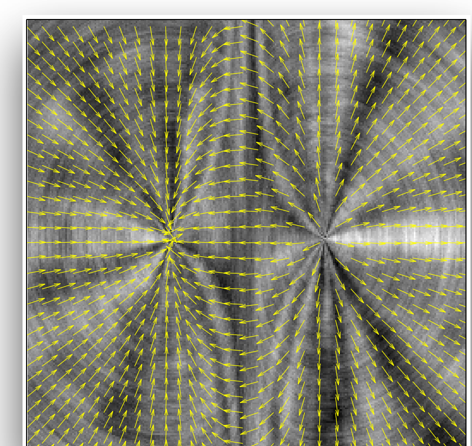
Outils pour le traitement d'image

- Synthèse de texture (2013 […] 2015)
- Super-résolution de lignes (2014-2015)
- Analyse de textures par TO monogène (2016)
- Super-résolution de sinusoïdes par TO monogène (2016)

Thèse de **Kévin Polisano** (LJK)
Marianne Clausel, Valérie Perrier (LJK)
Laurent Condat (Gipsa-Lab)

→ **TEXTURE MODELING BY GAUSSIAN FIELDS WITH PRESCRIBED LOCAL ORIENTATION**



ANR ASTRES - ENS Lyon, le 19 novembre 2015

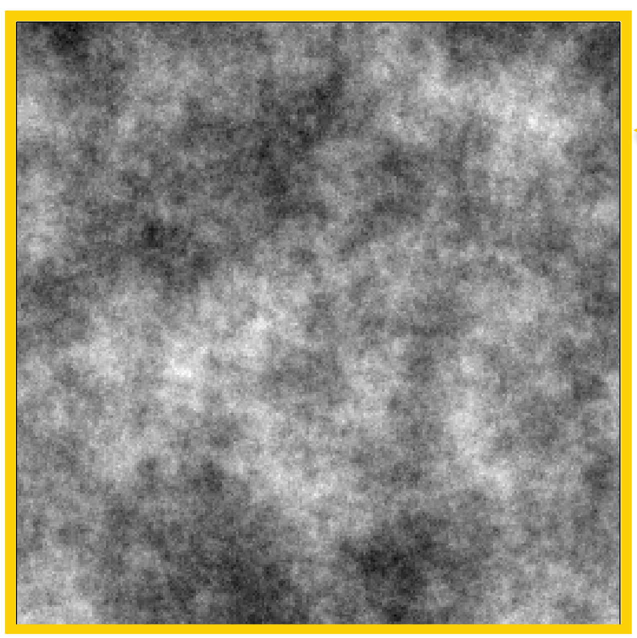
The basic component : Fractional Brownian Field (FBF)

- Harmonizable representation [Samorodnitsky,Taqqu,1997]

$$B^H(\mathbf{x}) = \int_{\mathbb{R}^2} \frac{e^{i\mathbf{x}\cdot\xi} - 1}{\|\xi\|^{H+1}} \, d\widehat{W}(\xi)$$

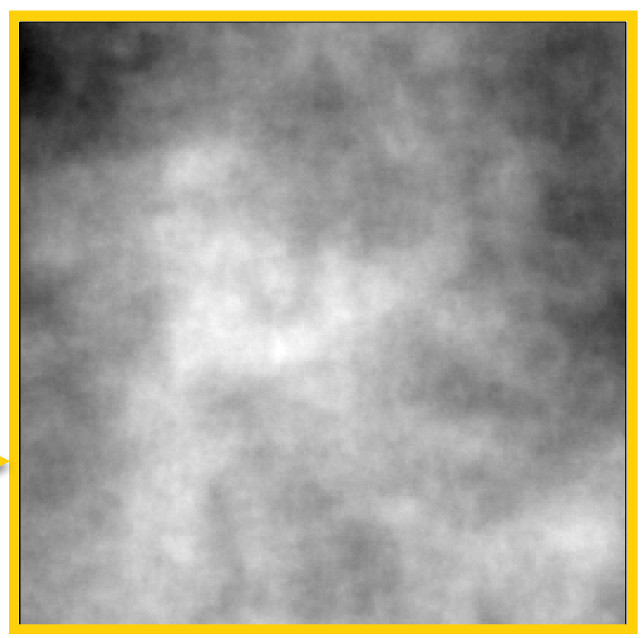
roughness indicator

complex Brownian measure



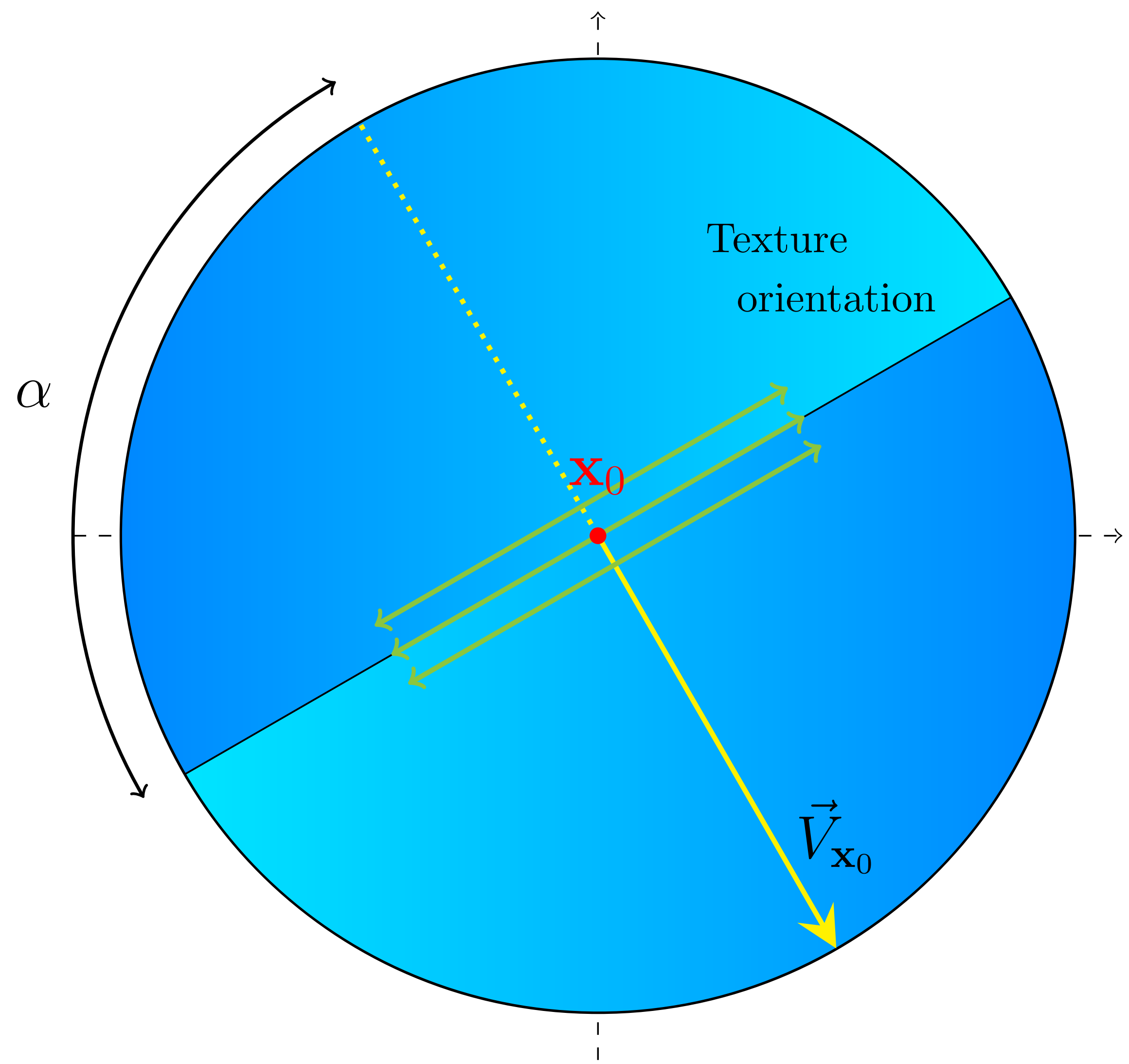
H=0.2

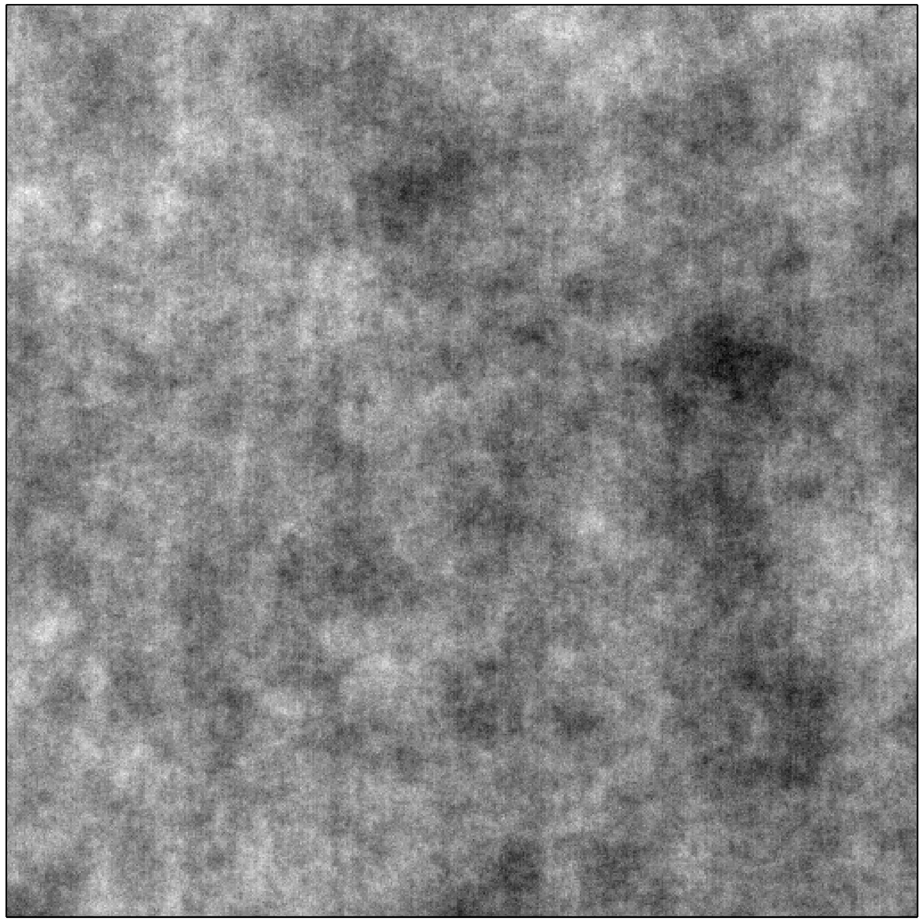
H=0.7



Elementary field (Bonami-Estrade 2003, Biermé-Moisan- Richard 2015)

$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1)\frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0)}{\|\xi\|^{H+1}} d\widehat{W}(\xi)$$

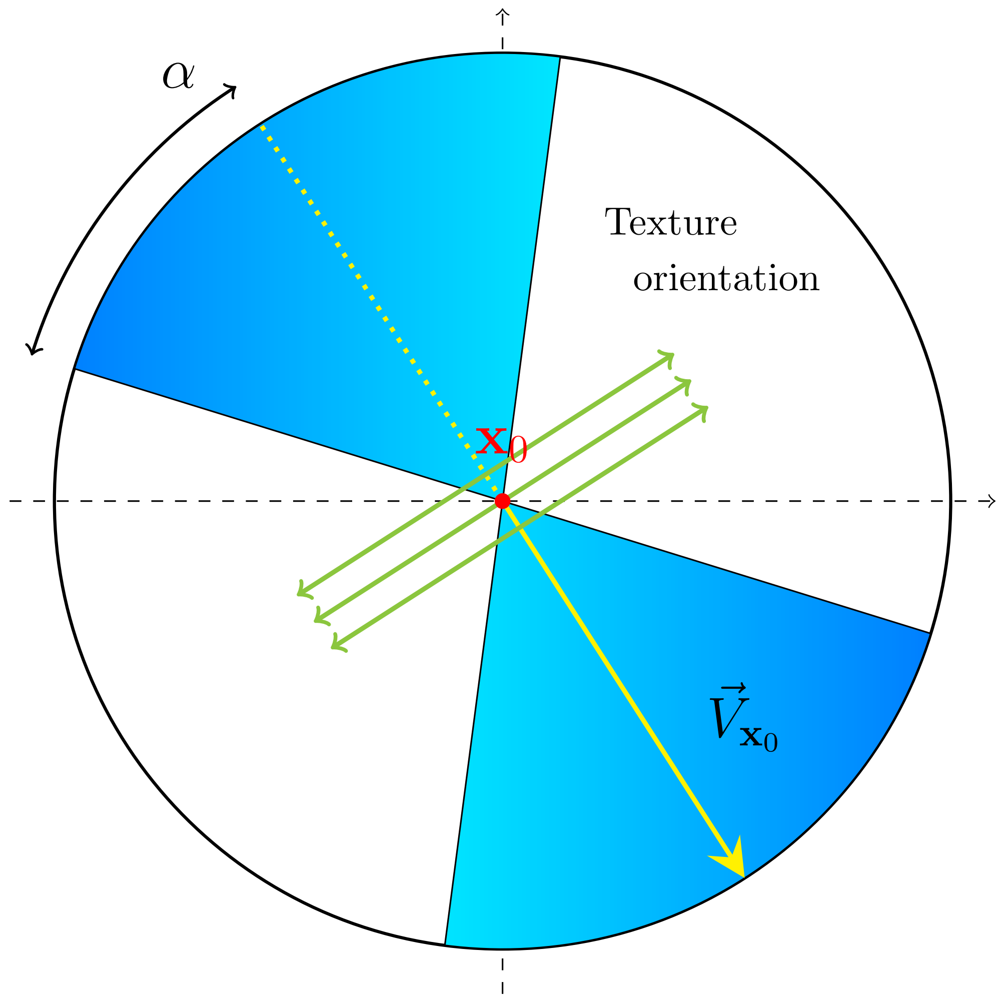


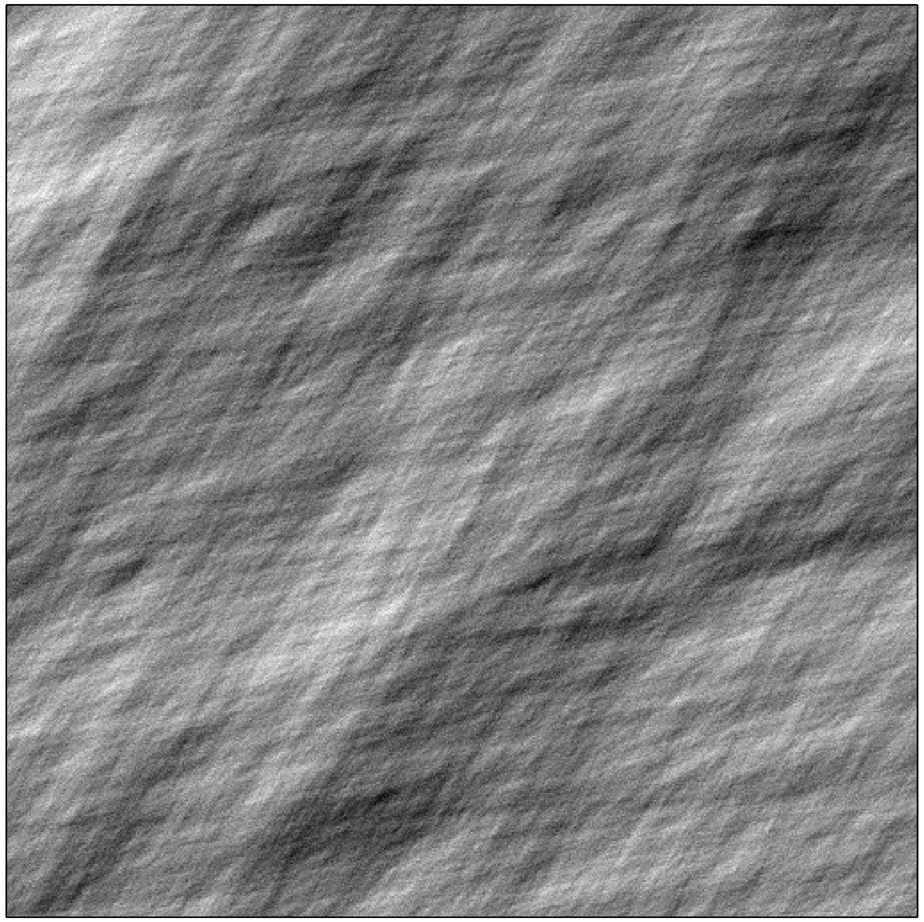


$$\alpha_0 = 0 \qquad \alpha = -\frac{\pi}{2}$$

Elementary field

$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\boldsymbol{\xi}} - 1) \frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg \boldsymbol{\xi} - \alpha_0)}{\|\boldsymbol{\xi}\|^{H+1}} d\widehat{W}(\boldsymbol{\xi})$$

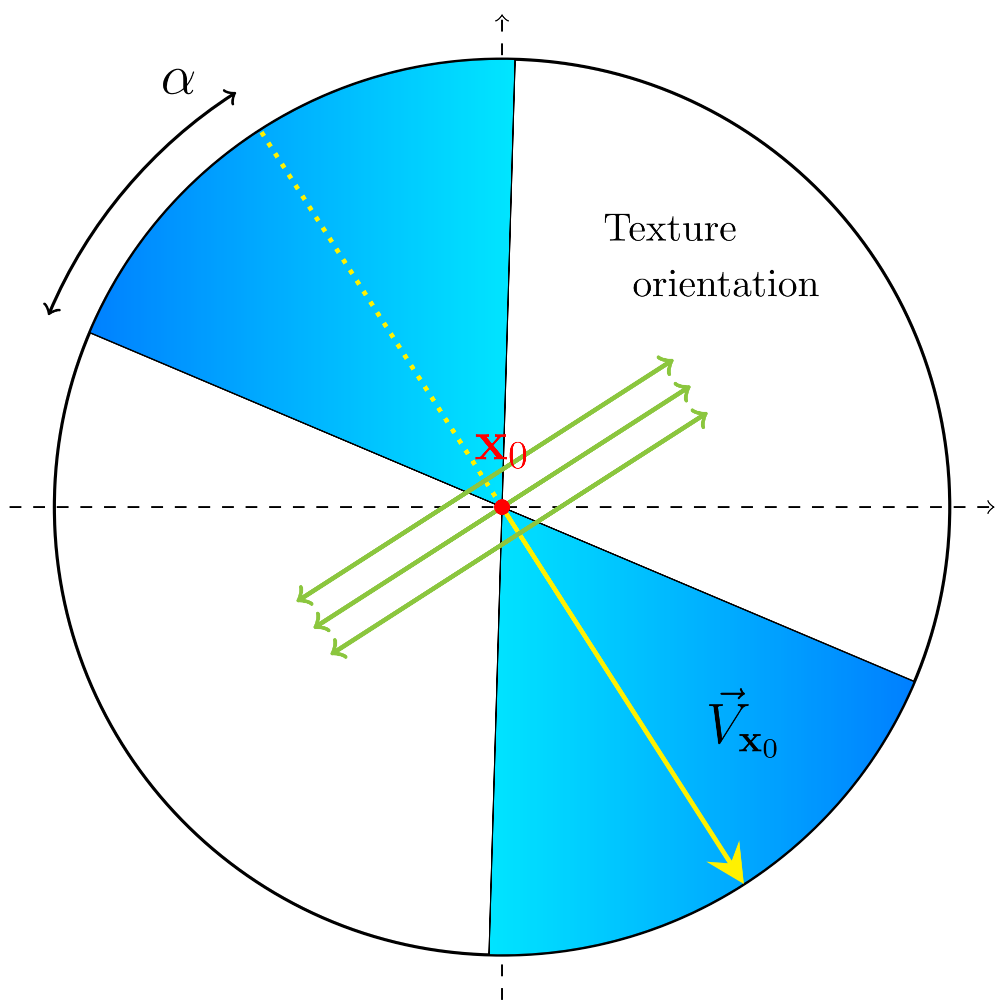


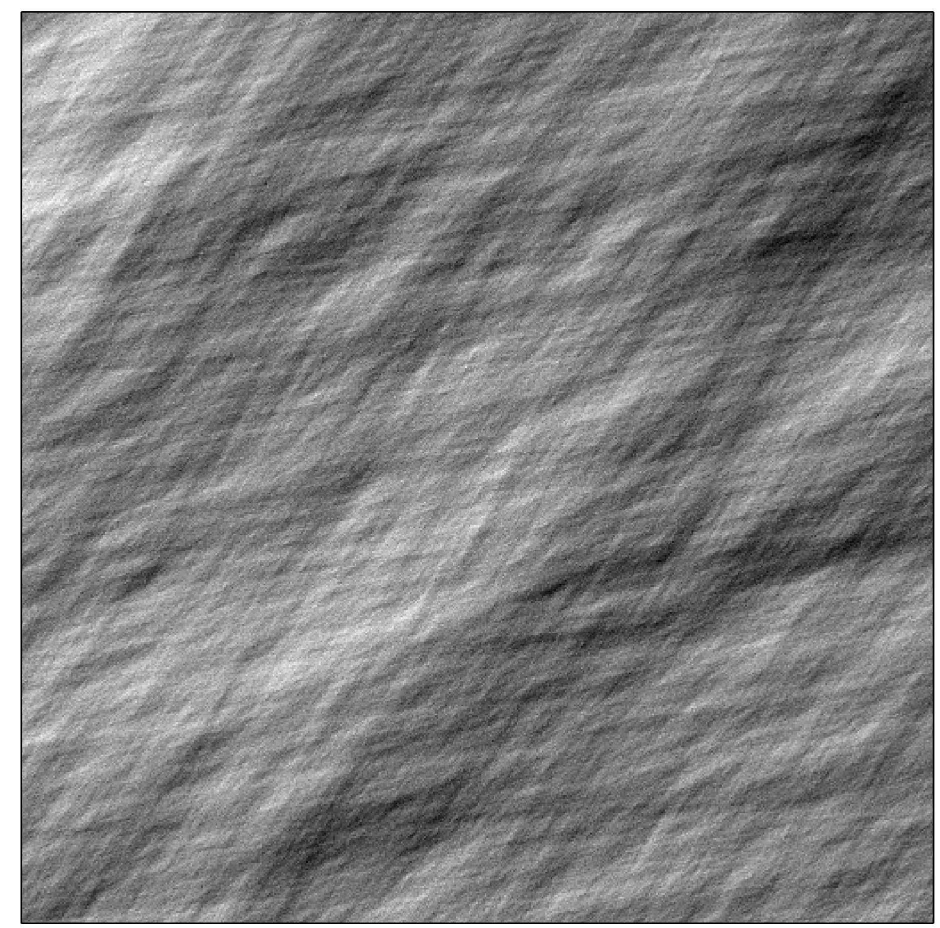


$$\alpha_0 = -\frac{\pi}{3} \qquad \alpha = 0.7$$

Elementary field

$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) \frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg \xi - \alpha_0)}{\|\xi\|^{H+1}} d\widehat{W}(\xi)$$

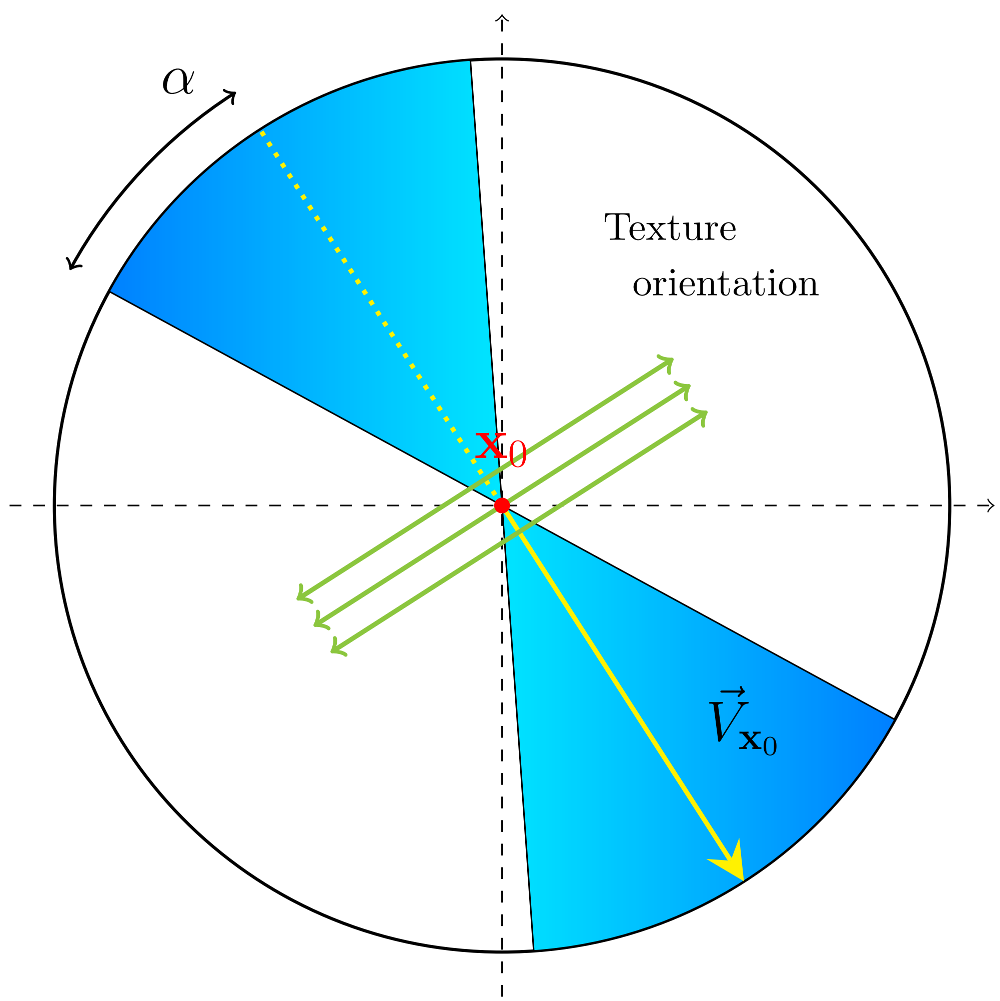


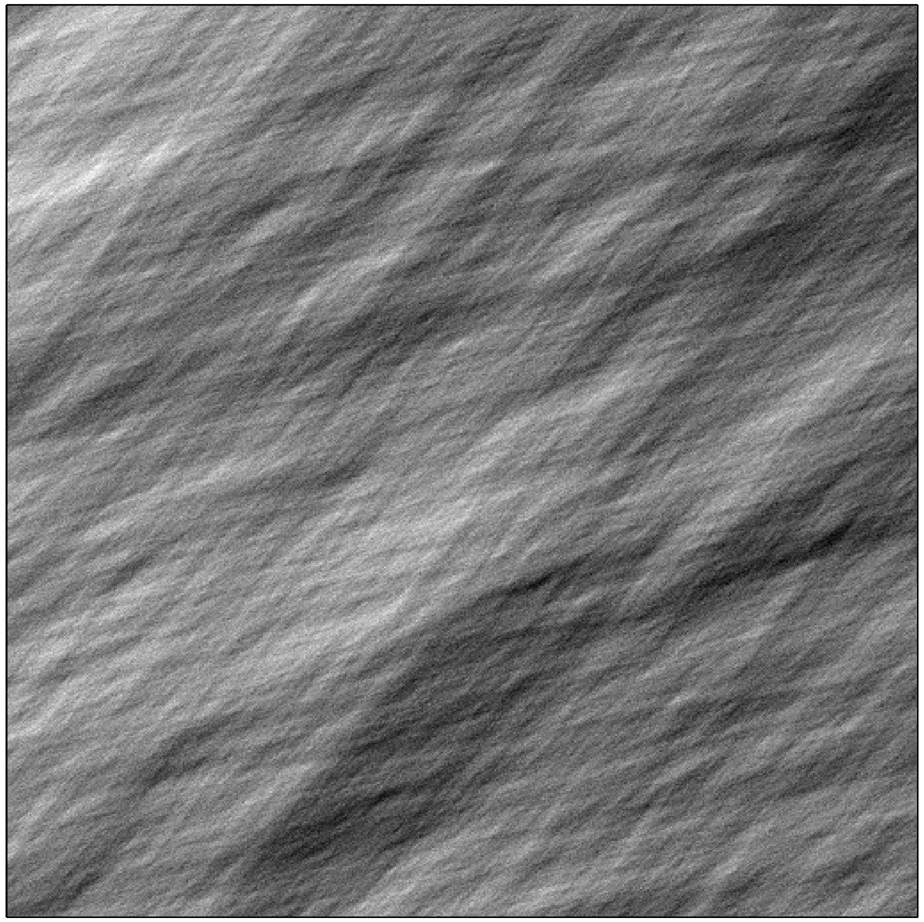


$$\alpha_0 = -\frac{\pi}{3} \qquad \alpha = 0.6$$

Elementary field

$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1)\frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0)}{\|\xi\|^{H+1}} d\widehat{W}(\xi)$$

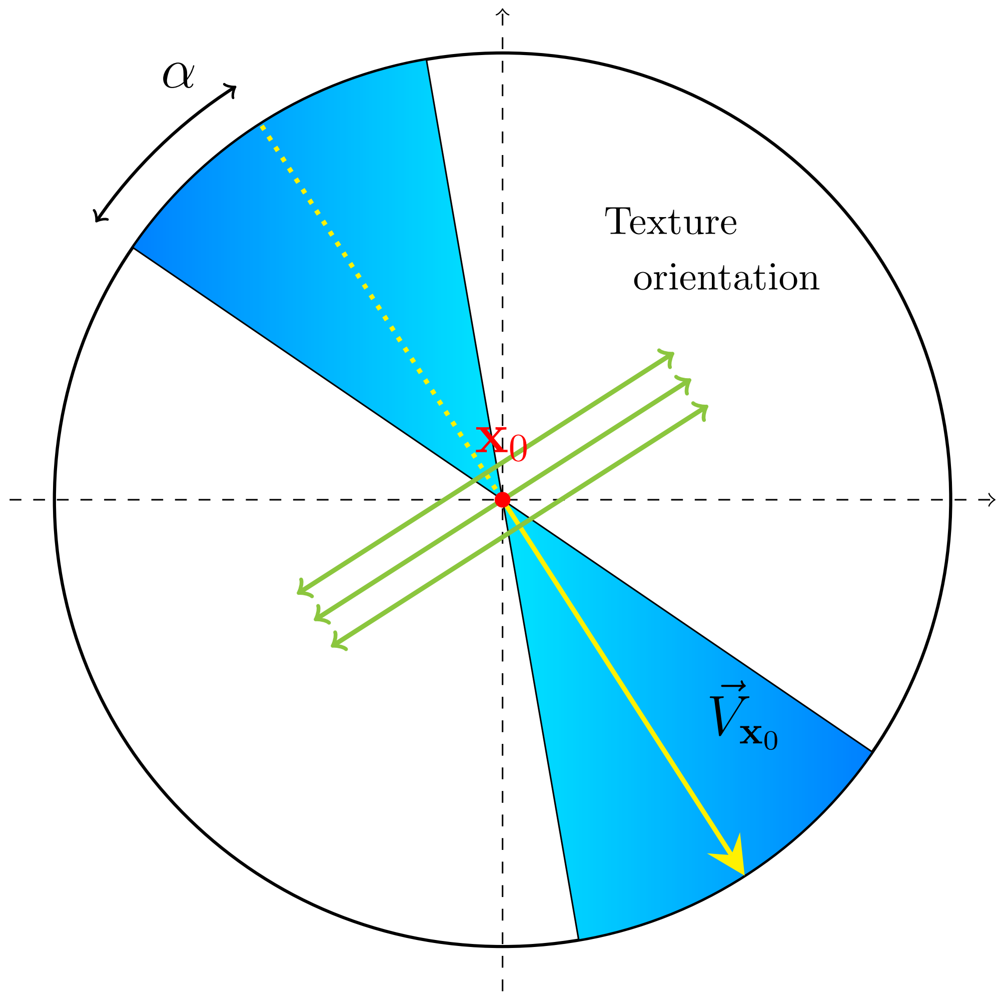


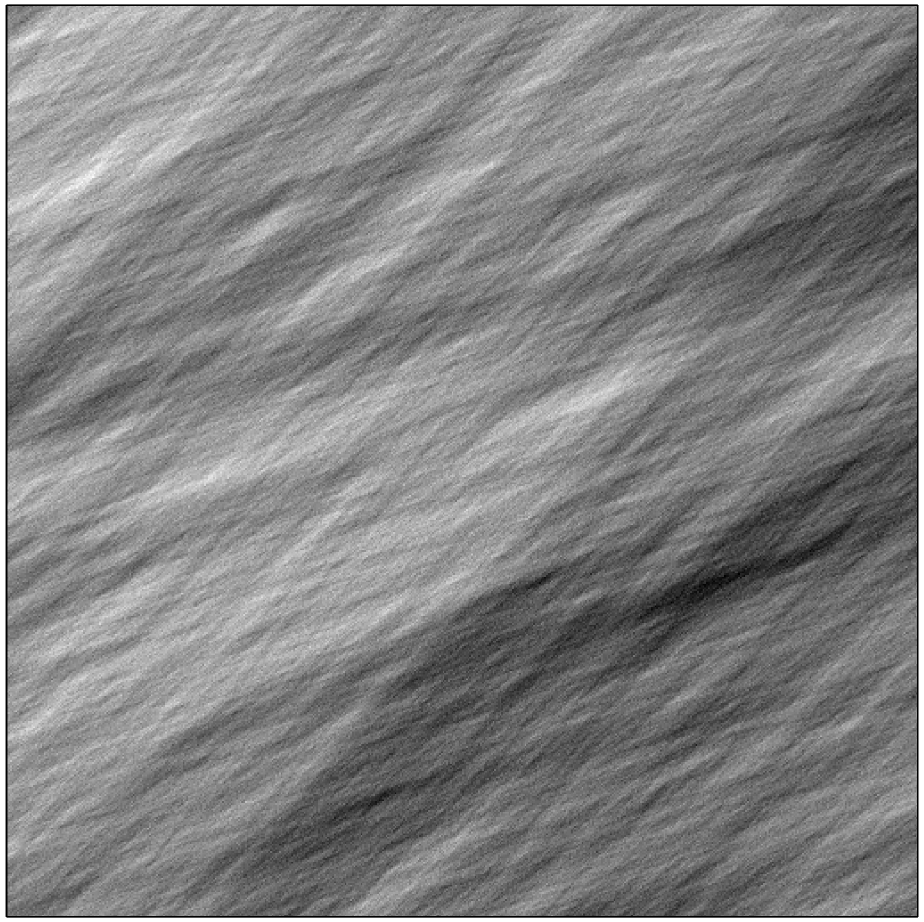


$$\alpha_0 = -\frac{\pi}{3} \qquad \alpha = 0.5$$

Elementary field

$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1)\frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0)}{\|\xi\|^{H+1}} d\widehat{W}(\xi)$$

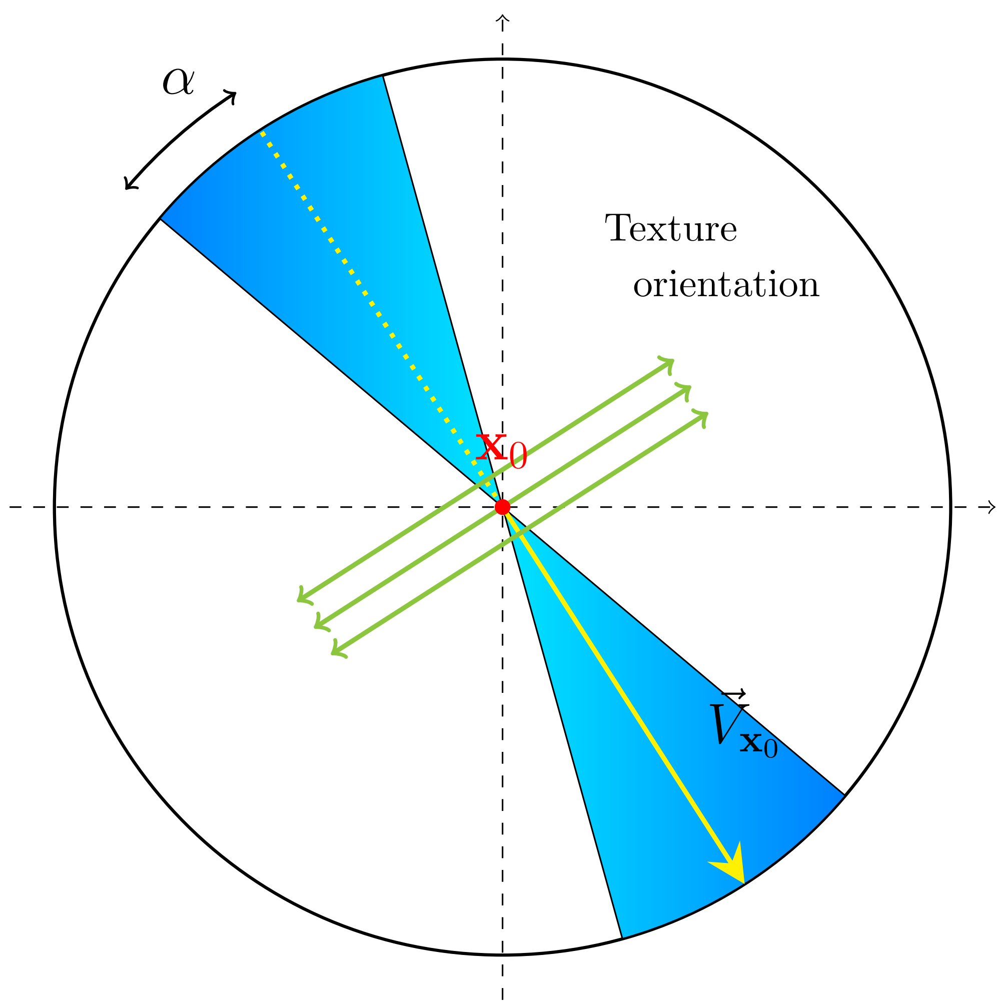




$$\alpha_0 = -\frac{\pi}{3} \quad \alpha = 0.4$$

Elementary field

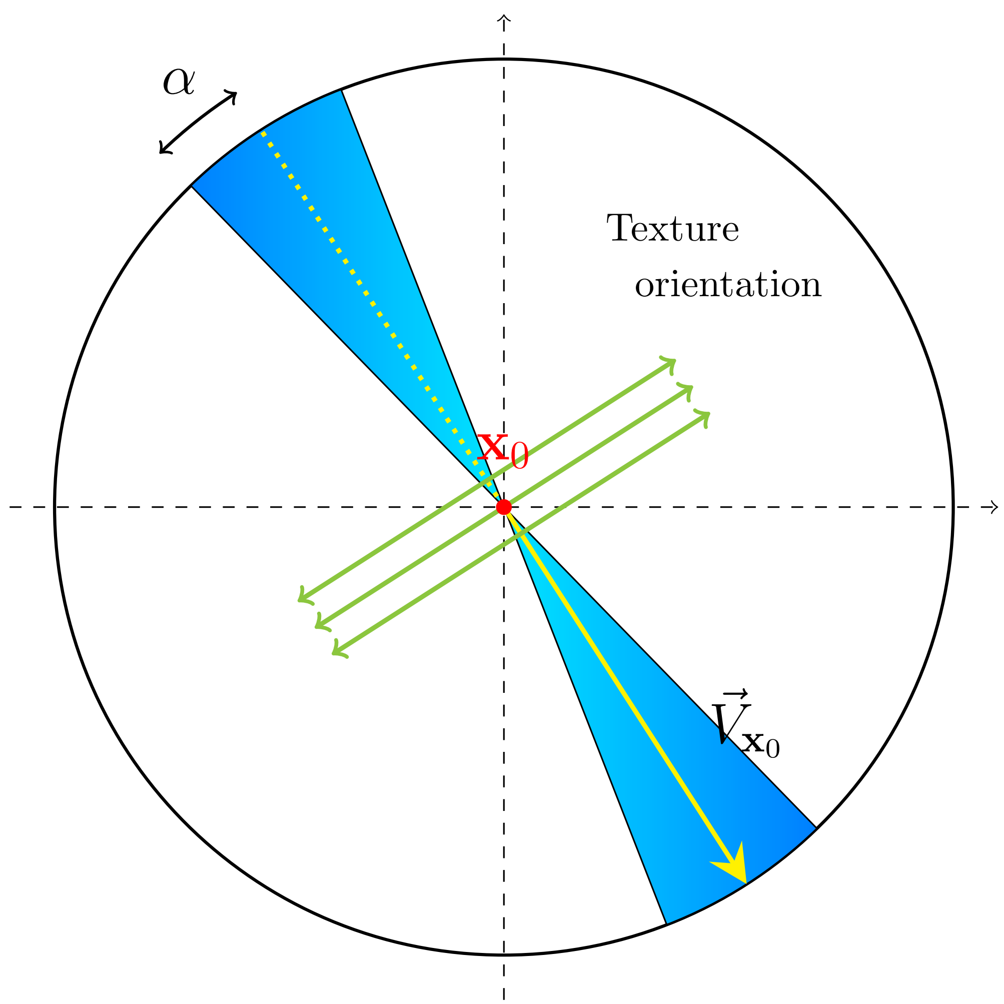
$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) \frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg \xi - \alpha_0)}{\|\xi\|^{H+1}} d\widehat{W}(\xi)$$

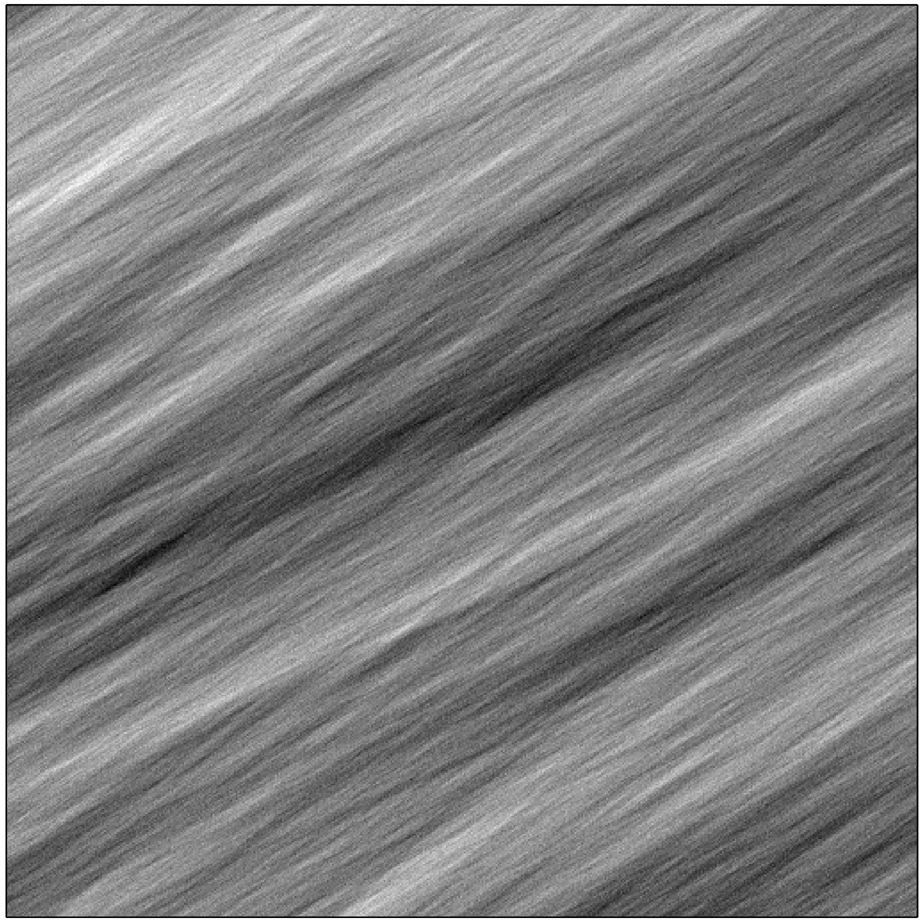


$$\alpha_0 = -\frac{\pi}{3} \qquad \alpha = 0.3$$

Elementary field

$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1)\frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0)}{\|\xi\|^{H+1}} d\widehat{W}(\xi)$$

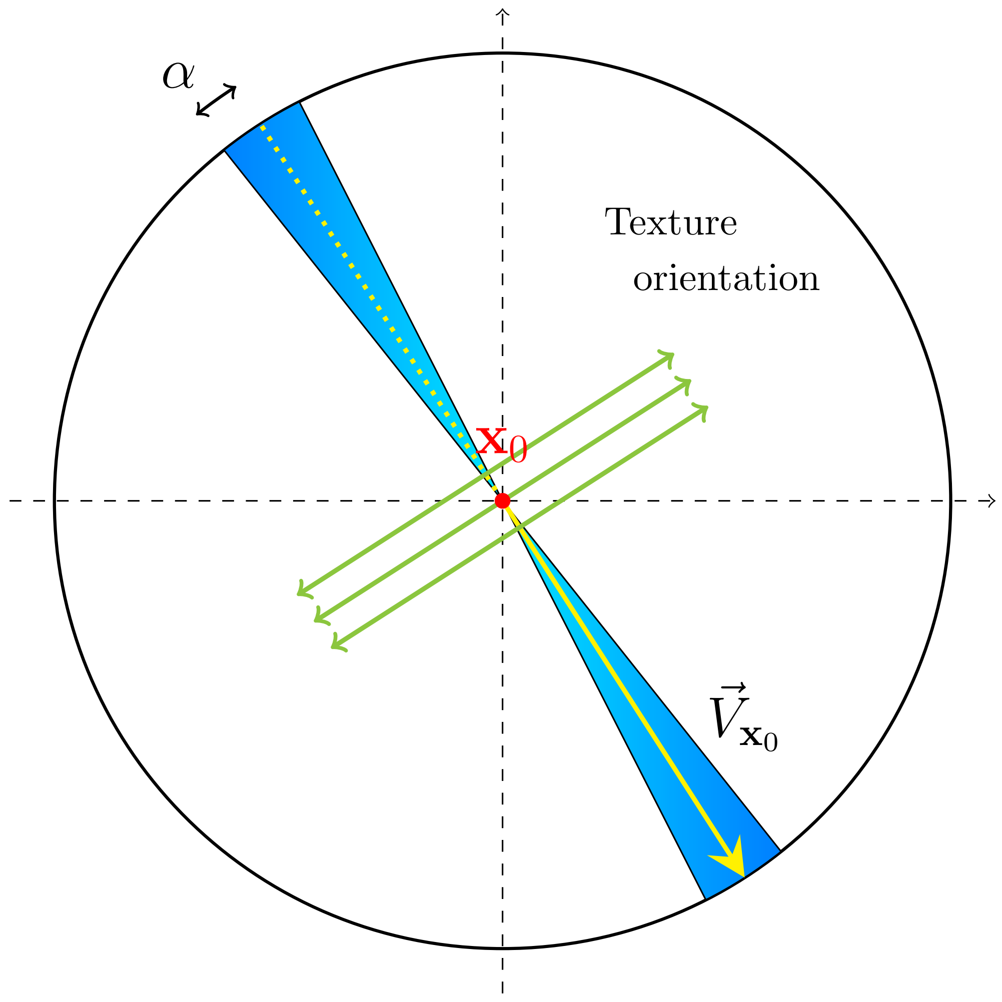


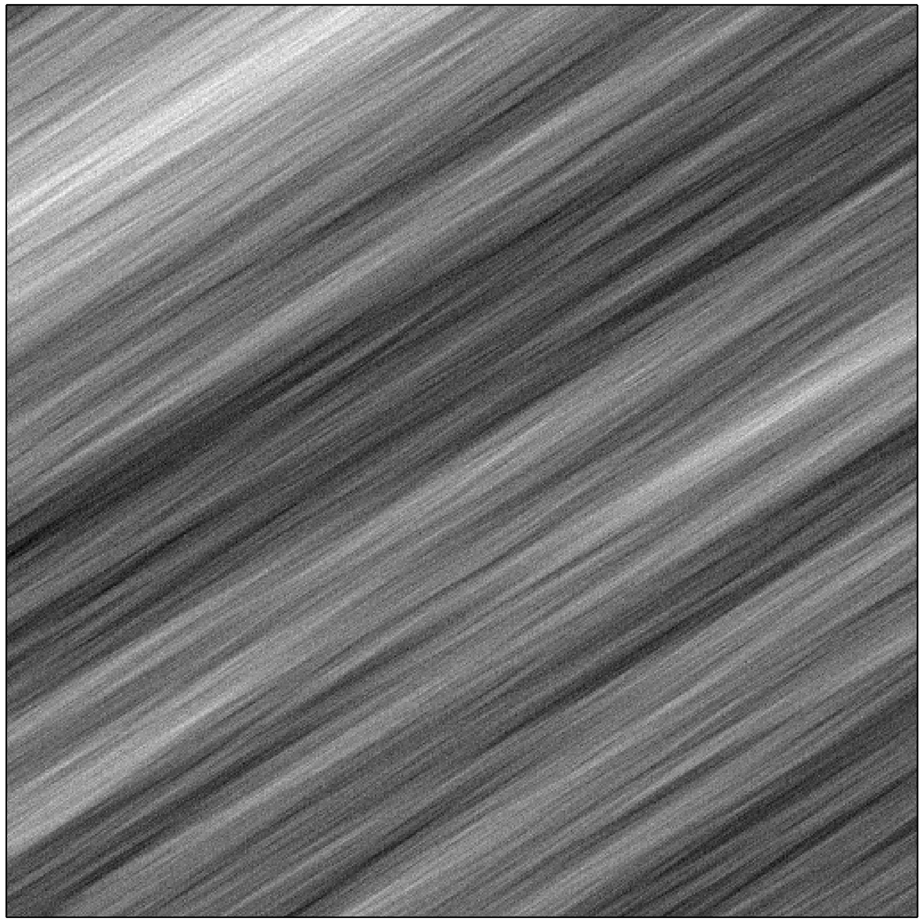


$$\alpha_0 = -\frac{\pi}{3} \qquad \alpha = 0.2$$

Elementary field

$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1)\frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0)}{\|\xi\|^{H+1}} d\widehat{W}(\xi)$$

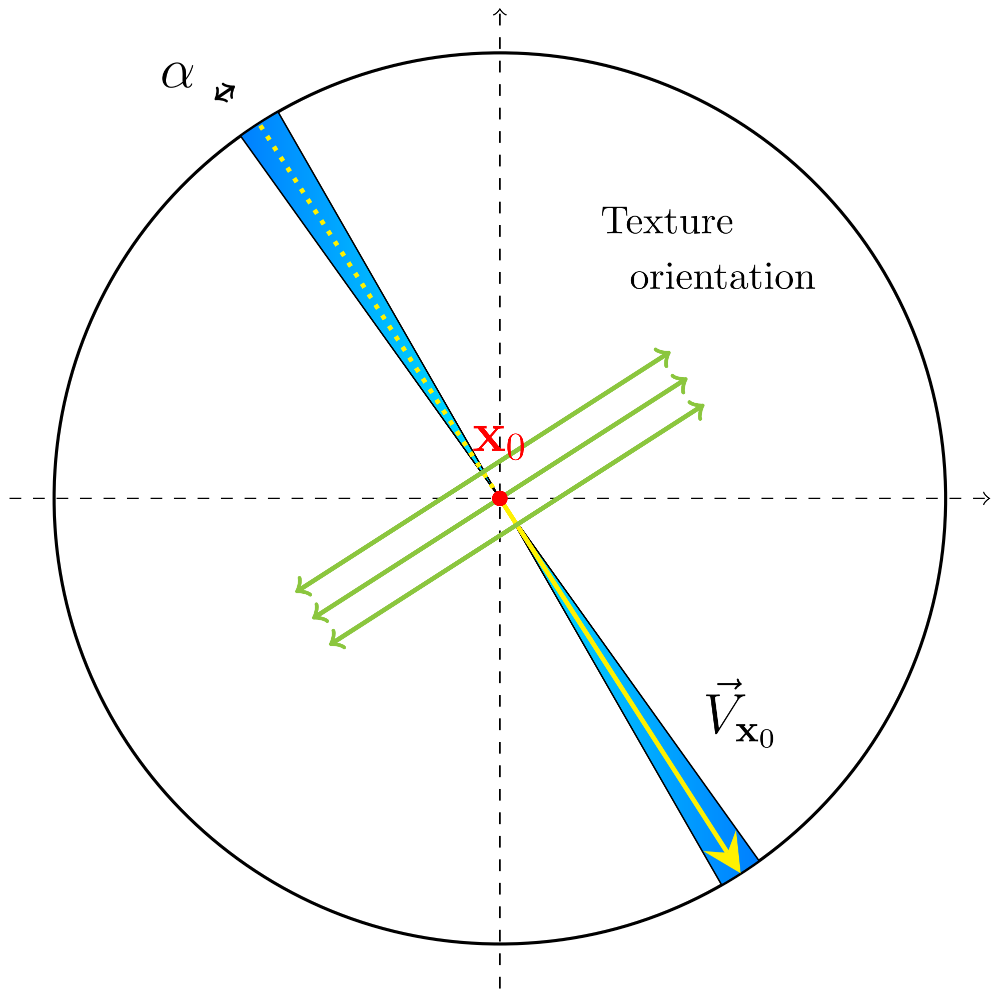


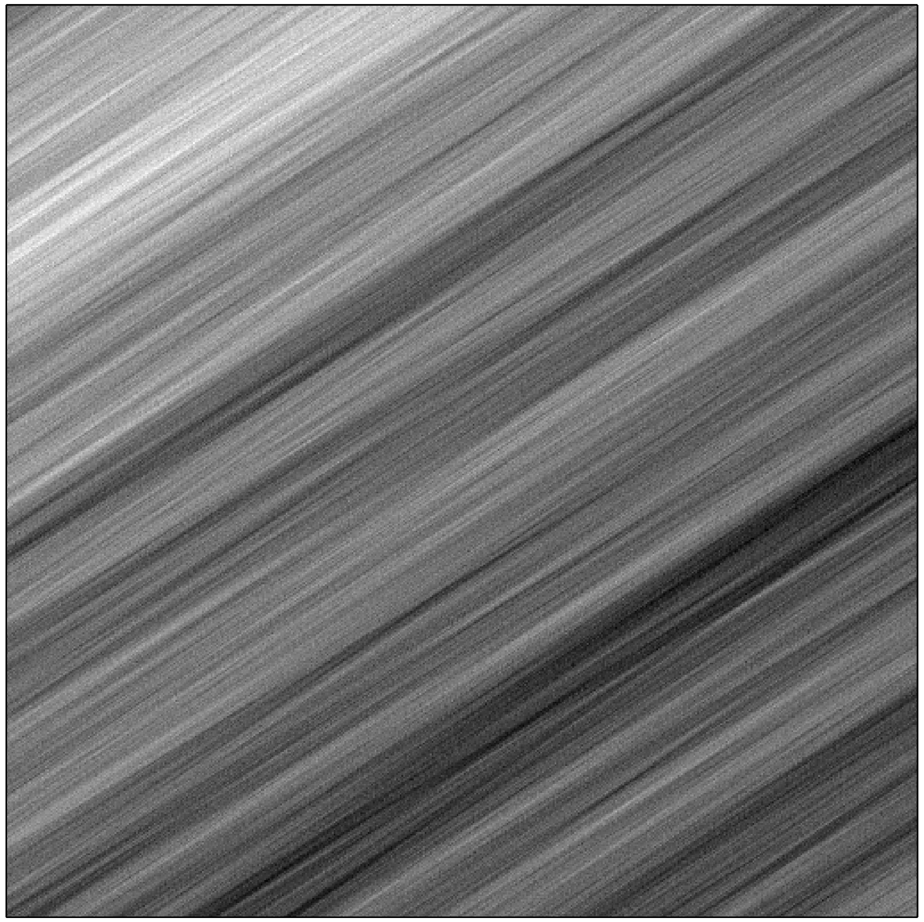


$$\alpha_0 = -\frac{\pi}{3} \qquad \alpha = 0.1$$

Elementary field

$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) \frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg \xi - \alpha_0)}{\|\xi\|^{H+1}} d\widehat{W}(\xi)$$





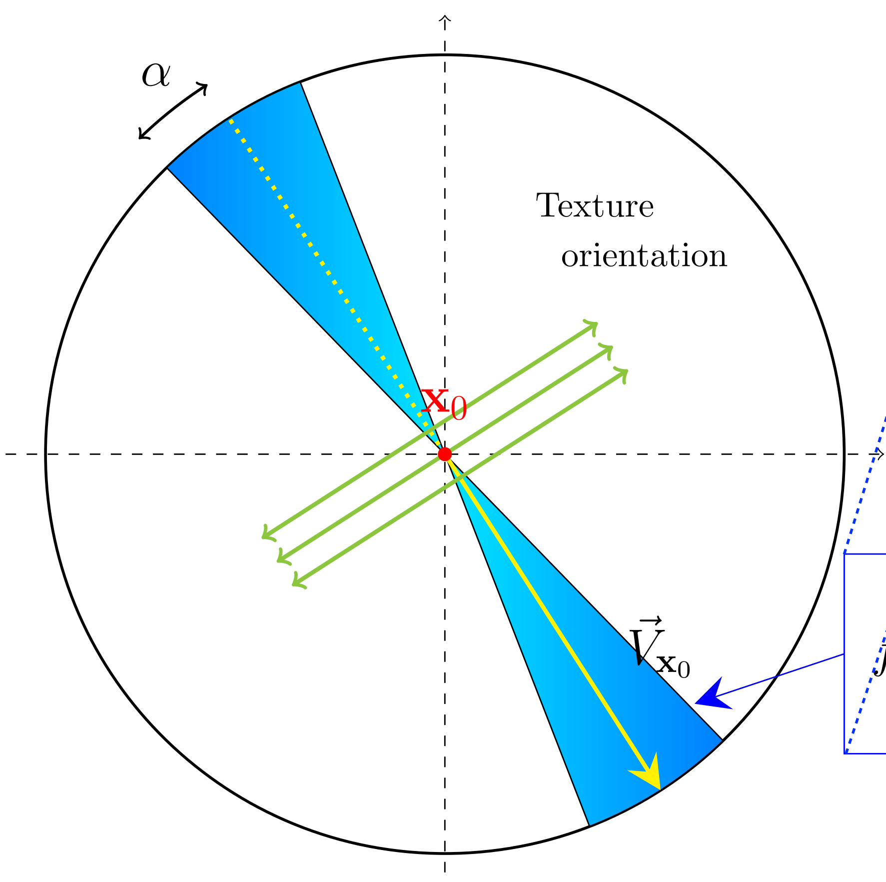
$$\alpha_0 = -\frac{\pi}{3} \qquad \alpha = 0.05$$

Locally Anisotropic Fractional Brownian Field (LAFBF)

- **Definition:** Our new Gaussian model LAFBF is a local version of the elementary field

$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} \left(e^{i\mathbf{x}\cdot\boldsymbol{\xi}} - 1\right) \frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg\boldsymbol{\xi} - \alpha_0(\mathbf{x}))}{\|\boldsymbol{\xi}\|^{H+1}} \, d\widehat{W}(\boldsymbol{\xi})$$

[Polisano et al.,2014]



$$f^{1/2}(\mathbf{x}_0,\boldsymbol{\xi}) = \frac{c_{\alpha_0,\alpha}(\mathbf{x}_0,\arg\boldsymbol{\xi})}{\|\boldsymbol{\xi}\|^{H+1}}$$

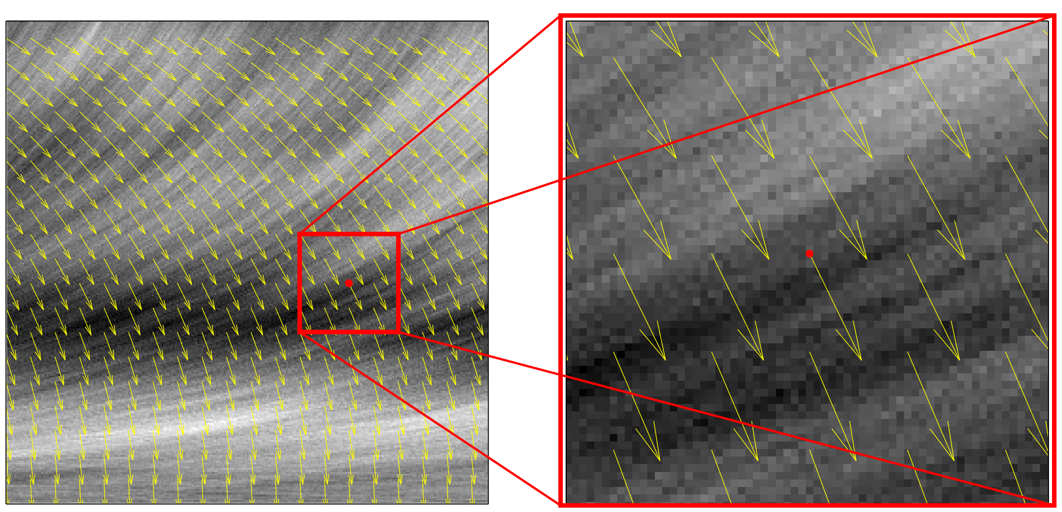
The orientation may varies spatially. α_0 is now a differentiable function on $\mathbb{R}^2$

Locally Anisotropic Fractional Brownian Field (LAFBF)

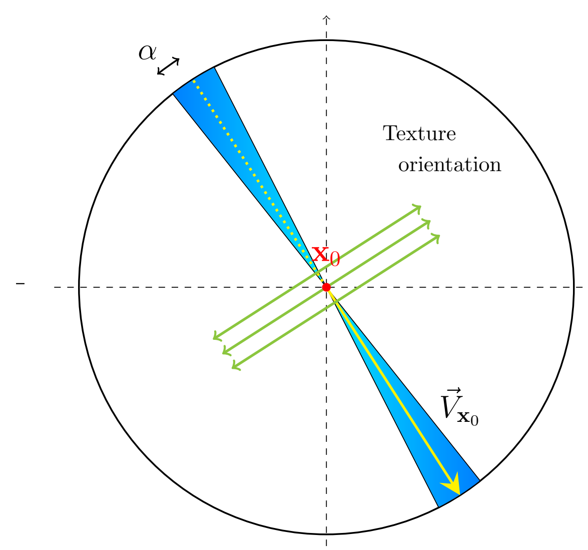
- **Definition:** Our new Gaussian model LAFBF is a local version of the elementary field

$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1)\frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg \xi - \alpha_0(\mathbf{x}))}{\|\xi\|^{H+1}} d\widehat{W}(\xi)$$

[Polisano et al.,2014]



(a) (b)



Tangent field

$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) \frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0(\mathbf{x}))}{\|\xi\|^{H+1}} d\widehat{W}(\xi)$$

Tangent field.

For a random field X locally asymptotically self-similar of order H,

$$\frac{X(\mathbf{x}_0 + \rho\mathbf{h}) - X(\mathbf{x}_0)}{\rho^H} \xrightarrow[\rho\to 0]{\mathcal{L}} Y_{\mathbf{x}_0}$$

→ $Y_{\mathbf{x}_0}$: tangent field of X at point $\mathbf{x}_0 \in \mathbb{R}^2$

[Benassi,1997]
[Falconer,2002]

Taylor's expansion ⟷ Tangent field

Deterministic case — *Stochastic case*

Tangent field

$$B^H_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1)\frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0(\mathbf{x}))}{\|\xi\|^{H+1}} d\widehat{W}(\xi)$$

■ **Theorem.** The LAFBF $B^H_{\alpha_0,\alpha}$ admits for tangent field $Y_{\mathbf{x}_0}$:

$$Y_{\mathbf{x}_0}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1)\frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0(\mathbf{x}_0))}{\|\xi\|^{H+1}} d\widehat{W}(\xi)$$

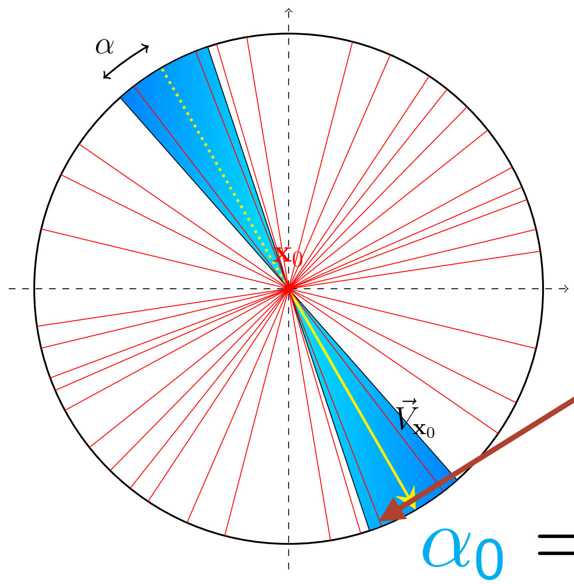
$\alpha_0(\mathbf{x}_0)$ → constant

→ $Y_{\mathbf{x}_0}$ *elementary field* with global orientation $\alpha_0(\mathbf{x}_0)$

⇨ $B^H_{\alpha_0,\alpha}(\mathbf{x}_0) \approx Y_{\mathbf{x}_0}(x = \mathbf{x}_0)$

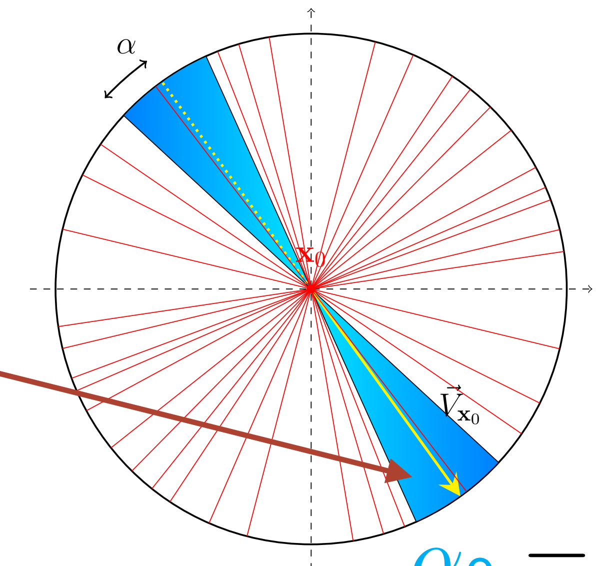
Elementary field : simulation by turning bands

$$Y_{\mathbf{x}_0}{}^{[n]}(\mathbf{x}) = \gamma(H)^{\frac{1}{2}} \sum_{i=1}^{n} \sqrt{\lambda_i c_{\alpha_0,\alpha}(\mathbf{x}_0, \theta_i)} B_i^H(\mathbf{x} \cdot \mathbf{u}(\theta_i))$$

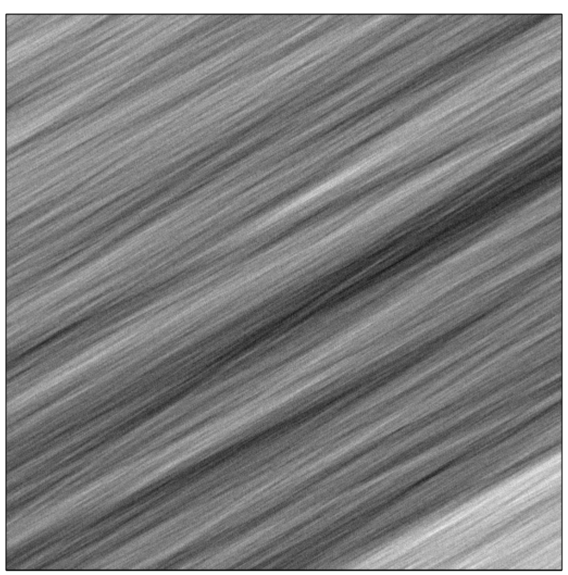


$$\alpha_0 = -\frac{\pi}{3}$$

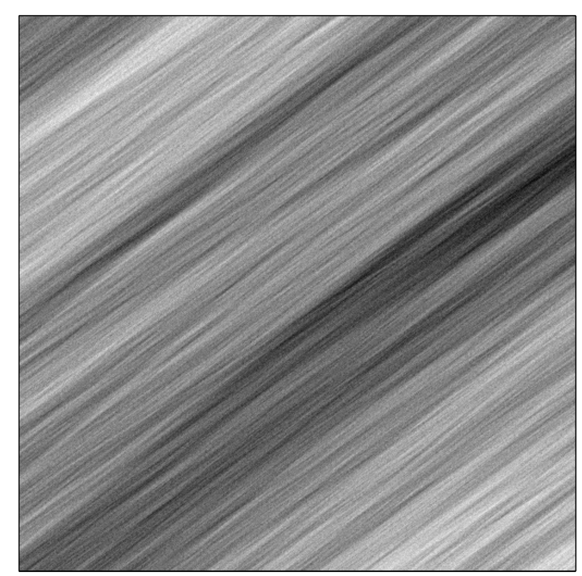
Different bands (1D FBM) are implied in the sum



$$\alpha_0 = -\frac{\pi}{3} + 0.1$$



The grayscale repartition completely changes between two close angles



$$\alpha = 0.1$$

LAFBF simulation by tangent fields [T.B]

$B^{H}_{\alpha_0,\alpha}(\mathbf{x}_0) \approx Y_{\mathbf{x}_0}(x = \mathbf{x}_0)$

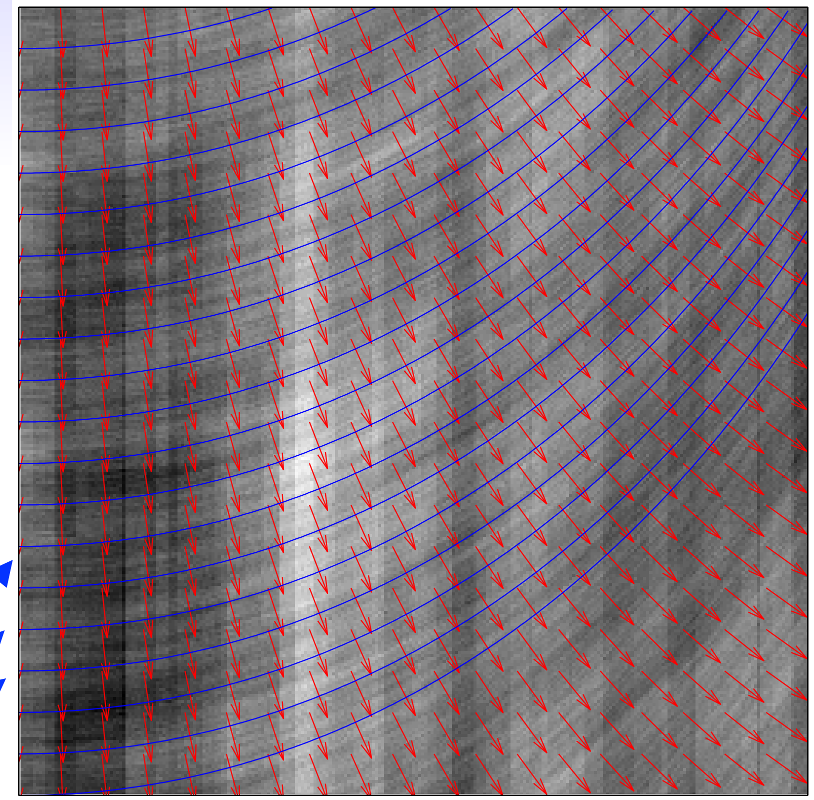
Vector field :

$$\alpha_0(x) = -\frac{\pi}{2} + x$$

Orientation curves :

$$\ln\left|\frac{1}{\cos x}\right| + y_0$$

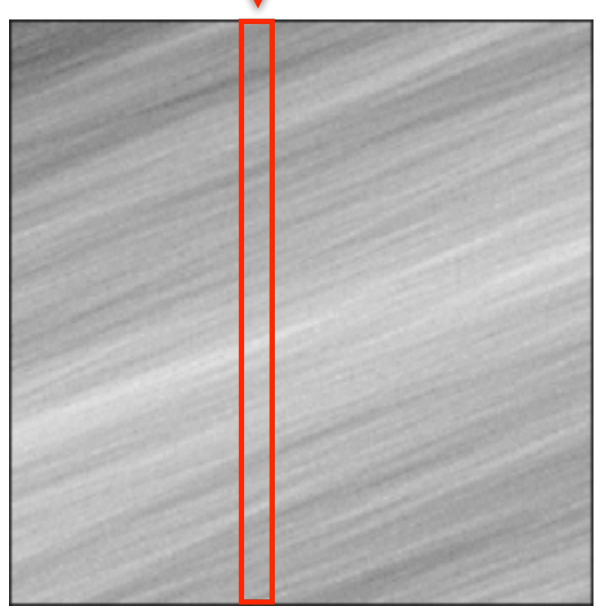
(y’(x)=tan α(x))

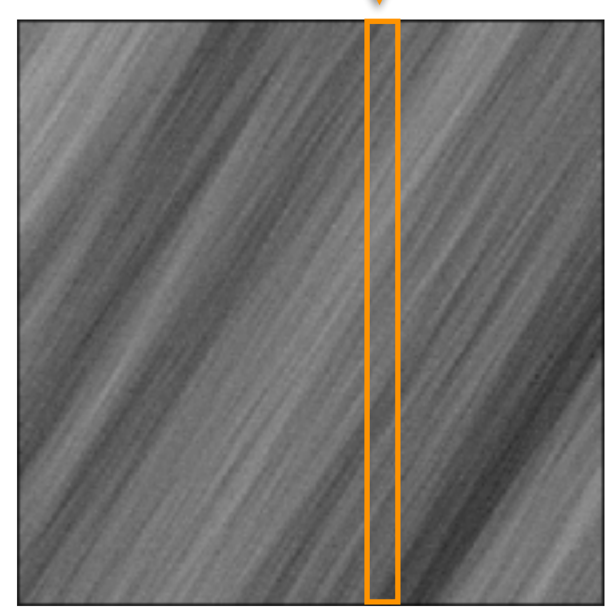


The texture is properly oriented

We observe bands artifacts

$\alpha_0 = -67°$





$\alpha_0 = -55°$

The grayscale repartition completely changes between two close angles

Elementary field : simulation by Cholesky

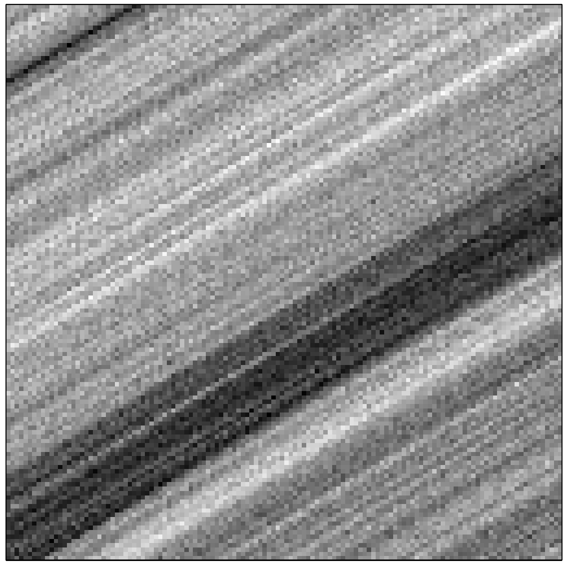
- **Variance and covariance of an elementary field.**

$$v_{H,\theta_1,\theta_2}(x) = 2^{2H-1}\gamma(H) C_{H,\theta_1,\theta_2}(\arg x)\|x\|^{2H}$$

$$\mathrm{Cov}\left(Y_{H,\theta_1,\theta_2}(x), Y_{H,\theta_1,\theta_2}(y)\right) = v_{H,\theta_1,\theta_2}(x) + v_{H,\theta_1,\theta_2}(y) - v_{H,\theta_1,\theta_2}(x-y)$$

- **Cholesky method.** $\Sigma = LL^T \quad Z \sim \mathcal{N}(0,1) \quad Y \leftarrow LZ$

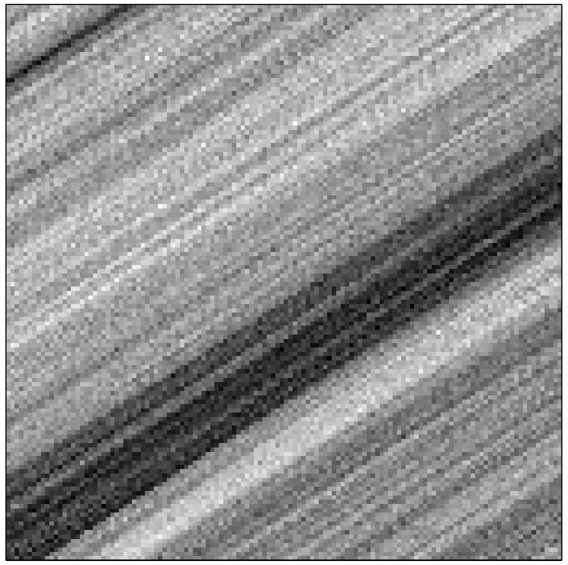
[Drawback : very time and memory consuming]



$\alpha_0 = -\frac{\pi}{3}$

The grayscale repartition is now the quite the same for two close angles

$\alpha = 0.1$

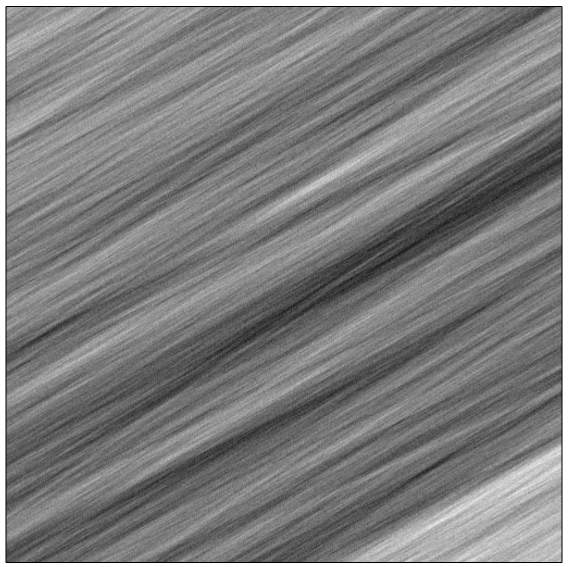


$\alpha_0 = -\frac{\pi}{3} + 0.1$

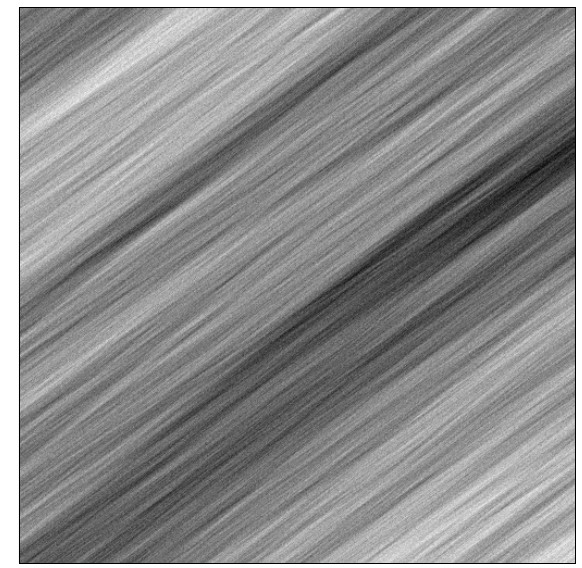
use the same random vector for the computation of all tangent field Y

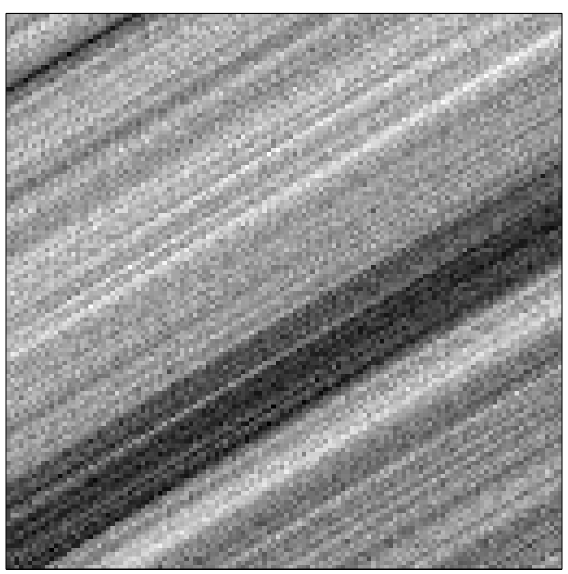
Elementary field simulation

Comparison Turning bands VS. Cholesky method

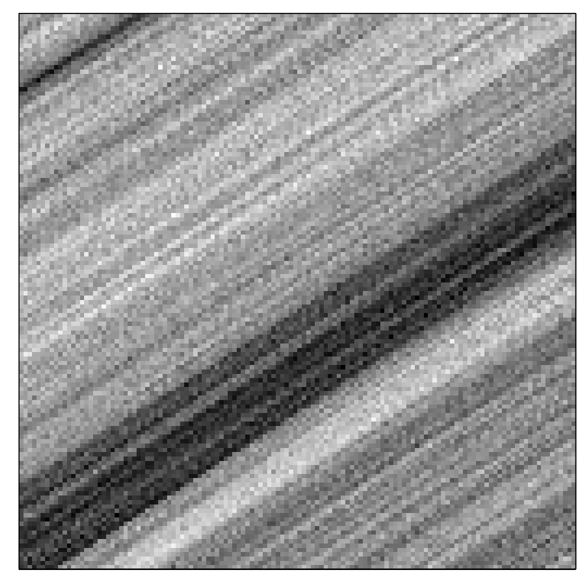


The grayscale repartition completely changes between two close angles





The grayscale repartition is now the quite the same for two close angles



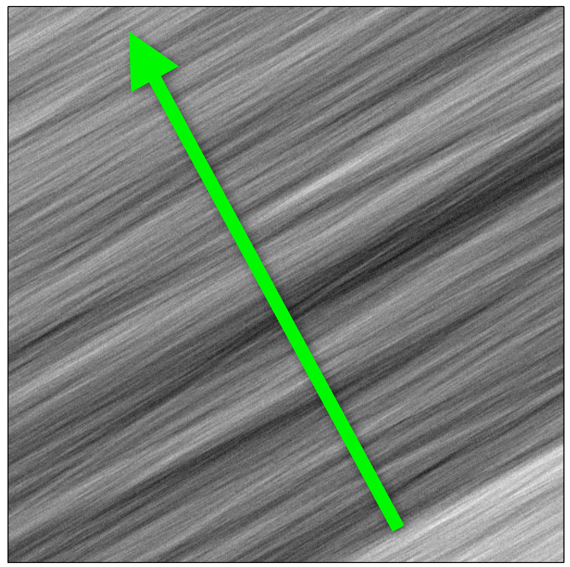
$$\alpha_0 = -\frac{\pi}{3}$$

$$\alpha = 0.1$$

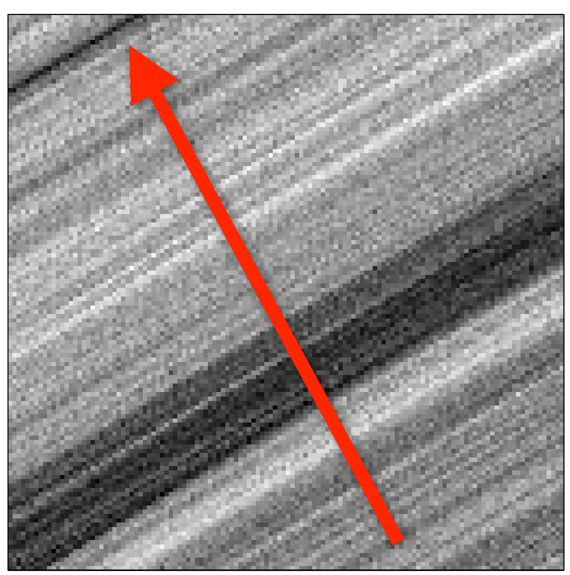
$$\alpha_0 = -\frac{\pi}{3} + 0.1$$

Elementary field simulation

- **Comparison Turning bands VS. Cholesky method**



The « transversal » variance varies



The « transversal » variance doesn't vary enough

$$\alpha_0 = -\frac{\pi}{3} \quad \alpha = 0.1$$

LAFBF simulation by tangent fields [Chol]

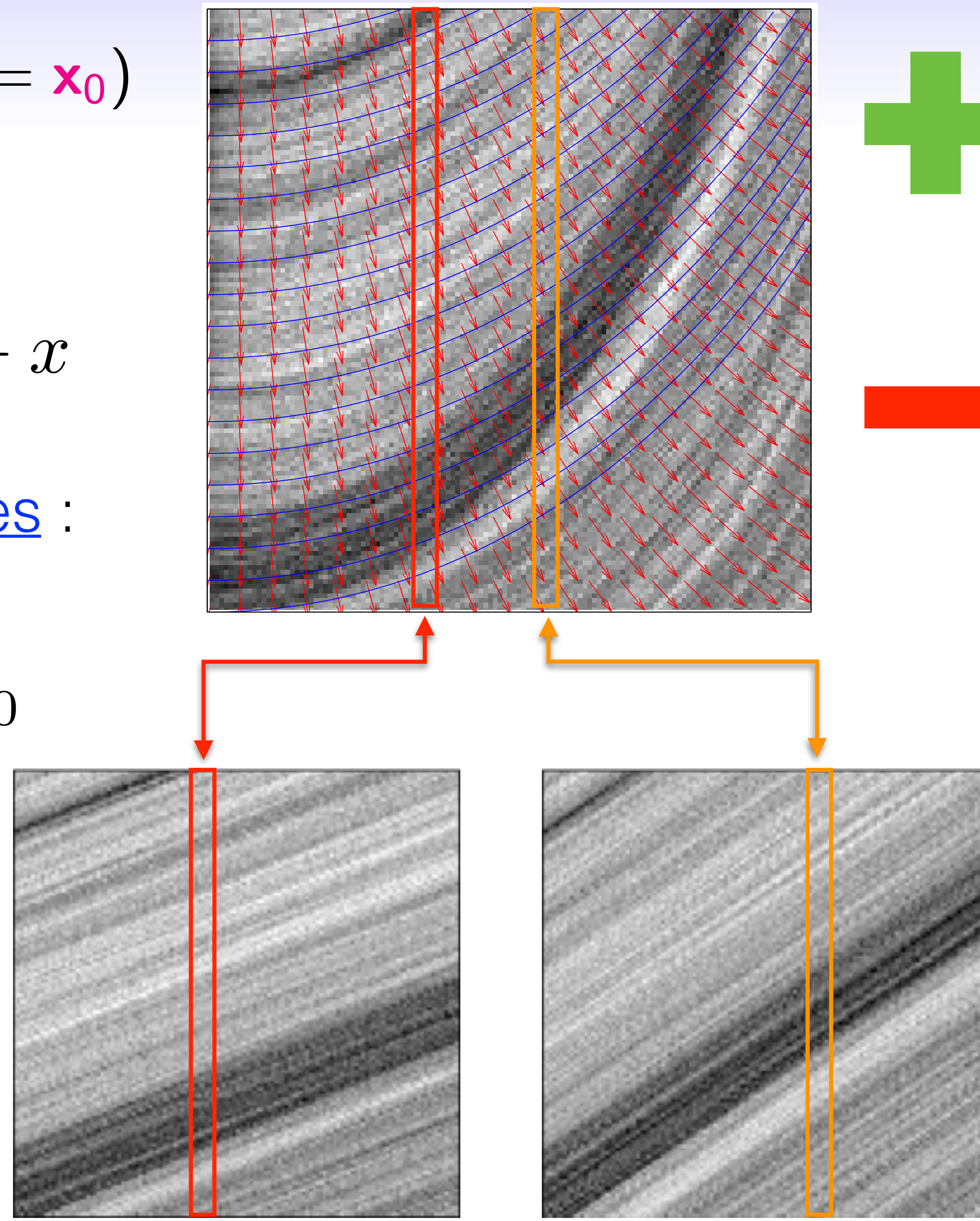
$B^{H}_{\alpha_0,\alpha}(\mathbf{x}_0) \approx Y_{\mathbf{x}_0}(x = \mathbf{x}_0)$

Vector field :

$$\alpha_0(x) = -\frac{\pi}{2} + x$$

Orientation curves :

$$\ln\left|\frac{1}{\cos x}\right| + y_0$$



$\alpha_0 = -67^\circ$

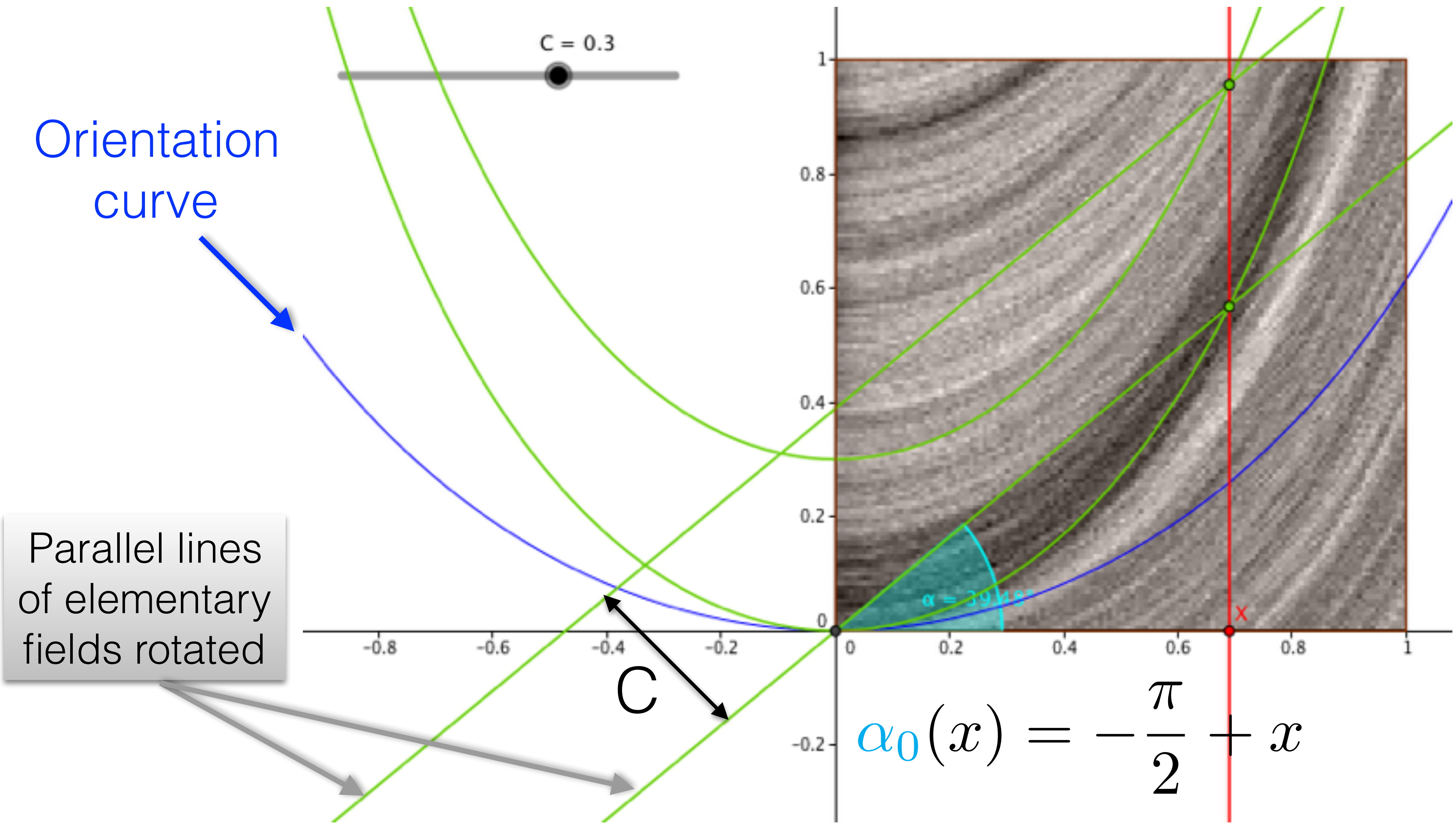
$\alpha_0 = -55^\circ$

Bands artifacts disappear

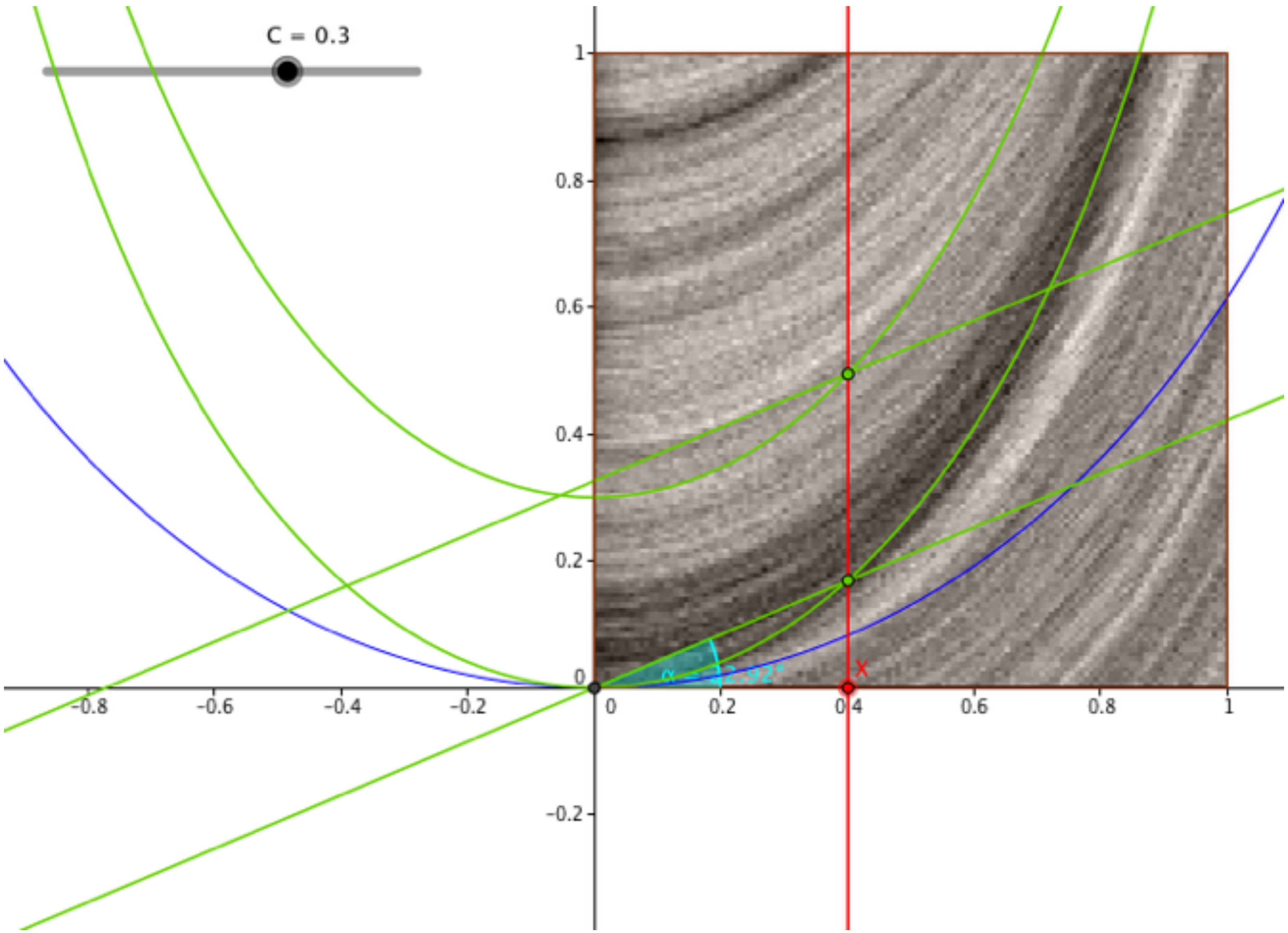
The grayscales don't fit the orientation as we expected

The grayscale repartition is now the quite the same for two close angles

Why the grayscale curves are like that ?



Why the grayscale curves are like that ?



Equations of the grayscale curves ?

$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}_0) \approx Y_{\mathbf{x}_0}(x = \mathbf{x}_0)$$

Vector field :

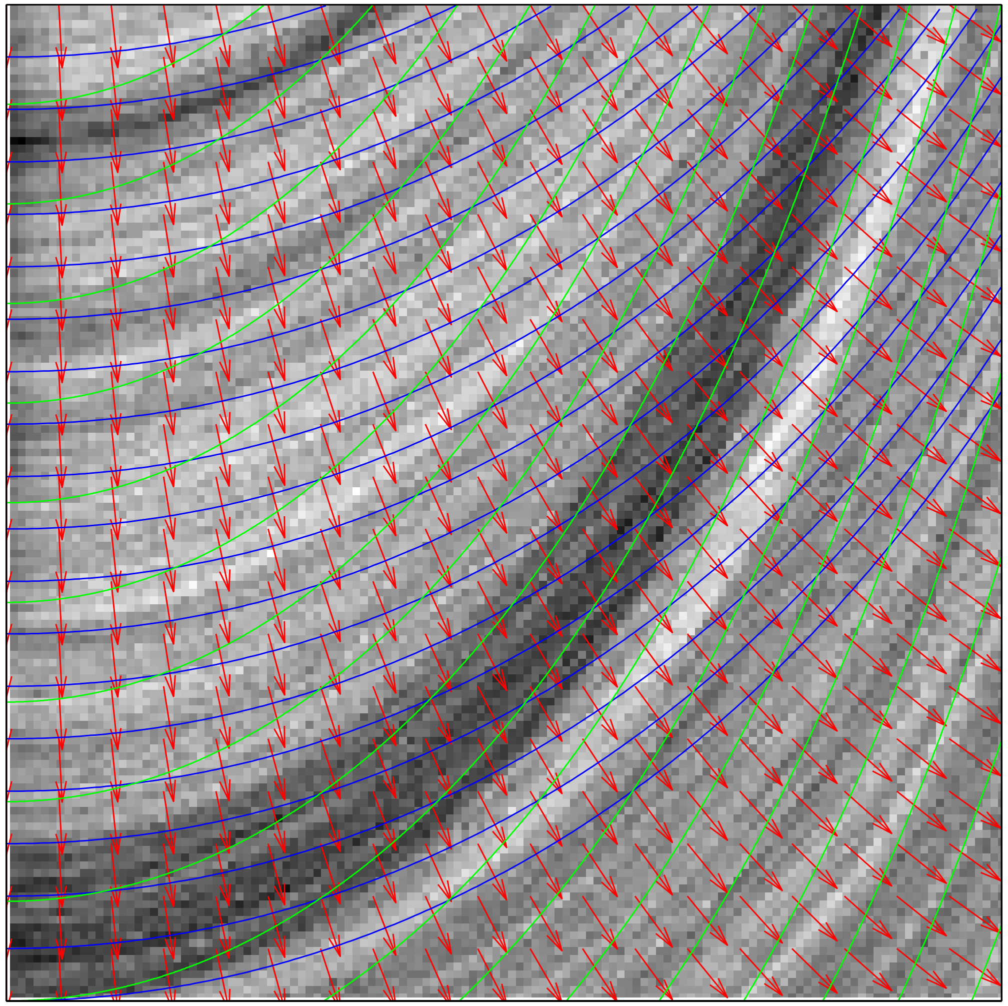
$$\alpha_0(x) = -\frac{\pi}{2} + x$$

Orientation curves :

$$\ln\left|\frac{1}{\cos x}\right| + y_0$$

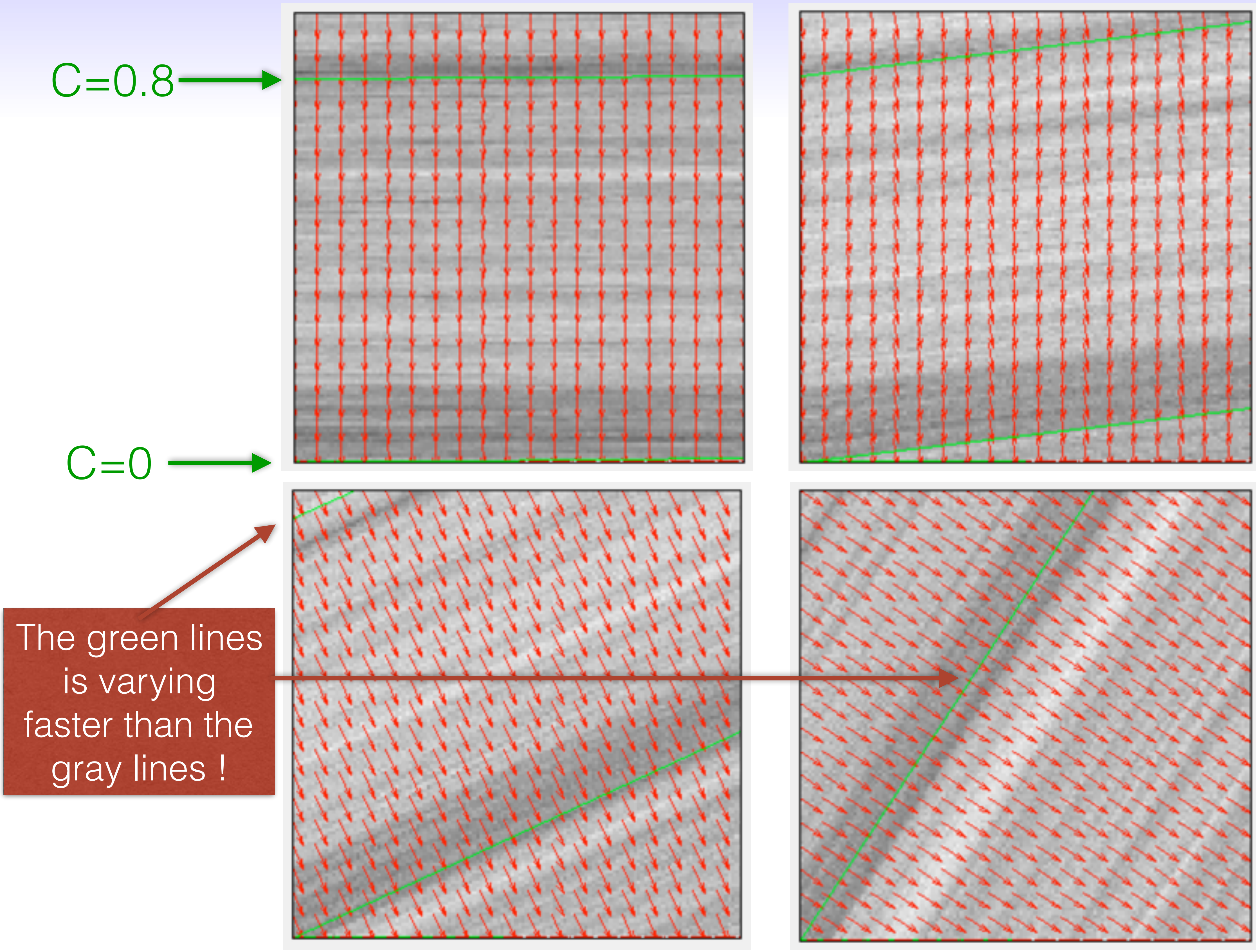
Theoritical grayscale curves :

$$\frac{C}{\cos x} + x\tan(x)$$



We observe the green curve look likes better but does not fit exactly the grayscale variations either…

Why does it not fit exactly ?



LAFBF simulation by tangent fields [Chol]

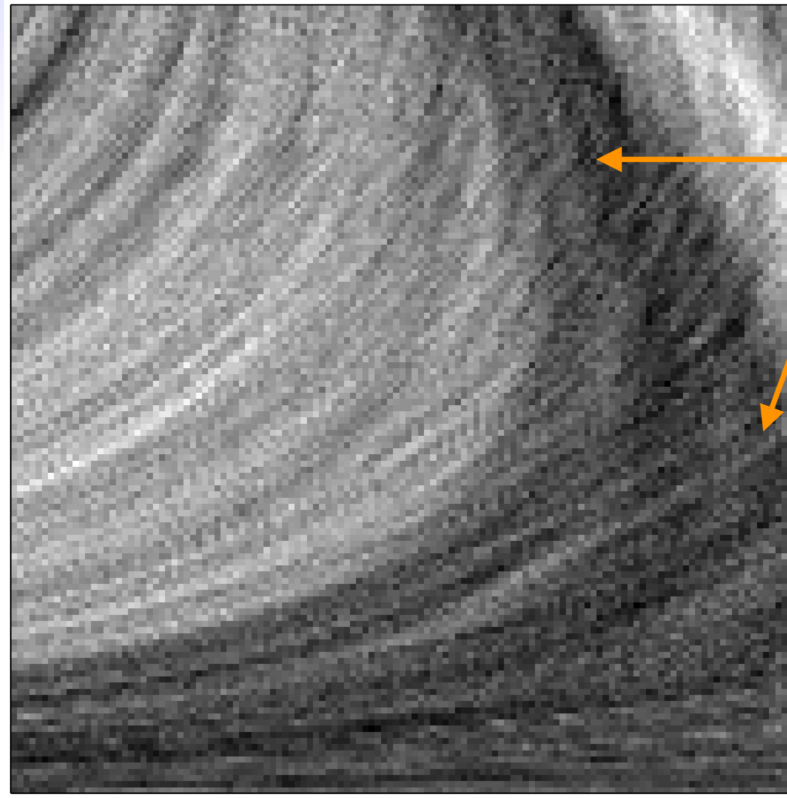
$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}_0) \approx Y_{\mathbf{x}_0}(x = \mathbf{x}_0)$$

Vector field :

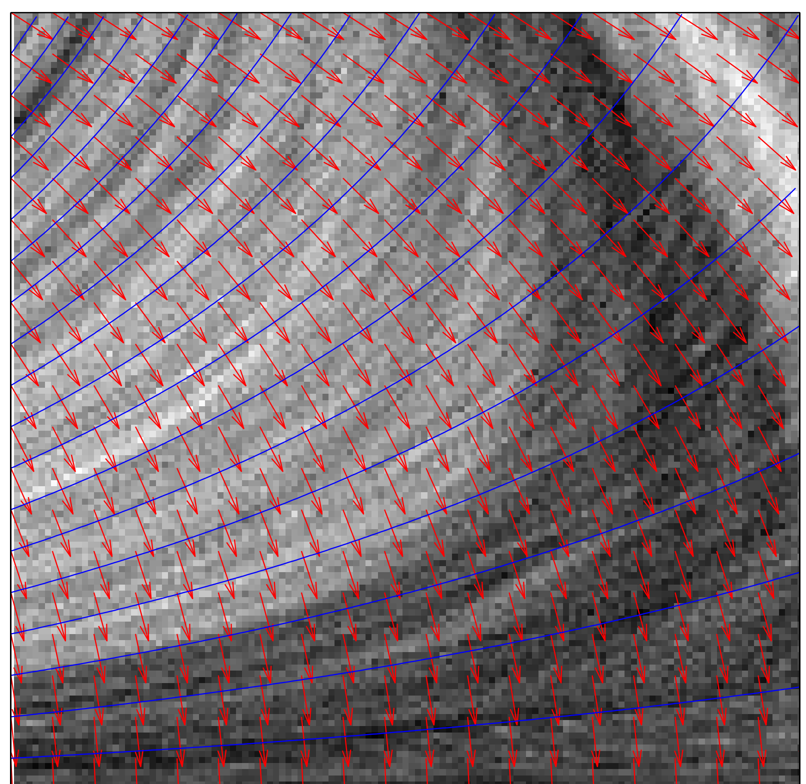
$$\alpha_0(x) = -\frac{\pi}{2} + y$$

Orientation curves :

$$\arcsin(\sin(y_0)e^{x})$$



We observe the accurate orientations but they seem to appear for high frequencies



- **Definition** of orientation in high frequencies ?
- **Wavelet** decomposition ?
- **Tangent field** formulation is well adapted to the simulation of this oriented model ?

LAFBF simulation by tangent fields [Chol]

$B^{H}_{\alpha_0,\alpha}(\mathbf{x}_0) \approx Y_{\mathbf{x}_0}(x = \mathbf{x}_0)$

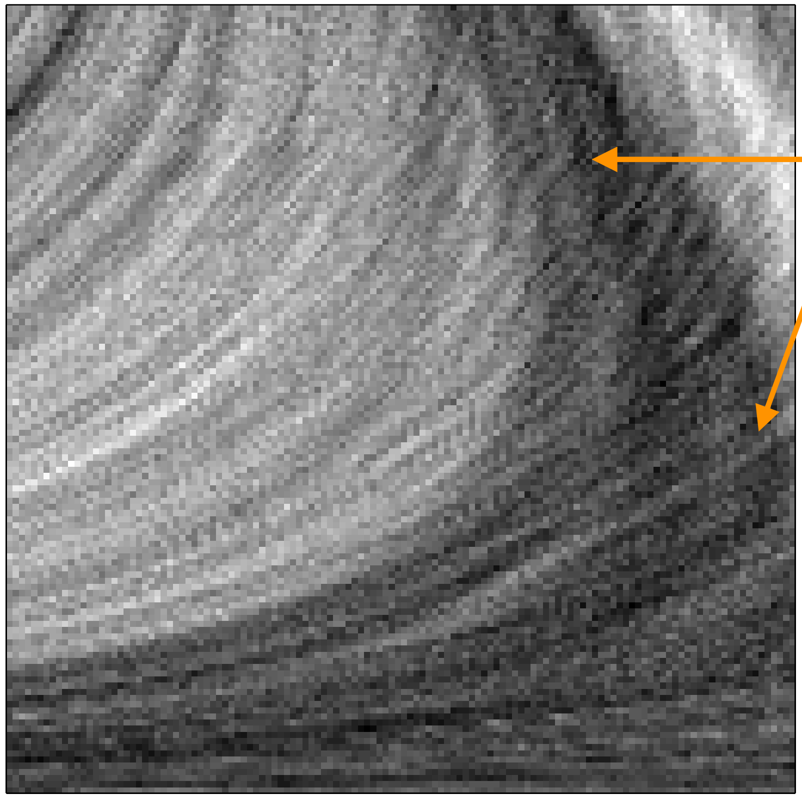
Vector field :

$$\alpha_0(x) = -\frac{\pi}{2} + y$$

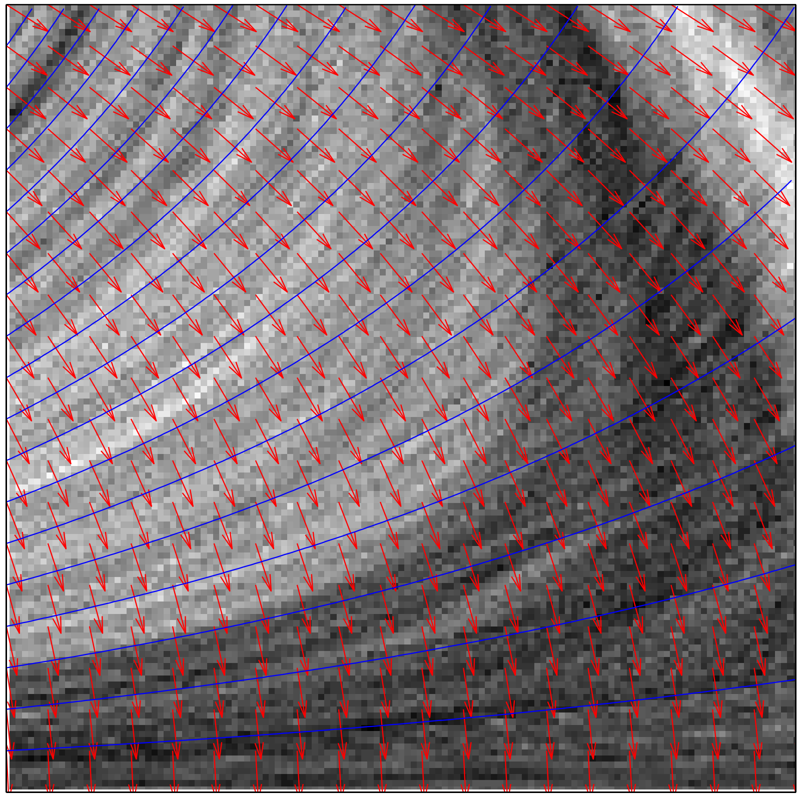
Orientation curves :

$$\arcsin(\sin(y_0)e^{x})$$

- **Orientation** contenue dans les hautes fréquences (le champ tangent est HF)
- **Définir l'orientation locale** comme orientation du champ tangent

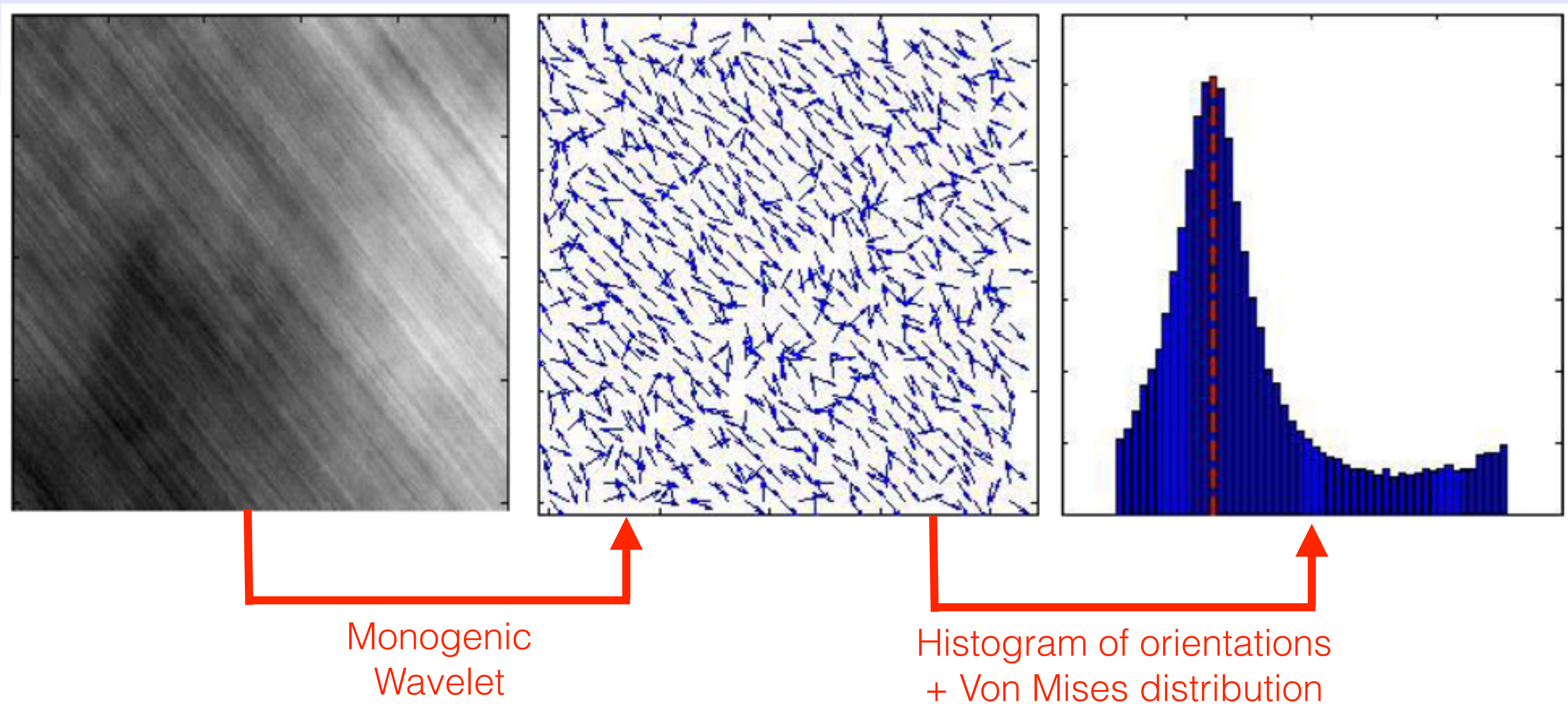


We observe the accurate orientations but they seem to appear for high frequencies



- **Definition** of orientation in high frequencies ?
- **Wavelet** decomposition ?
- **Tangent field** formulation is well adapted to the simulation of this oriented model ?

Local orientations estimation



Working on empirical covariance ?

- Olhede S., *Detection directionality in random fields using the monogenic signal*, preprint 2013.
- Richard F., *Tests of isotropy for rough textures of trended images,* preprint 2014.