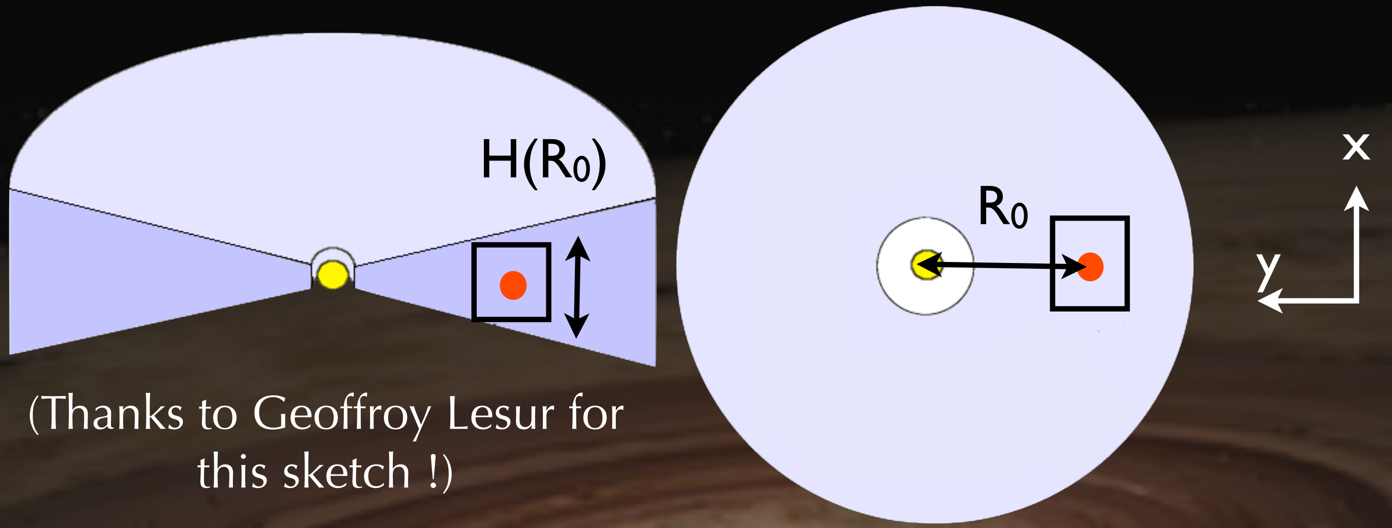
Local approaches to rotating shear fows



The shearing sheet

[Goldreich and Lynden-Bell, MNRAS 130, 125 (1965)]

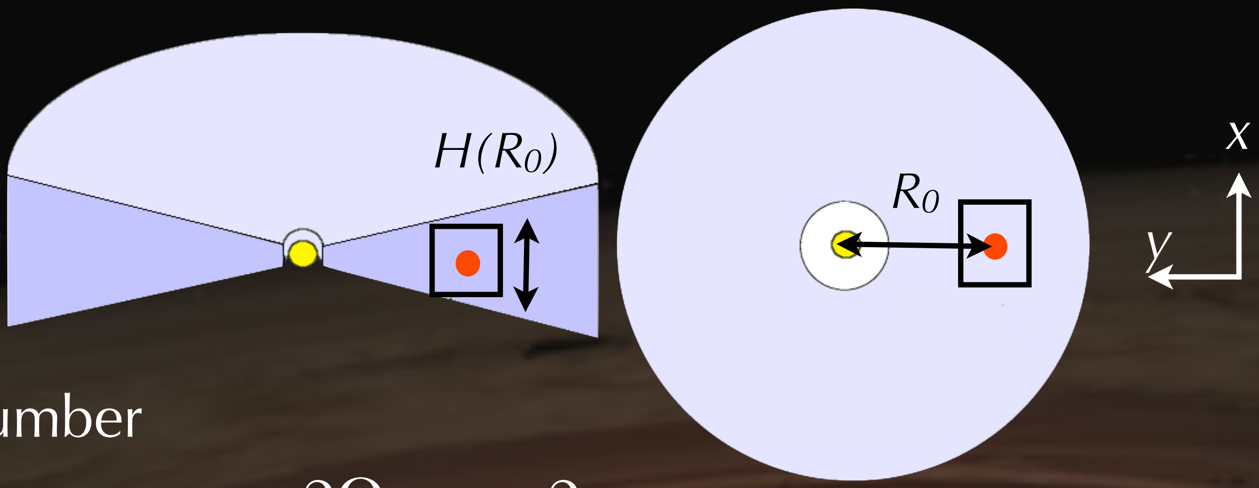
- Discs are complicated objects
  - curved geometry
  - lots of physical processes not essential to instabilities and turbulence
- Devise simplified frameworks
  - local cartesian approximation: $x,y,z << R$



(Thanks to Geoffroy Lesur for this sketch !)

Differential rotation on a cylinder again

- Locally, differential rotation on a cylinder combines
  - pure shear (velocity gradient): $Sy\,\mathbf{e}_x$
  - pure rotation: $\Omega\,\mathbf{e}_z$
- For a generic differential rotation profile $\Omega(R) \sim R^{-q}$
  - Around R=R_0, $\Omega = \Omega(R_0) = C^{st},\ S = q\Omega$



  - Rotation number

$$R_\Omega = -\frac{2\Omega}{S} = -\frac{2}{q}$$

Incompressible shearing sheet equations

- In the rotating frame, solve for

$$\begin{aligned}\partial_t \mathbf{u} + Sy\partial_x \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} &= -\nabla p - 2\Omega u_x \mathbf{e_y} + (2\Omega - S)u_y \mathbf{e_x} \\ &\quad +\mathbf{B}\cdot\nabla\mathbf{B} + \nu\Delta\mathbf{u} \\ \partial_t \mathbf{B} + Sy\partial_x \mathbf{B} &= SB_y \mathbf{e_x} + \nabla\times(\mathbf{u}\times\mathbf{B}) + \eta\Delta\mathbf{B}\end{aligned}$$

$$\nabla\cdot\mathbf{u} = 0, \quad \nabla\cdot\mathbf{B} = 0$$

- Important dimensionless numbers
  - Rotation number $R_\Omega = -\frac{2\Omega}{S} = -\frac{2}{q}$
  - Standard MHD things

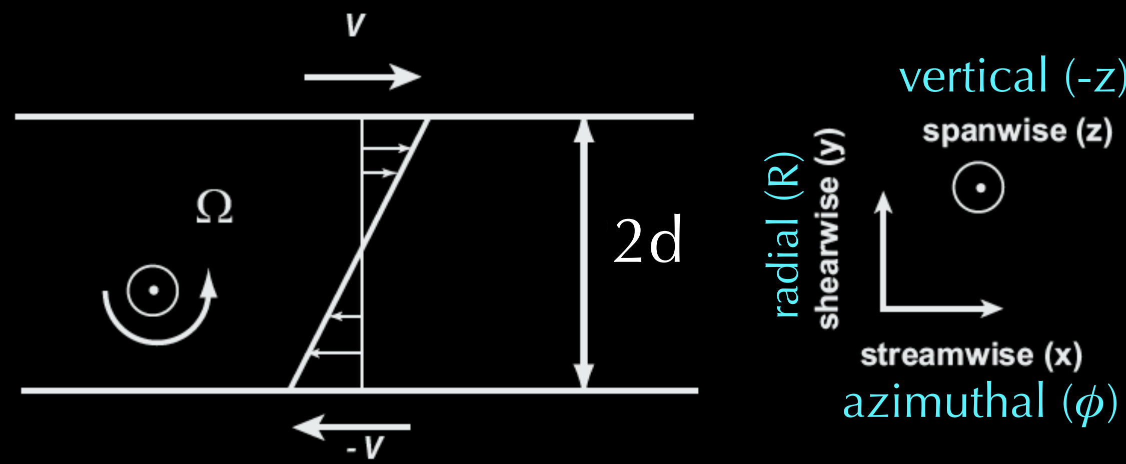
$$\mathrm{Re} \sim \frac{c_s H}{\nu} \sim \frac{|S|L^2}{\nu} \qquad \mathrm{Rm} \sim \frac{c_s H}{\eta} \sim \frac{|S|L^2}{\eta} \qquad \mathrm{Pm} = \frac{\nu}{\eta}$$

Rotating plane Couette flow

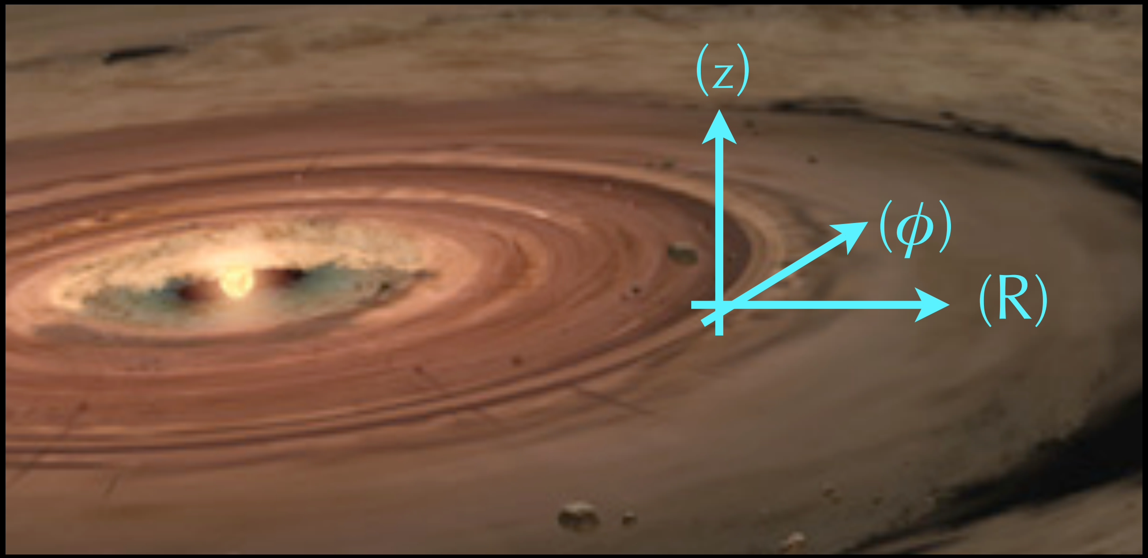
- Simplest available rotating shear flow

$$\mathrm{Re} = \frac{|S|d^2}{\nu}$$

$$\mathrm{Rm} = \frac{|S|d^2}{\eta}$$



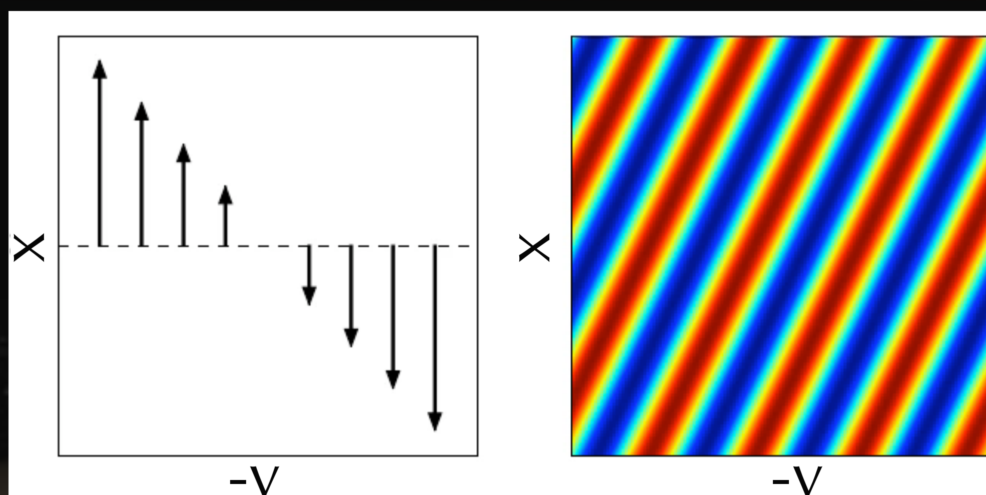
- Wall-bounded flow
  - Non-axisymmetric global modes
  - Same linear stability as shearing sheet
- But disks don't have walls !

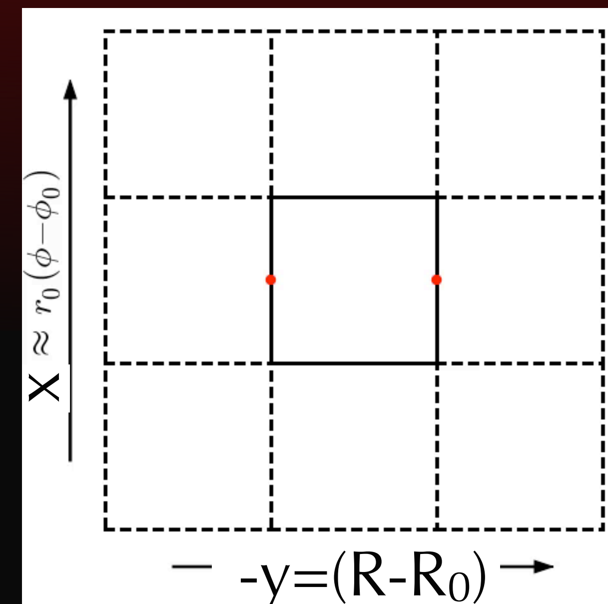


Local numerical approach: the shearing box

- Assume background shear flow $\mathbf{U}_x = Sy\,\mathbf{e}_x$
- Shearing-periodic boundary conditions
  - Plane waves get sheared according to

$$\mathbf{k}(t) = k_x\mathbf{e_x} + (k_y^{(0)} - Stk_x)\mathbf{e_y} + k_z\mathbf{e_z}$$



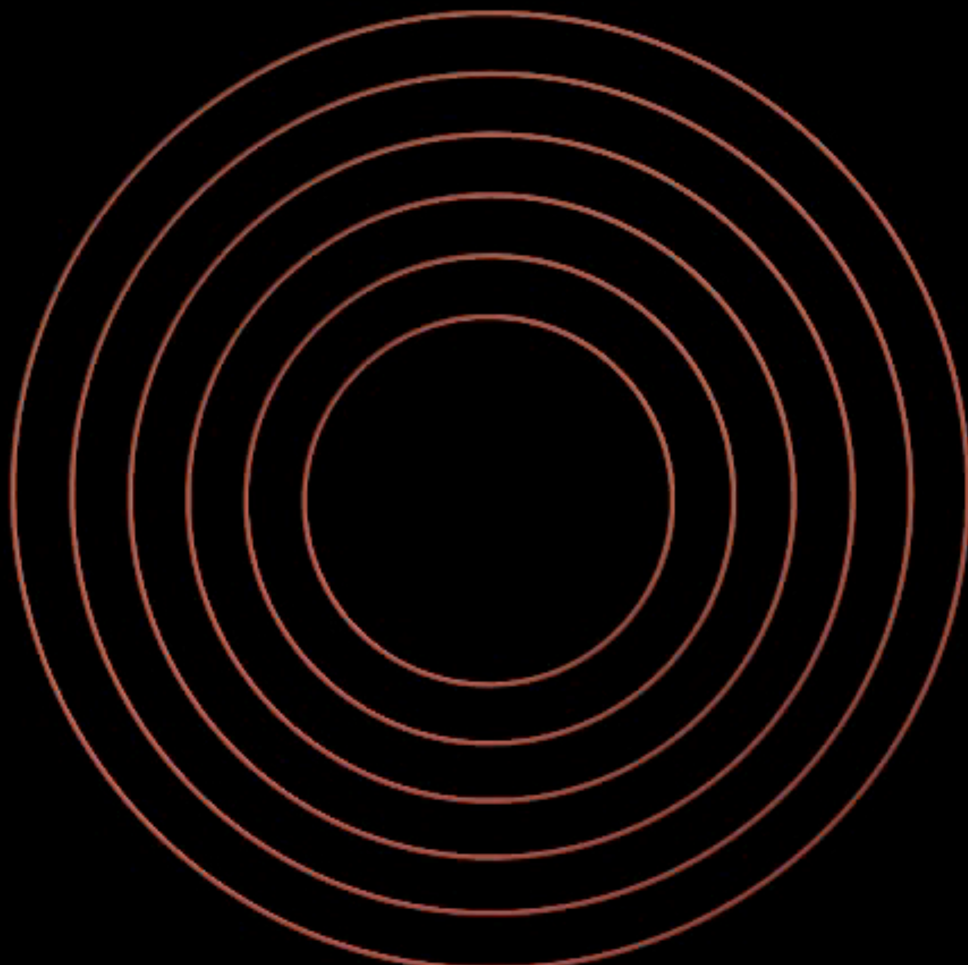


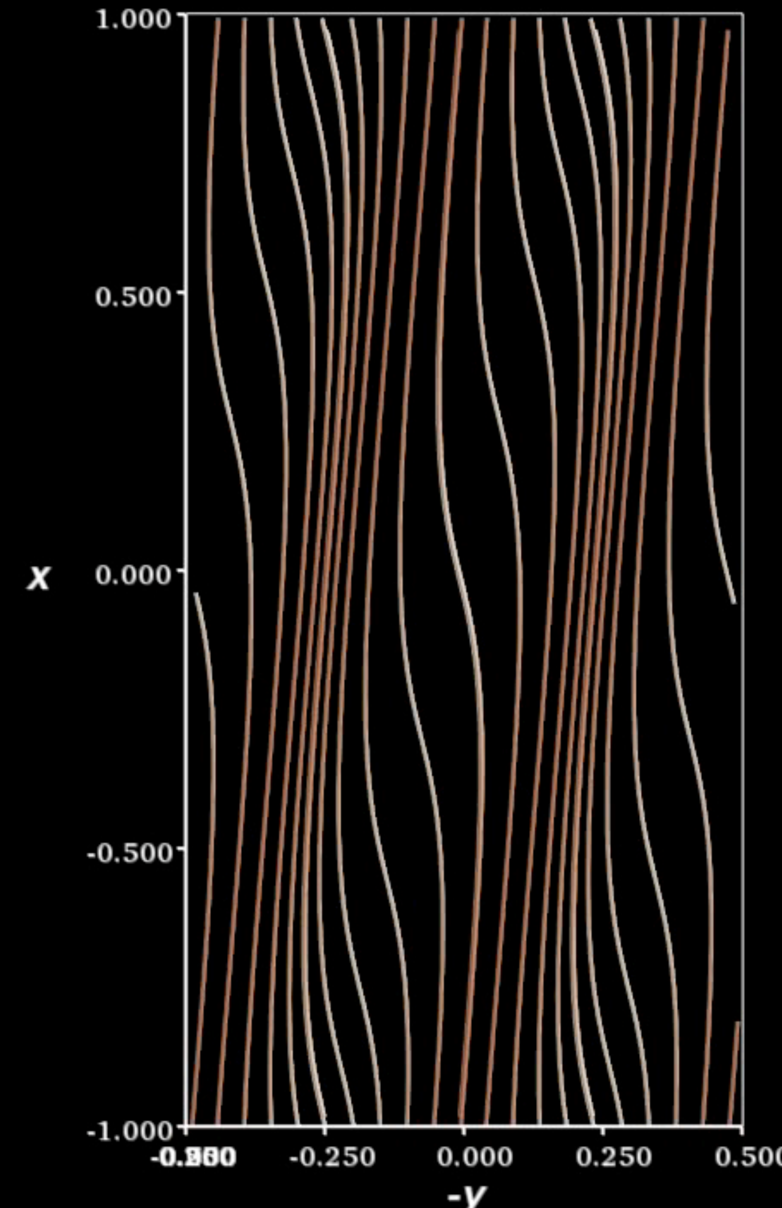
Courtesy G. Lesur
& T. Heinemann

  - Fields can be represented spectrally in a sheared Lagrangian frame ($x'=x-Syt$)

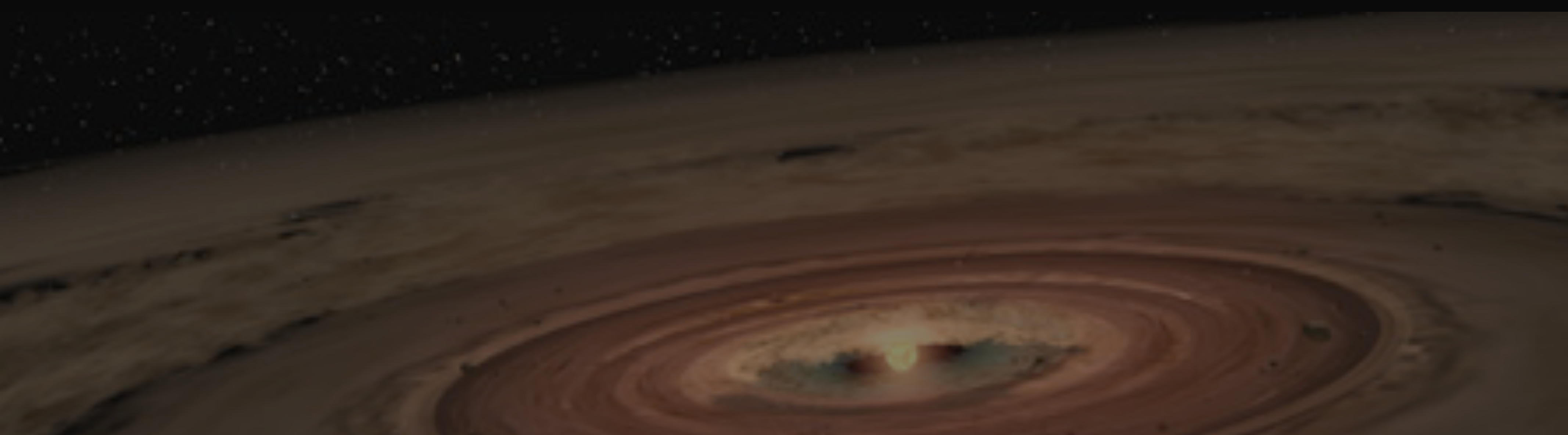
$$\mathbf{Q}(x,y,z,t) = \sum_{\mathbf{k}} \widehat{\mathbf{Q}}_{\mathbf{k}}(t)\exp\left[\exp(i\mathbf{k}(t)\cdot\mathbf{x})\right]$$

Example : toroidal MRI in the shearing box





Nonlinear hydrodynamic stability in fluid discs (short version)

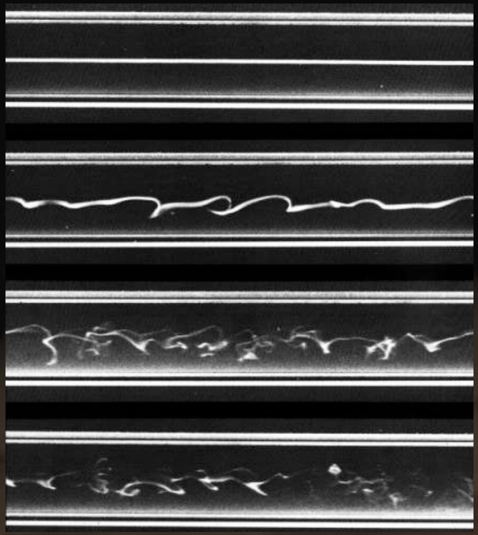


A long-standing question

- Is hydrodynamic transport possible and efficient in discs ?
  - Linear instability in "thick disks", but only for $R_\Omega > -1.15$ ($q > \sqrt{3}$) [Papaloizou & Pringle, MNRAS 208, 721 (1984)]
  - Strato-rotational instability [Dubrulle et al., A&A 429, 1 (2005)]
    - several authors have argued that it is a global instability requiring walls [Brandenburg & Dintrans, A&A 450, 437, 2006 - Geoffroy Lesur's PhD (2007)]
  - Thermal convection
    - 1990's numerics indicated inwards transport [Kley et al., ApJ 416, 679 (1993) Cabot, ApJ 465, 874 (1996), Stone & Balbus, ApJ 464, 364 (1996)]
    - recently revisited at high resolution: outwards but weak transport [Lesur, A&A (2010)]
  - Transport by vortices, subcritical baroclinic instabilities
    - nothing really convincing so far that could rival MRI
- But, given $Re > 10^{10}$ in discs...
  - subcritical shear turbulence has long been the #1 suspect

Hydro transition in non-rotating shear flows

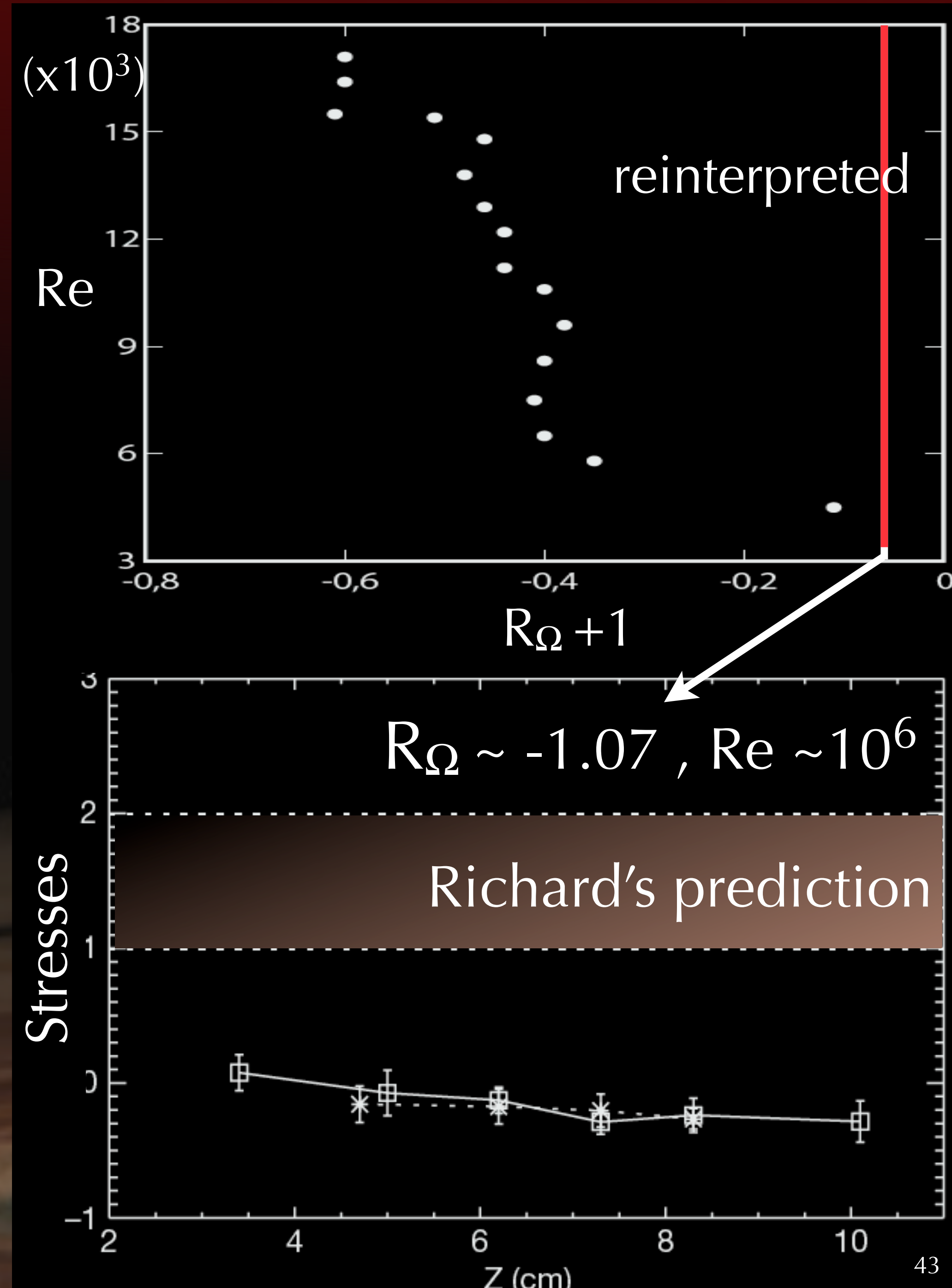
- Many standard non-rotating shear flows are
  - linearly stable
  - but nonlinearly unstable
- For instance, consider turbulence in pipe flow
  - A one century-old problem ! (O. Reynolds, 1883)



- So maybe Keplerian discs behave in the same way ?

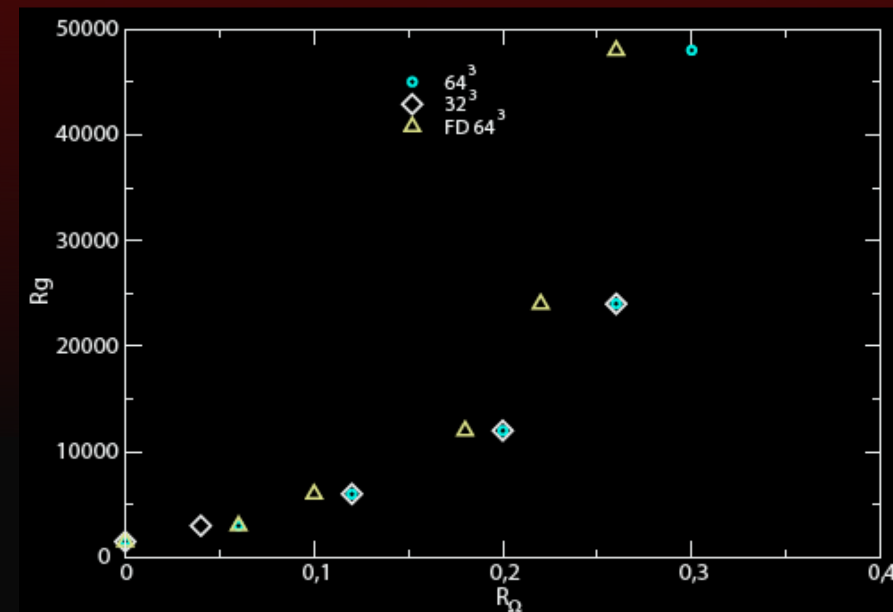
Experiments in Taylor-Couette & rotating Couette

- Richard & Zahn
  - Wendt & Taylor experiments
    [A&A 347, 734 (1999)
    New results (2001)]
  - Turbulence at $R_\Omega < -1$ for Re ~10^4
- Tillmark & Alfredsson (1996)
  - Turbulence dies at $R_\Omega \sim -1$
- Ji et al. [Nature 444, 343 (2006)]
  - Nothing up to Re ~10^6
  - Minimize stresses at end caps
    - Avoids Ekman circulation

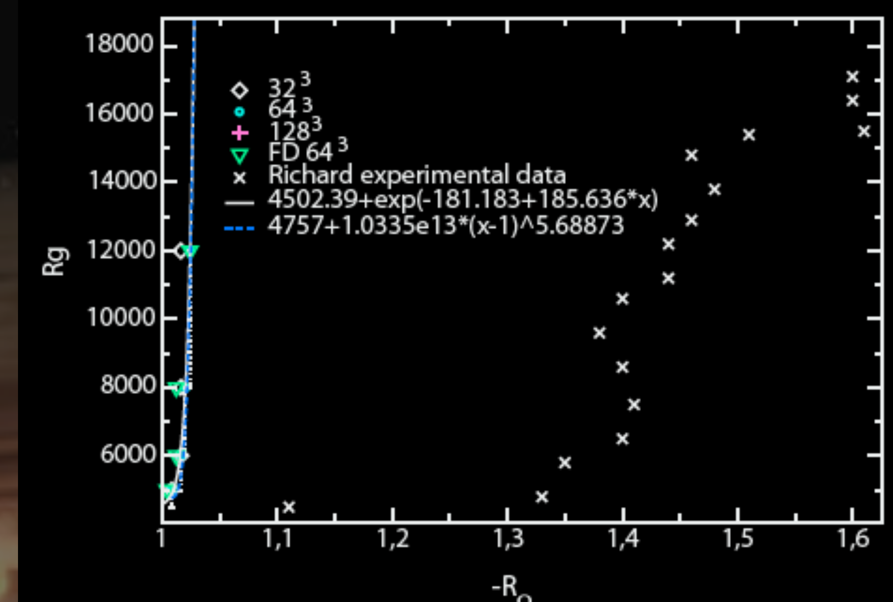


...and numerical results

- Hawley et al. [ApJ 518, 394 (1999)]
  - Shearing box simulations
  - Turbulence dies out at $R_\Omega = -1.03$
  - Criticized by Longaretti [ApJ 576, 587 (2002)]
    - Resolution & dissipation issues...



- Lesur & Longaretti [A&A 444, 25 (2005)]
  - Spectral & FD simulations
    - Couette & shearing box
  - Turbulence dies out at $R_\Omega = -1.03$
  - Nothing in Couette for $R_\Omega < -1$



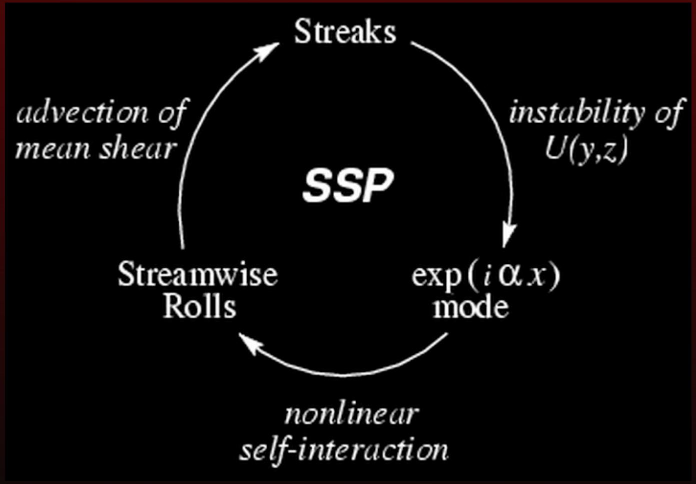
Back to zero rotation: the self-sustaining process

- Streamwise-averaged equations

$$\overline{\mathbf{u}}_{\mathrm{p}} = \nabla \times (\psi \mathbf{e}_x), \;\; \omega_x = -\Delta\psi$$

$$\frac{\partial \overline{u}_x}{\partial t} = -\overline{u}_y - \mathbf{e}_x \cdot \left(\overline{\mathbf{u}\cdot\nabla\mathbf{u}}\right) + \frac{1}{\mathrm{Re}}\Delta\,\overline{u}_x$$

$$\frac{\partial \omega_x}{\partial t} - \frac{\partial(\psi,\omega)}{\partial(y,z)} = -\mathbf{e}_x\cdot\nabla\times\;\left(\overline{\mathbf{u}'\cdot\nabla\mathbf{u}'}\right) + \frac{1}{\mathrm{Re}}\Delta\,\omega_x$$

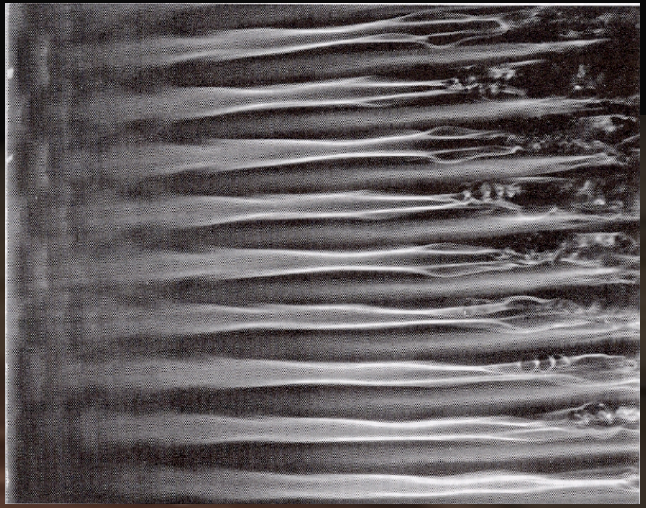


- Key element: the lift-up effect

  [Ellingsen & Palm, Phys .Fluids 1975 - Landahl, JFM 1980]

  - Transient algebraic growth of streaks
  - Streamwise-independent linear process

$$\partial_t \overline{u}_x + \overline{u}_y = 0 \qquad \Rightarrow \qquad \overline{u}_x \sim \overline{u}_y t$$



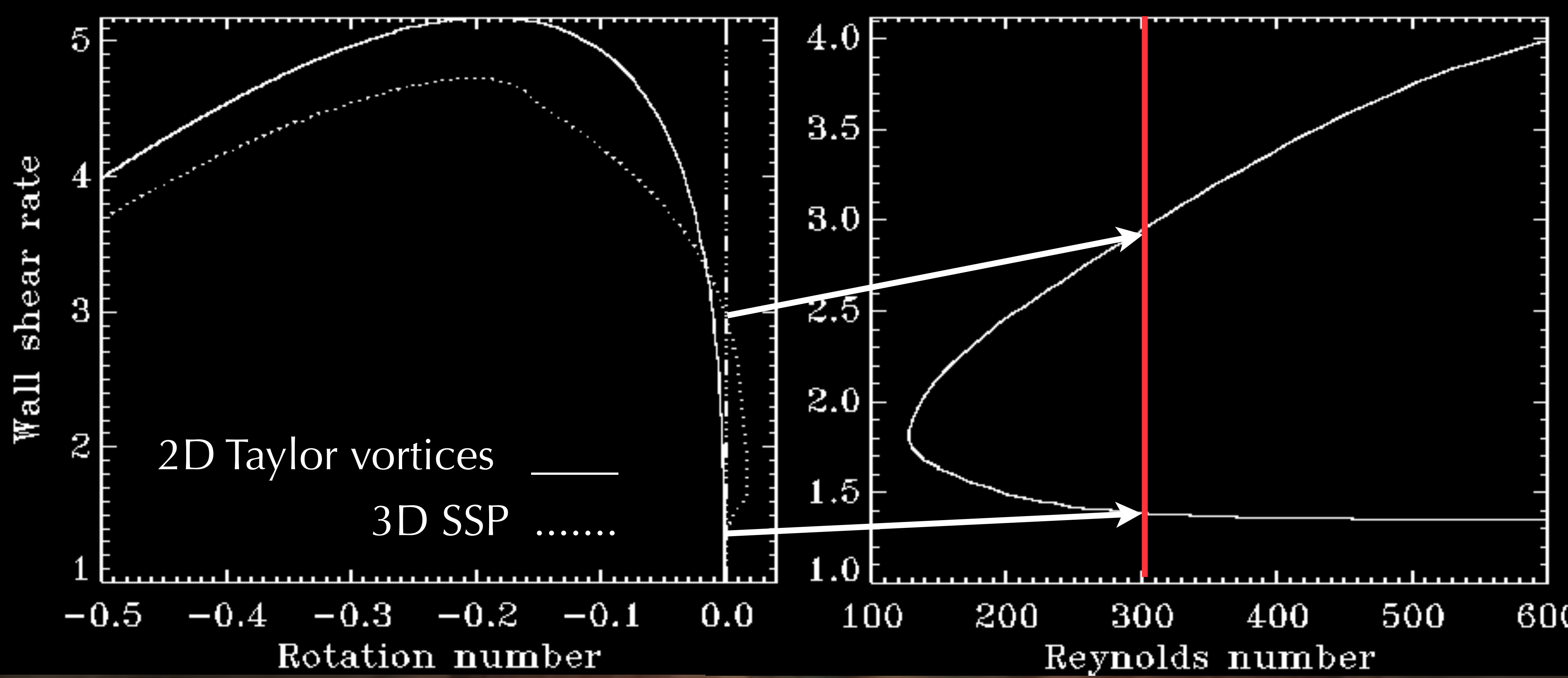
[Hamilton et al., PF 1995 - Waleffe, SAM 1995, PF 1997, PRL 1998, JFM 2001, PF 2003 - Schmiegel 1999 - Faisst & Eckardt, PRL 2003, Wedin & Kerswell, JFM 2004]

Extension to linearly stable cyclonic regimes

- $R_\Omega \geq 0$ solutions connect to centrifugal solutions at $R_\Omega < 0$

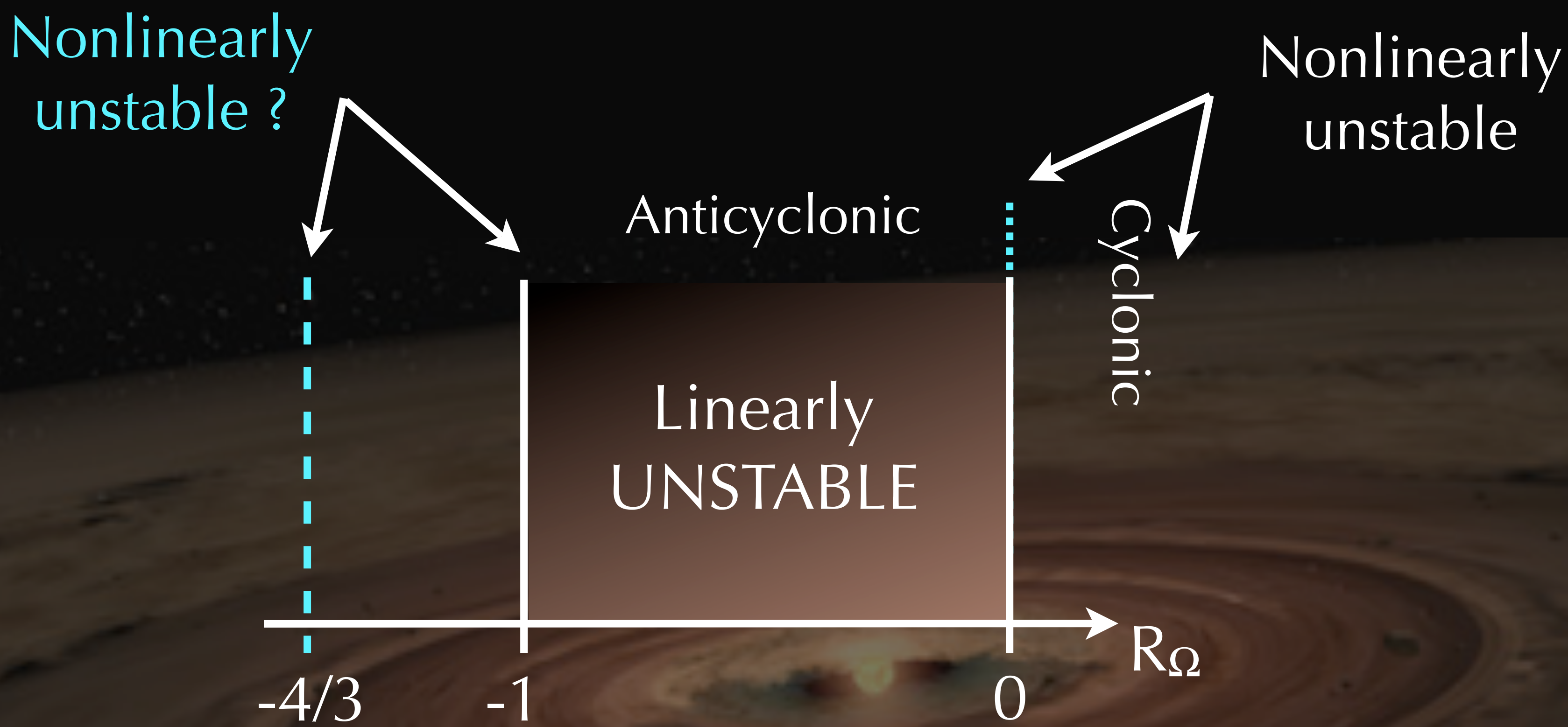
[Nagata, JFM 188, 858 (1988), JFM 217, 519 (1990) - Rincon, Ogilvie & Cossu, A&A 463, 817 (2007)]

Re=300 $R_\Omega = 0$



Subcritical transition in anticyclonic, Rayleigh-Stable shear flows ?

- Cyclonic or non-rotating regime are nonlinearly unstable
  - Is it also true for the anticyclonic, linearly stable regime ?
  - Could there be a self-sustaining process on the Rayleigh line ?

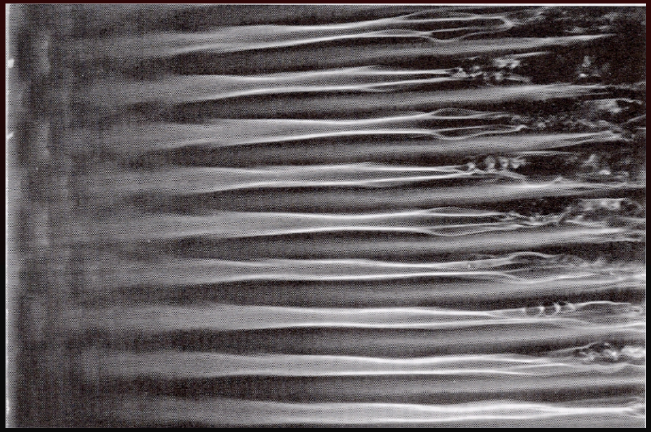


Transient axisymmetric growth

- Linear equation for transient "axisymmetric" amplification

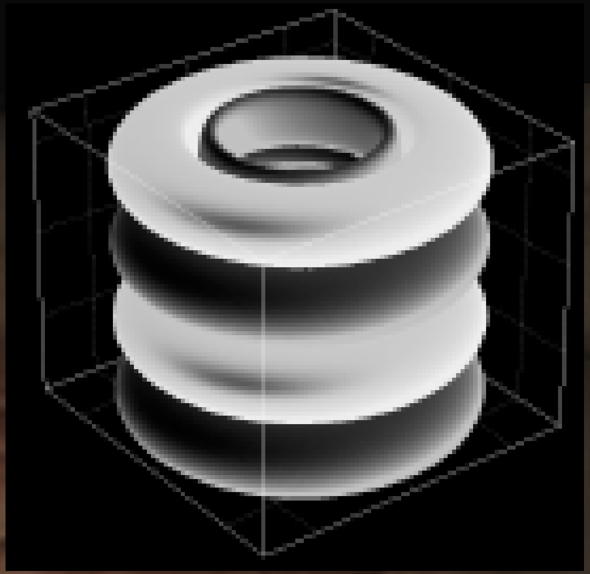
$$\frac{\partial u_x}{\partial t} = -(R_\Omega + 1)u_y + \frac{1}{\mathrm{Re}}\Delta u_x$$

$$\frac{\partial \omega_x}{\partial t} = -R_\Omega \frac{\partial u_x}{\partial z} + \frac{1}{\mathrm{Re}}\Delta \omega_x$$



- $R_\Omega = 0$: no source for ω_x
  - Rolls transiently generate strong streaks
  - Instability of streaks leads to SSP

- $R_\Omega = -1$: no source for u_x
  - Streaks transiently generate strong rolls
  - Instability of rolls... ?



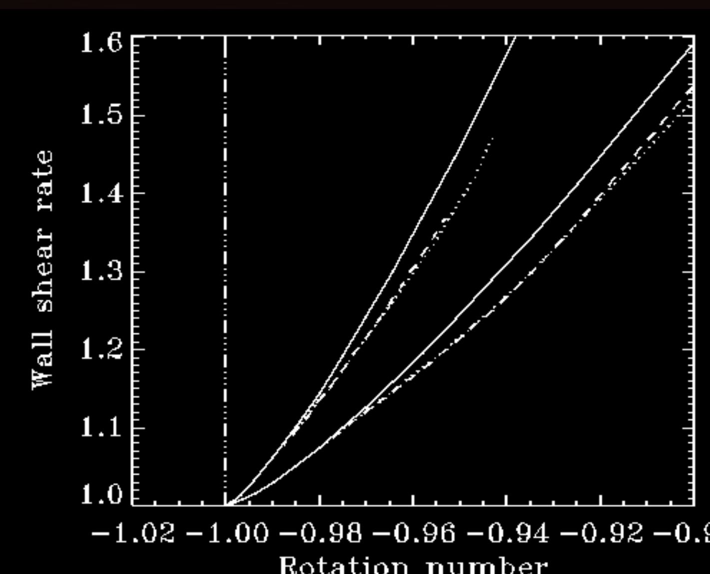
[Antkowiak et Brancher, JFM 2007]

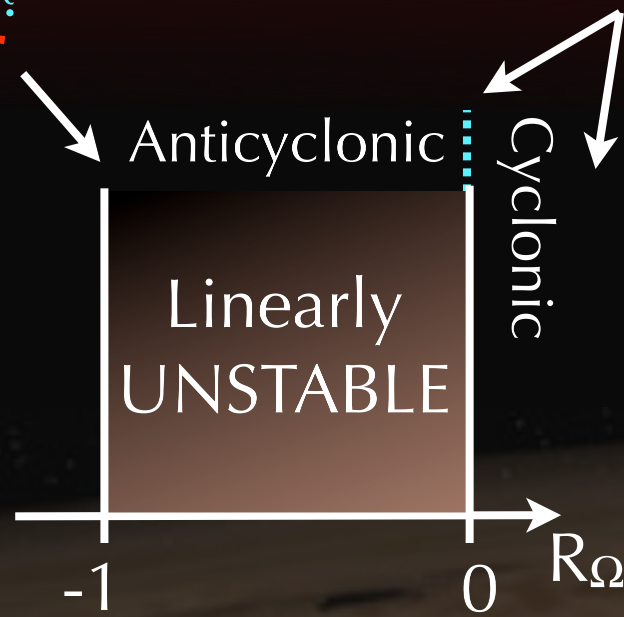
Solutions close to the Rayleigh line

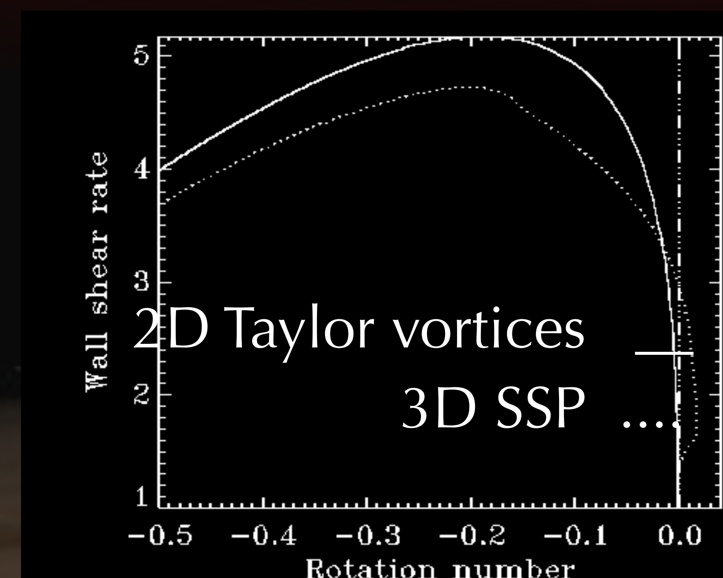
- Nonlinear 3D solutions for R_Ω -> -1^+
  - Result from instability of 2D Taylor vortices [Nagata, JFM 169, 229 (1986)]
  - But do not cross the Rayleigh line: no possible continuation to $R_\Omega \leq -1$

~~Nonlinearly unstable ?~~

Nonlinearly unstable



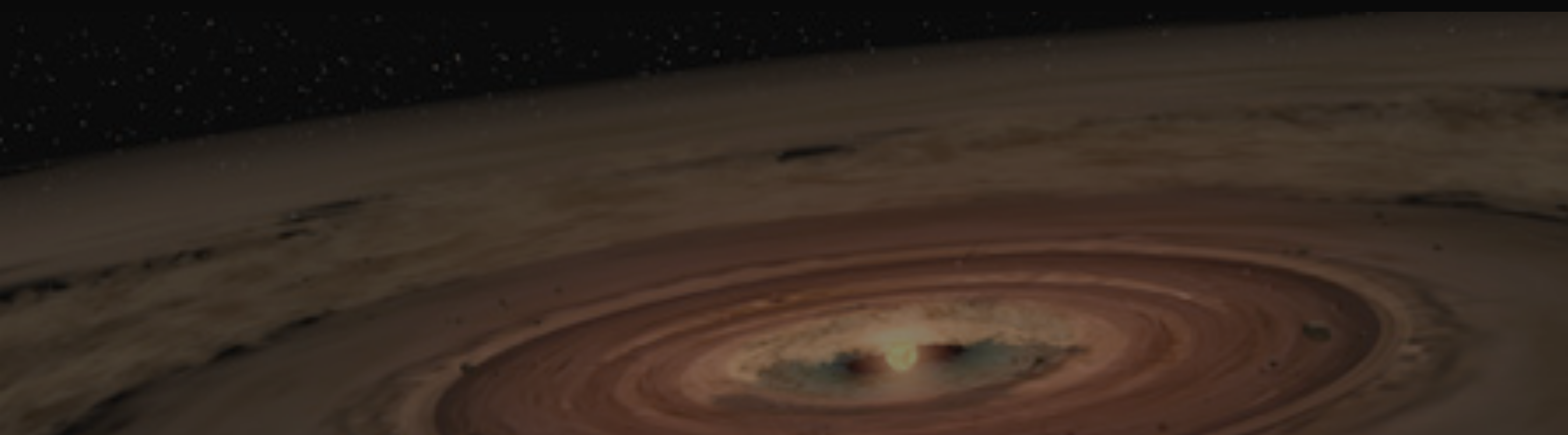




Phenomenology of non-rotating and cyclonic shear flow transition does not carry over to anticyclonic shear flows like Keplerian flow

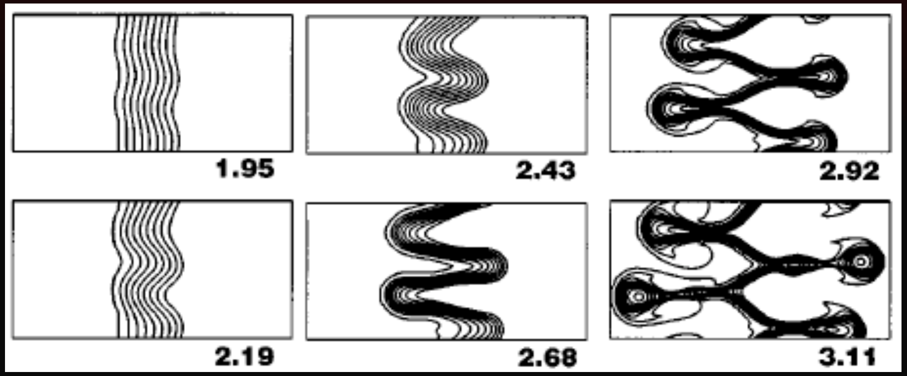
[Rincon, Ogilvie & Cossu, A&A 463, 817 (2007)]

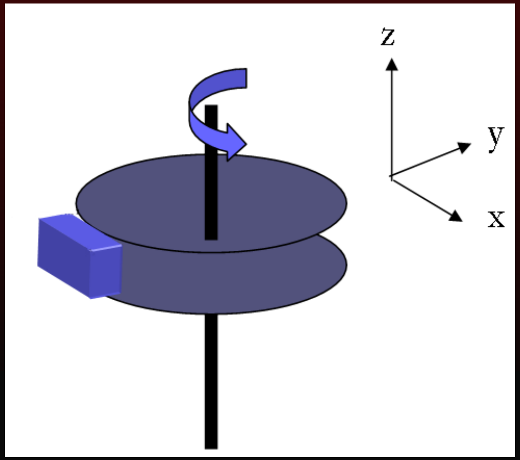
MHD turbulence in fluid discs



Magnetorotational turbulence: classic papers

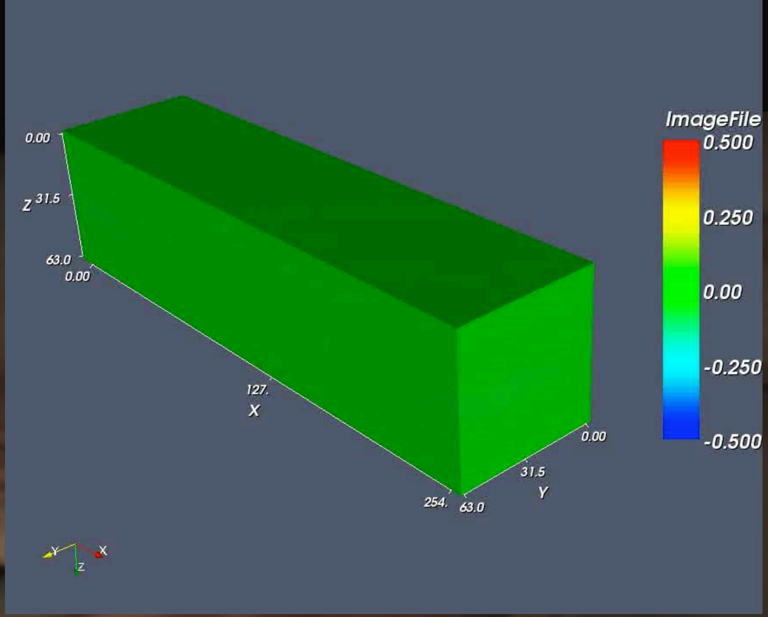
- MRI with imposed vertical field ("net-flux")
  - 2D: Hawley & Balbus [ApJ 376, 223 (1991)]
    - Channel modes in the shearing box





  - 3D: Hawley et al. [ApJ 440, 742 (1995)]
    - All field geometries are unstable
    - Breaks down into MHD turbulence
  - Apparently efficient at transporting angular momentum (α~0.1, but...)

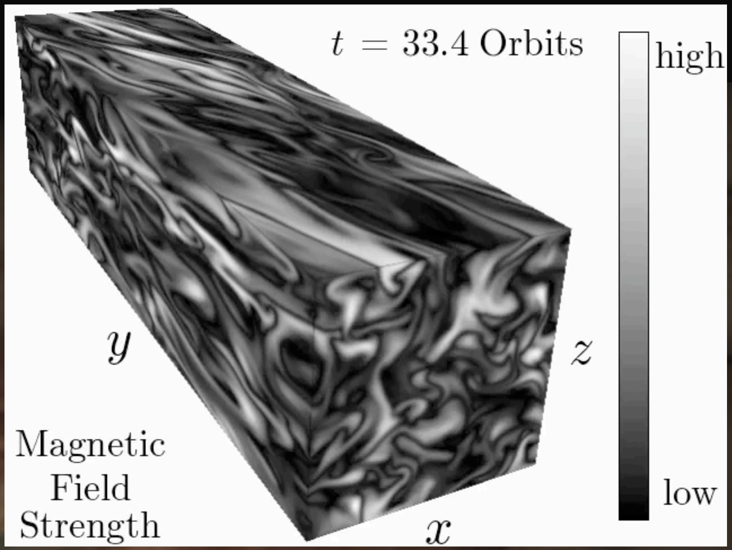
[Lesur & Longaretti, MNRAS 2007]



What happens without a mean field ?

- Discs may be able to develop “their own field”
  [observational evidence in protostellar disk claimed by Donati et al., Nature 438, 466 (2005)]
  - MRI dynamo action may be possible
  - And we may learn a lot on dynamos from this problem...(more later)

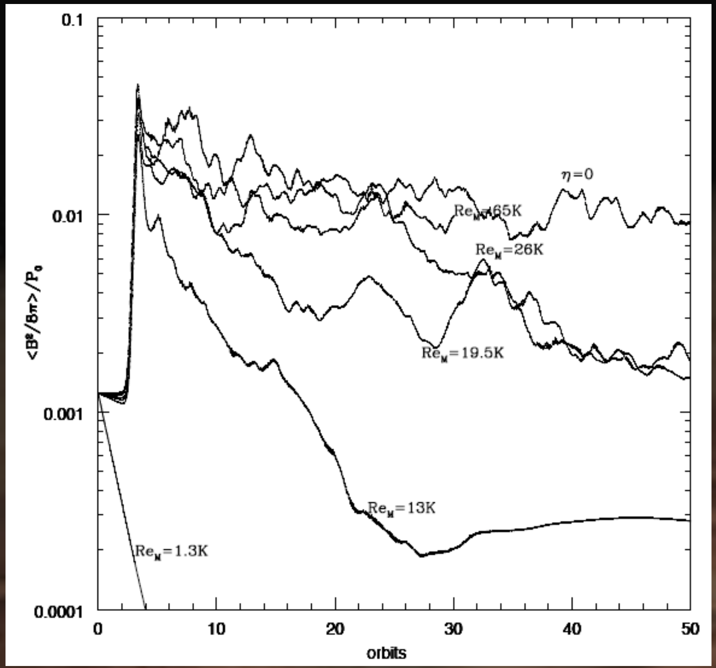
- Early SB simulations for <B>=0 showed that
  [Brandenburg et al., ApJ 446, 741 (1995), Hawley et al. ApJ 464, 690 (1996)]
  - “MRI” dynamo action is possible
  - Only the 3D case is sustained
  - MRI dynamo requires Lorentz force
    - Not a kinematic dynamo !
  - Transport apparently much smaller than in the net-flux problem



[Heinemann & Papaloizou, MNRAS 2009]

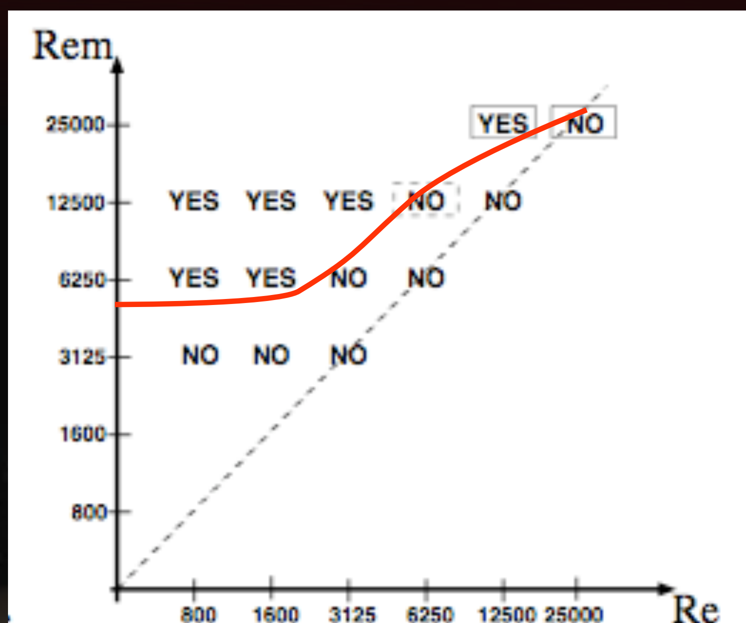
MRI turbulence and dissipation

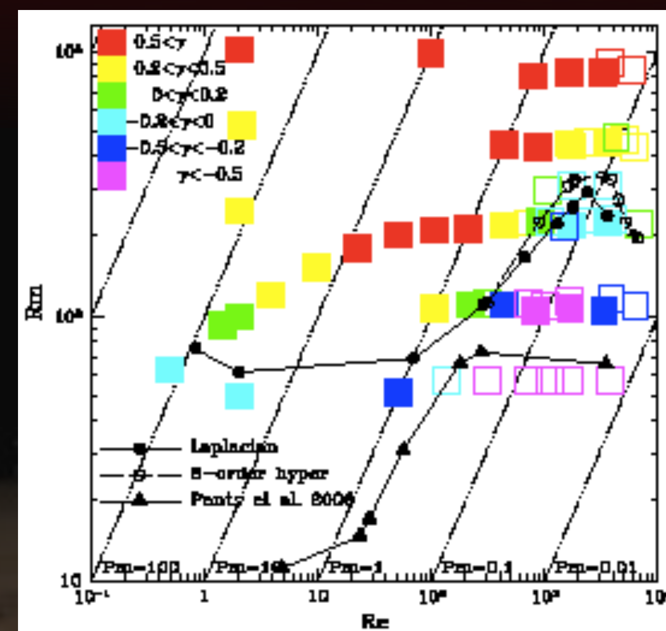
- "Classic" MRI simulations relied fully on grid dissipation...
  - No explicit viscosity or resistivity: so no Re, no Rm, no Pm
  - The disc community only started to worry about this recently
- First hints of a possible problem: inclusion of resistivity
  - No MRI dynamo below $Rm=10^4$! [Fleming et al., ApJ 530, 464 (2000)]



Critical threshold for MRI dynamo action

- Consider MRI dynamo with well defined Re, Rm, and Pm
  [Fromang et al., A&A 476, 1123 (2007), A&A 514, L5 (2010)]
  - MHD turbulence dies out at Pm=O(1), even for Re, Rm=25000 !



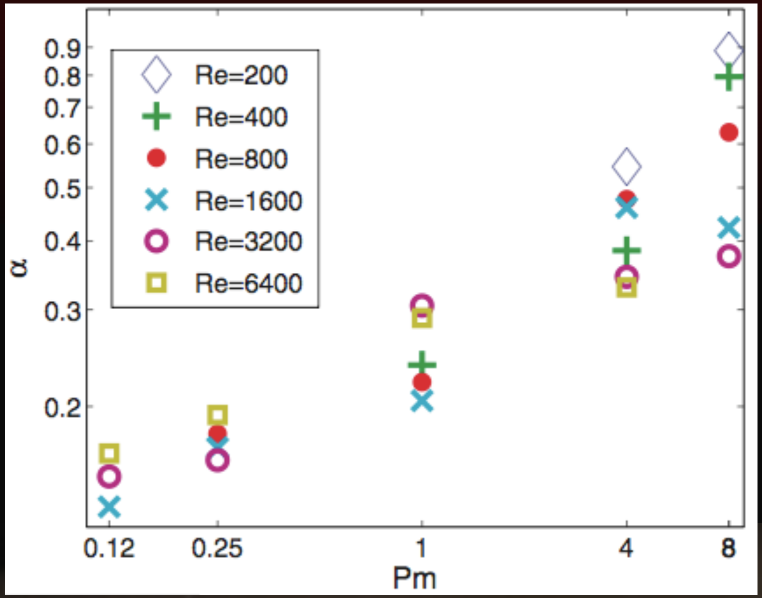


Lots in common with the fluctuation dynamo problem apparently...
but no time to talk about that today, see Scheckochihin et al. [NJP 9, 300 (2007)]

Net-flux case revisited

- It's not only the dynamo set-up that causes trouble...
  - Let's look at the mean-field case again

- Lesur & Longaretti
  [MNRAS 378, 1471 (2007)]
  - Spectral, full control on dissipation
  - max(Re)=6400, max(Rm)=25000
  - Transport gets small at low Pm



- Fromang et al. [A&A 476, 1113 (2007)]
  - Typical astro codes relying on grid dissipation have "effective" Pm = 2-3
  - Correspondingly, reasonable transport coefficients are found: α~0.2-0.5
  - But this value is really dependent on your numerics...no good !

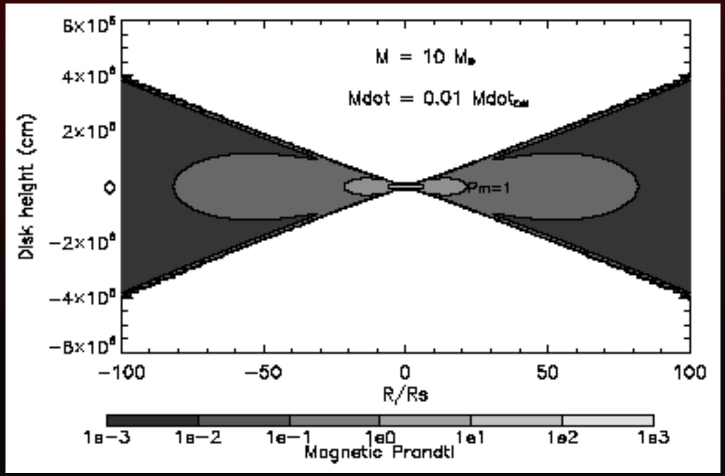
Is this important ?

- For a pure hydrogen plasma [Te=Ti (T in K, n in SI)],
  [more elaborate estimates for discs in [Balbus & Henri, ApJ 674, 408 (2008)]

$$\nu = 1.4 \times 10^{11} \frac{T^{5/2}}{n \ln \Lambda_{ii}} \,\mathrm{m}^2\,\mathrm{s}^{-1}$$

$$\eta = 5.2 \times 10^{7} \frac{\ln \Lambda_{ie}}{T^{3/2}} \,\mathrm{m}^2\,\mathrm{s}^{-1}$$

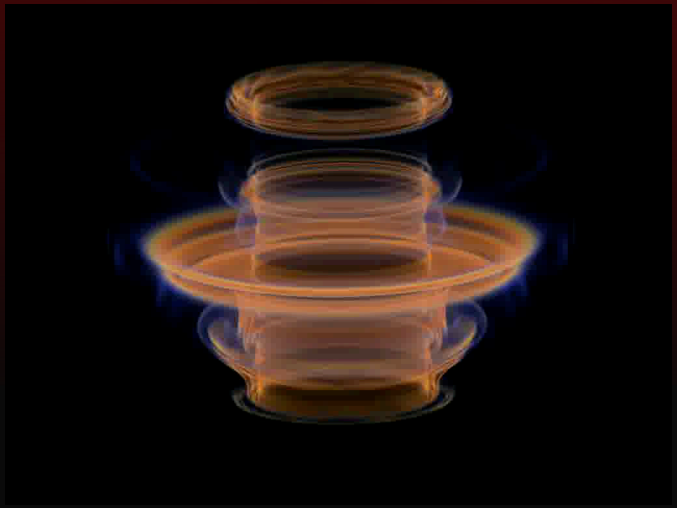
$$\mathrm{Pm} = \frac{\nu}{\eta} \simeq 2.5 \times 10^{3} \frac{T^4}{n \ln \Lambda^2}$$



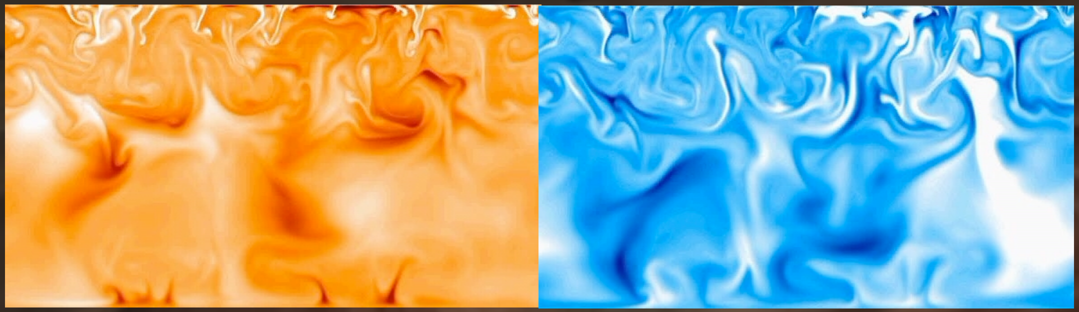
- Questions that suddenly popped up
  - Is MRI turbulence really the key to transport in all types of discs ? (YSOs)
  - Is there any MRI dynamo at low Pm ?
- Maybe everything will be fine in the end...
  - ...but right now, the problem turns out to be much more complex (and interesting !) than thought for a long time

Recent numerical progress

- Obabko & Cattaneo's phantom simulation
  - Taylor-Couette MRI set-up
  - Pm down to 0.5
  - Re, Rm up to 60000 - spectral elements



- Small-scale dynamo action ?
  - Claim that the results are the same with/without net-flux
    - Need to go to large enough Re/Rm ?
  - Also claim that everything looks like the turbulent convection dynamo

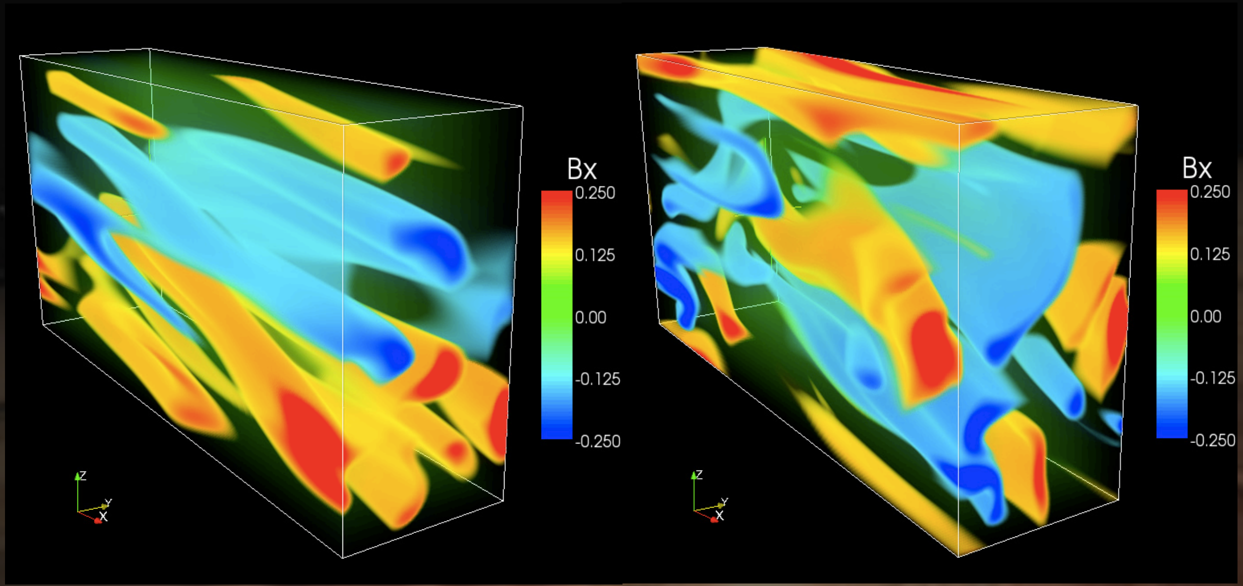


Even more recent numerical progress

- MRI dynamo [Fromang, A&A 514, L5 (2010)]
  - transport independent of Re at Re>>1,Pm=4, $\alpha \sim 7 \times 10^{-3}$
  - preliminary study of spectral scalings (closer to IK than GS ?)
- Net flux [Longaretti & Lesur, A&A 516, A51 (2010)]
  - transport scales with Pm at Pm > 1 and Re >> 1
  - transport scales with Rm at Pm < 1 and Rm >> 1
- Study of spectral energy transfers in MRI turbulence [Lesur & Longaretti, A&A 528, A17 (2011)]
  - Non-local interactions seem to be important
  - May have some important implications for the α-Pm relationship ?
- Lots still to be done....

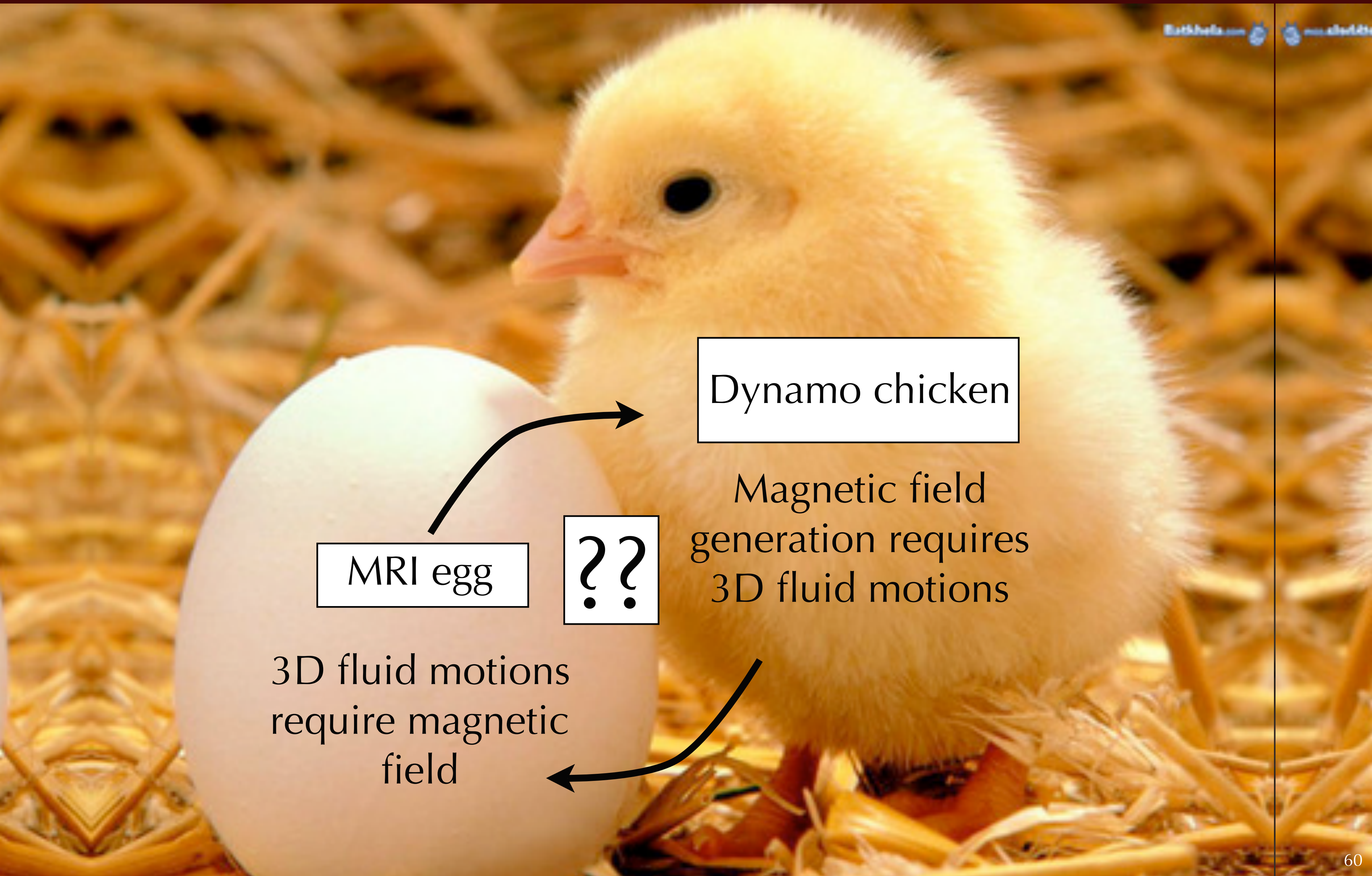
Connexion between MRI & Dynamos

- Consider the MRI dynamo problem in more detail
  - 2D case decays (consistend with antidynamo theorem)
  - If you switch off the Lorentz force, everything decays: not kinematic !
    - Strongly hints that the MRI "spring" is playing a role
- Pseudo-cyclic MRI dynamos reported in several papers
  - Is this a mean-field dynamo or something else ?



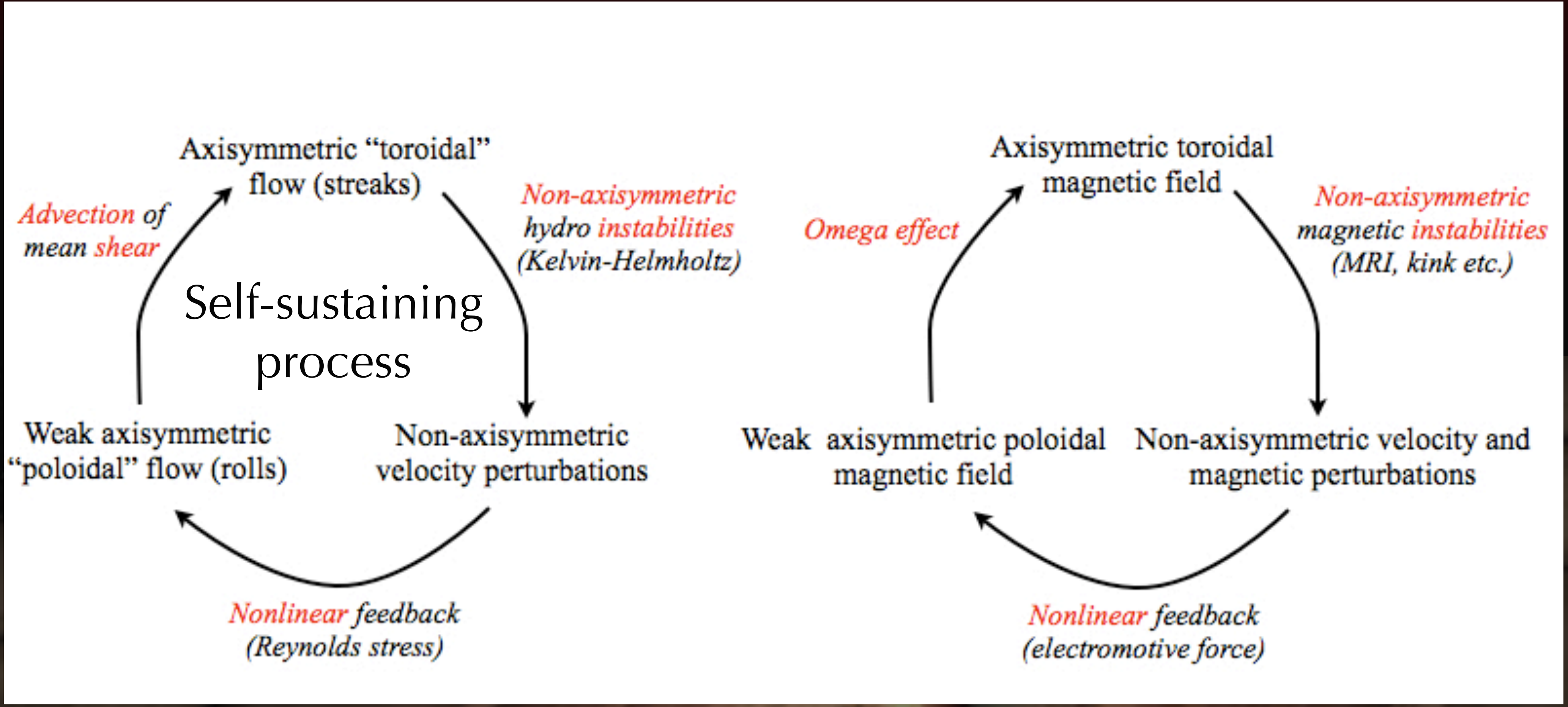
[Brandenburg et al., ApJ 446, 741 (1995), Lesur & Ogilvie, A&A 488, 451 (2008), Davis, Stone & Pessah, ApJ 713, 52 (2010)]

The chicken and egg dynamo



Subcritical shear dynamos

A perfect analog to hydrodynamic transition to turbulence in non-rotating shear flows ?



Self-sustaining dynamo processes

- Axisymmetric part of induction equation
  - Analogy with rolls & streaks equations in non-rotating shear flow
    - transient algebraic growth of b_x: the Ω effect in dynamo theory

$$\frac{\partial b_x}{\partial t} = \bar{b}_y + \mathbf{e}_x \cdot \overline{\nabla \times (\mathbf{u} \times \mathbf{b})} + \frac{1}{\mathrm{Rm}} \Delta \bar{b}_x \ ,$$

$$\frac{\partial \chi}{\partial t} - \frac{\partial (\psi, \chi)}{\partial (y, z)} = \overline{(\mathbf{u}' \times \mathbf{b}')} \cdot \mathbf{e}_x + \frac{1}{\mathrm{Rm}} \Delta \chi$$

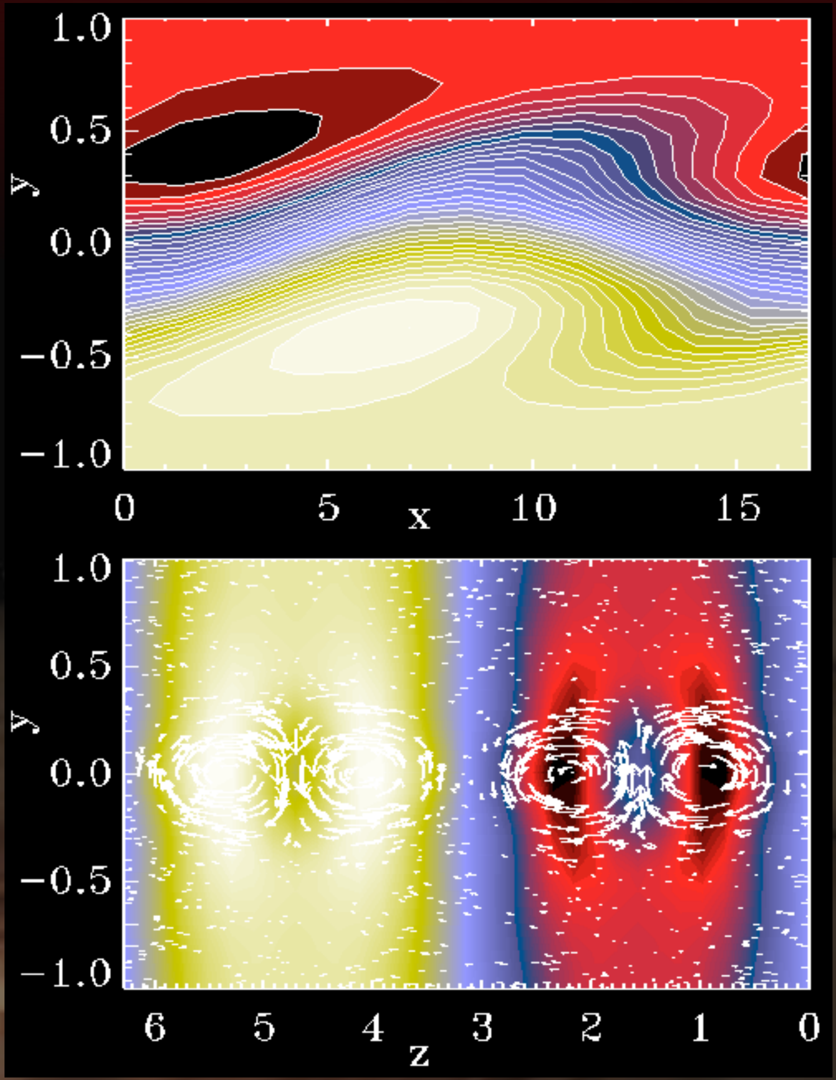
$$\mathbf{b}_\mathrm{p} = \nabla \times (\chi \mathbf{e}_x)$$

- 3D non-linear feedback is required to sustain poloidal field
  - Cowling's theorem that 2D dynamo action does not exist

La chasse aux papillons (butterfly hunting)

- Attempt to capture coherent MRI dynamo structures
  - Nonlinear equilibria, travelling waves
  - Dynamo limit cycles ?
  - Strange attractors ?

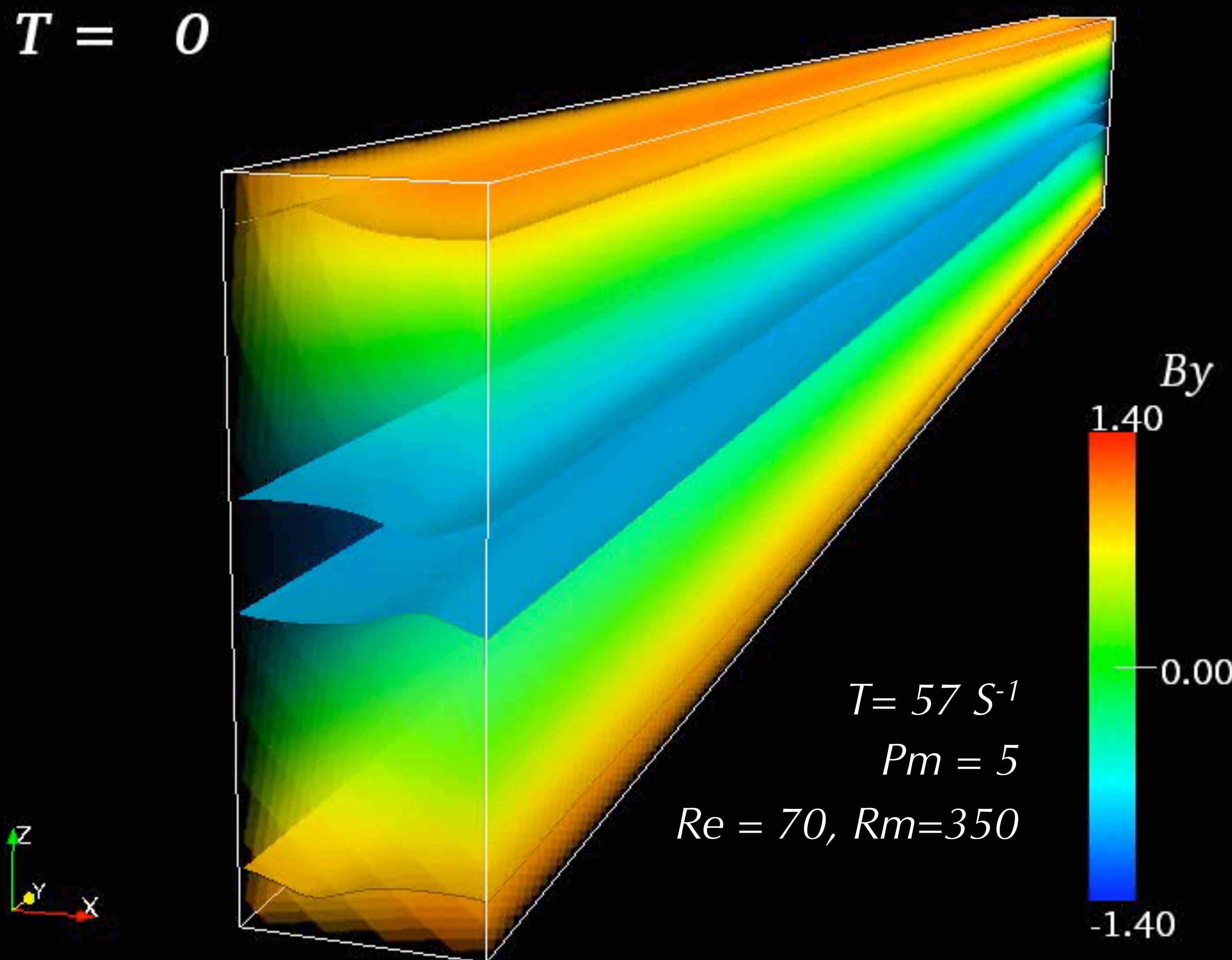
- Search for fixed points
  - Homotopy a la Waleffe/Nagata
  - Continuation using Newton solver



Nonlinear MRI dynamo equilibrium
in Keplerian Couette flow with
conducting walls

[Rincon, Ogilvie & Proctor, PRL 98, 254502 (2007)]

A nonlinear MRI dynamo limit cycle !



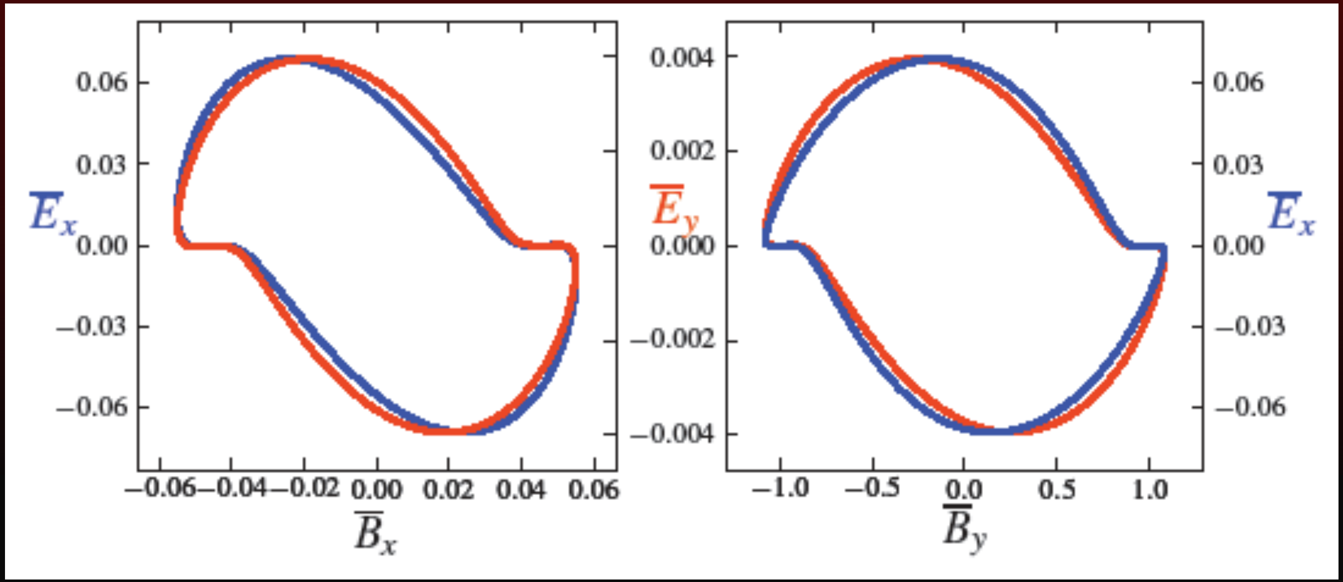
T= 57 S^{-1}

Pm = 5

Re = 70, Rm=350

[Herault, Rincon, Cossu et al., submitted]

EMF vs B: not a mean field dynamo !



- So, this is really a new, funky animal in the dynamo zoo
  - Fully 3D, nonlinear, non-kinematic and time-periodic
  - Guaranteed with zero mean-field content
  - Zero kinetic and magnetic helicity
- Prototype of instability-driven dynamos in shear flows

Plasma physics and turbulence in hot accretion flows

[Notation change: in this section, **u** denotes the total fluid velocity, including differential rotation, while **v** denotes individual particle velocities]

Plasma turbulence in hot accretion flows

- Recap of local conditions
  - Very hot, low density plasma: $T_i \sim 10^{11\text{-}12}$ K, $n \sim 10^{12}$ m^{-3}
  - Low collisionality, 2-temperature plasma: m.f.p. ~ 10^{10} km ~ R_{gas}
  - A very important extra natural ordering : Larmor radius vs m.f.p.
    - B ~ 30G, $\rho_i \sim 10^{-1}$ km << m.f.p. !!
- What happens to "large-scale, fluid" instabilities ?
  - MRI, thermal convection...
  - New instabilities ?
- Astrophysical implications ?
  - Heating: viscosity, thermal conductivity, etc.
  - Relevance of fluid models ? Transport theories/coefs ?

A major actor: pressure anisotropy

- Qualitative picture
  - For m.f.p. >> ρ_i , conservation of $\mu_s = m_s v_\perp^2 / 2B$
    - Field-stretching motions naturally generate pressure anisotropy
    - Relaxed by "weak" collisions

$$\frac{1}{p_\perp}\frac{\mathrm{d}p_\perp}{\mathrm{d}t} \sim \frac{1}{B}\frac{\mathrm{d}B}{\mathrm{d}t} - \nu_{ii}\frac{p_\perp - p_\|}{p}$$

$$\frac{1}{B}\frac{\mathrm{d}B}{\mathrm{d}t} = \hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla\mathbf{u} \qquad p_\perp - p_\| \sim (\rho v_{\mathrm{th}}^2/\nu_{ii})\hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla\mathbf{u} \equiv \rho v_{\mathrm{th}}^2 \Delta$$

- Theory must capture this effect: isotropic MHD not enough
  - Does this require a full kinetic description ? Let's try simpler things first...

Equations for L >> m.f.p. >> ρ_i

- Use scale-separations with fast cyclotron motions
  - MHD-like equations

$$\rho \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = -\nabla \left(p_\perp + \frac{B^2}{8\pi} \right) + \nabla \cdot \left[\hat{\mathbf{b}}\hat{\mathbf{b}} \left(p_\perp - p_\| \right) \right] + \frac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi}$$

$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} = \mathbf{B} \cdot \nabla \mathbf{u}$$

- Collisional limit (L >> m.f.p. >> ρ_i)
  - Fluid equations with anisotropic transport coefficients [Braginskii, Rev. Plasma Phys. 1, 205 (1965)]
  - Braginskii viscosity: $p_\perp - p_\| = 3\rho\nu_B \hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla\mathbf{u} \equiv \rho v_{\mathrm{th}}^2 \Delta$
  - Only damps field-stretching motions, not Alfvén wave-polarized fluctuations

$$\frac{1}{B}\frac{\mathrm{d}B}{\mathrm{d}t} = \hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla\mathbf{u}$$

Kinetic MHD

- Low-frequency, long wavelength approximation
  - $\omega << \Omega_i$, $L >> \rho_i$ [Kulsrud, Handbook of plasma physics (1983)]
  - MHD-like equations but m.f.p. > $L >> \rho_i$ allowed

$$\rho \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = -\nabla\left(p_\perp + \frac{B^2}{8\pi}\right) + \nabla\cdot\left[\hat{\mathbf{b}}\hat{\mathbf{b}}\left(p_\perp - p_\|\right)\right] + \frac{\mathbf{B}\cdot\nabla\,\mathbf{B}}{4\pi}$$

$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} = \mathbf{B}\cdot\nabla\,\mathbf{u}$$

- Solve drift-kinetic equation to obtain pressures

$$\frac{\mathrm{d}f_s}{\mathrm{d}t} + v_\|\nabla_\| f_s + \frac{v_\perp}{2}\left(\hat{\mathbf{b}}\hat{\mathbf{b}}:\nabla\mathbf{u} - \nabla\cdot\mathbf{u} + v_\|\frac{\nabla_\| B}{B}\right)\frac{\partial f_s}{\partial v_\perp} - \left[\hat{\mathbf{b}}\cdot\left(\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} + v_\|\nabla_\|\mathbf{u}\right) + \frac{v_\perp^2}{2}\frac{\nabla_\| B}{B}\right]\frac{\partial f_s}{\partial v_\|} = \left.\frac{\partial f_s}{\partial t}\right|_{\mathrm{coll}}$$

$$p_\perp = 2\pi\sum_s m_s\int_0^\infty dv_\perp v_\perp\int_{-\infty}^{+\infty} dv_\|\frac{v_\perp^2}{2}f_s \qquad p_\| = 2\pi\sum_s m_s\int_0^\infty dv_\perp v_\perp\int_{-\infty}^{+\infty} dv_\| v_\|^2 f_s$$

Fluid closures

- To avoid kinetic equation, one can use closure theories
  - For instance, take the 2nd order moment of drift-kinetic equation
  - Pressure equations

$$\rho B \frac{d}{dt}\left(\frac{p_\perp}{\rho B}\right) = -\nabla\cdot(\hat{\mathbf{b}}\,q_\perp) - q_\perp \nabla\cdot\hat{\mathbf{b}}$$

$$\frac{\rho^3}{B^2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{p_{||}B^2}{\rho^3}\right) = -\nabla\cdot(\hat{\mathbf{b}}\,q_{||}) + 2\,q_\perp \nabla\cdot\hat{\mathbf{b}}$$

- Closure: prescription of heat fluxes q
  - Simplest choice: q=0 (CGL, double adiabatic)...very limited validity
  - More elaborate Landau fluid models include
    - Landau resonance physics
      [Snyder, Hammett & Dorland, Phys. Plasmas 4, 3974 (1997)]
    - Finite Larmor Radius physics (FLR)
      [Passot & Sulem, Phys. Plasmas 14, 082502 (2007)]

The kinetic MRI

- Perturbed anisotropic MHD equations in a Keplerian flow

$$\rho\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = -\nabla\left(p_\perp + \frac{B^2}{8\pi}\right) + \nabla\cdot\left[\hat{\mathbf{b}}\hat{\mathbf{b}}\left(p_\perp - p_\|\right)\right] + \frac{\mathbf{B}\cdot\nabla\,\mathbf{B}}{4\pi}$$

$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} = \mathbf{B}\cdot\nabla\,\mathbf{u}$$

$$\mathbf{u} = R\,\Omega(r)\mathbf{e}_\phi + \delta\mathbf{u}$$

$$\mathbf{B} = B_\phi\mathbf{e}_\phi + B_z\mathbf{e}_z + \delta\mathbf{B}$$

$$p_\| = p_0 + \delta p_\|$$

$$p_\perp = p_0 + \delta p_\perp$$

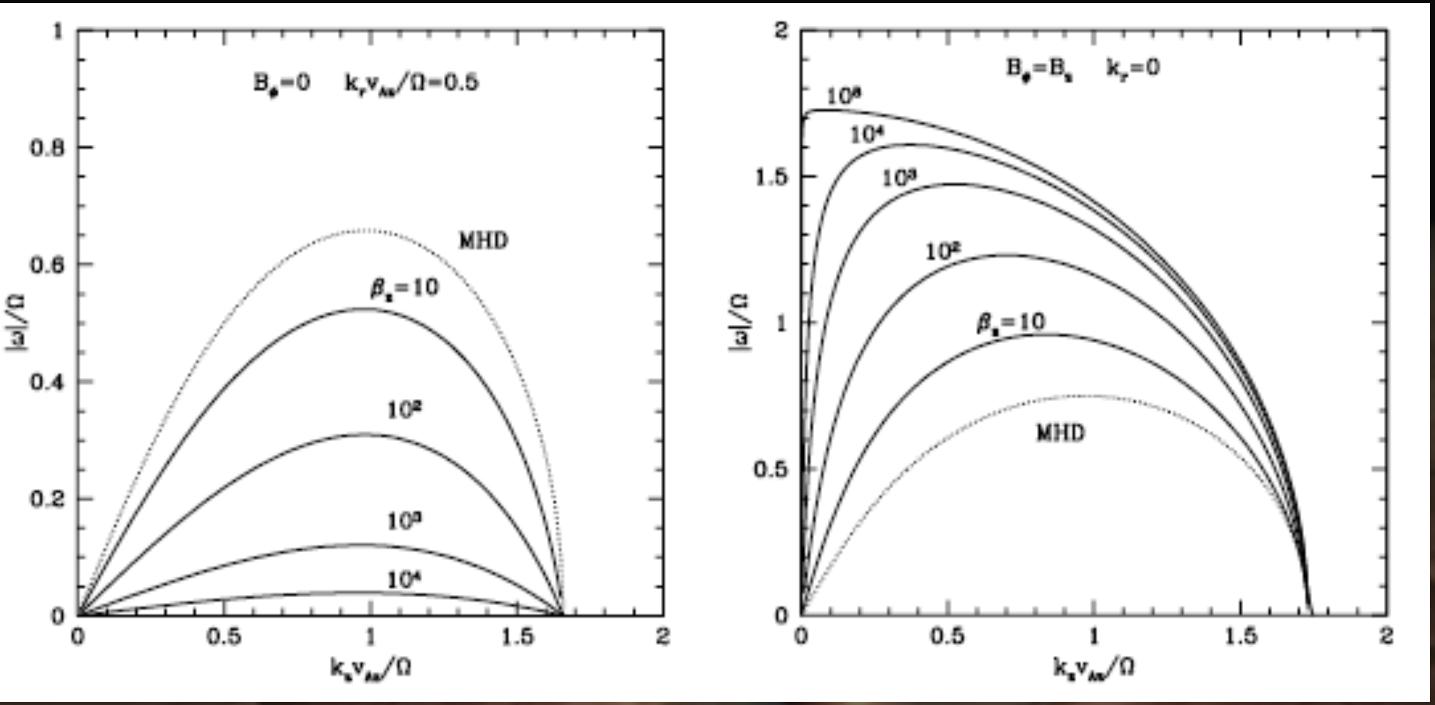
$$\rho = \rho_0 + \delta\rho$$

- Use your favorite closure for pressure perturbations
  - Kinetic MHD / Landau fluid approach
    - no collisions at all [Quataert et al., ApJ 577, 524 (2002)]
    - collisionless / collisional transition [Sharma et al., ApJ 596, 1121 (2003)]
  - Braginskii MHD approach [Balbus, ApJ 616, 857 (2004)]

Specificities of linear kinetic MRI

[Quataert et al., ApJ 577, 524 (2002)]

- Growth rates now depend on plasma β
- Background toroidal field plays an important role
  - with $B_\phi = 0$, kinetic growth rates < MHD ones
  - with $B_\phi \neq 0$, kinetics growth rates > MHD ones, larger-scale modes



Underlying physics (simple version)

- Anisotropic MHD equations

$$\rho\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t}=-\nabla\left(p_\perp+\frac{B^2}{8\pi}\right)+\nabla\cdot\left[\hat{\mathbf{b}}\hat{\mathbf{b}}\,(p_\perp-p_\|)\right]+\frac{\mathbf{B}\cdot\nabla\,\mathbf{B}}{4\pi}$$

- Pressure anisotropy naturally provides a viscous torque

$$\frac{\partial(\rho\,R\,u_\phi)}{\partial t}+\nabla\cdot\left[R\left(\underbrace{\rho\,u_\phi\mathbf{u}-\frac{B_\phi\,\mathbf{B}}{4\pi}}_{\propto W_{R\phi}}\left(1-\underbrace{\frac{4\pi(p_\|-p_\perp)}{B^2}}_{\propto A_{R\phi}}\right)+\left(P+\frac{B^2}{8\pi}\right)\mathbf{e}_\phi\right)\right]\;=\;0$$

  - Collisional limit: easily interpreted in terms of Braginskii viscosity

$$p_\perp-p_\|=3\rho\nu_B\hat{\mathbf{b}}\hat{\mathbf{b}}:\nabla\mathbf{u}$$

  - The new term only depends on the orientation of B, not on its magnitude
    - Competition with magnetic tension: expect β dependence in the results

Anisotropic pressure: an extra “spring”

- Azimuthal restoring force is at the heart of MRI
  - magnetic tension in the MHD case
- Here, we have an extra anisotropic pressure term

$$(\nabla \cdot \mathbf{P})_{\mathrm{anisotropic}} = -\nabla \cdot \left[\hat{\mathbf{b}}\hat{\mathbf{b}}(p_\perp - p_\|) \right]$$

  - Perturbed azimuthal pressure force even in the axisymmetric case

$$-\mathbf{e}_\phi \cdot (\nabla \cdot \delta \mathbf{P}) = i b_\phi b_z k_z \delta(p_\perp - p_\|)$$

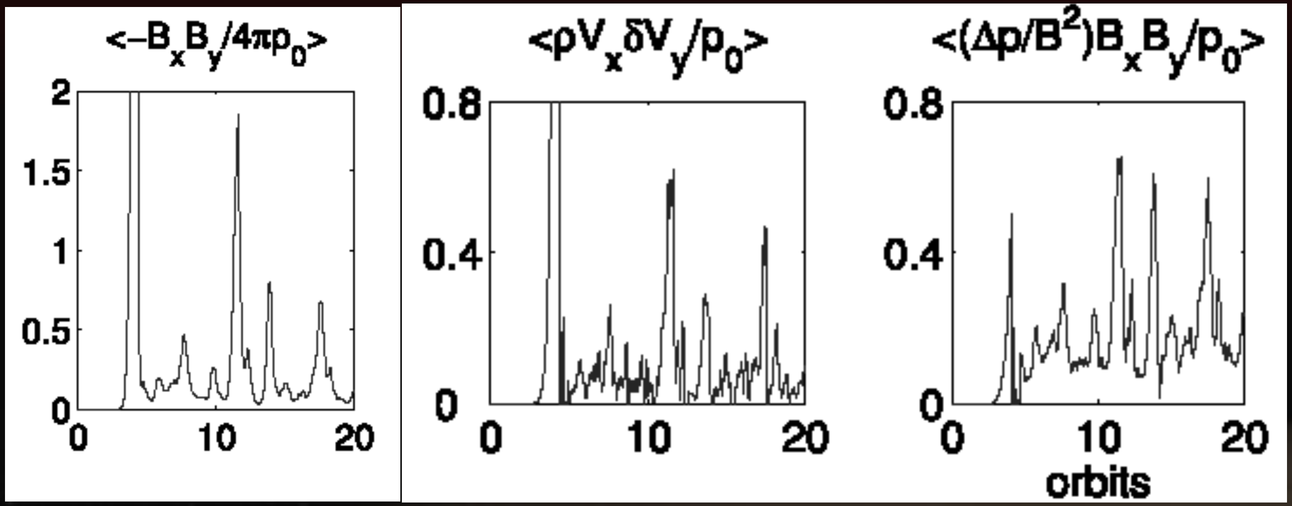
- Pressure response
  - Not quite as simple as the CGL prescription

$$\frac{\delta p_\perp}{p_0} = \frac{\delta \rho}{\rho_0} + \frac{\delta B}{B} \qquad \frac{\delta p_\|}{p_0} = 3\frac{\delta \rho}{\rho_0} - 2\frac{\delta B}{B}$$

  - Low frequency instability: clearly not adiabatic
  - Full derivation of various limits in Sharma et al. [ApJ 596, 1121 (2003)]

Numerical demonstration - transport

- Only simulations so far are with modified fluid code ZEUS
  [Sharma et al., ApJ 637, 952 (2006)]
  - Integration of a simplified form of Landau fluid MHD equations
    [Snyder, Hammett & Dorland, Phys. Plasmas 4, 3974 (1997)]



- Kinetic effects are dynamically important
  - Anisotropic pressure stress $A_{R\phi} \propto \left\langle \frac{B_z B_\phi}{B^2} (p_\| - p_\perp) \right\rangle$ ~ Maxwell stress
  - Moderately enhanced transport compared to MHD case
  - $p_\perp > p_\|$ on average

The magnetoviscous instability (MVI)

[Balbus, ApJ 616, 857 (2004)]

- Consider Braginskii viscosity $p_\perp - p_\| = 3\rho\nu_B \hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla \mathbf{u}$
- For a differentially rotating flow, $\mathbf{u}_0 = R\,\Omega(R)\mathbf{e}_\phi$
- Perturb $(\nabla \cdot \mathbf{P})_{\text{anisotropic}} = -\nabla \cdot \left[\hat{\mathbf{b}}\hat{\mathbf{b}}(p_\perp - p_\|)\right]$
  - Azimuthal component of perturbed pressure force

$$-\mathbf{e}_\phi \cdot (\nabla \cdot \delta\mathbf{P}) = i b_\phi b_z k_z \delta(p_\perp - p_\|)$$

  - Contribution from differential rotation

$$(\delta p_\perp - \delta p_\|)_\Omega = 3\rho\nu_B b_\phi \delta b_r \frac{d\Omega}{d\ln R}$$

- This term leads to a new MRI-like fluid instability, the MVI
  - Unlike for MRI, magnetic tension is not needed for instability !
  - The background field is only responsible for anisotropy
  - The condition for instability is still $\dfrac{d\Omega^2}{d\ln R} < 0$

ok, but...this is not the whole story

- Take shear Alfvén waves with pressure anisotropy

$$\omega^2 = \frac{k_\parallel^2 v_{\rm th}^2}{2} (\Delta + 2/\beta)$$

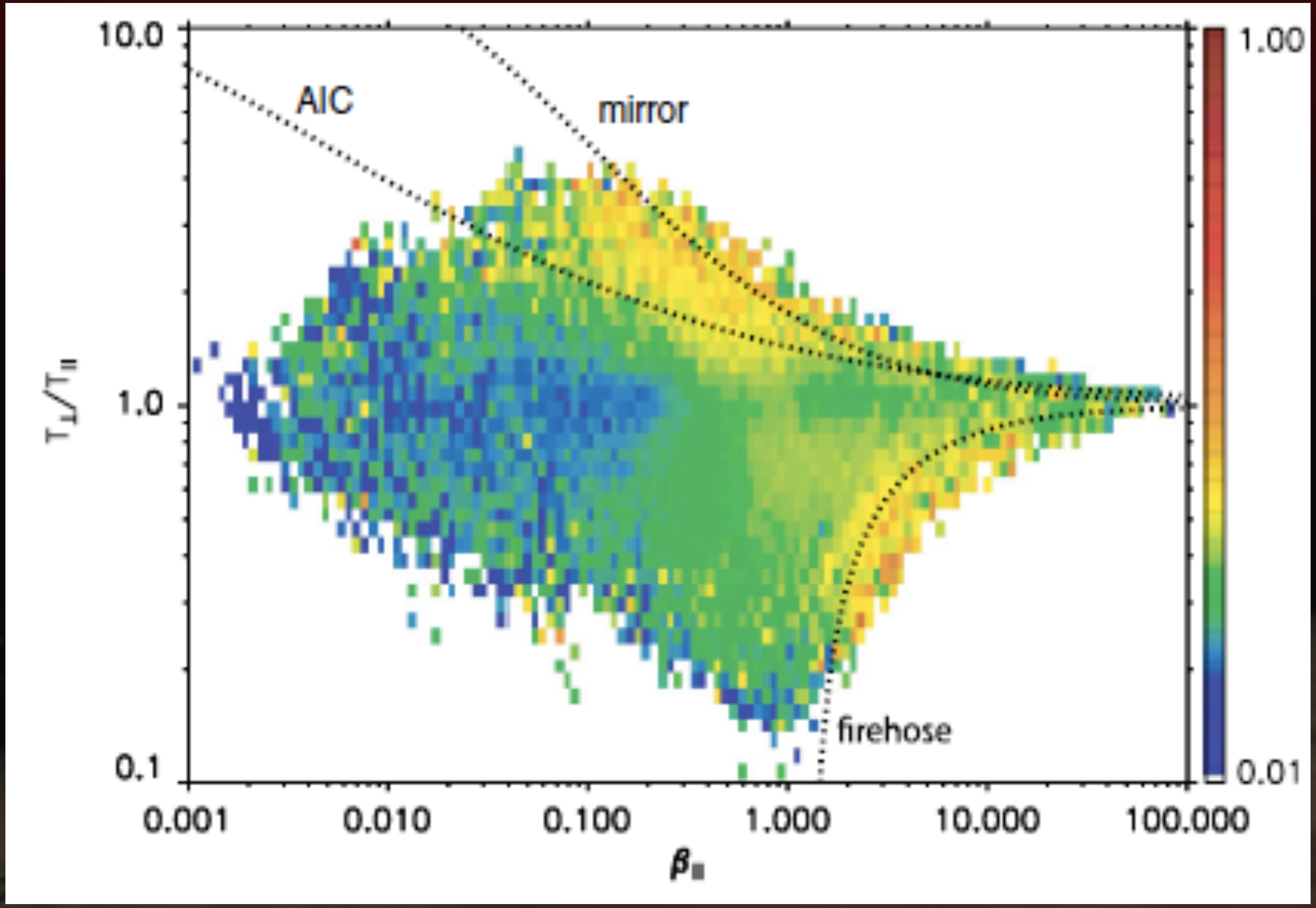
  - Firehose instability for $\Delta \equiv \frac{p_\perp - p_\parallel}{p_\perp} < -2/\beta$
  - The smallest scales are the most unstable ones !

- High-β, anisotropic plasmas are unstable down to ρ_i scale
  - Firehose instability for $p_\perp < p_\parallel$
  - Mirror instability for $p_\perp > p_\parallel$

[Rosenbluth (1956), Chandrasekhar et al., PRSL 245, 435 (1958), Parker, Phys. Rev. 109, 1874 (1958), Vedenov & Sagdeev, Sov. Phys. 3, 278 (1958), Hellinger, PoP 14, 082105 (2007)]

Observational evidence

- Temperature anisotropy measurements in the solar wind



[Bale et al., Phys. Rev. Lett. 103, 211101 (2009)]

Coupling to large scale dynamics

- $\mu = mv_\perp^2/2B$ conservation implies instability everywhere
  - local increase of B => increase of $p_\perp$
    - Mirror unstable
  - local decrease of B => decrease of $p_\perp$
    - Firehose unstable

- Whenever m.f.p. >> ρ_i, they develop extremely quickly
  - Affect transport properties: conductivity, viscosity
  - Dynamical feedback on large-scale dynamics

[Schekochihin et al., ApJ 629, 139 (2005)]

Sharma et al.'s cure

- Keep microscale instabilities at marginal level
  - Very useful, as there are no FLR corrections in their Landau fluid model
  - Relaxation terms for pressure anisotropy in the pressure equations

$$\frac{\partial p_\perp}{\partial t} = \cdots - \nu(p_\perp, p_\parallel, \beta)\,[p_\perp - p_\parallel] \qquad \frac{\partial p_\parallel}{\partial t} = \cdots - \nu(p_\perp, p_\parallel, \beta)\,[p_\parallel - p_\perp]$$

- Physical arguments
  - In the solar wind, measured anisotropies are marginal
  - Microscale instabilities produce a foam of magnetic fluctuations
    - These fluctuations induce particle scattering ~ effective collisions
- Important caveats
  - Isotropisation of P = "more collisions" = smaller m.f.p. and viscosity !
    - Grid dissipation still likely to play some dirty tricks to the dynamics...
  - Marginal stability may not result from enhanced effective collisions [Rosin et al, MNRAS in press (2011)]

Application: heating in hot accretion flows

- Radiative efficiency of an accretion flow is $\mathcal{L} \sim \eta \dot{M} c^2$
  - Low luminosity AGNs: small accretion rate or low radiative efficiency ?
- Most of the radiation comes from electrons
  - Need to quantify how much energy they receive from viscous heating
- Viscous heating in accretion flow
  - pressure anisotropy levels are critical

Note: $\Delta p_s \equiv p_{\|,s} - p_{\perp,s}$

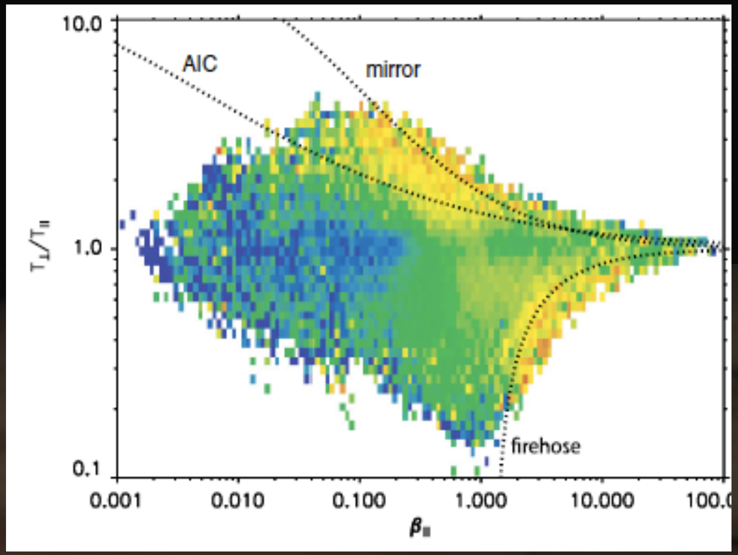
$$q_s^+ = -\Delta p_s \, \hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla \mathbf{u}$$

  - Collisional Braginskii estimate: $q_s^+ \propto m_s^{1/2} T_s^{5/2} (\hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla \mathbf{u})^2$
    - For $T_i \geq T_e$, ion heating is much larger
  - Collisionless limit: what is the pressure anisotropy ?

Impact of microscale instabilities

[Sharma et al., ApJ 667, 714 (2007)]

- Split viscous heating
  - background flow component: $q^+_{1,s} = -\left(\frac{d\Omega}{d\ln R}\right)\Delta p_s b_R b_\phi$
  - fluctuation component: $q^+_{2,s} = -\Delta p_s \, \hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla\delta\mathbf{u}$
- Assume system is marginal with respect to anisotropy-driven instabilities
  - Ion and electron firehose: $1 - \frac{p_{\perp,s}}{p_{\parallel,s}} < \frac{2}{\beta_{\parallel,s}}$
  - Ion cyclotron: $\frac{T_{\perp,i}}{T_{\parallel,i}} - 1 > \frac{S_i}{\beta_{\parallel,i}^{\alpha_i}}$
  - Electron whistler: $\frac{T_{\perp,e}}{T_{\parallel,e}} - 1 > \frac{S_e}{\beta_{\parallel,i}^{\alpha_e}}$

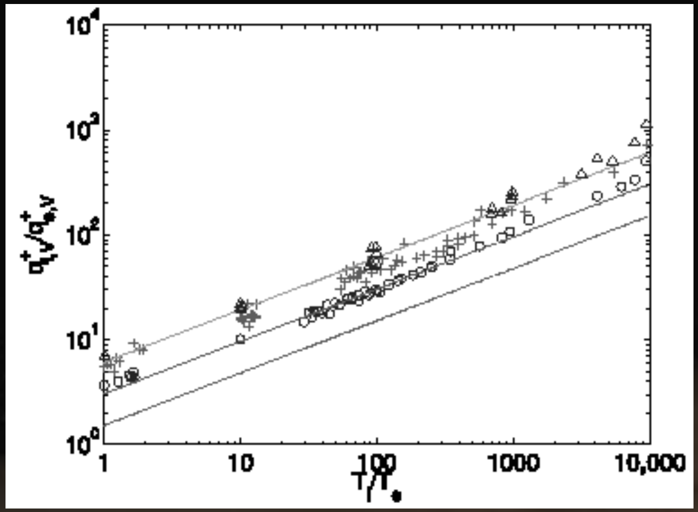


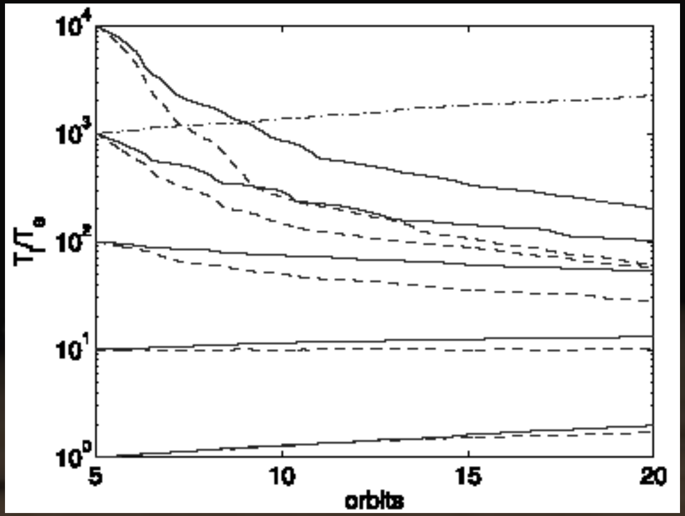
- Implications
  - Viscous heating independent of mass ratio !

$$q^+_{1,s} \propto S_s \beta_s^{1-\alpha_s} \propto T_s^{1/2} \quad \text{(for IC and whistler)}$$

Numerical simulations of kinetic MRI

- Results (same model as before) [Sharma et al., ApJ 667, 714 (2007)]
  - pressure anisotropy is on average < 0 in simulations
  - heating dominated by background flow term $q^+_{1,s} = -\left(\frac{d\Omega}{d\ln R}\right)\Delta p_s b_R b_\phi$
  - $(T_i/T_e)^{1/2}$ dependence of heating ratio makes sense
  - 2-temperature plasma regimes can be sustained

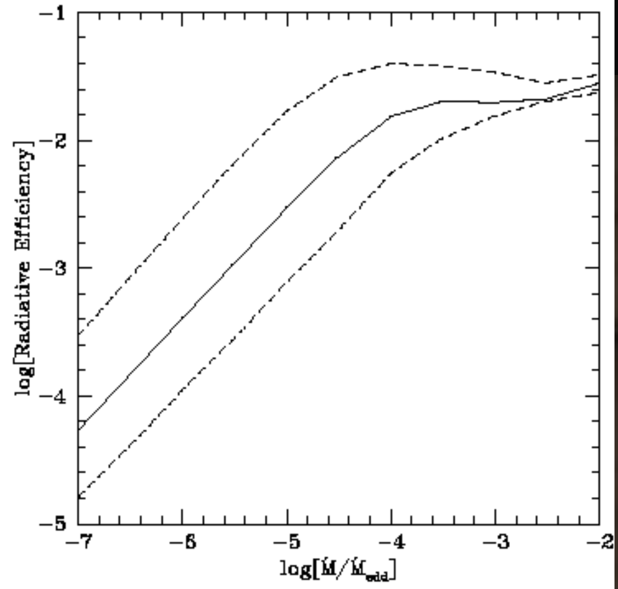




- Limitations
  - grid dissipation is strong $q^+_{2,s} = -\Delta p_s\, \hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla\delta\mathbf{u}$
  - contribution of small-scale fluctuations may be underestimated

Implications for accretion

- Astrophysics that can be done using this kind of results
  - try to understand the luminosity properties of AGNs or Sgr A*
- Use a “realistic” 1D, 2-temperature RIAF model
  - plug in heating ratio and temperature ratio, alpha transport coefficient obtained from simulations
  - detailed calculation of emission processes gives you radiative efficiency
- Results
  - significant radiative efficiency (> 0.5%, not that small -- usual values ~1-10%)
  - may imply that only a small fraction of the mass available is actually accreted



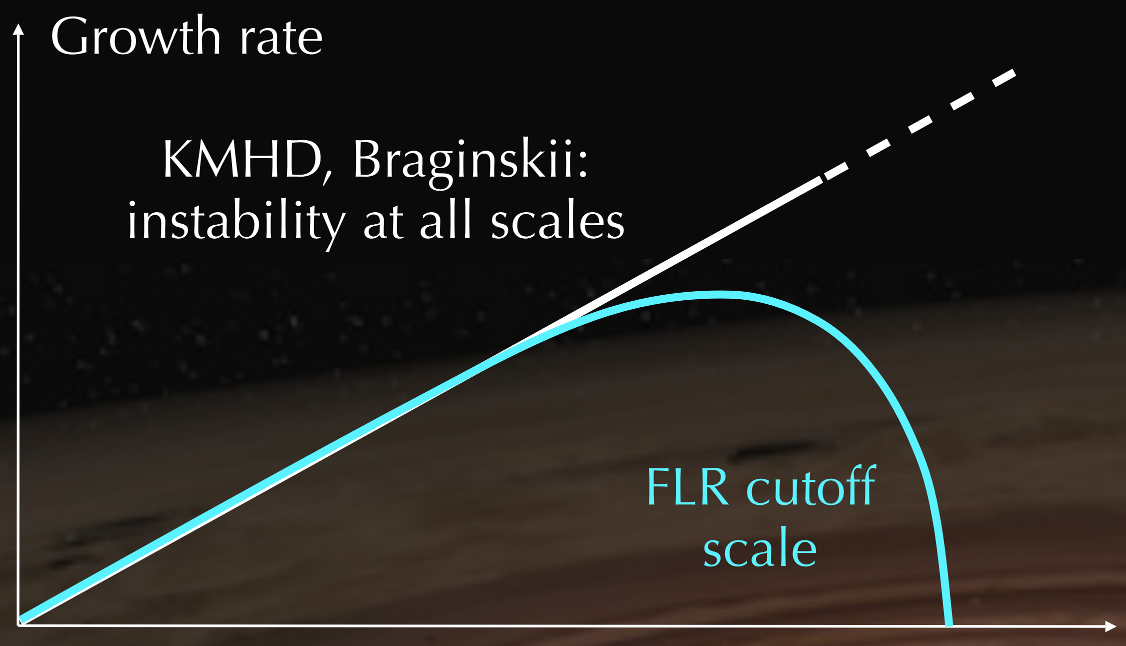
More elaborate cures to microscale instabilities ?

- Full treatment would require
  - letting these instabilities develop and saturate
  - ...but this requires including ρ_i scale physics
- $\rho_i \ll$ MRI & system scales ~ m.f.p. in hot accretion flows
  - scale separation is numerically very hard to deal with
- Attempt to construct transport theories (“closures”)
  - linear theory with FLR tells us that most unstable modes are at $k\rho_i \sim 1$
    - neither Braginskii nor KMHD can do the job
  - study the nonlinear development of these instabilities
    - asymptotic methods: calculate effective transport coefficients
  - formulate “simple” equations for the large-scale dynamics

Preliminary efforts: the parallel firehose

[Rosin et al, MNRAS in press (2011)]

- Easiest case to deal with
  - Purely 1D, no Landau resonance
- Linear dispersion relation
  - Finite Larmor-radius correction stabilizes instability at small scales



$$k_{\|}\rho_i = 2\sqrt{2}|\Delta + 2/\beta|^{1/2}$$

Vlasov-Maxwell equations

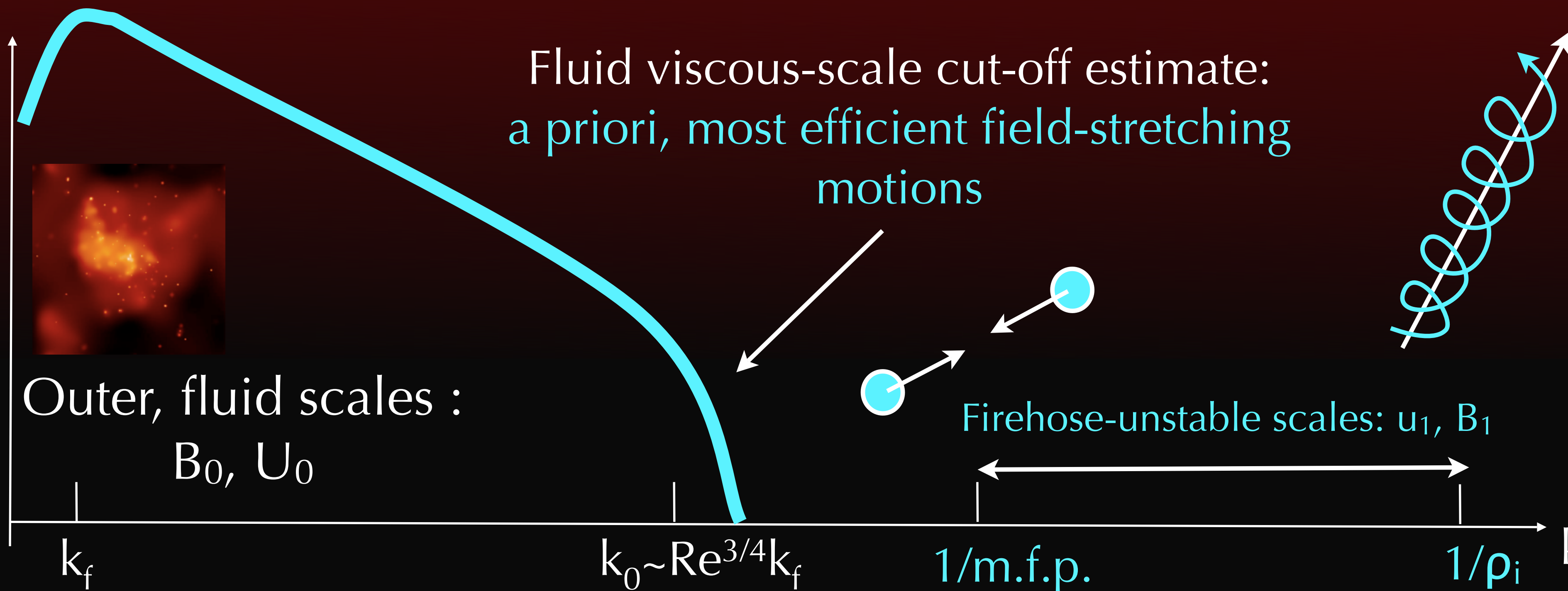
- Solve equations using asymptotic orderings

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \nabla f_s + \frac{Z_s e_s}{m_s} \left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = \left(\frac{\partial f_s}{\partial t} \right)_{\text{coll}}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -c \nabla \times \mathbf{E} \qquad \nabla \times \mathbf{B} = \frac{4\pi \mathbf{j}}{c}$$

- Electron equation: use m_e/m_i ordering
  - Simple Ohm's law, isothermal electrons
- Secondary ordering based on slow/fast motions
  - Ion and electromagnetic fluctuations dynamics
  - Transport equations

Scale ordering



$$\varepsilon \equiv M\mathrm{Re}^{-1/4} = u_0/v_{\mathrm{th}i} = \lambda_{\mathrm{mfp}i}/\ell_0$$

$$\frac{1}{B_0}\frac{\mathrm{d}B_0}{\mathrm{d}t} = \hat{\mathbf{b}}_0\hat{\mathbf{b}}_0 : \nabla\mathbf{u}_0 = \gamma_0 < 0$$

What you get after some algebra

- Wave equation with time-dependent frequency

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\frac{\mathbf{B}_1}{B_0} = \nabla_\parallel^2\left[\frac{v_{\mathrm{th}i}^2}{2}\left(\Delta(t)+\frac{2}{\beta_{0i}}\right)\frac{\mathbf{B}_1}{B_0} + \frac{\rho_i v_{\mathrm{th}i}}{2}\frac{\mathrm{d}}{\mathrm{d}t}\frac{\mathbf{B}_1}{B_0}\times\hat{\mathbf{b}}_0\right]$$

- Solve simultaneously for the pressure anisotropy
  - use simple pitch-angle scattering collision operator

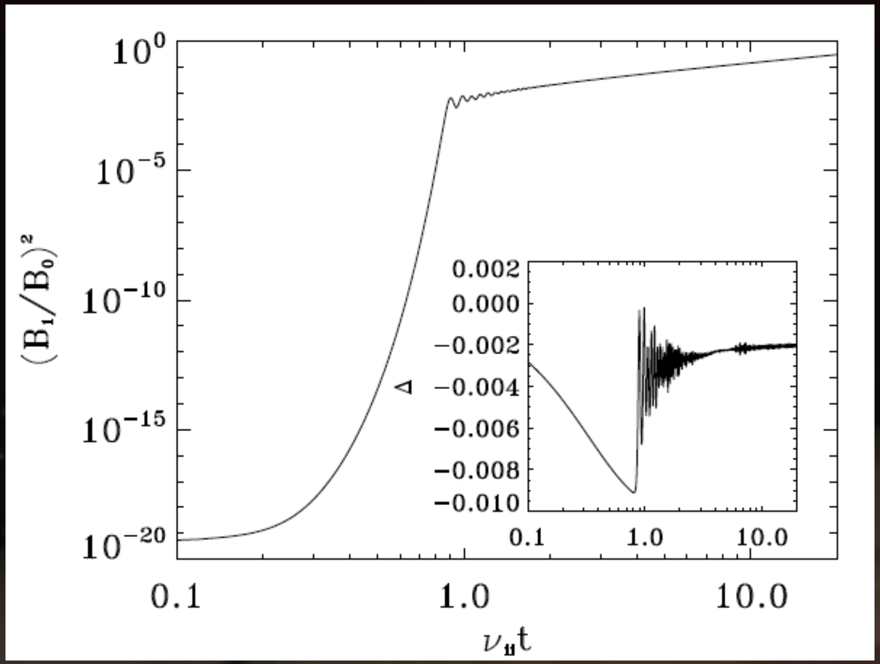
$$\Delta(t) = 3\int_0^t \mathrm{d}t' e^{-3\nu_{ii}(t-t')}\overline{\hat{\mathbf{b}}\hat{\mathbf{b}}:\nabla\mathbf{u}}(t') = -\frac{|\gamma_0|}{\nu_{ii}}\left(1-e^{-3\nu_{ii}t}\right) + \frac{3}{2}\int_0^t \mathrm{d}t' e^{-3\nu_{ii}(t-t')}\frac{\mathrm{d}}{\mathrm{d}t}\frac{\overline{B_1^2(t')}}{B_0^2}$$

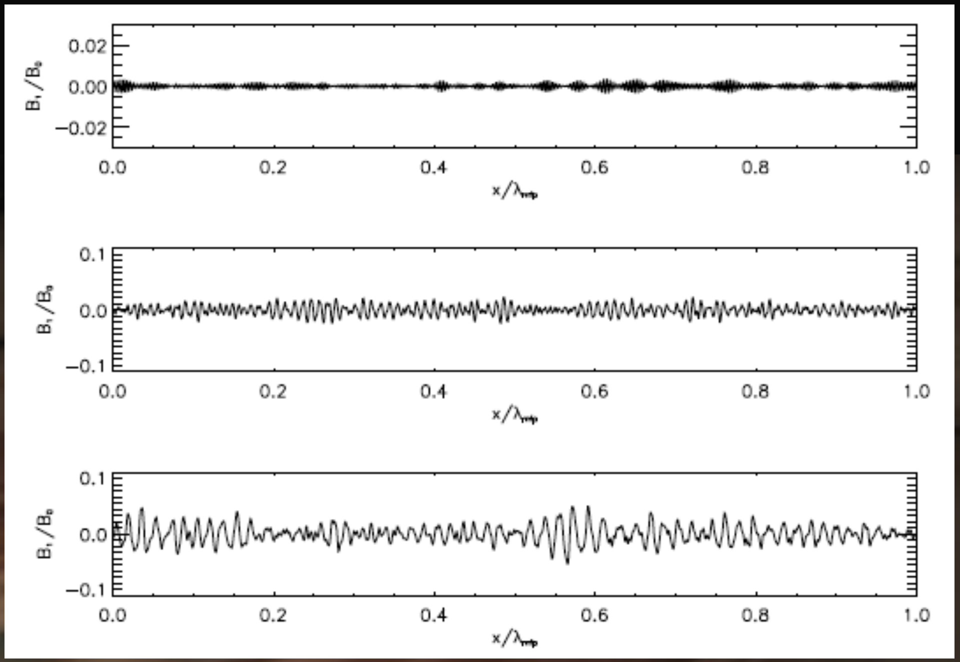
Numerics: linear stage

- Pressure anisotropy builds up due to field stretching

$$\Delta(t) = 3\int_0^t \mathrm{d}t' e^{-3\nu_{ii}(t-t')}\overline{\hat{\mathbf{b}}\hat{\mathbf{b}}:\nabla\mathbf{u}(t')} = -\frac{|\gamma_0|}{\nu_{ii}}\left(1-e^{-3\nu_{ii}t}\right)$$

$$+ \frac{3}{2}\int_0^t \mathrm{d}t' e^{-3\nu_{ii}(t-t')}\frac{\mathrm{d}}{\mathrm{d}t}\frac{\overline{B_1^2(t')}}{B_0^2}$$



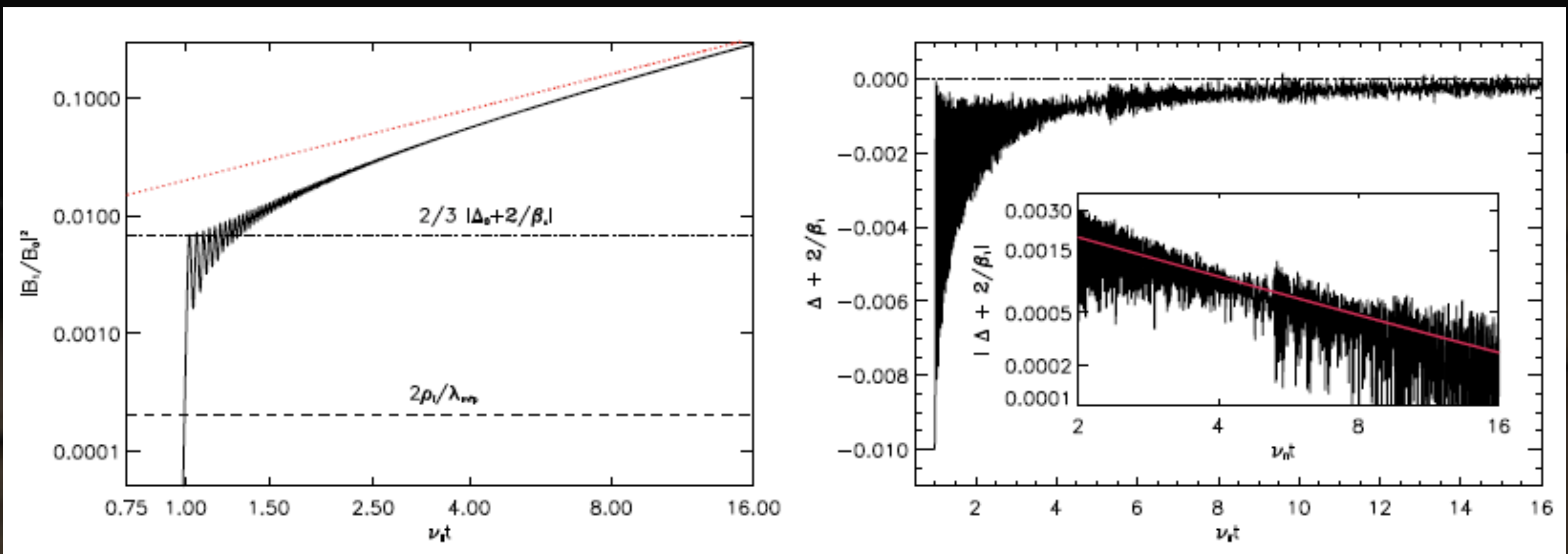


- Exponential firehose growth

Nonlinear stages

- Pressure anisotropy driven to $\Delta \sim -2/\beta$

$$\Delta(t) = 3\int_0^t \mathrm{d}t' e^{-3\nu_{ii}(t-t')}\overline{\hat{\mathbf{b}}\hat{\mathbf{b}}:\nabla\mathbf{u}}(t') = -\frac{|\gamma_0|}{\nu_{ii}}\left(1-e^{-3\nu_{ii}t}\right) + \frac{3}{2}\int_0^t \mathrm{d}t' e^{-3\nu_{ii}(t-t')}\frac{\mathrm{d}}{\mathrm{d}t}\frac{\overline{B_1^2(t')}}{B_0^2}$$



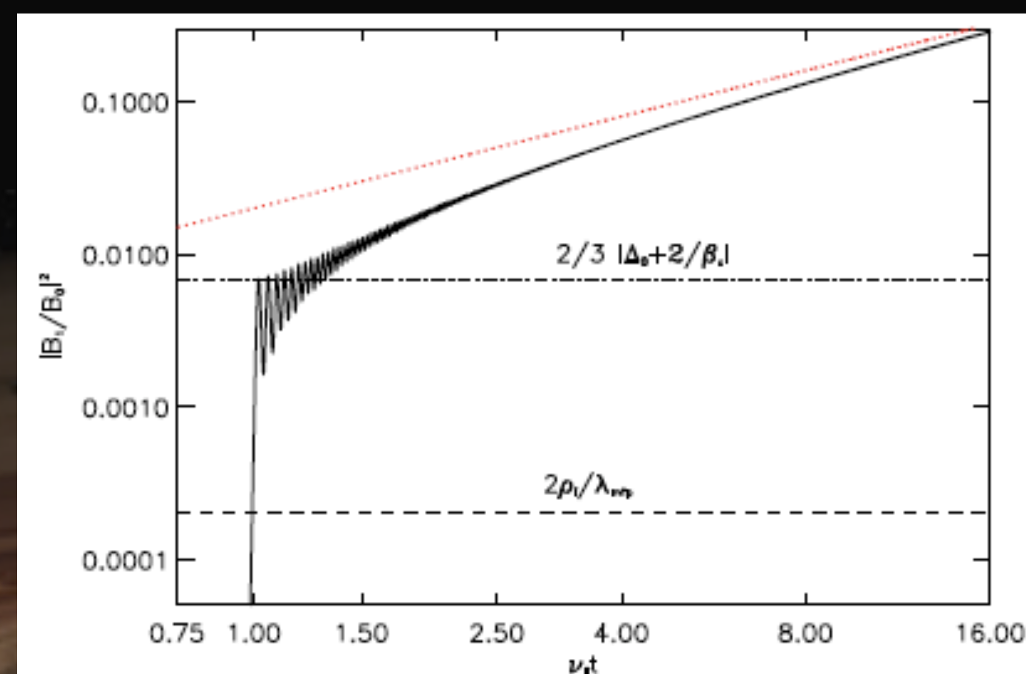
Nonlinear stages

- Nonlinear secular growth

[Schekochihin et al., PRL 100, 081301 (2008)
Rosin et al., MNRAS in press (2011)]

$$\frac{1}{B_0^2}\frac{\mathrm{d}\overline{B_1^2}}{\mathrm{d}t} = -\frac{1}{B_0^2}\frac{\mathrm{d}B_0^2}{\mathrm{d}t} = |\gamma_0| \qquad (B_1/B_0)^2 \sim |\gamma_0| t$$

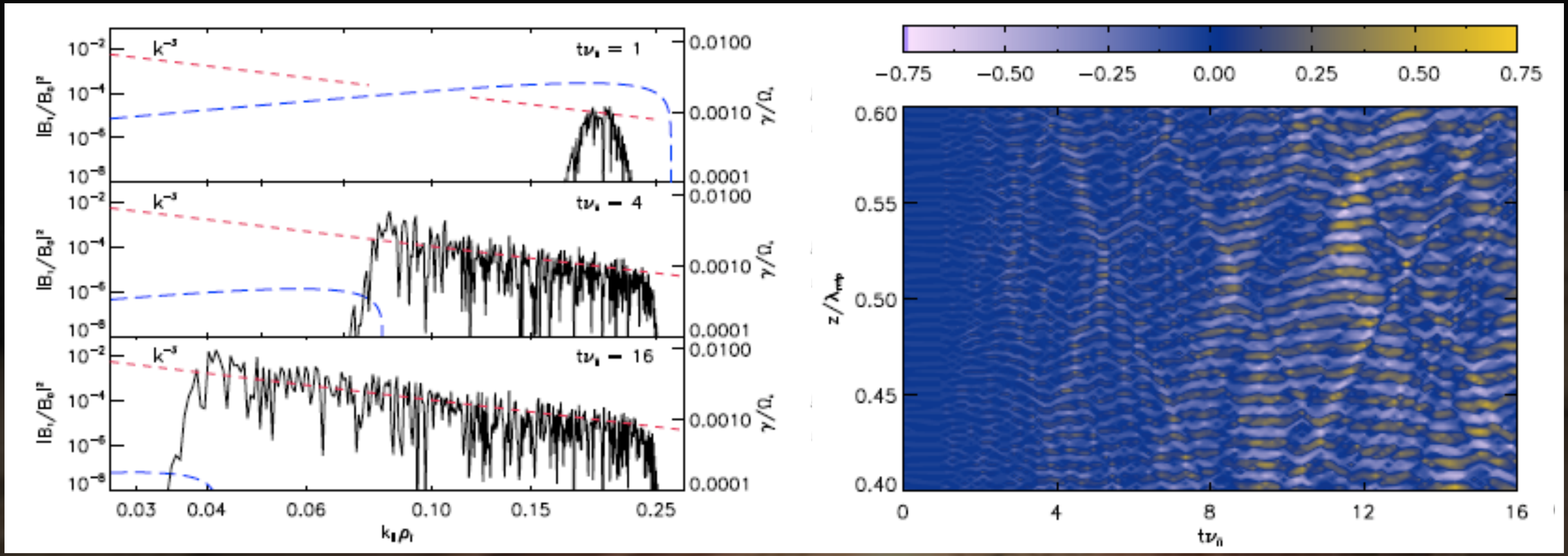
- Magnetic fluctuations ultimately reach $\delta B/B \sim 1$
  - Dynamically significant !



Nonlinear stages

- Return to marginality stabilizes small scales
- Energy spectrum with ever larger scales
  - Ultimately, loss of scale-separation

$$k_{\|}^{(\mathrm{peak})}\rho_i = 2|\Delta + 2/\beta|^{1/2} \sim 1/\sqrt{t}$$



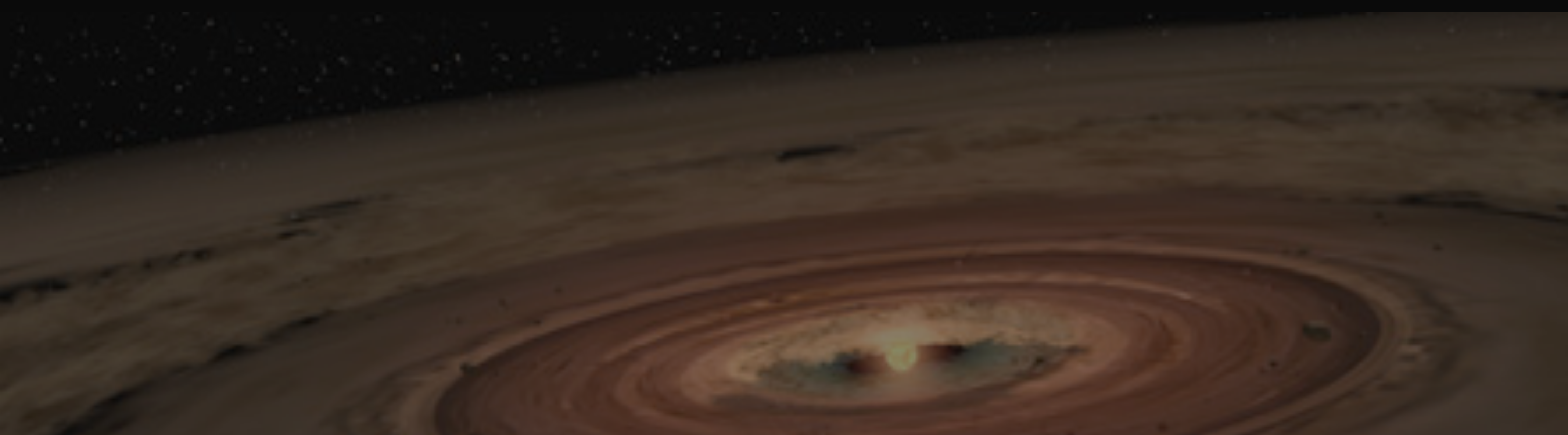
Possible dynamical consequences of FH

- Large-scale motions follow Euler equation with no field ?

$$\rho \frac{\mathrm{d}\mathbf{u}_0}{\mathrm{d}t} = -\nabla \left(p_{\perp 2} + \frac{B_0^2}{8\pi} \right) + \nabla \cdot \left[\hat{\mathbf{b}}_0 \hat{\mathbf{b}}_0 (p_{\perp 2} - p_{\parallel 2}) \right] + \frac{\mathbf{B}_0 \cdot \nabla \mathbf{B}_0}{4\pi}$$

  - Magnetic tension locally vanishes
    - some evidence for this in collisionless reconnection simulations by J. Drake...
  - Actual viscous scale far smaller than fluid estimate ?
    - large-scale motions would penetrate down to ever smaller scales
    - loss of scale separation...breakdown of the asymptotics
- But...real physics is undoubtedly much more complex
  - Other instabilities (mirror, IC, GTI) come into play
    - different polarizations, scales, thresholds...different asymptotics ?
    - how do we mix everything ?

Conclusions and future directions



Conclusions

- Accretion discs display a wide range of physical phenomena
  - Some of them are now quite well understood
    - basic phenomenology and instabilities, disc structure, radiation processes
  - Some of them are only partly or even very crudely understood
    - nonlinear MRI dynamics, saturation and transport: fluid & kinetic
    - magnetic field generation
    - heating, accretion efficiency, luminosity
- Exciting new developments in the past ten years
  - MRI turbulence and dynamo
    - parametric dependences, saturation, transport, dynamo threshold
    - coherent MRI dynamo structures & subcritical dynamos in shear flows
  - Groundbreaking ideas on the plasma physics of hot accretion flows
    - role of pressure anisotropy, MVI, kinetic MRI, heating processes

The future of MHD research on discs

- MRI turbulence and transport
  - Look for asymptotic scalings for nonlinear saturation and transport
    - high resolution DNS, LES, spectral energy transfers, theory ?
  - Coupling of large-scale models and 3D DNS/LES in a self-consistent way
    - we still don't know if an alpha prescription quantitatively makes sense
- Coherent MRI dynamo structures
  - The backbone of MRI turbulence ?
    - may be key to understand better parametric dependences and transport
    - interactions with fluctuation dynamo ?
  - A possible guide to understand dynamo action in stellar environments
    - magnetic buoyancy dynamo, Spruit dynamo, magnetoshear dynamo

The future of plasma research on discs

- Hard work on the dynamics of β>1 magnetised plasmas
  - Dynamical and thermodynamic impact of Larmor-scale instabilities
  - Experiments: Cary@Wisconsin
  - More elaborate simulations: what is the most promising avenue ?
    - PIC codes, Landau fluids models, gyrokinetics ?
- Various astrophysical playgrounds for this research
  - Accretion in AGNs: heating & luminosity [Sharma et al., ApJ 667, 714 (2007)]
  - Galaxy clusters: ICM thermal structure [Kunz & Schekochihin, MNRAS 410, 2446 (2011)]
  - Magnetic field generation in the early Universe [Schekochihin & Cowley, Ast. Nach. 327, 599 (2006]
  - Magnetic structures in the solar wind [Califano et al., JGR 113, A08219 (2008)]

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  - Sharma et al., ApJ 667, 714 (2007)