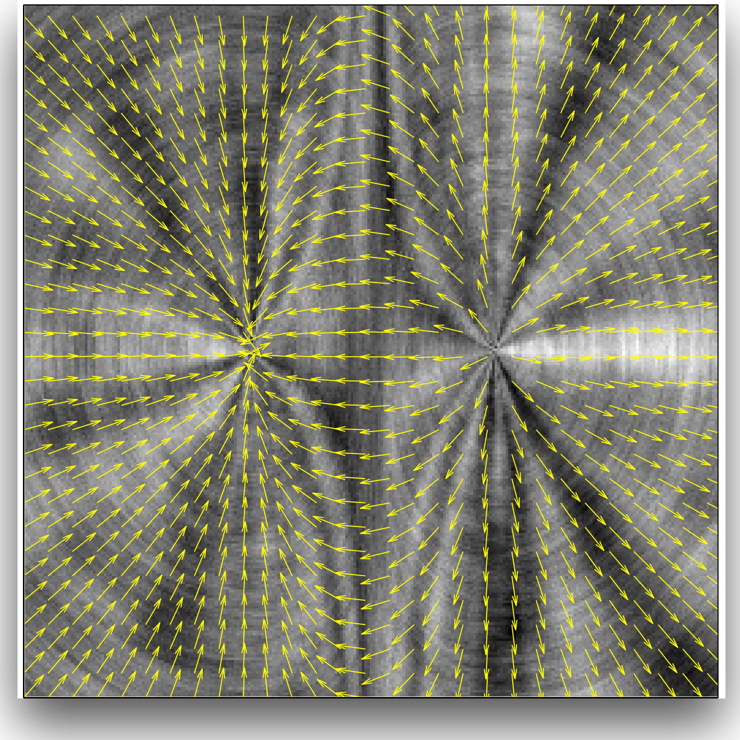
TEXTURE MODELING BY GAUSSIAN FIELDS WITH PRESCRIBED LOCAL ORIENTATION

Kévin Polisano / kevin.polisano@imag.fr

joint work with

Marianne Clausel
Valérie Perrier
Laurent Condat



IEEE ICIP 2014 : International Conference on Image Processing. Lecture session CNIT Paris, October 27-30, 2014

Outline

- Introduction
  - Motivation
  - General probabilistic framework
- Our new stochastic model
  - Definition: Locally Anisotropic Fractional Brownian Field
  - Notion of tangent field
- Synthesis methods
  - Tangent field simulation by a turning bands method
  - LAFBF simulation via tangent field formulation
- Numerical experiments
- Conclusion and future work

How to synthesize natural random textures ?

Mathematical
model ?

How to synthesize natural random textures ?

Mathematical model ?

How to synthesize natural random textures ?

Randomness
Self-similarity

Mathematical
model ?

How to synthesize natural random textures ?

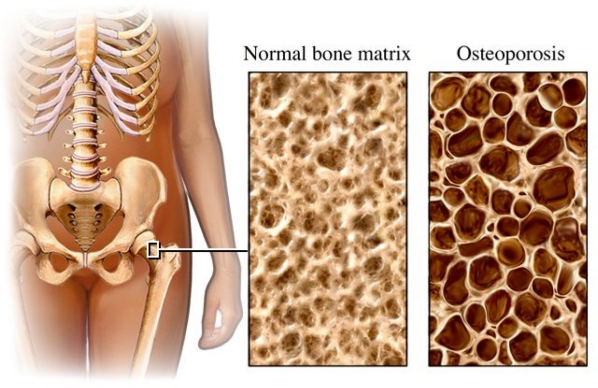
Randomness
Self-similarity

Mathematical
model ?

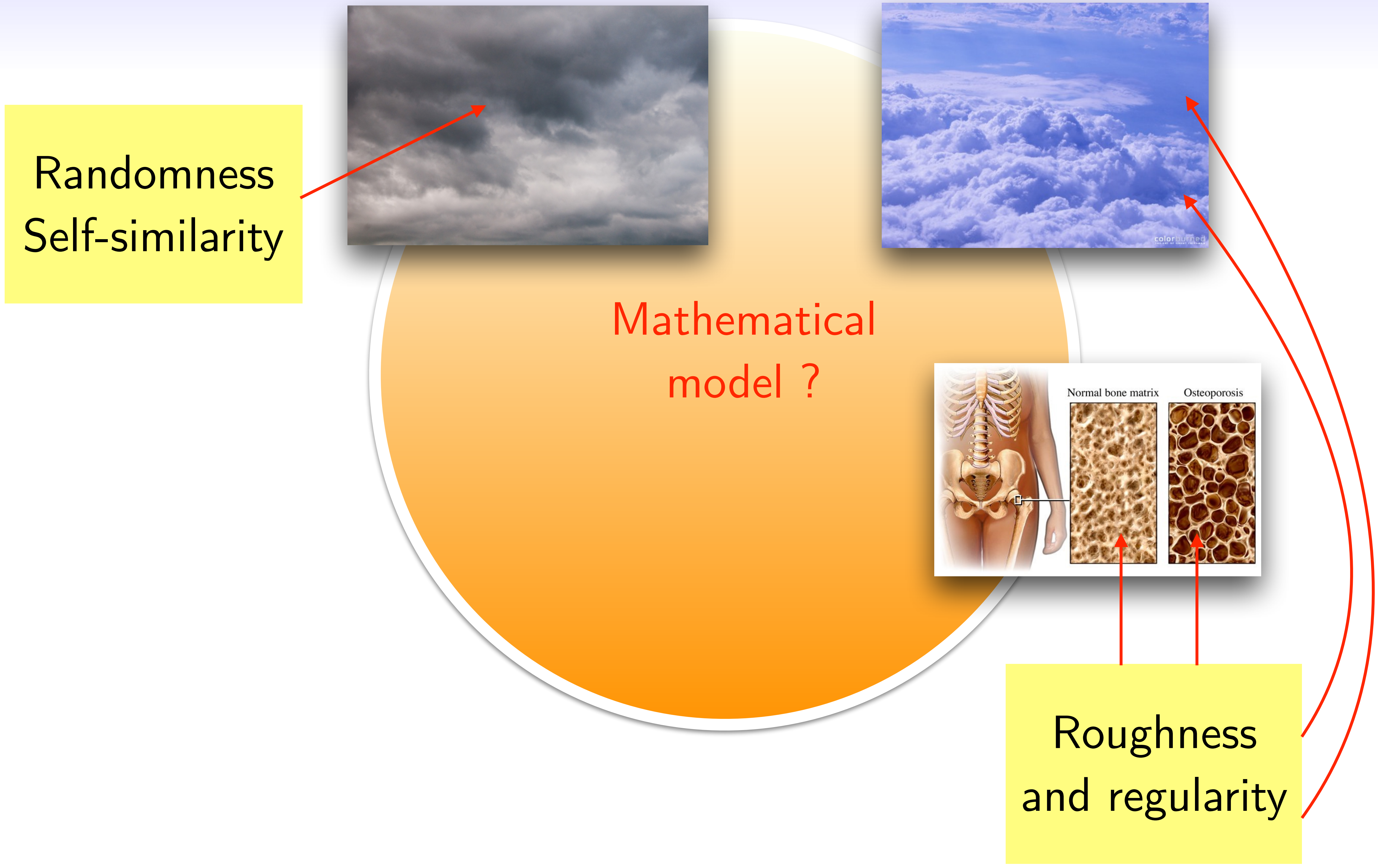
How to synthesize natural random textures ?

Randomness
Self-similarity

Mathematical model ?



How to synthesize natural random textures ?



Randomness
Self-similarity

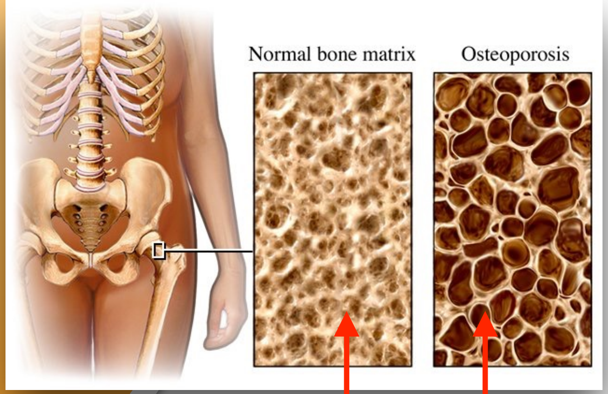
Mathematical
model ?

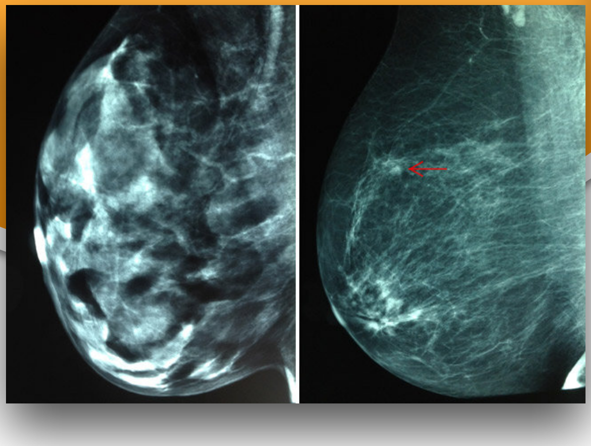
Roughness
and regularity

How to synthesize natural random textures ?

Randomness
Self-similarity

Mathematical
model ?



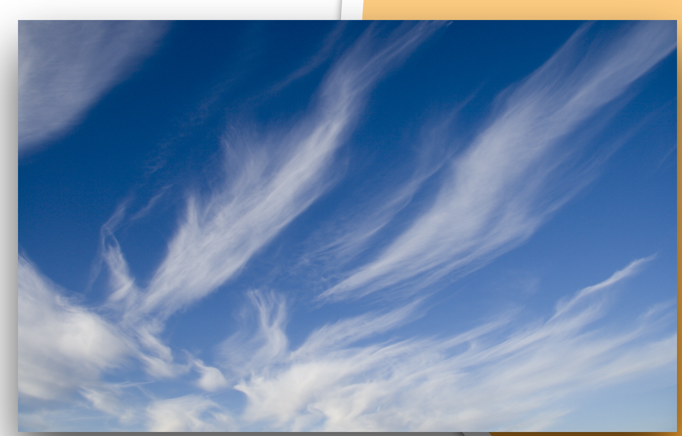


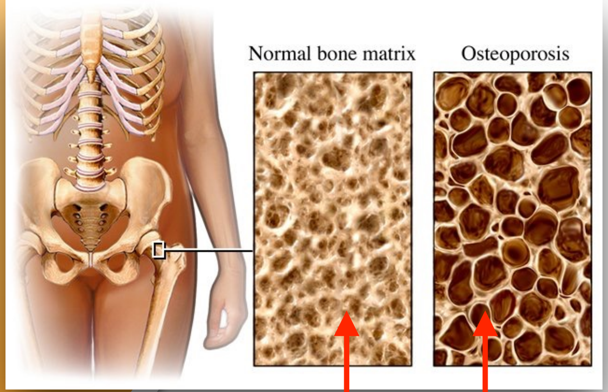
Roughness
and regularity

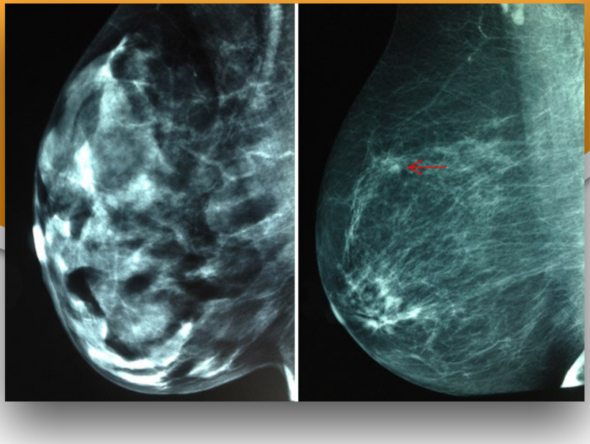
How to synthesize natural random textures ?

Randomness
Self-similarity

Mathematical
model ?

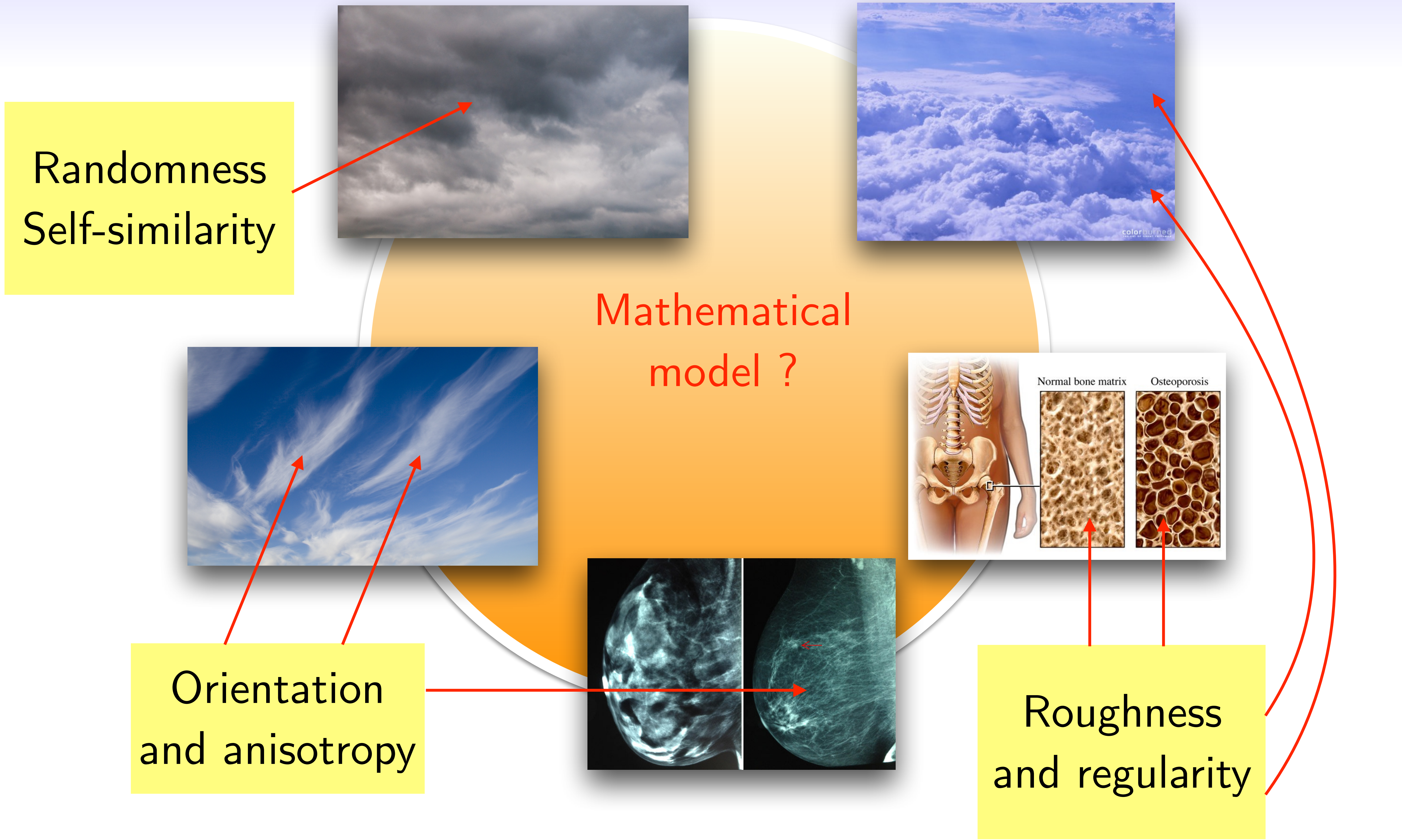






Roughness
and regularity

How to synthesize natural random textures ?



Randomness
Self-similarity

Mathematical
model ?

Orientation
and anisotropy

Roughness
and regularity

The basic component :

Fractional Brownian Field (FBF)

- B^H FBF with Hurst index $0 < H < 1$ [Mandelbrot,Van Ness,1968]
  - stationary increments : $B^H(\cdot + \mathbf{z}) - B^H(\mathbf{z}) \overset{\mathcal{L}}{=} B^H(\cdot) - B^H(0)$
  - self-similar : $B^H(\lambda\cdot) \overset{\mathcal{L}}{=} \lambda^H B^H(\cdot)$
  - isotropic : $B^H \circ R_\theta \overset{\mathcal{L}}{=} B^H$
- The covariance is given by

$$\mathrm{Cov}(B^H(\mathbf{x}), B^H(\mathbf{y})) = c_H(\|\mathbf{x}\|^{2H} + \|\mathbf{y}\|^{2H} - \|\mathbf{x} - \mathbf{y}\|^{2H})$$

The basic component :

Fractional Brownian Field (FBF)

- Harmonizable representation [Samorodnitsky,Taqqu,1997]

$$\boxed{B^{H}(\mathbf{x}) = \int_{\mathbb{R}^2} \frac{e^{i\mathbf{x}\cdot\xi} - 1}{\|\xi\|^{H+1}} \, d\widehat{W}(\xi)}$$

The basic component :

Fractional Brownian Field (FBF)

- Harmonizable representation [Samorodnitsky,Taqqu,1997]

$$\boxed{B^{H}(\mathbf{x}) = \int_{\mathbb{R}^2} \frac{e^{i\mathbf{x}\cdot\xi} - 1}{\|\xi\|^{H+1}}\, d\widehat{W}(\xi)}$$

complex Brownian measure

The basic component :

Fractional Brownian Field (FBF)

- Harmonizable representation

[Samorodnitsky,Taqqu,1997]

$$\boxed{B^H(\mathbf{x}) = \int_{\mathbb{R}^2} \frac{e^{i\mathbf{x}\cdot\xi} - 1}{\|\xi\|^{H+1}} \, d\widehat{W}(\xi)}$$

roughness indicator

complex Brownian measure

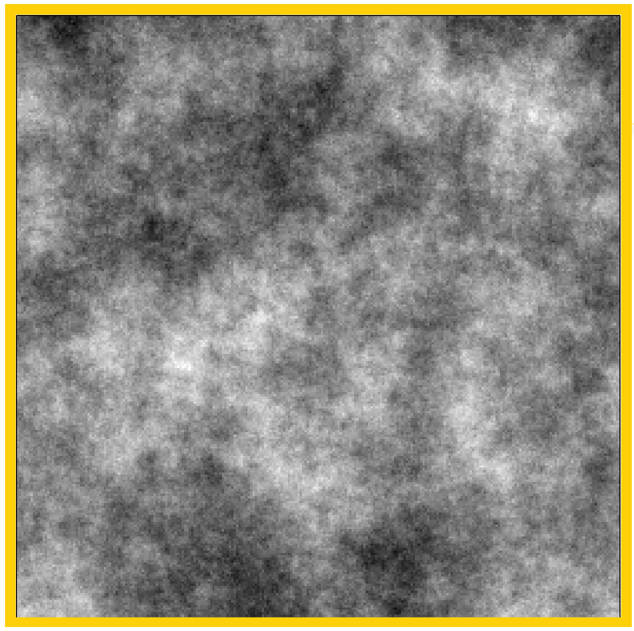
The basic component :
Fractional Brownian Field (FBF)

- Harmonizable representation [Samorodnitsky,Taqqu,1997]

$$\boxed{B^{H}(\mathbf{x}) = \int_{\mathbb{R}^2} \frac{e^{i\mathbf{x}\cdot\xi} - 1}{\|\xi\|^{H+1}} d\widehat{W}(\xi)}$$

roughness indicator

complex Brownian measure



H=0.2

The basic component :

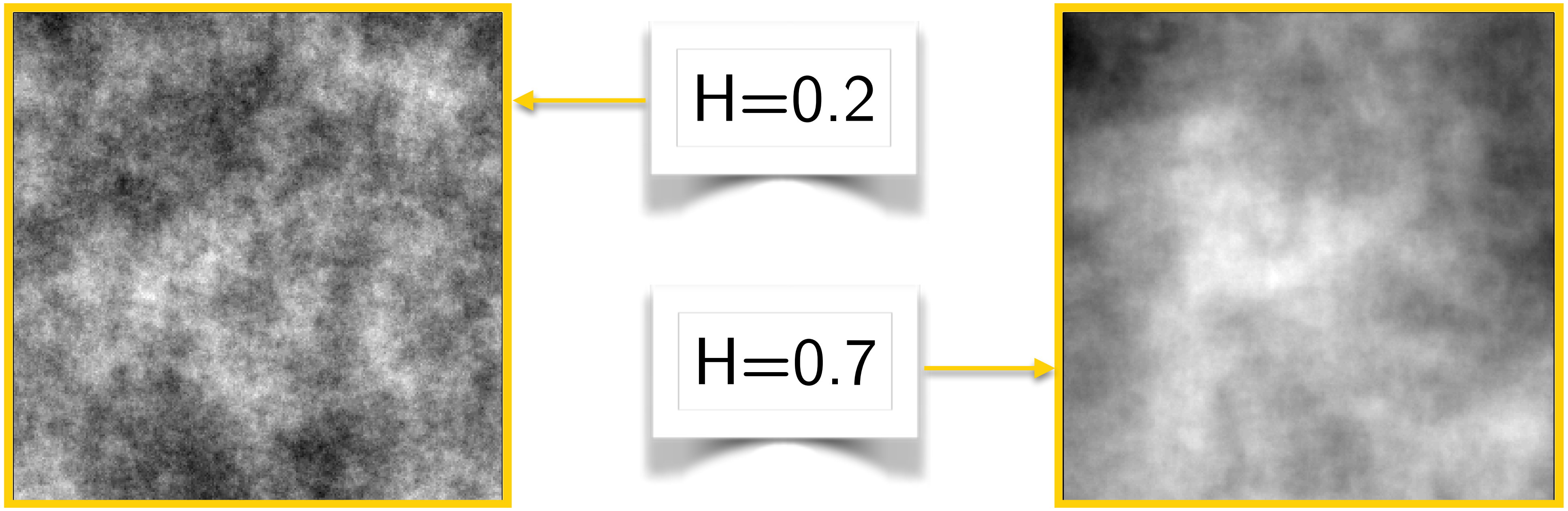
Fractional Brownian Field (FBF)

- Harmonizable representation [Samorodnitsky,Taqqu,1997]

$$B^{H}(\mathbf{x}) = \int_{\mathbb{R}^2} \frac{e^{i\mathbf{x}\cdot\xi} - 1}{\|\xi\|^{H+1}} \, d\widehat{W}(\xi)$$

roughness indicator

complex Brownian measure



General model :

anisotropic self-similar Gaussian fields

$$X(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\boldsymbol{\xi}} - 1) f^{1/2}(\mathbf{x}, \boldsymbol{\xi}) d\widehat{W}(\boldsymbol{\xi})$$

General model :

anisotropic self-similar Gaussian fields

$$X(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) f^{1/2}(\mathbf{x}, \xi) d\widehat{W}(\xi)$$

spectral density

$$f^{1/2}(\mathbf{x}, \xi) = c(\mathbf{x}, \xi) \|\xi\|^{-h(\mathbf{x},\xi)-1}$$

General model :

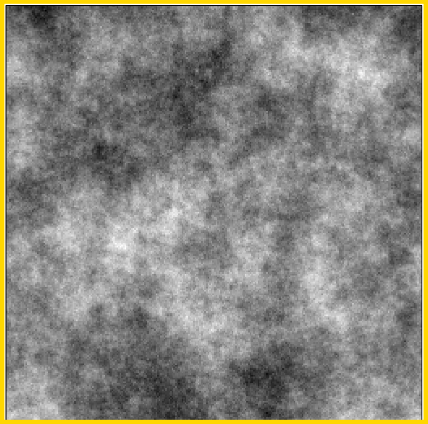
anisotropic self-similar Gaussian fields

$$X(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) f^{1/2}(\mathbf{x},\xi) d\widehat{W}(\xi)$$

spectral density

$$f^{1/2}(\mathbf{x},\xi) = c(\mathbf{x},\xi)\|\xi\|^{-h(\mathbf{x},\xi)-1}$$

- $c(\mathbf{x},\xi) \equiv 1$ and $h(\mathbf{x},\xi) \equiv H \Rightarrow X = B^H$ [Mandelbrot,Van Ness,1968]



General model :

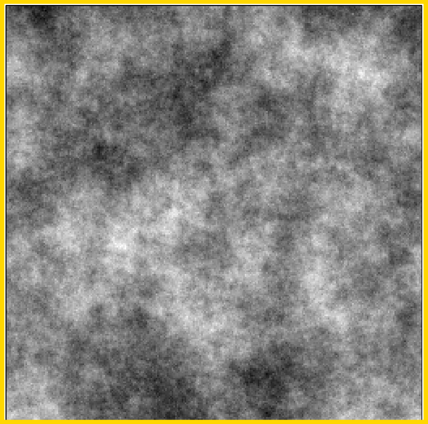
anisotropic self-similar Gaussian fields

$$X(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) \boxed{f^{1/2}(\mathbf{x}, \xi)} d\widehat{W}(\xi)$$

spectral density

$$\boxed{f^{1/2}(\mathbf{x}, \xi) = c(\mathbf{x}, \xi) \|\xi\|^{-h(\mathbf{x},\xi)-1}}$$

- $c(\mathbf{x}, \xi) \equiv 1$ and $h(\mathbf{x}, \xi) \equiv H \Rightarrow X = B^H$ [Mandelbrot,Van Ness,1968]
- $c(\mathbf{x}, \xi) \equiv c(\arg \xi)$ and $h(\mathbf{x}, \xi) \equiv h(\arg \xi) \Rightarrow X = AFBF$ [Bonami,Estrade,2003]



General model :

anisotropic self-similar Gaussian fields

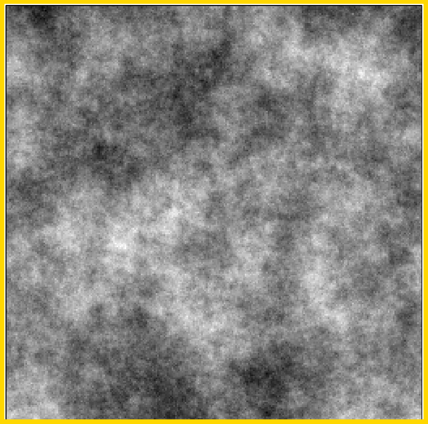
$$X(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) f^{1/2}(\mathbf{x},\xi) d\widehat{W}(\xi)$$

spectral density

$$f^{1/2}(\mathbf{x},\xi) = c(\mathbf{x},\xi)\|\xi\|^{-h(\mathbf{x},\xi)-1}$$

- $c(\mathbf{x},\xi) \equiv 1$ and $h(\mathbf{x},\xi) \equiv H \Rightarrow X = B^H$ [Mandelbrot,Van Ness,1968]
- $c(\mathbf{x},\xi) \equiv c(\arg\xi)$ and $h(\mathbf{x},\xi) \equiv h(\arg\xi) \Rightarrow X = AFBF$ [Bonami,Estrade,2003]
  - Example : *elementary fields* $c(\arg\xi) = \mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0)$

[Bierme,Richard,Moisan,2012]



General model :

anisotropic self-similar Gaussian fields

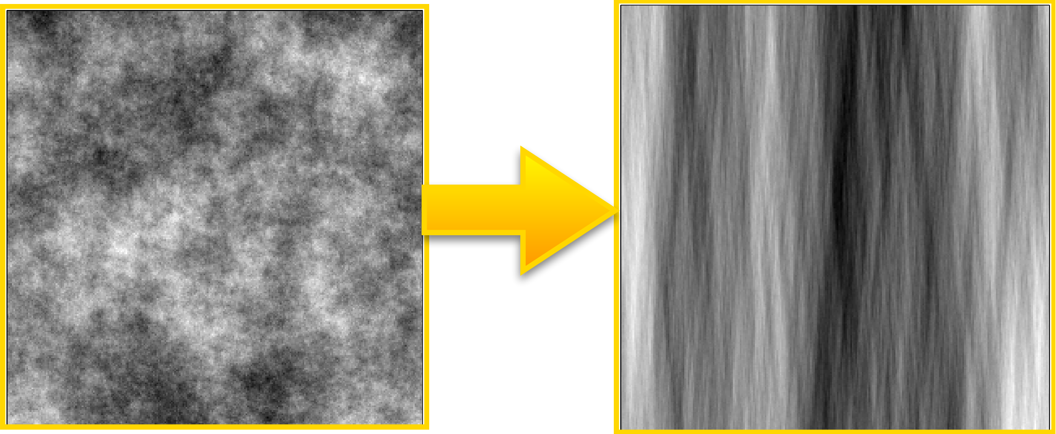
$$X(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) \boxed{f^{1/2}(\mathbf{x},\xi)} \, d\widehat{W}(\xi)$$

spectral density

$$\boxed{f^{1/2}(\mathbf{x},\xi) = c(\mathbf{x},\xi)\|\xi\|^{-h(\mathbf{x},\xi)-1}}$$

- $c(\mathbf{x},\xi) \equiv 1$ and $h(\mathbf{x},\xi) \equiv H \Rightarrow X = B^H$ [Mandelbrot,Van Ness,1968]
- $c(\mathbf{x},\xi) \equiv c(\arg\xi)$ and $h(\mathbf{x},\xi) \equiv h(\arg\xi) \Rightarrow X = AFBF$ [Bonami,Estrade,2003]
  - Example : *elementary fields* $c(\arg\xi) = \mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0)$

[Bierme,Richard,Moisan,2012]



General model :

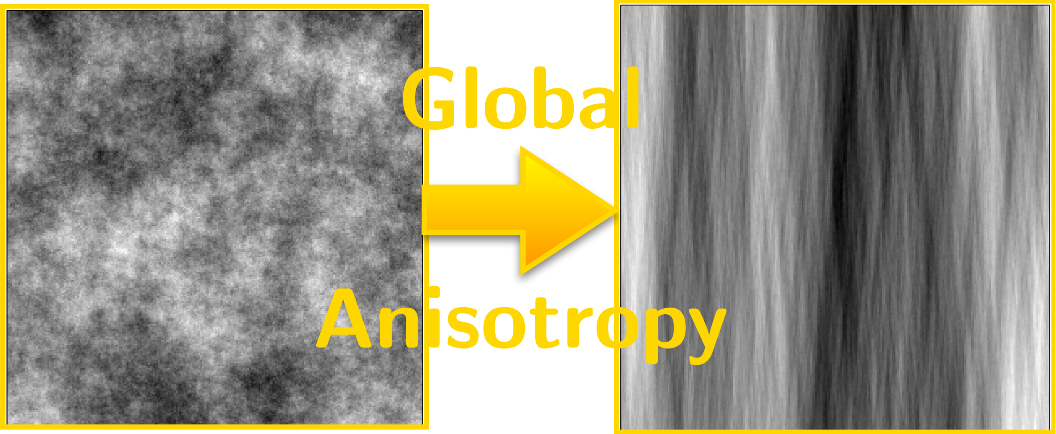
anisotropic self-similar Gaussian fields

$$X(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) f^{1/2}(\mathbf{x},\xi) d\widehat{W}(\xi)$$

spectral density

$$\boxed{f^{1/2}(\mathbf{x},\xi) = c(\mathbf{x},\xi)\|\xi\|^{-h(\mathbf{x},\xi)-1}}$$

- $c(\mathbf{x},\xi) \equiv 1$ and $h(\mathbf{x},\xi) \equiv H \Rightarrow X = B^H$ [Mandelbrot,Van Ness,1968]
- $c(\mathbf{x},\xi) \equiv c(\arg\xi)$ and $h(\mathbf{x},\xi) \equiv h(\arg\xi) \Rightarrow X = AFBF$ [Bonami,Estrade,2003]
  - Example : *elementary fields* $c(\arg\xi) = \mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0)$ [Bierme,Richard,Moisan,2012]



General model :

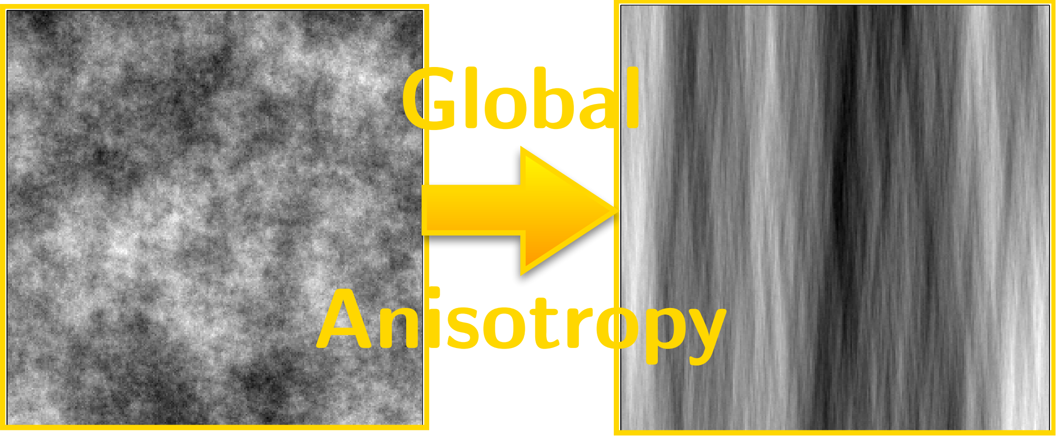
anisotropic self-similar Gaussian fields

$$X(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) f^{1/2}(\mathbf{x},\xi)\, d\widehat{W}(\xi)$$

spectral density

$$\boxed{f^{1/2}(\mathbf{x},\xi) = c(\mathbf{x},\xi)\|\xi\|^{-h(\mathbf{x},\xi)-1}}$$

- $c(\mathbf{x},\xi) \equiv 1$ and $h(\mathbf{x},\xi) \equiv H \Rightarrow X = B^H$ [Mandelbrot,Van Ness,1968]
- $c(\mathbf{x},\xi) \equiv c(\arg\xi)$ and $h(\mathbf{x},\xi) \equiv h(\arg\xi) \Rightarrow X = AFBF$ [Bonami,Estrade,2003]
  - Example : *elementary fields* $c(\arg\xi) = \mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0)$ [Bierme,Richard,Moisan,2012]
- $c(\mathbf{x},\xi) \equiv c(\mathbf{x},\arg\xi)$ and $h(\mathbf{x},\xi) \equiv h(\mathbf{x})$ [Polisano,Clausel,Perrier,Condat,2014]



General model :

anisotropic self-similar Gaussian fields

$$X(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) f^{1/2}(\mathbf{x},\xi) d\widehat{W}(\xi)$$

spectral density

$$\boxed{f^{1/2}(\mathbf{x},\xi) = c(\mathbf{x},\xi)\|\xi\|^{-h(\mathbf{x},\xi)-1}}$$

- $c(\mathbf{x},\xi) \equiv 1$ and $h(\mathbf{x},\xi) \equiv H \Rightarrow X = B^H$ [Mandelbrot,Van Ness,1968]
- $c(\mathbf{x},\xi) \equiv c(\arg\xi)$ and $h(\mathbf{x},\xi) \equiv h(\arg\xi) \Rightarrow X = AFBF$ [Bonami,Estrade,2003]
  - Example : *elementary fields* $c(\arg\xi) = \mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0)$
- $c(\mathbf{x},\xi) \equiv c(\mathbf{x},\arg\xi)$ and $h(\mathbf{x},\xi) \equiv h(\mathbf{x})$ [Bierme,Richard,Moisan,2012] [Polisano,Clausel,Perrier,Condat,2014]

General model :

anisotropic self-similar Gaussian fields

$$X(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) f^{1/2}(\mathbf{x},\xi)\, d\widehat{W}(\xi)$$

spectral density

$$\boxed{f^{1/2}(\mathbf{x},\xi) = c(\mathbf{x},\xi)\|\xi\|^{-h(\mathbf{x},\xi)-1}}$$

- $c(\mathbf{x},\xi) \equiv 1$ and $h(\mathbf{x},\xi) \equiv H \Rightarrow X = B^H$ [Mandelbrot,Van Ness,1968]
- $c(\mathbf{x},\xi) \equiv c(\arg\xi)$ and $h(\mathbf{x},\xi) \equiv h(\arg\xi) \Rightarrow X = AFBF$ [Bonami,Estrade,2003]
  - Example : *elementary fields* $c(\arg\xi) = \mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0)$ [Bierme,Richard,Moisan,2012]
- $c(\mathbf{x},\xi) \equiv c(\mathbf{x},\arg\xi)$ and $h(\mathbf{x},\xi) \equiv h(\mathbf{x})$ [Polisano,Clausel,Perrier,Condat,2014]

General model :

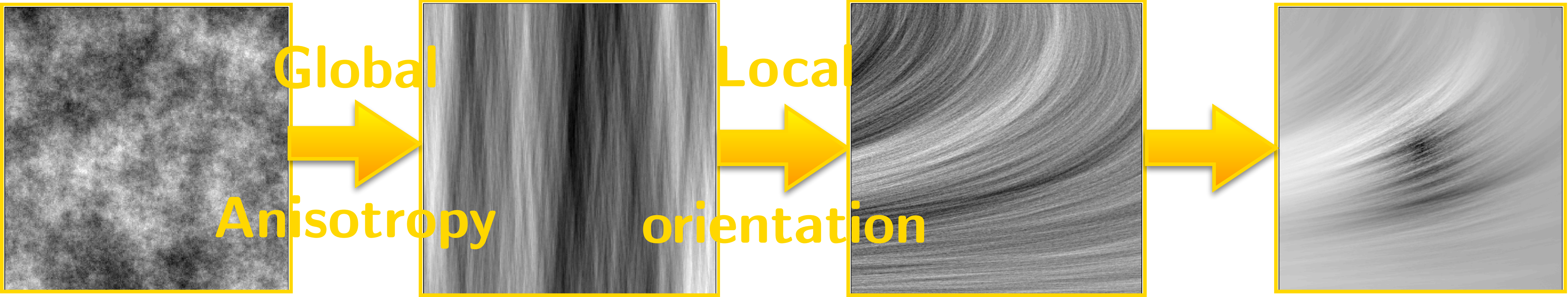
anisotropic self-similar Gaussian fields

$$X(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) f^{1/2}(\mathbf{x},\xi)\, d\widehat{W}(\xi)$$

spectral density

$$\boxed{f^{1/2}(\mathbf{x},\xi) = c(\mathbf{x},\xi)\|\xi\|^{-h(\mathbf{x},\xi)-1}}$$

- $c(\mathbf{x},\xi) \equiv 1$ and $h(\mathbf{x},\xi) \equiv H \Rightarrow X = B^H$ [Mandelbrot,Van Ness,1968]
- $c(\mathbf{x},\xi) \equiv c(\arg\xi)$ and $h(\mathbf{x},\xi) \equiv h(\arg\xi) \Rightarrow X = AFBF$ [Bonami,Estrade,2003]
  - Example : *elementary fields* $c(\arg\xi) = \mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0)$ [Bierme,Richard,Moisan,2012]
- $c(\mathbf{x},\xi) \equiv c(\mathbf{x},\arg\xi)$ and $h(\mathbf{x},\xi) \equiv h(\mathbf{x})$ [Polisano,Clausel,Perrier,Condat,2014]



General model :

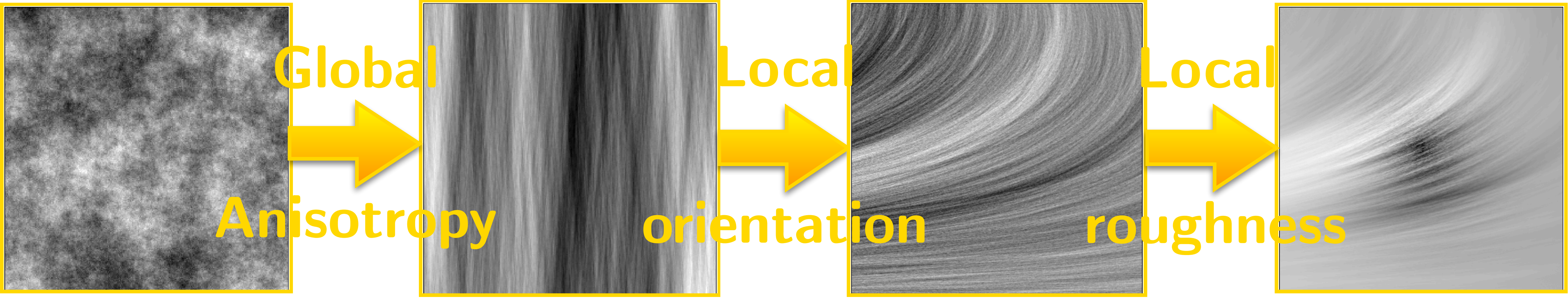
anisotropic self-similar Gaussian fields

$$X(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) f^{1/2}(\mathbf{x},\xi)\, d\widehat{W}(\xi)$$

spectral density

$$\boxed{f^{1/2}(\mathbf{x},\xi) = c(\mathbf{x},\xi)\|\xi\|^{-h(\mathbf{x},\xi)-1}}$$

- $c(\mathbf{x},\xi) \equiv 1$ and $h(\mathbf{x},\xi) \equiv H \Rightarrow X = B^H$ [Mandelbrot,Van Ness,1968]
- $c(\mathbf{x},\xi) \equiv c(\arg\xi)$ and $h(\mathbf{x},\xi) \equiv h(\arg\xi) \Rightarrow X = AFBF$ [Bonami,Estrade,2003]
  - Example : *elementary fields* $c(\arg\xi) = \mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0)$ [Bierme,Richard,Moisan,2012]
- $c(\mathbf{x},\xi) \equiv c(\mathbf{x},\arg\xi)$ and $h(\mathbf{x},\xi) \equiv h(\mathbf{x})$ [Polisano,Clausel,Perrier,Condat,2014]



General model :

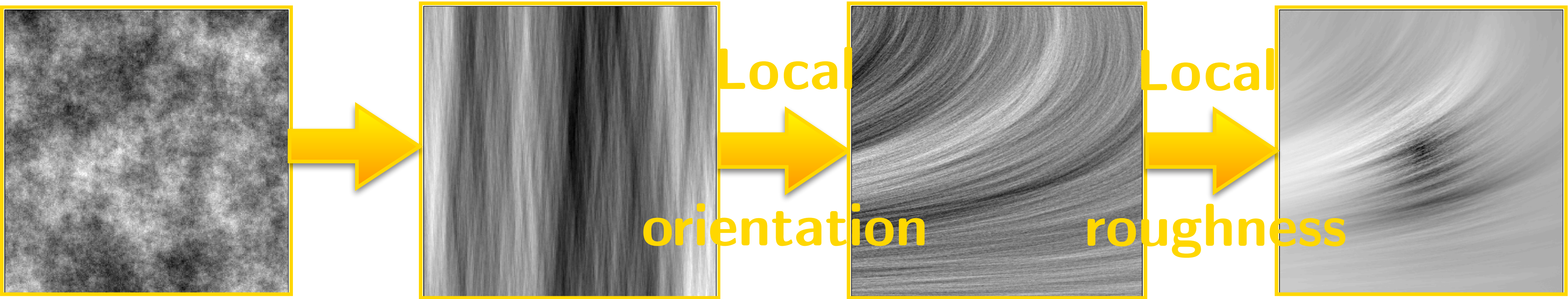
anisotropic self-similar Gaussian fields

$$X(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) f^{1/2}(\mathbf{x},\xi)\, d\widehat{W}(\xi)$$

spectral density

$$f^{1/2}(\mathbf{x},\xi) = c(\mathbf{x},\xi)\|\xi\|^{-h(\mathbf{x},\xi)-1}$$

- $c(\mathbf{x},\xi) \equiv 1$ and $h(\mathbf{x},\xi) \equiv H \Rightarrow X = B^H$ [Mandelbrot,Van Ness,1968]
- $c(\mathbf{x},\xi) \equiv c(\arg\xi)$ and $h(\mathbf{x},\xi) \equiv h(\arg\xi) \Rightarrow X = AFBF$ [Bonami,Estrade,2003]
  - Example : *elementary fields* $c(\arg\xi) = \mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0)$ [Bierme,Richard,Moisan,2012]
- $c(\mathbf{x},\xi) \equiv c(\mathbf{x},\arg\xi)$ and $h(\mathbf{x},\xi) \equiv h(\mathbf{x})$ [Polisano,Clausel,Perrier,Condat,2014]



General model :

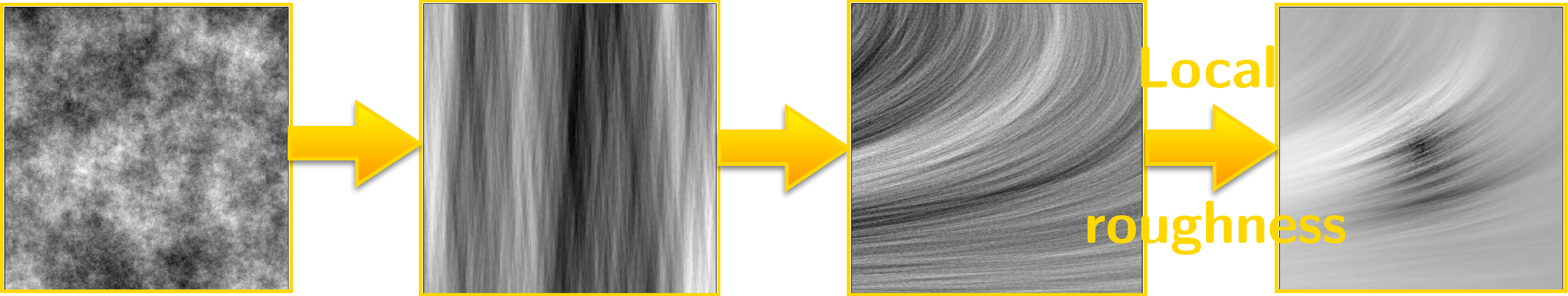
anisotropic self-similar Gaussian fields

$$X(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) f^{1/2}(\mathbf{x},\xi)\, d\widehat{W}(\xi)$$

spectral density

$$f^{1/2}(\mathbf{x},\xi) = c(\mathbf{x},\xi)\|\xi\|^{-h(\mathbf{x},\xi)-1}$$

- $c(\mathbf{x},\xi) \equiv 1$ and $h(\mathbf{x},\xi) \equiv H \Rightarrow X = B^H$ [Mandelbrot,Van Ness,1968]
- $c(\mathbf{x},\xi) \equiv c(\arg\xi)$ and $h(\mathbf{x},\xi) \equiv h(\arg\xi) \Rightarrow X = AFBF$ [Bonami,Estrade,2003]
  - Example : *elementary fields* $c(\arg\xi) = \mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0)$ [Bierme,Richard,Moisan,2012]
- $c(\mathbf{x},\xi) \equiv c(\mathbf{x},\arg\xi)$ and $h(\mathbf{x},\xi) \equiv h(\mathbf{x})$ [Polisano,Clausel,Perrier,Condat,2014]



General model :

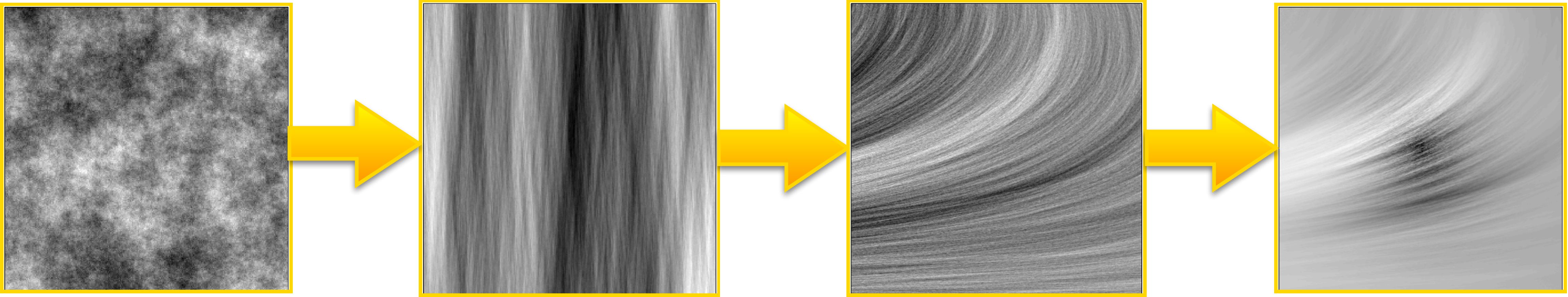
anisotropic self-similar Gaussian fields

$$X(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) f^{1/2}(\mathbf{x}, \xi) \, d\widehat{W}(\xi)$$

spectral density

$$\boxed{f^{1/2}(\mathbf{x}, \xi) = c(\mathbf{x}, \xi) \|\xi\|^{-h(\mathbf{x},\xi)-1}}$$

- $c(\mathbf{x}, \xi) \equiv 1$ and $h(\mathbf{x}, \xi) \equiv H \Rightarrow X = B^H$ [Mandelbrot,Van Ness,1968]
- $c(\mathbf{x}, \xi) \equiv c(\arg \xi)$ and $h(\mathbf{x}, \xi) \equiv h(\arg \xi) \Rightarrow X = AFBF$ [Bonami,Estrade,2003]
  - Example : *elementary fields* $c(\arg \xi) = \mathbb{1}_{[-\alpha,\alpha]}(\arg \xi - \alpha_0)$
- $c(\mathbf{x}, \xi) \equiv c(\mathbf{x}, \arg \xi)$ and $h(\mathbf{x}, \xi) \equiv h(\mathbf{x})$ [Bierme,Richard,Moisan,2012] [Polisano,Clausel,Perrier,Condat,2014]



General model :

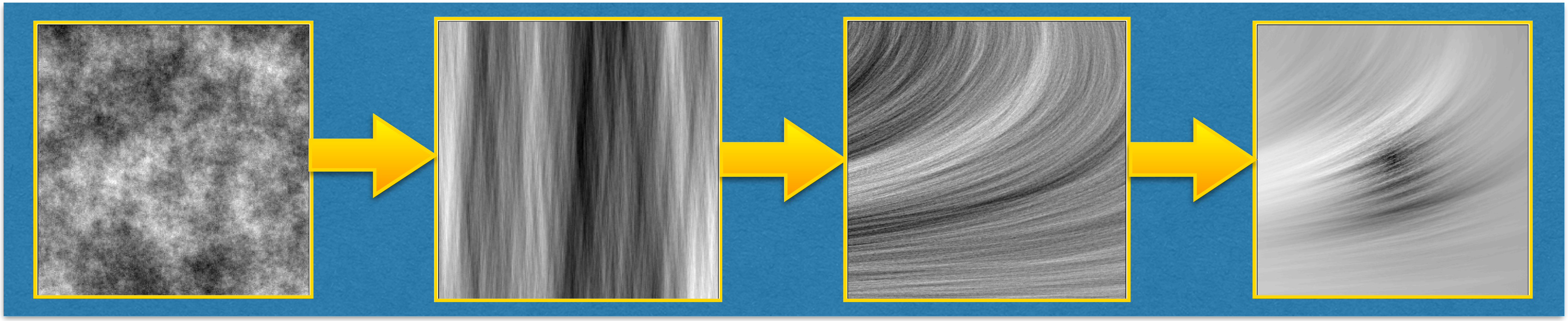
anisotropic self-similar Gaussian fields

$$X(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1)\boxed{f^{1/2}(\mathbf{x},\xi)}\, d\widehat{W}(\xi)$$

spectral density

$$\boxed{f^{1/2}(\mathbf{x},\xi) = c(\mathbf{x},\xi)\|\xi\|^{-h(\mathbf{x},\xi)-1}}$$

- $c(\mathbf{x},\xi) \equiv 1$ and $h(\mathbf{x},\xi) \equiv H \Rightarrow X = B^H$ [Mandelbrot,Van Ness,1968]
- $c(\mathbf{x},\xi) \equiv c(\arg\xi)$ and $h(\mathbf{x},\xi) \equiv h(\arg\xi) \Rightarrow X = AFBF$ [Bonami,Estrade,2003]
  - Example : *elementary fields* $c(\arg\xi) = \mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0)$ [Bierme,Richard,Moisan,2012]
- $c(\mathbf{x},\xi) \equiv c(\mathbf{x},\arg\xi)$ and $h(\mathbf{x},\xi) \equiv h(\mathbf{x})$ [Polisano,Clausel,Perrier,Condat,2014]



Locally Anisotropic Fractional Brownian Field (LAFBF)

- **Definition**: Our new Gaussian model LAFBF is a local version of the elementary field

$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1)\frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0(\mathbf{x}))}{\|\xi\|^{H+1}} d\widehat{W}(\xi)$$

[Polisano et al.,2014]

Locally Anisotropic Fractional Brownian Field (LAFBF)

- **Definition**: Our new Gaussian model LAFBF is a local version of the elementary field

$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) \frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg \xi - \boxed{\alpha_0(\mathbf{x})})}{\|\xi\|^{H+1}} d\widehat{W}(\xi)$$

[Polisano et al.,2014]

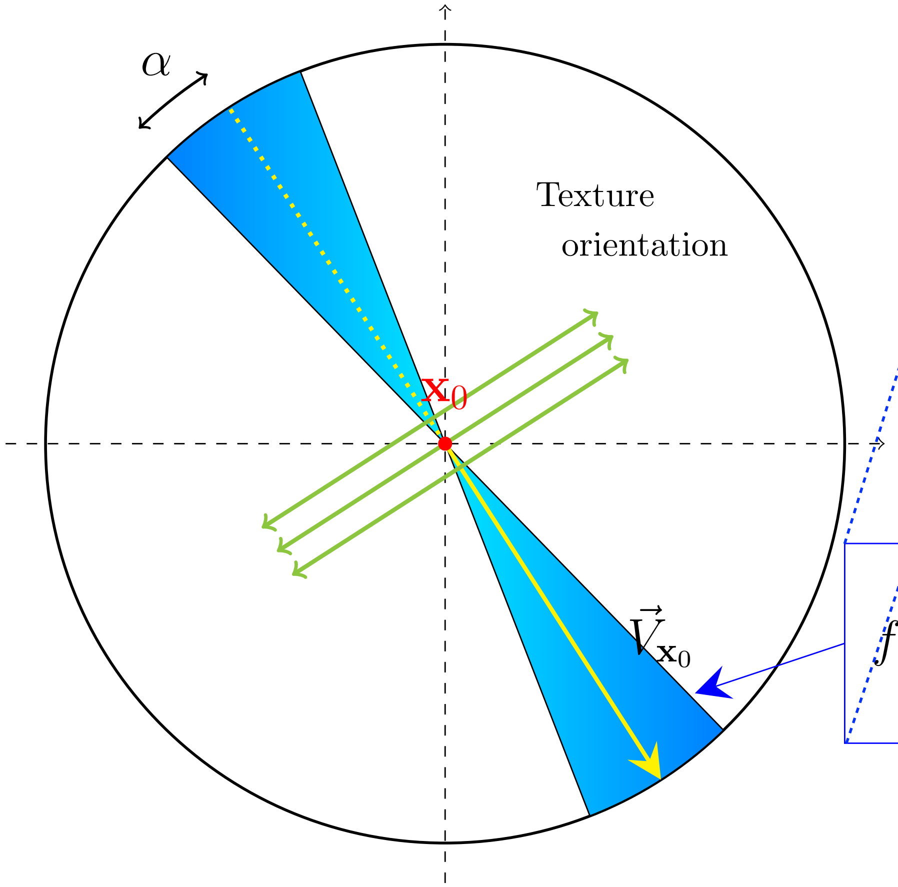
The orientation may varies spatially. α_0 is now a differentiable function on $\mathbb{R}^2$

Locally Anisotropic Fractional Brownian Field (LAFBF)

- **Definition**: Our new Gaussian model LAFBF is a local version of the elementary field

$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\boldsymbol{\xi}} - 1)\frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg\boldsymbol{\xi} - \alpha_0(\mathbf{x}))}{\|\boldsymbol{\xi}\|^{H+1}}\, d\widehat{W}(\boldsymbol{\xi})$$

[Polisano et al.,2014]

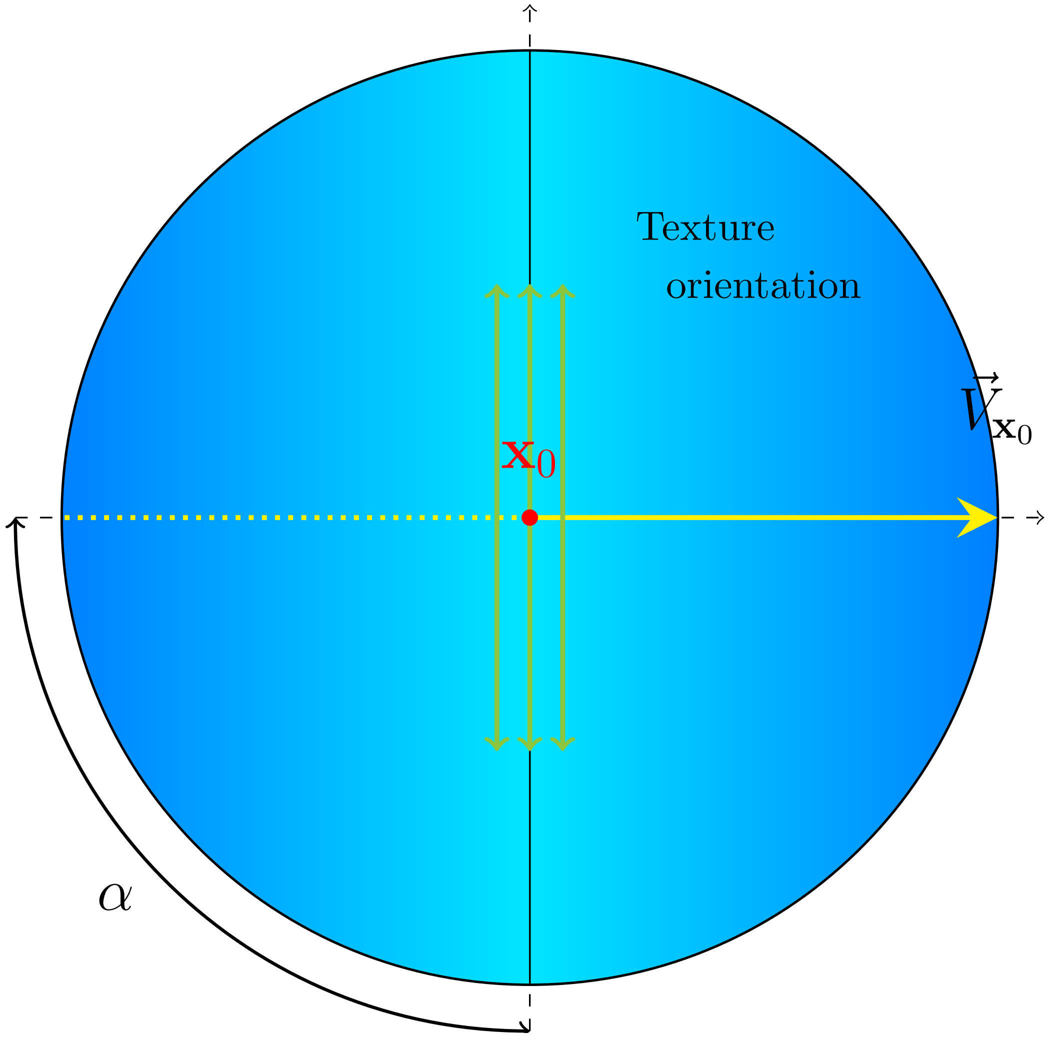


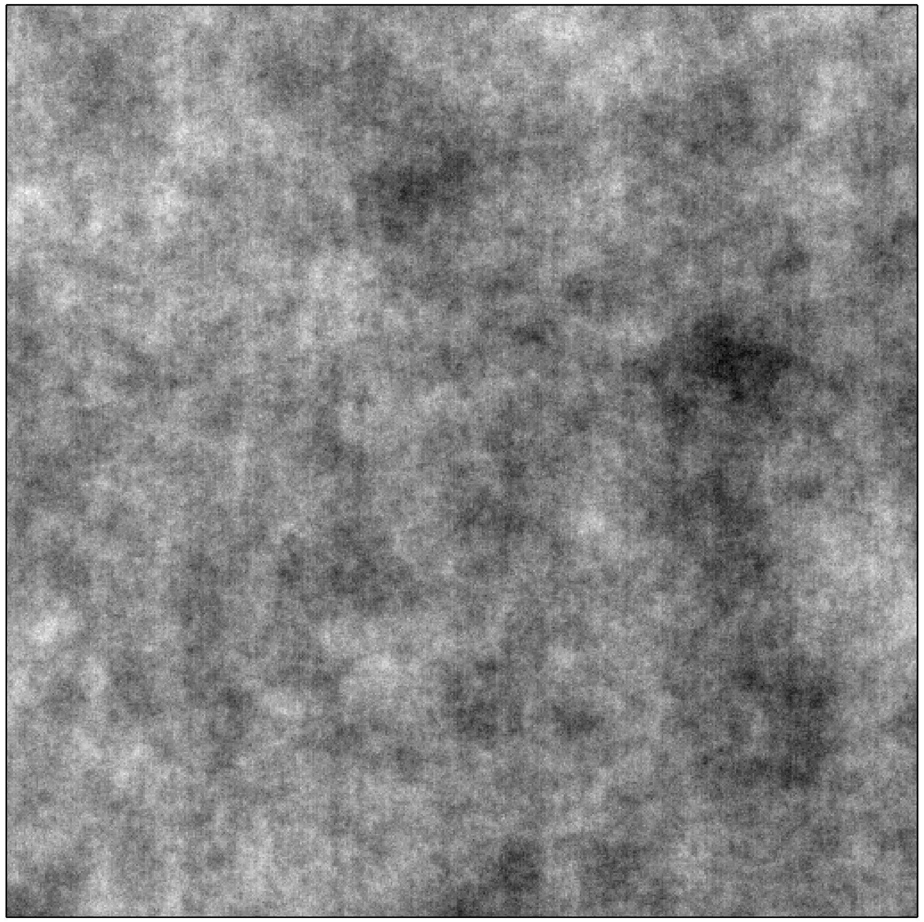
$$f^{1/2}(\mathbf{x}_0, \boldsymbol{\xi}) = \frac{c_{\alpha_0,\alpha}(\mathbf{x}_0, \arg\boldsymbol{\xi})}{\|\boldsymbol{\xi}\|^{H+1}}$$

The orientation may varies spatially. α_0 is now a differentiable function on $\mathbb{R}^2$

Elementary field

$$\alpha_0 = 0$$

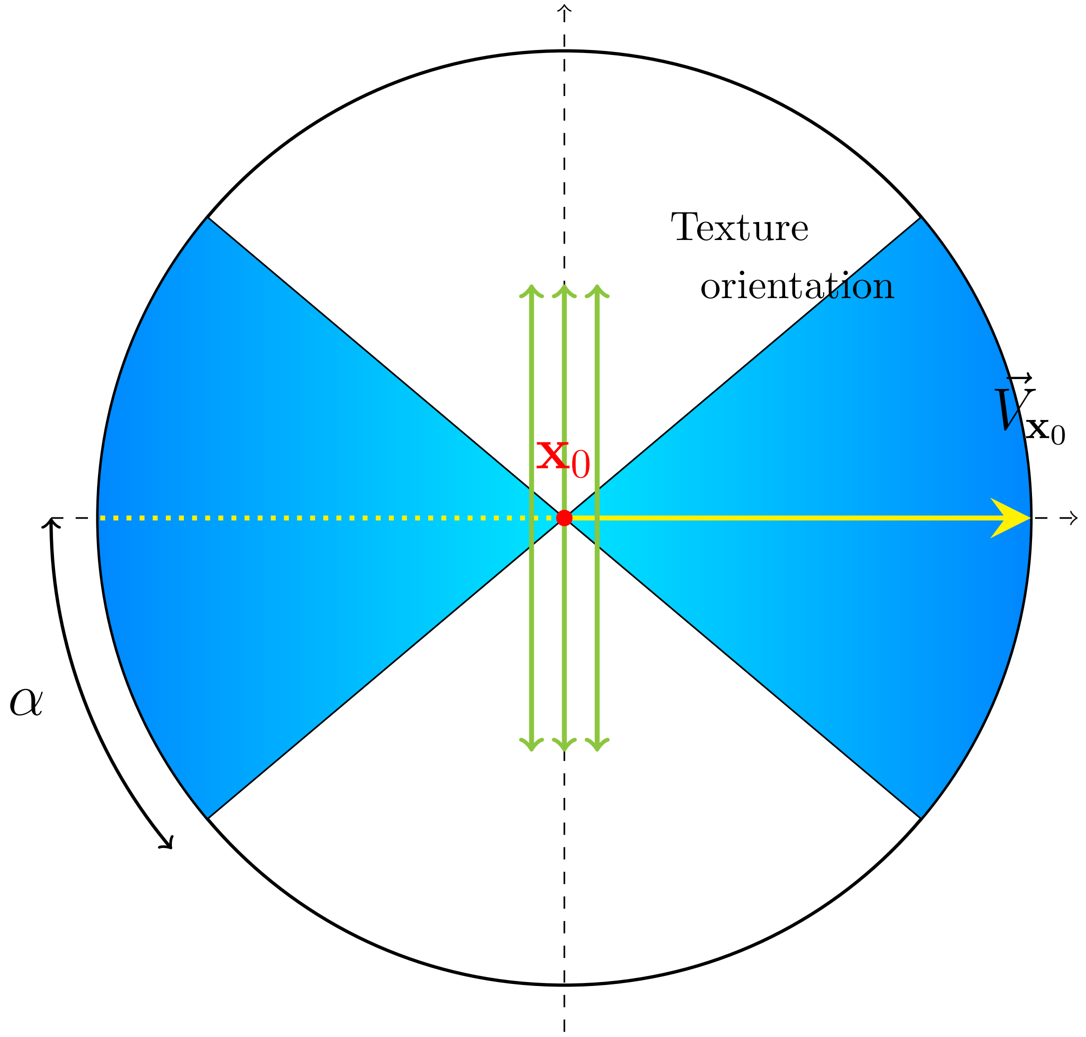


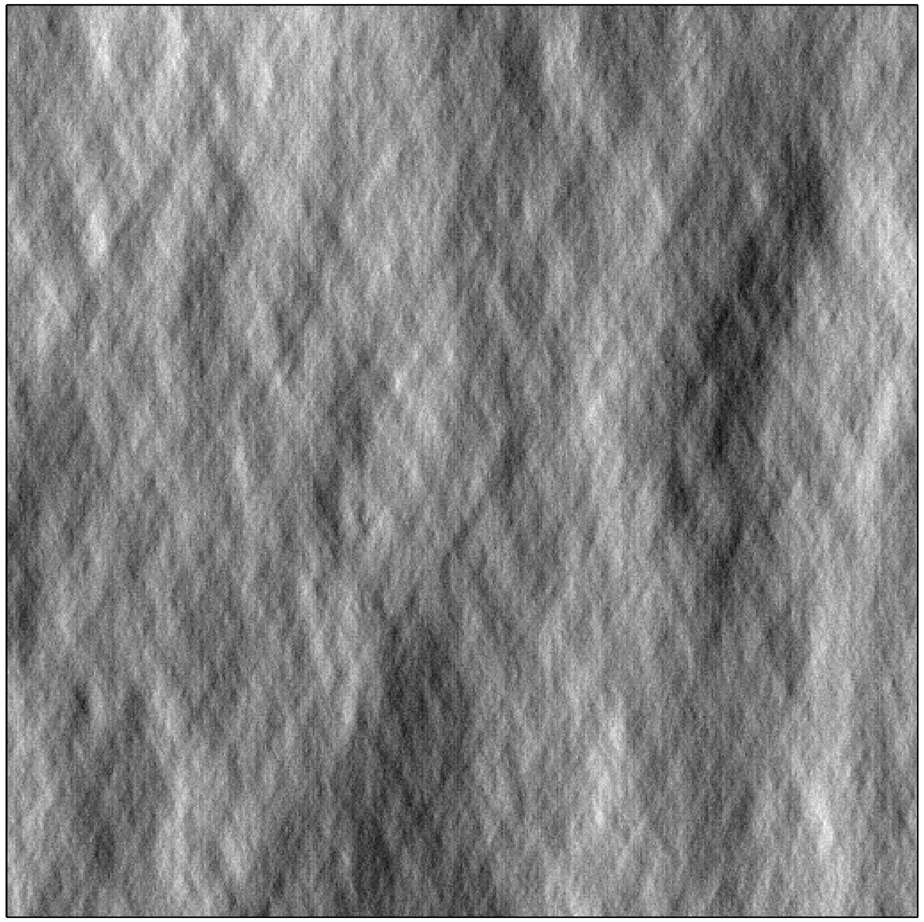


$$\alpha = -\frac{\pi}{2}$$

Elementary field

$\alpha_0 = 0$

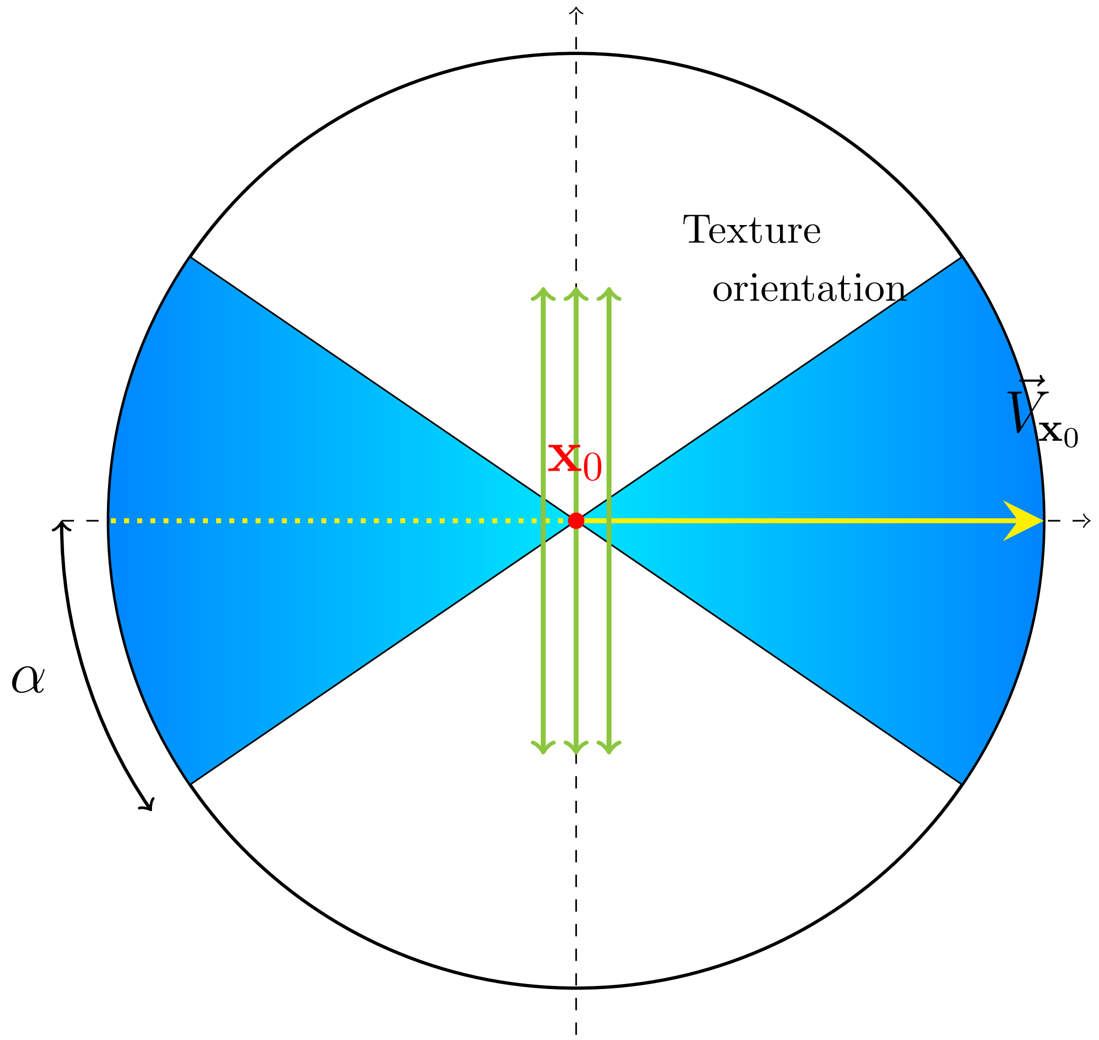


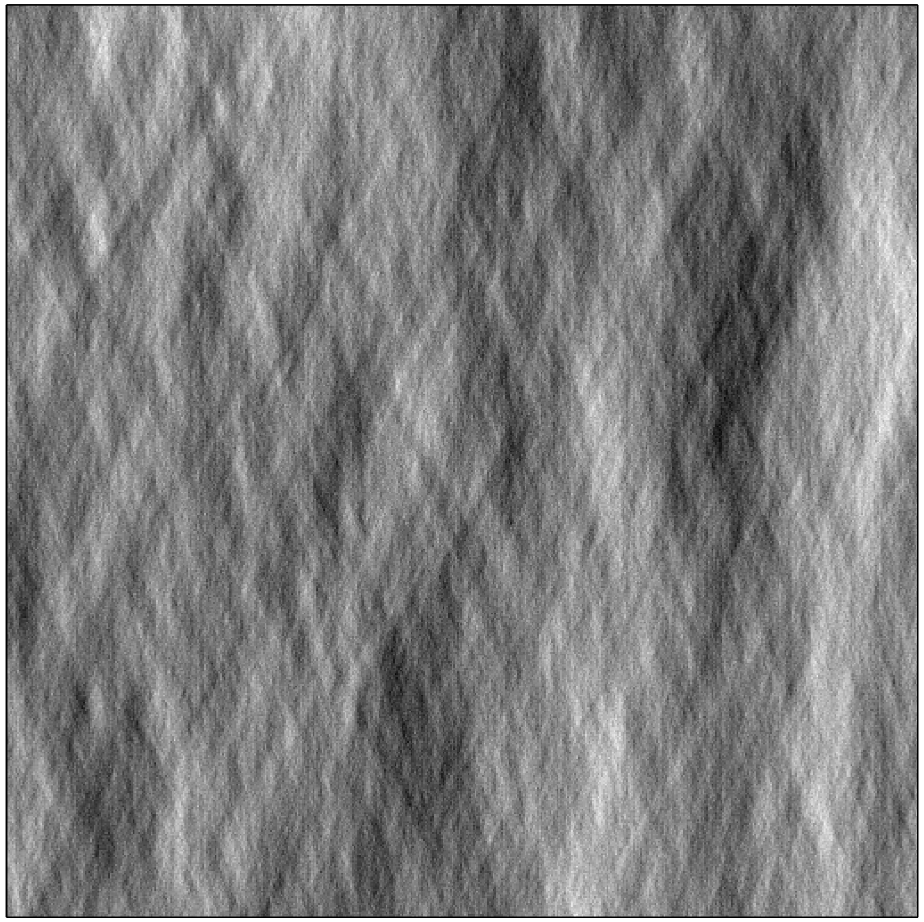


$\alpha = 0.7$

Elementary field

$\alpha_0 = 0$

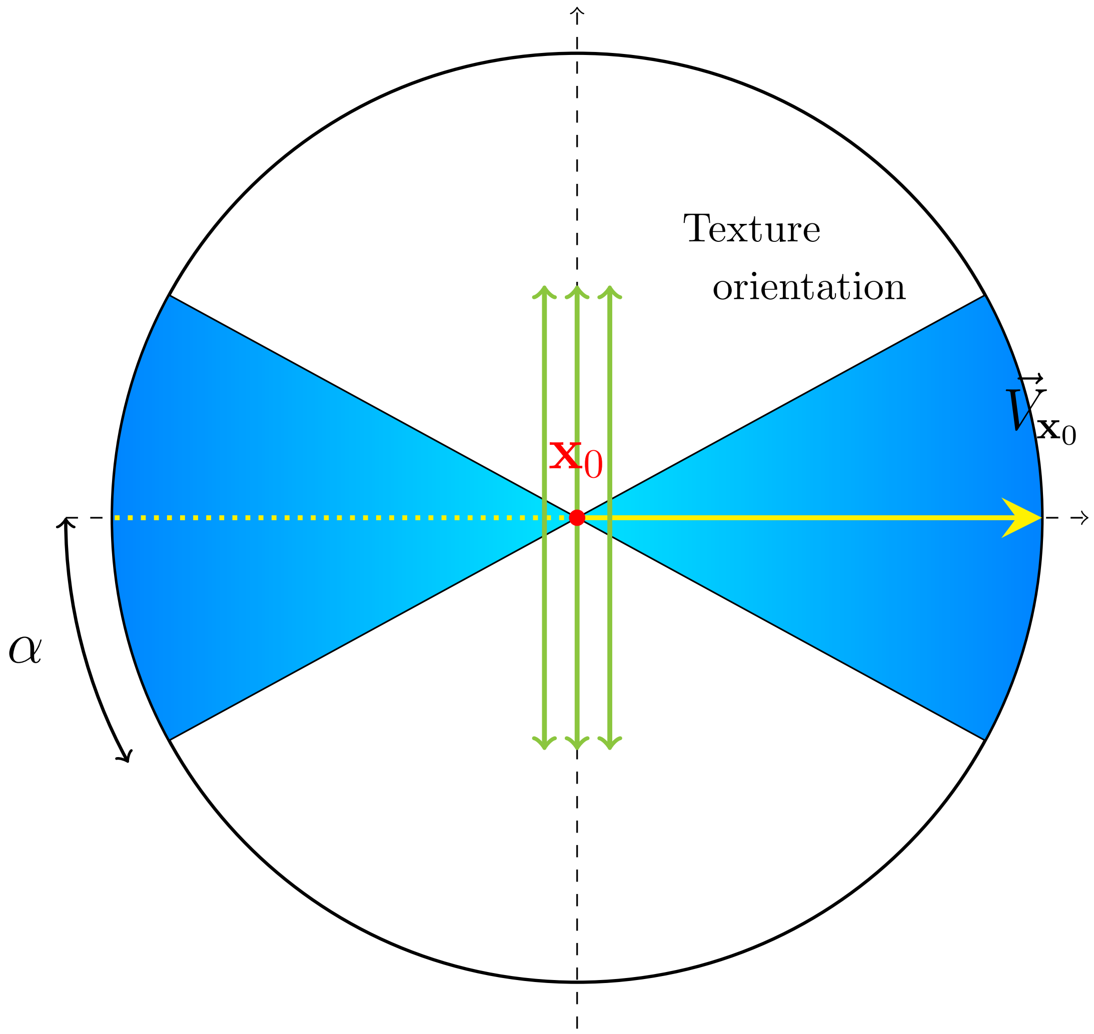


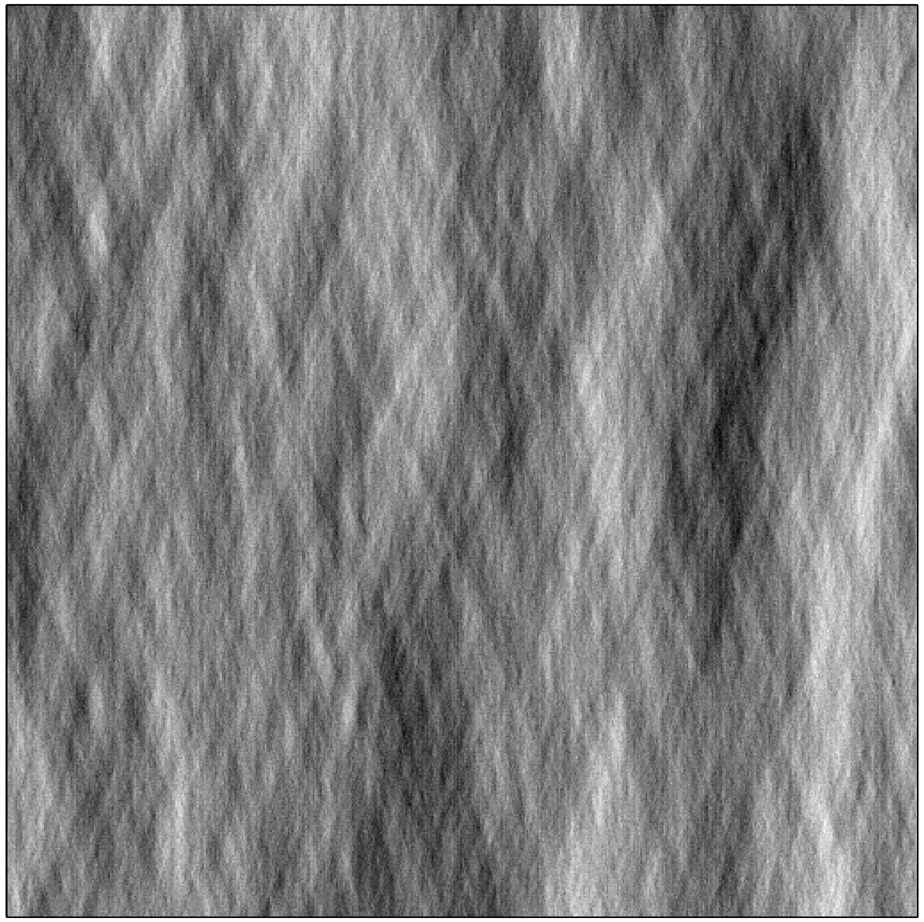


$\alpha = 0.6$

Elementary field

$\alpha_0 = 0$

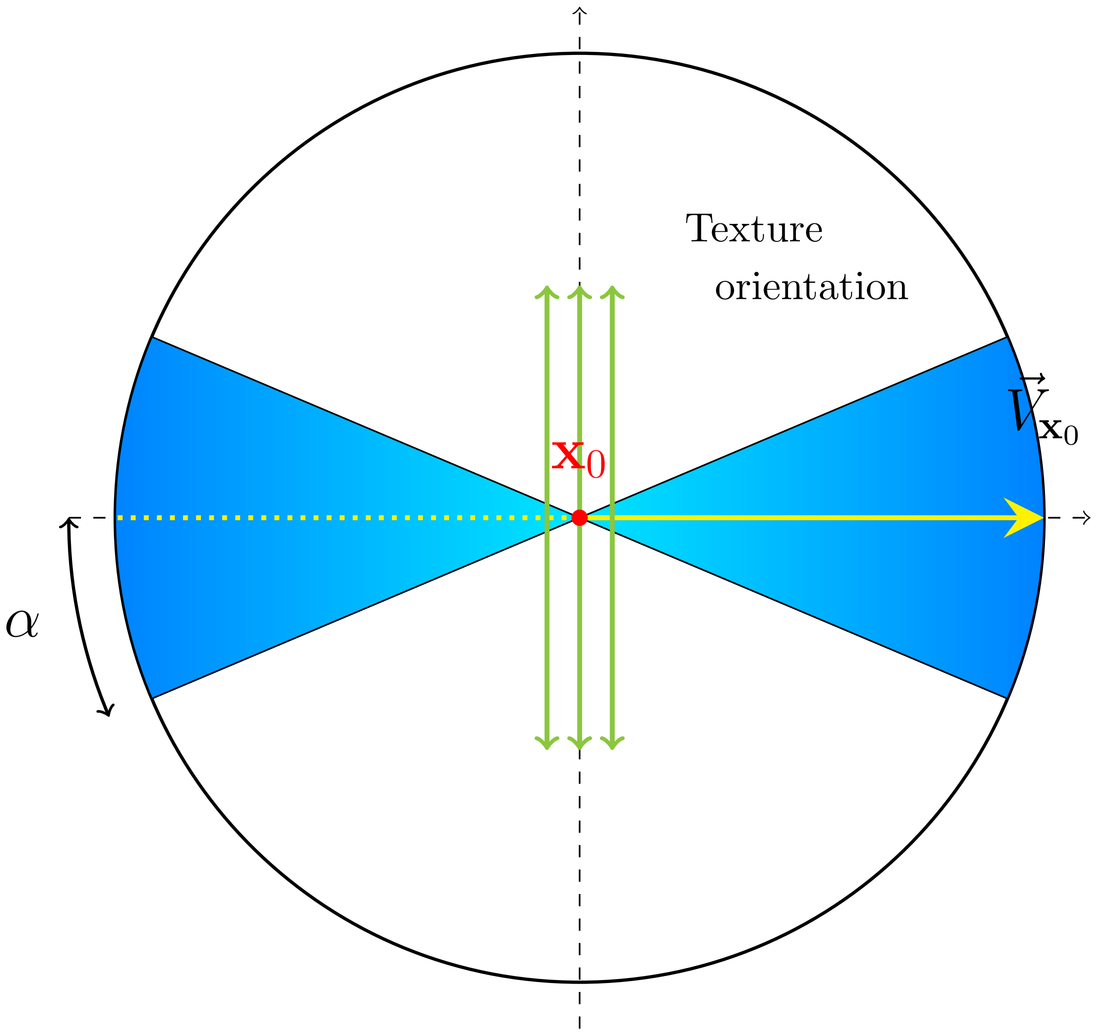


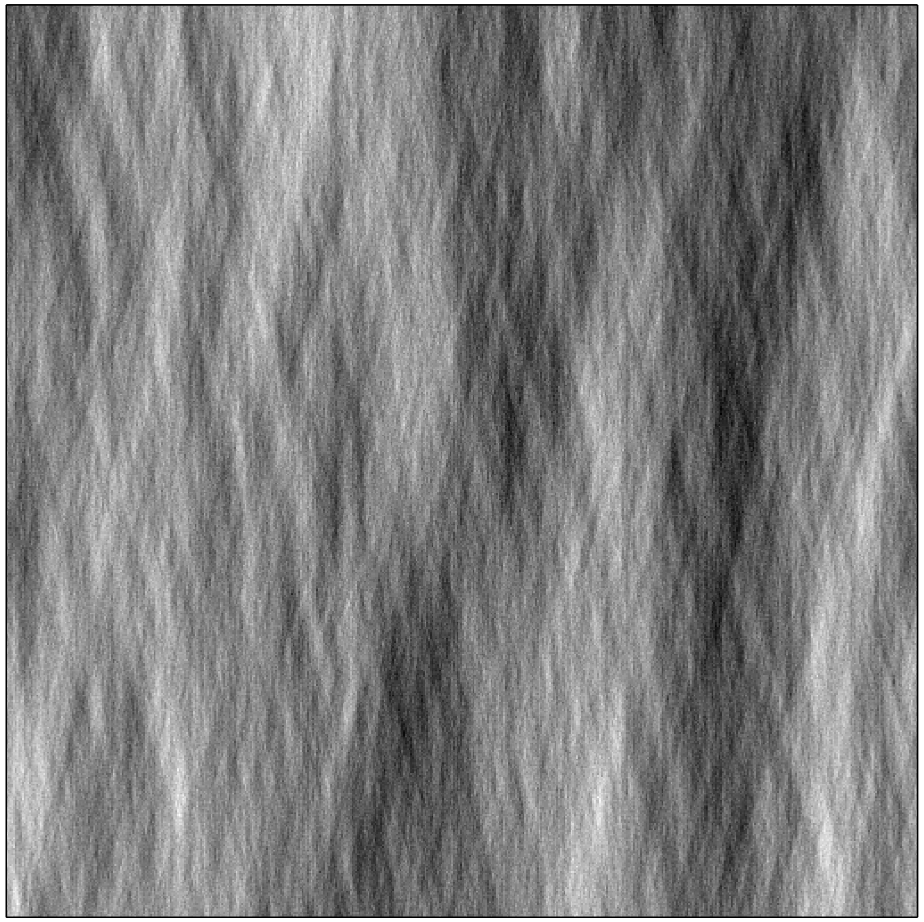


$\alpha = 0.5$

Elementary field

$\alpha_0 = 0$

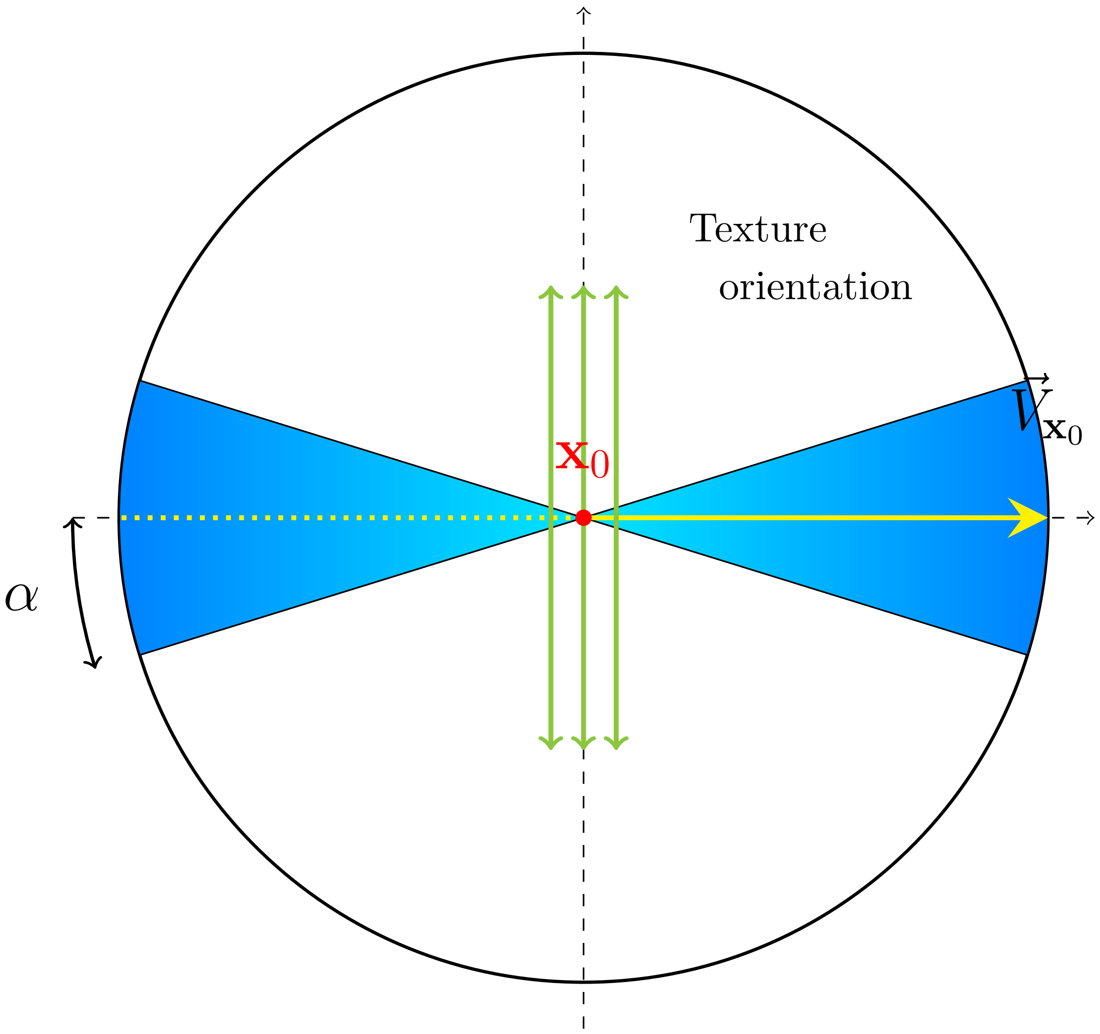


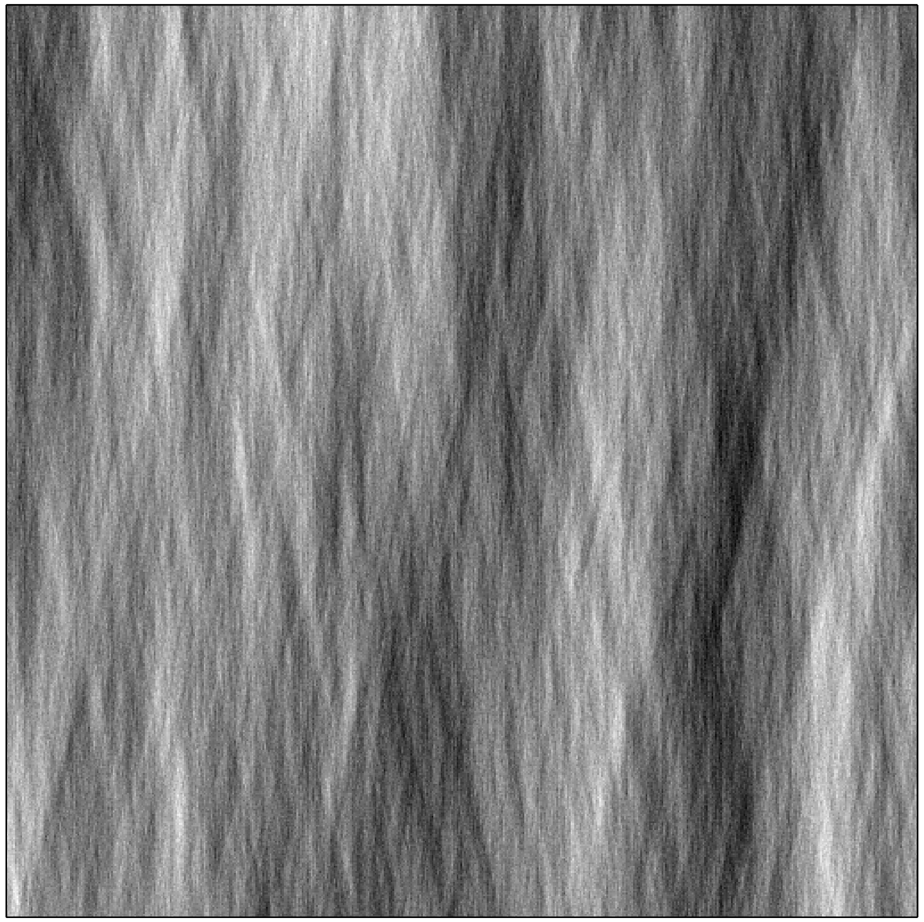


$\alpha = 0.4$

Elementary field

$\alpha_0 = 0$

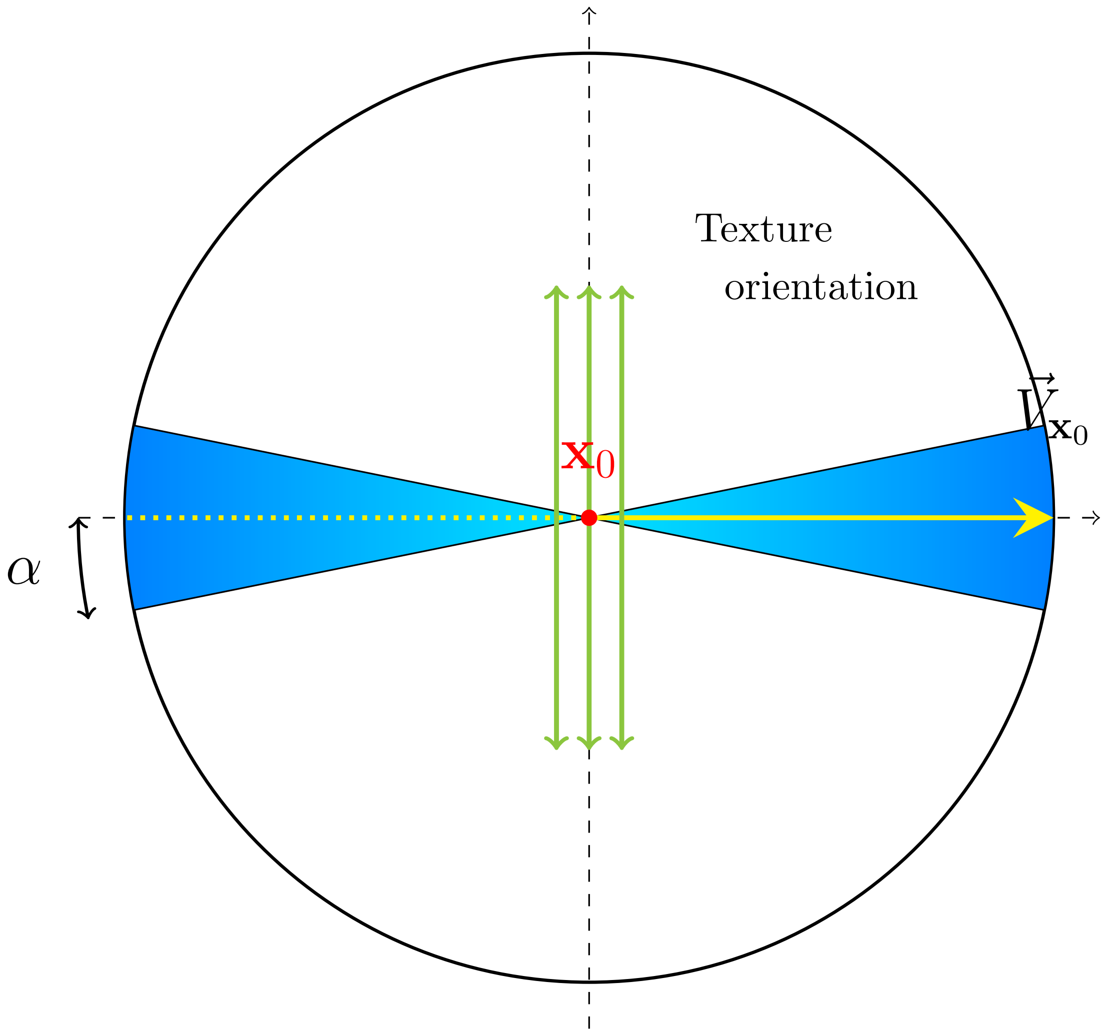




$\alpha = 0.3$

Elementary field

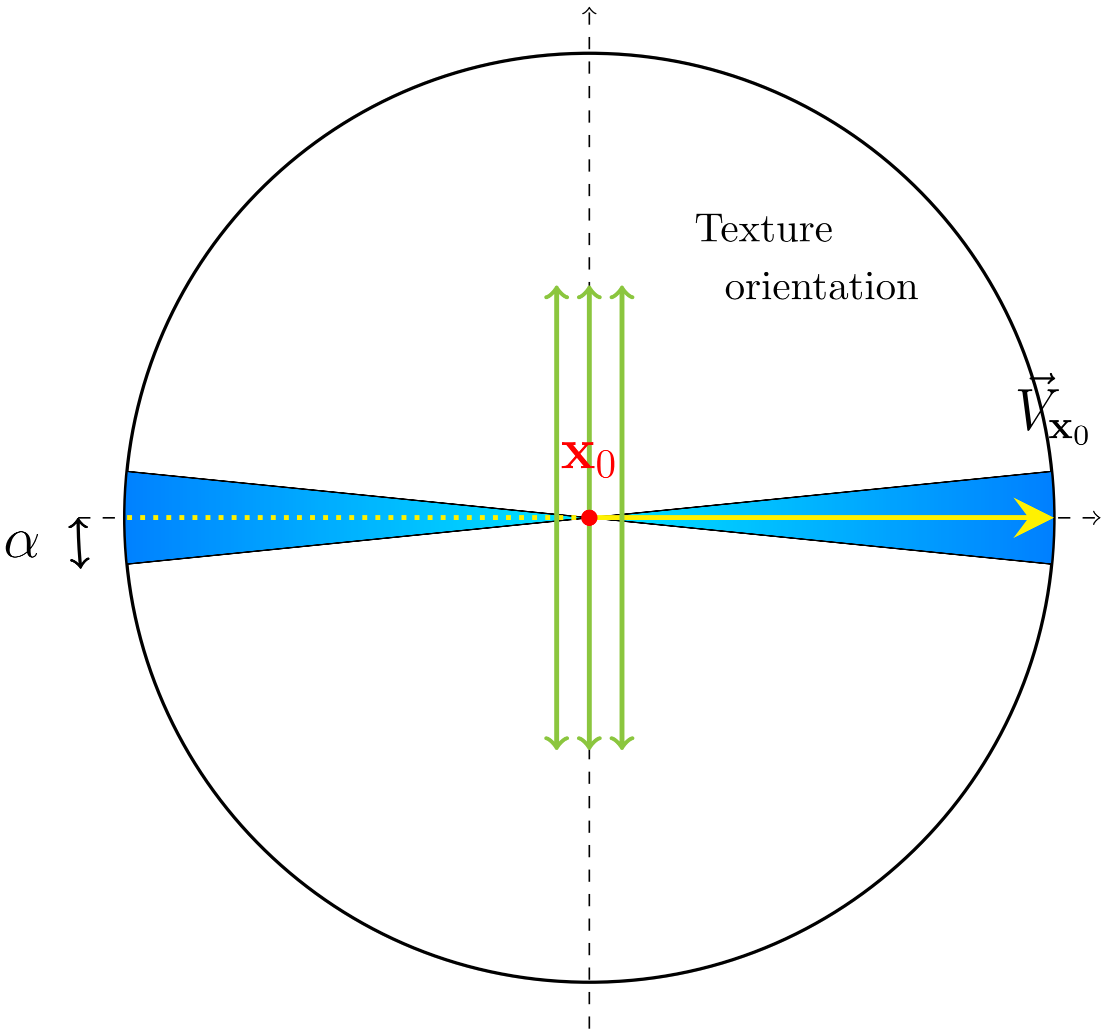
$\alpha_0 = 0$

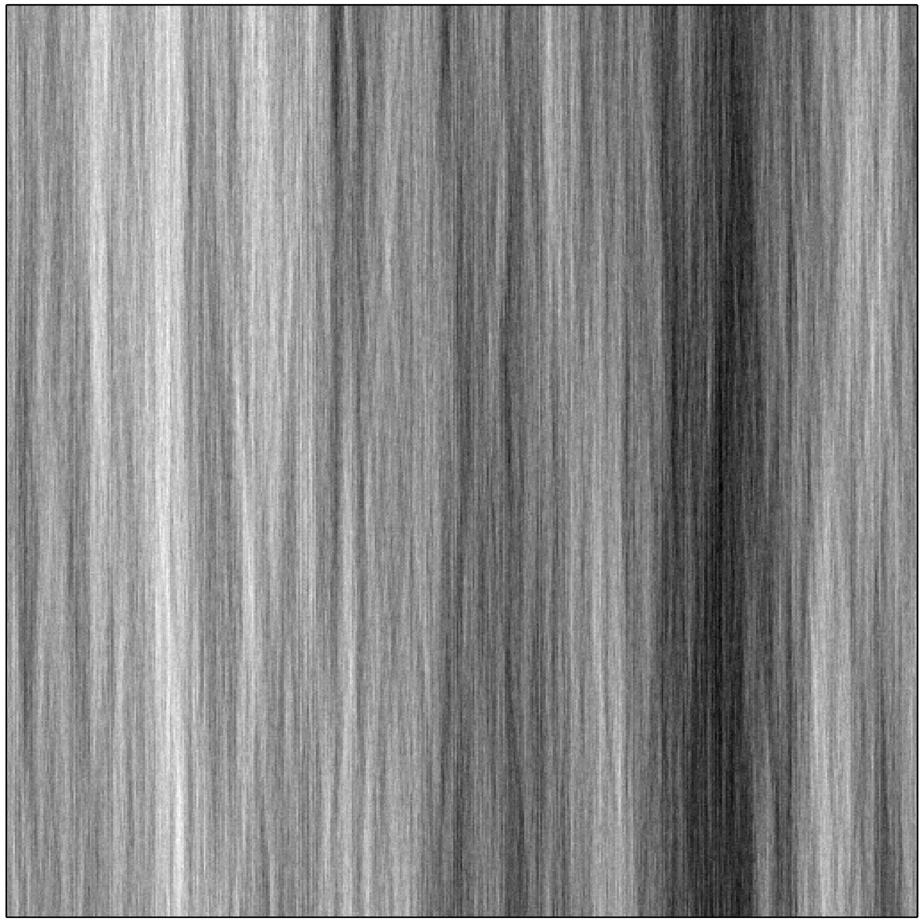


$\alpha = 0.2$

Elementary field

$\alpha_0 = 0$

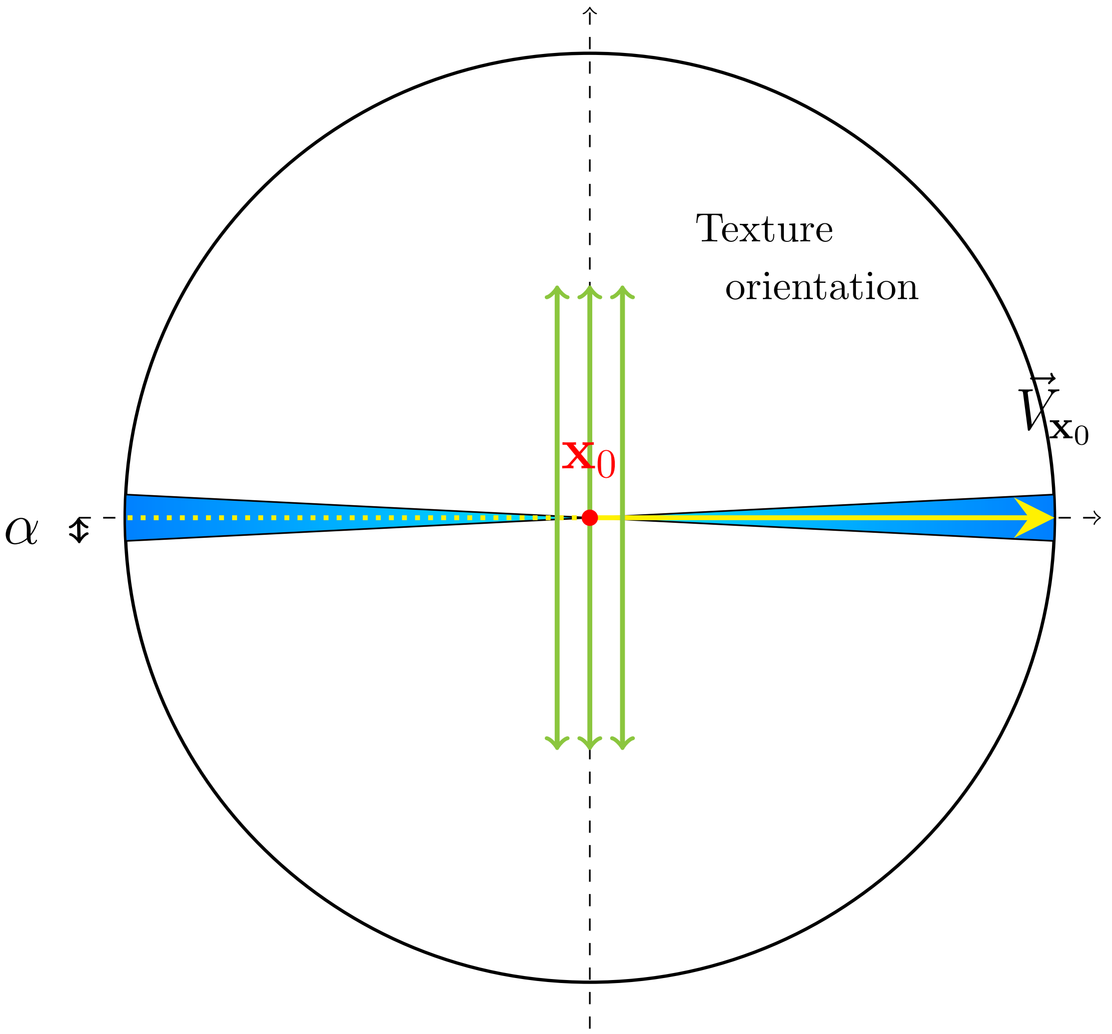


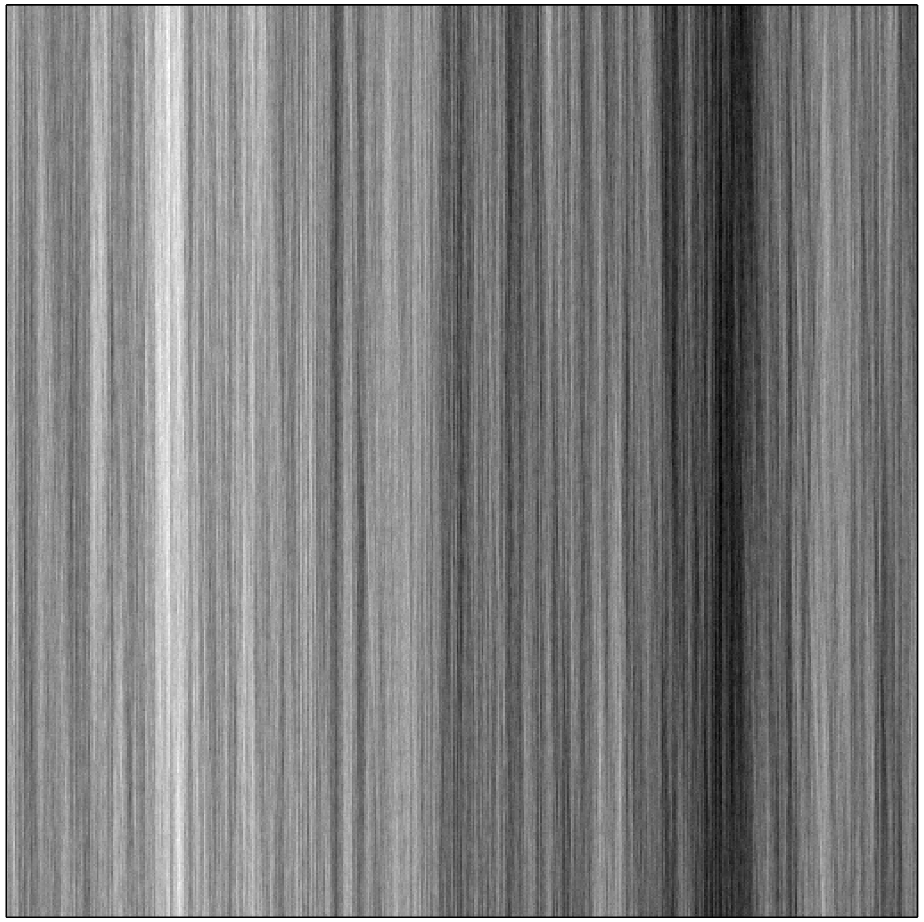


$\alpha = 0.1$

Elementary field

$\alpha_0 = 0$

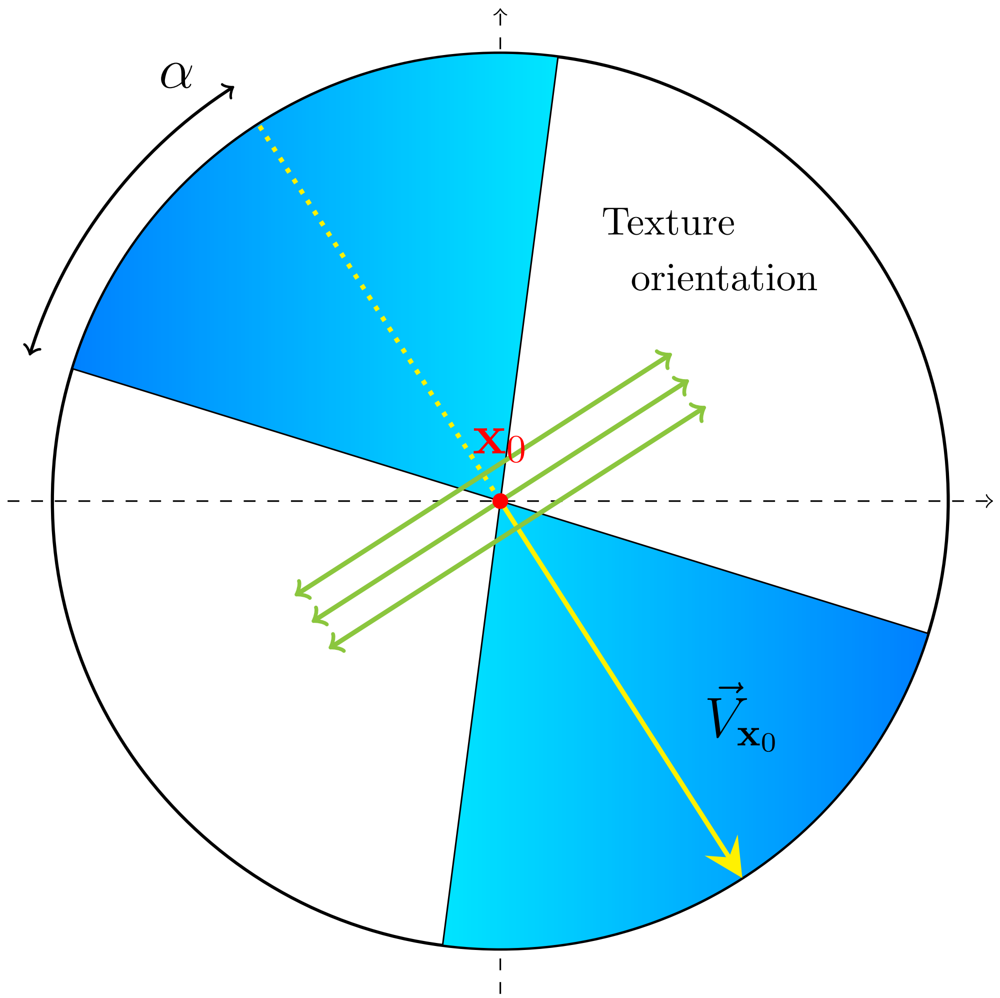


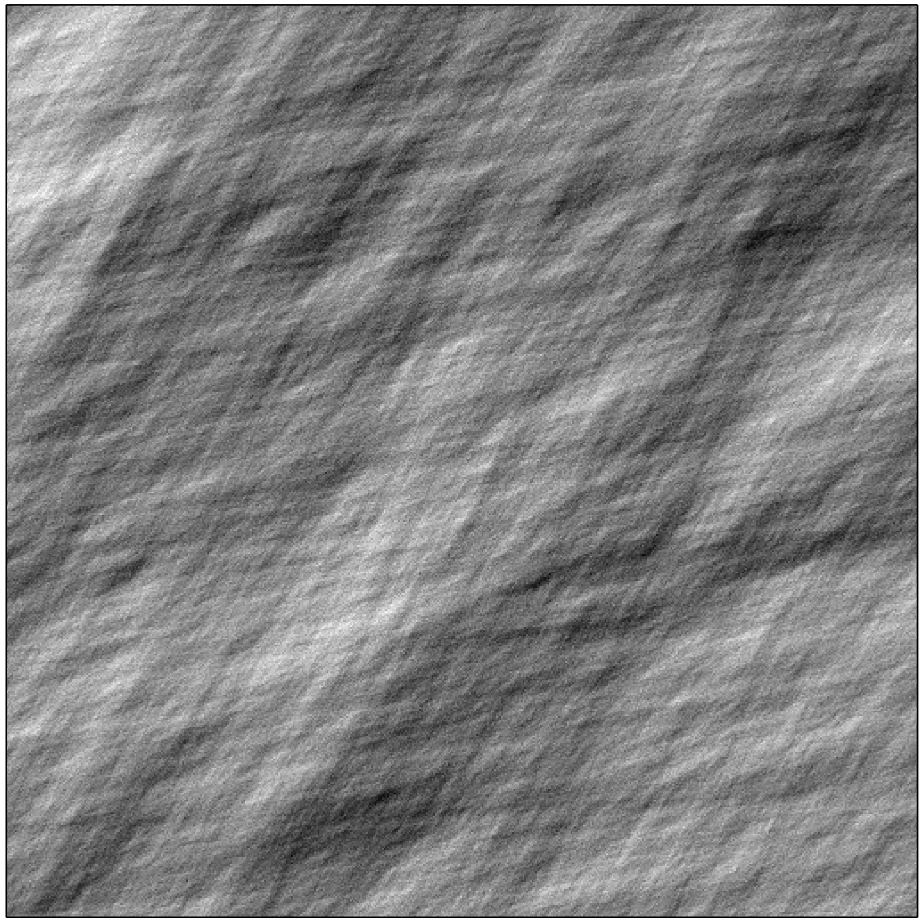


$\alpha = 0.05$

Elementary field

$$\alpha_0 = -\frac{\pi}{3}$$

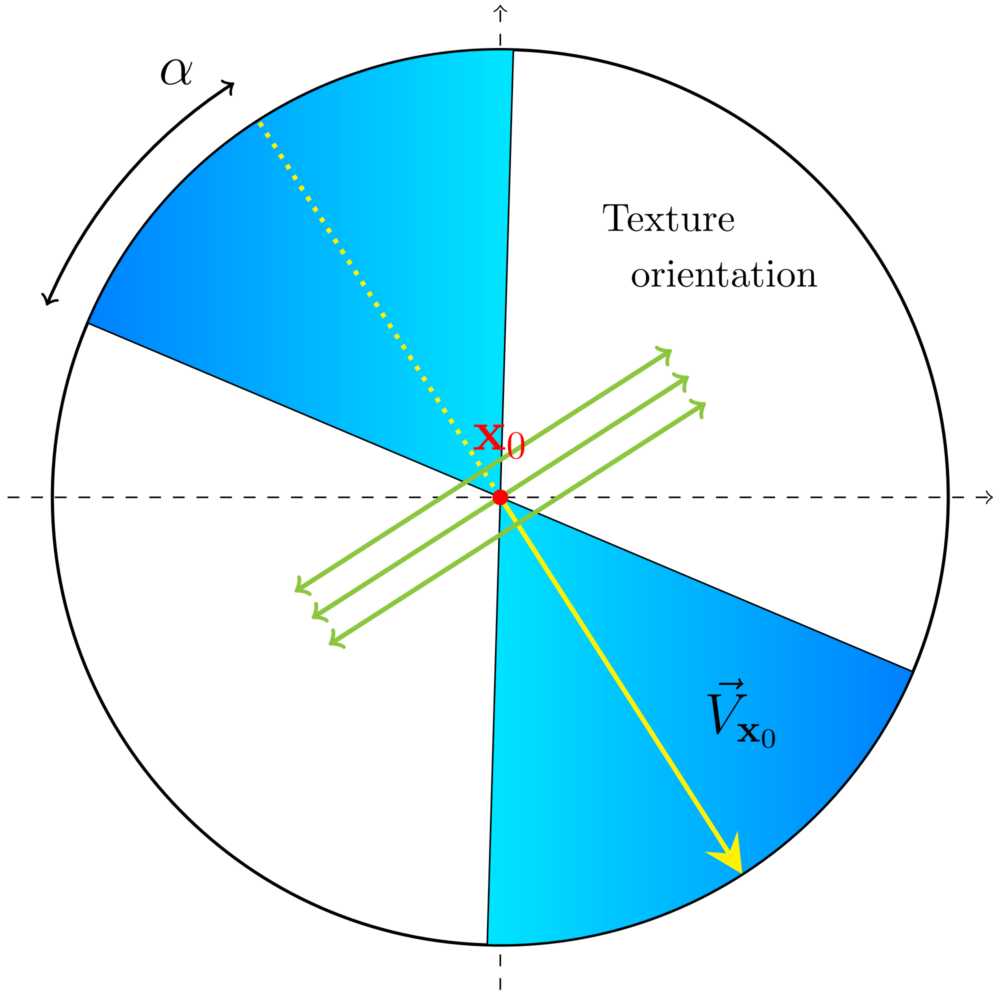


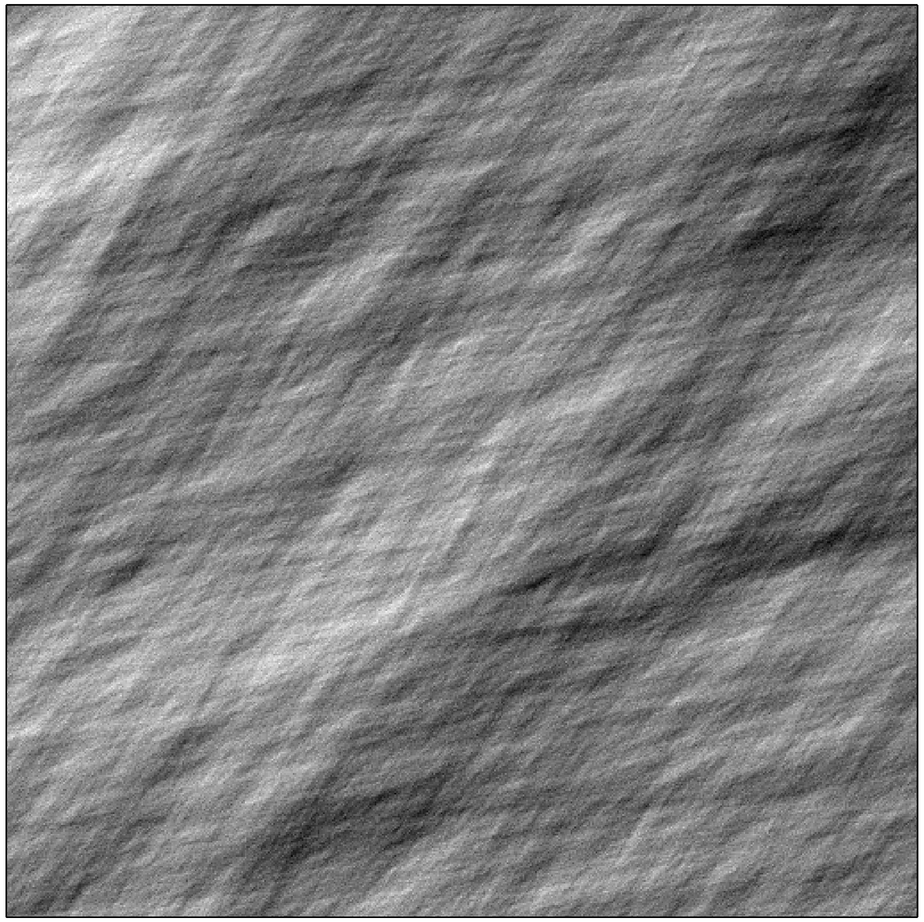


$$\alpha = 0.7$$

Elementary field

$$\alpha_0 = -\frac{\pi}{3}$$

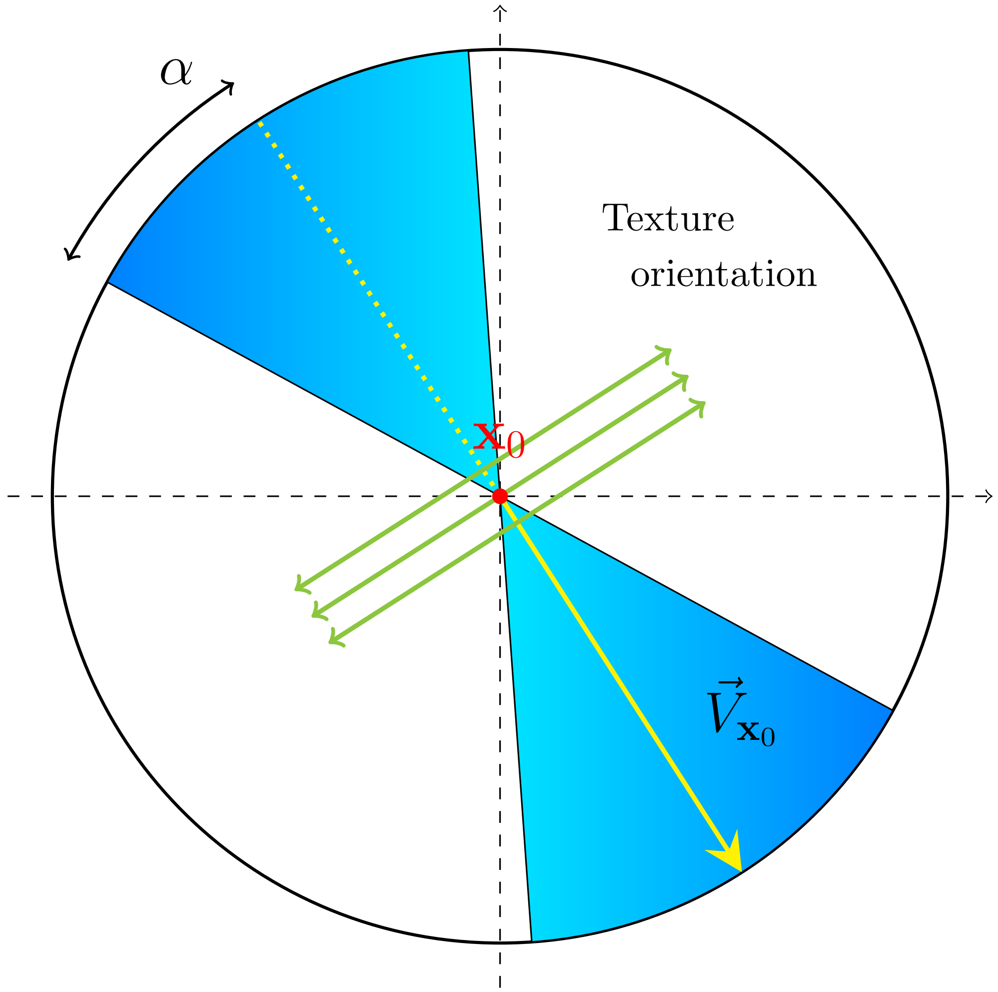


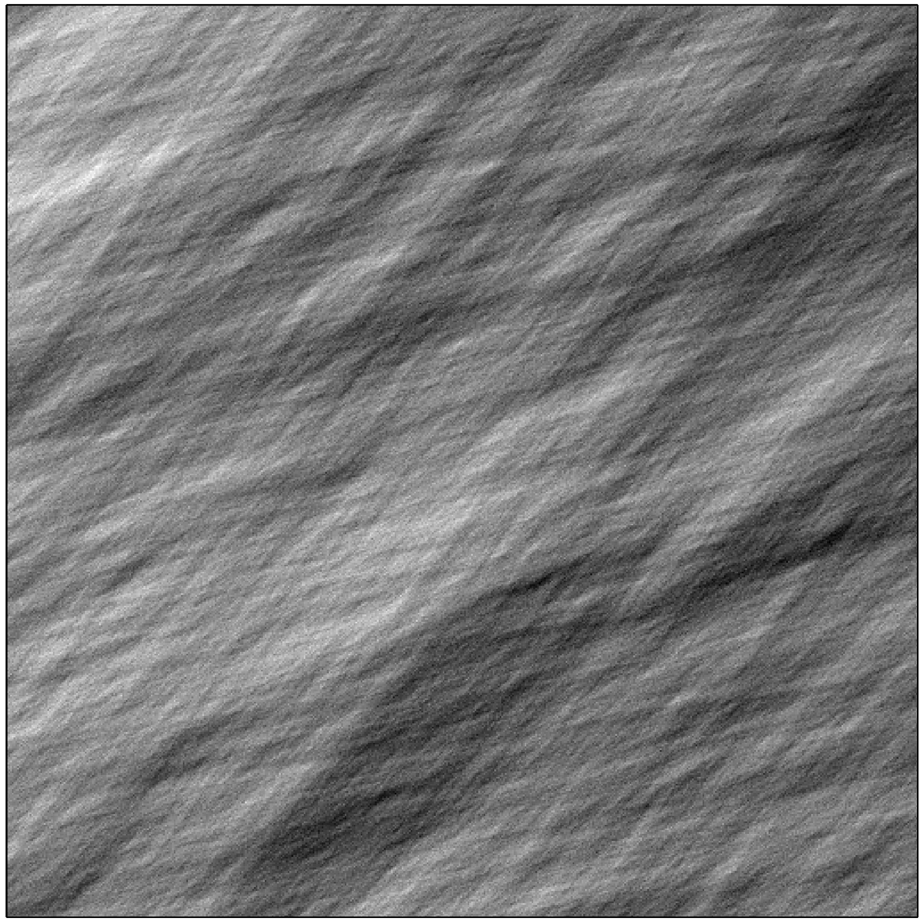


$$\alpha = 0.6$$

Elementary field

$$\alpha_0 = -\frac{\pi}{3}$$

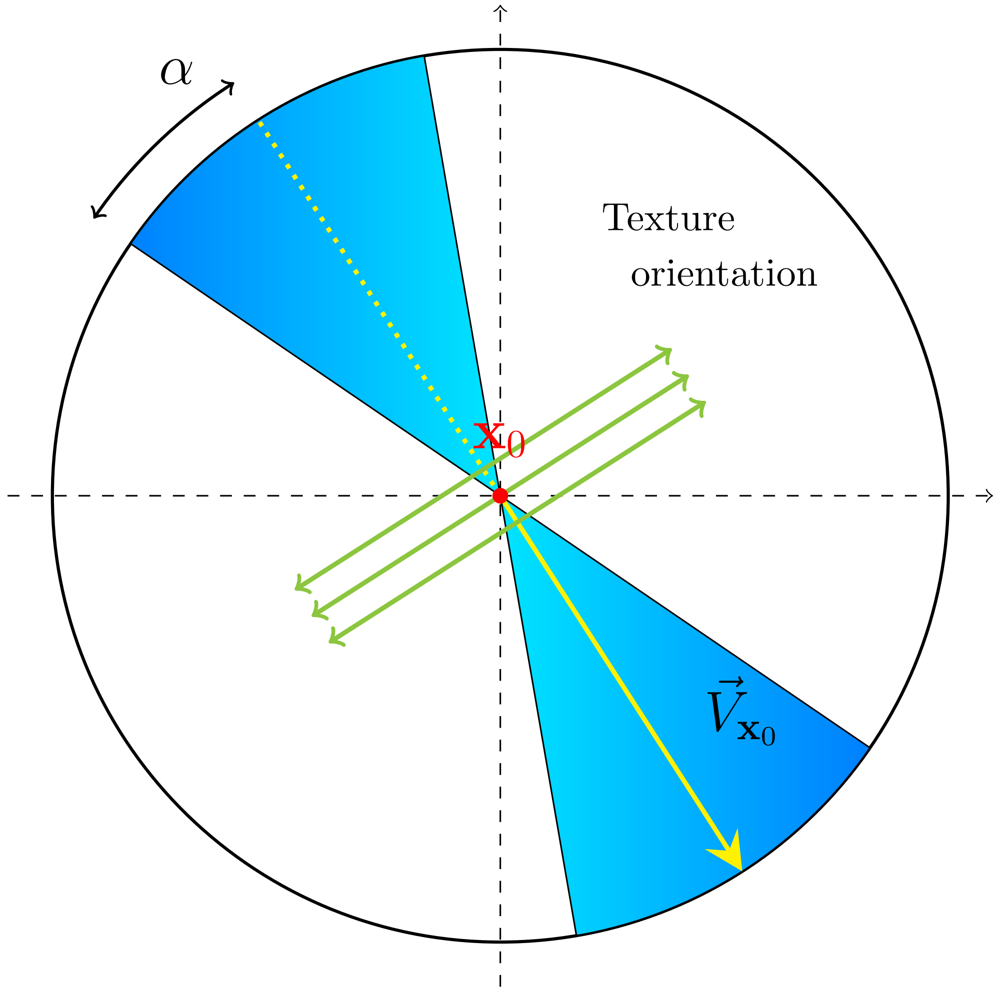


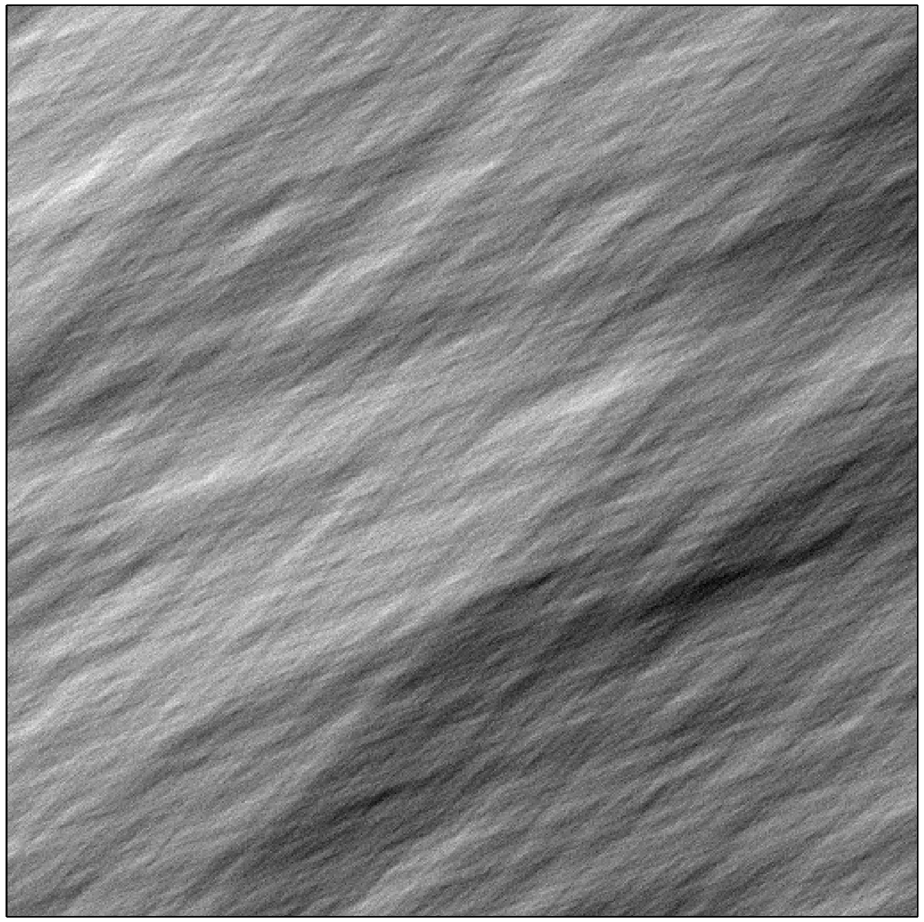


$$\alpha = 0.5$$

Elementary field

$$\alpha_0 = -\frac{\pi}{3}$$

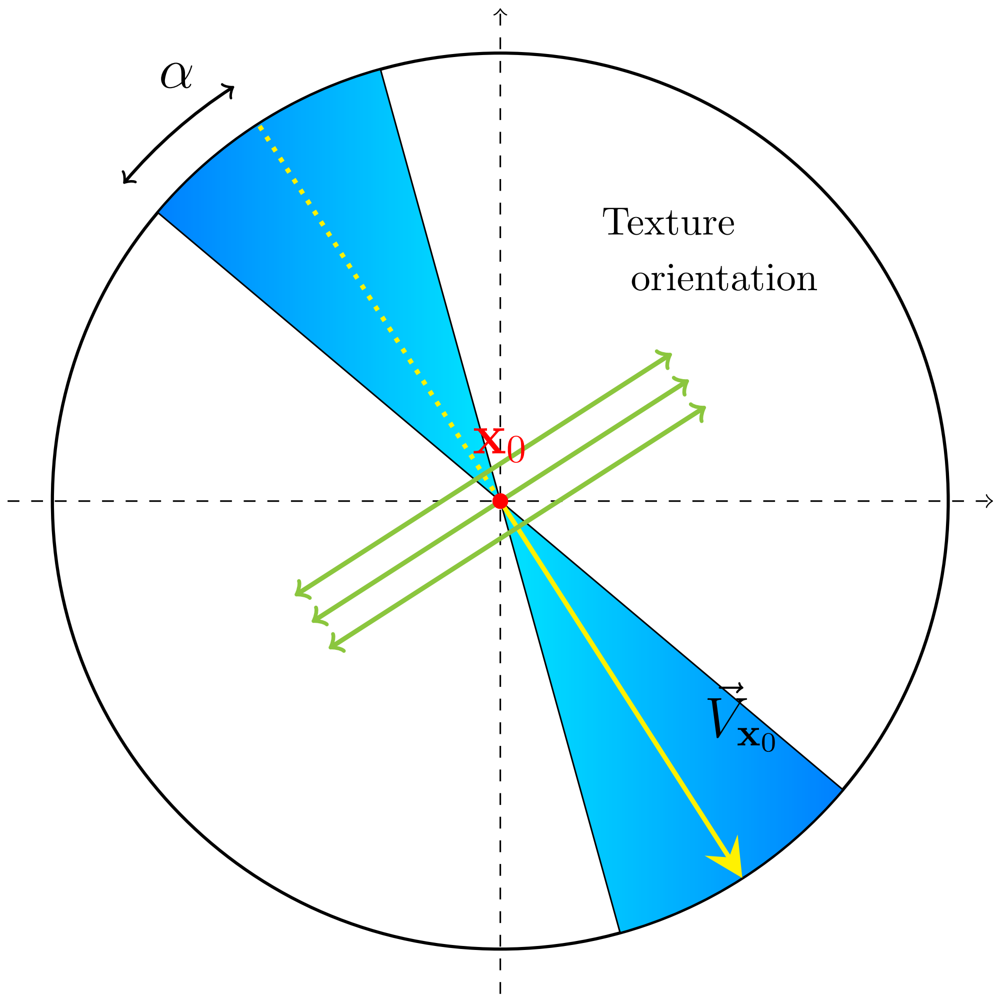




$$\alpha = 0.4$$

Elementary field

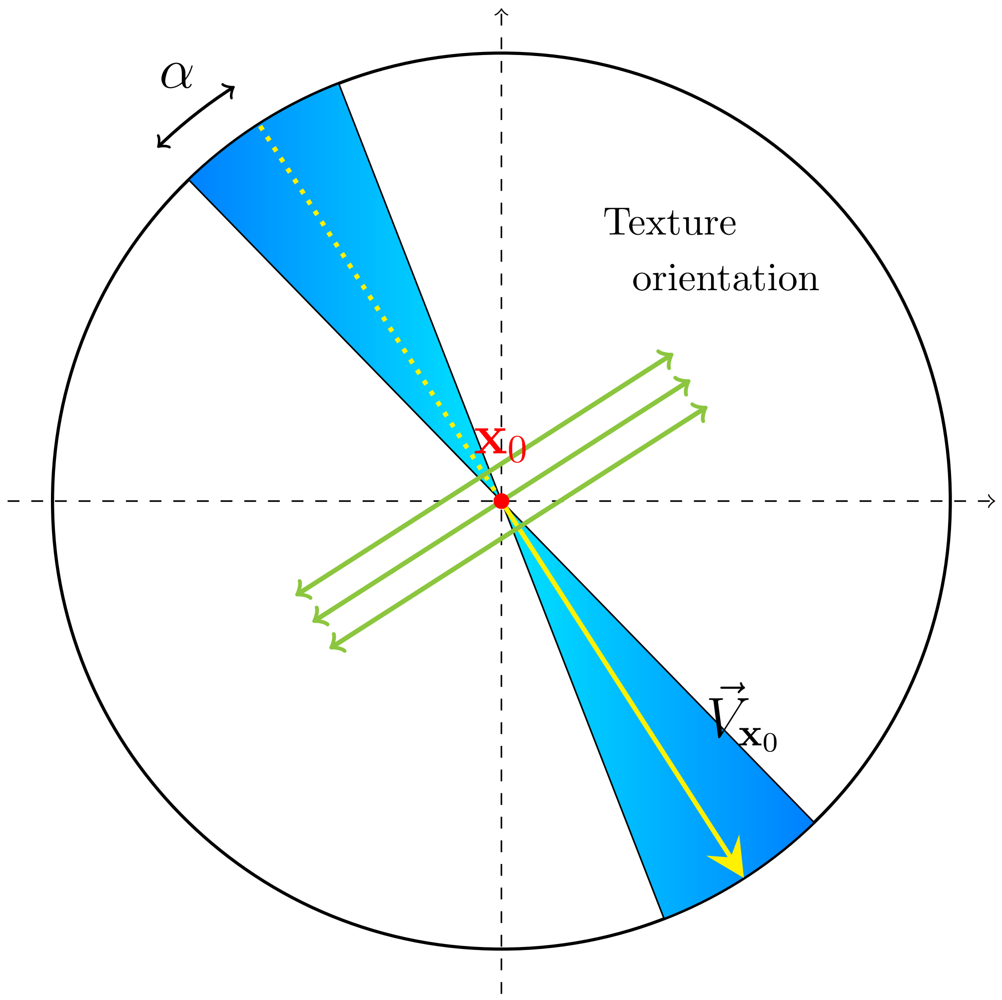
$$\alpha_0 = -\frac{\pi}{3}$$

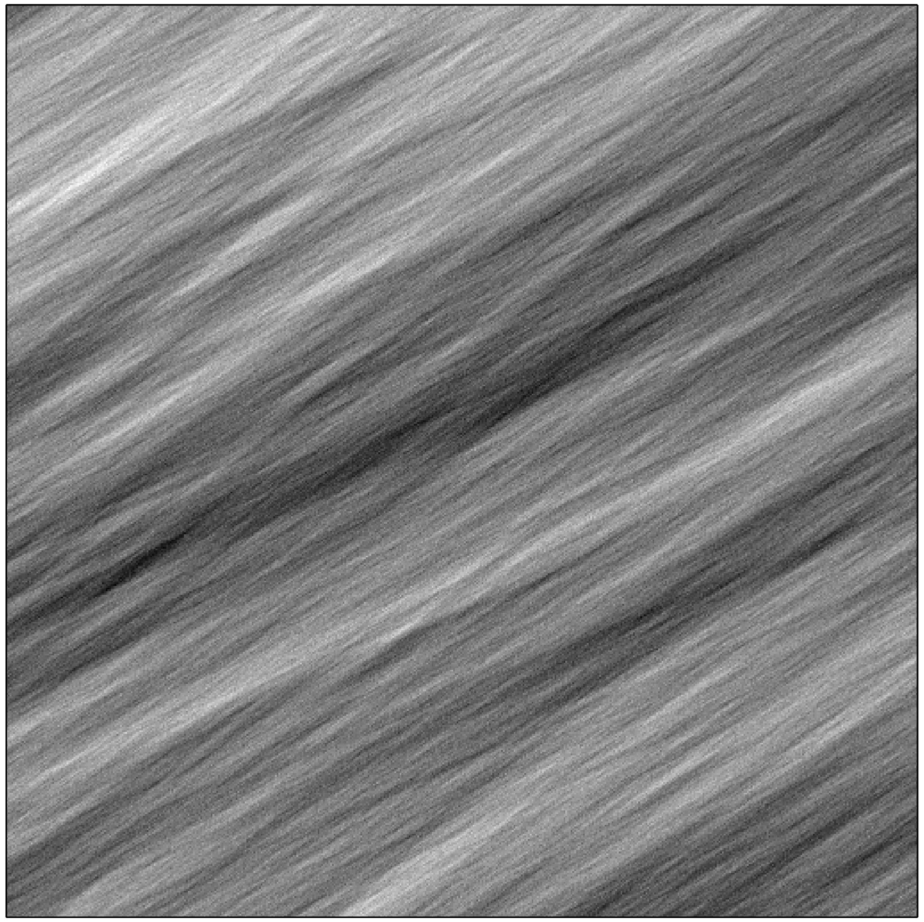


$$\alpha = 0.3$$

Elementary field

$$\alpha_0 = -\frac{\pi}{3}$$

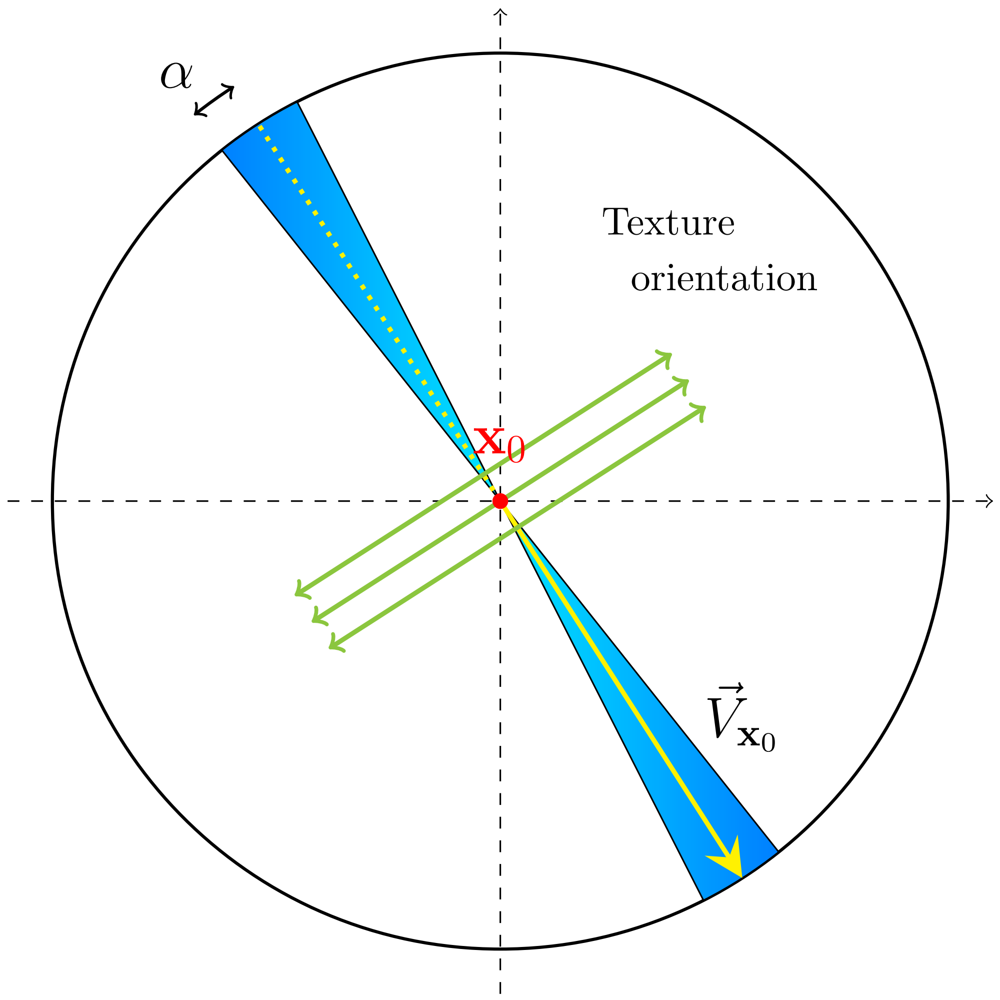


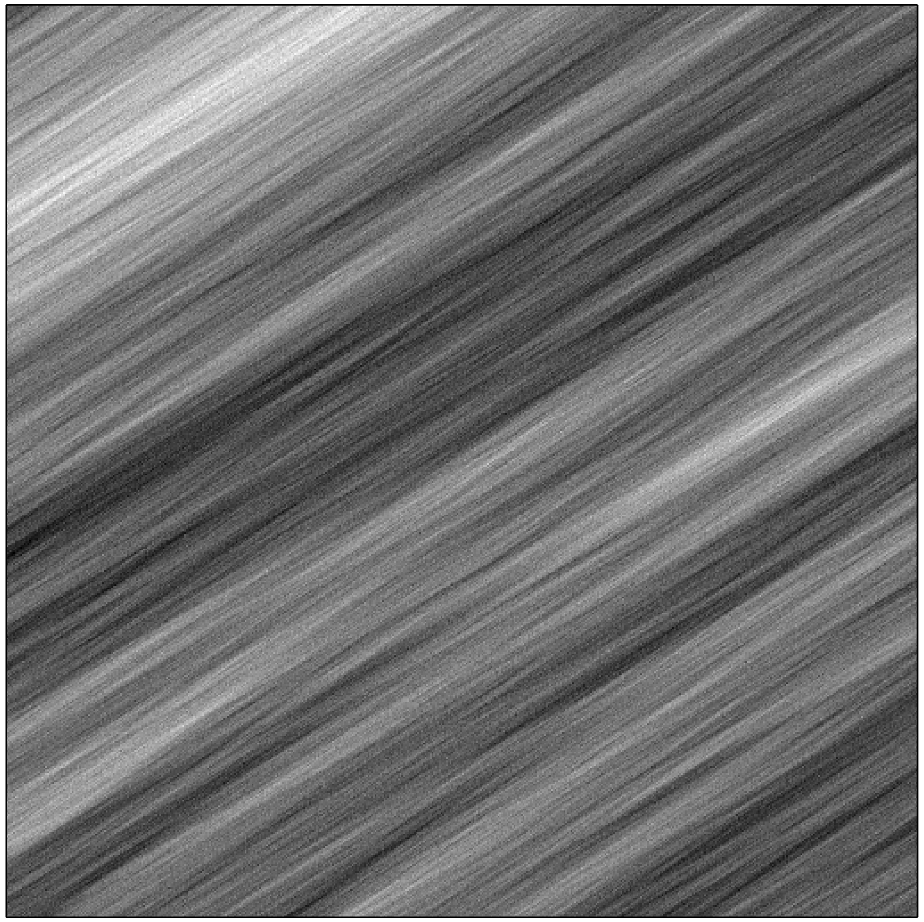


$$\alpha = 0.2$$

Elementary field

$$\alpha_0 = -\frac{\pi}{3}$$

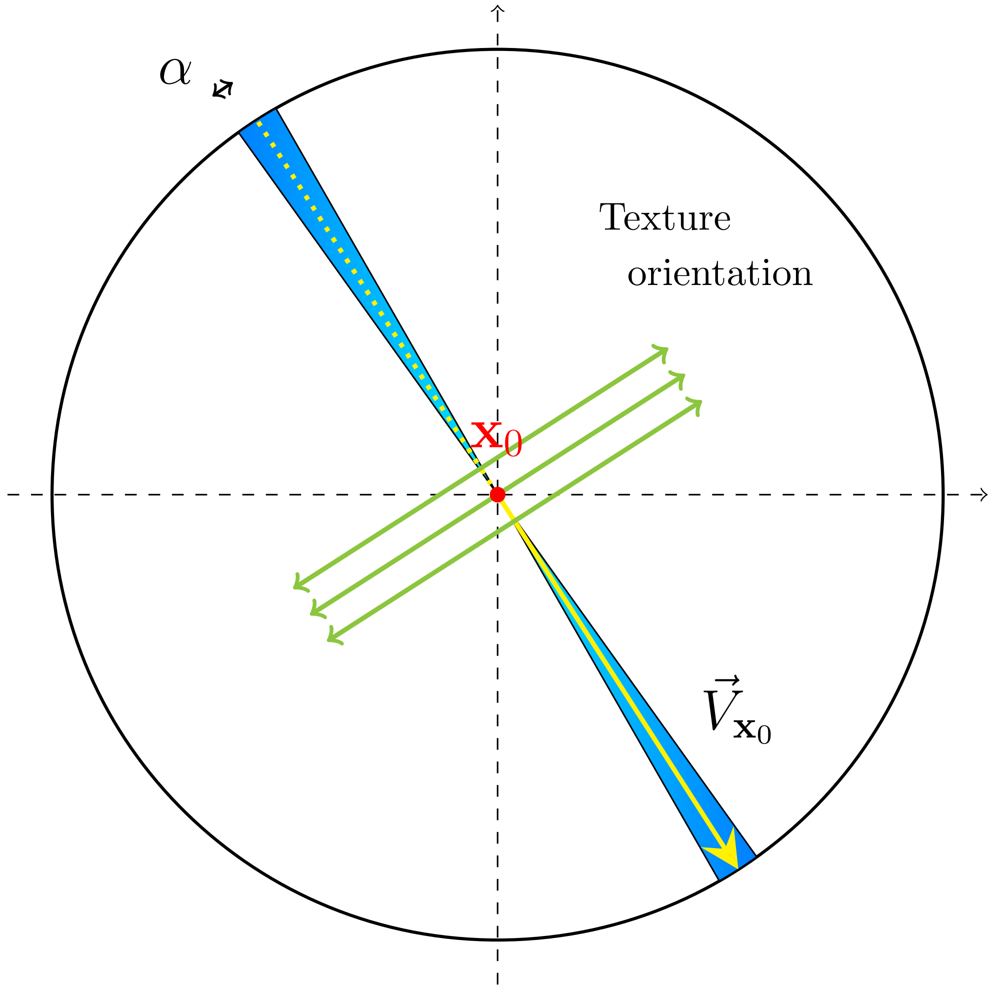


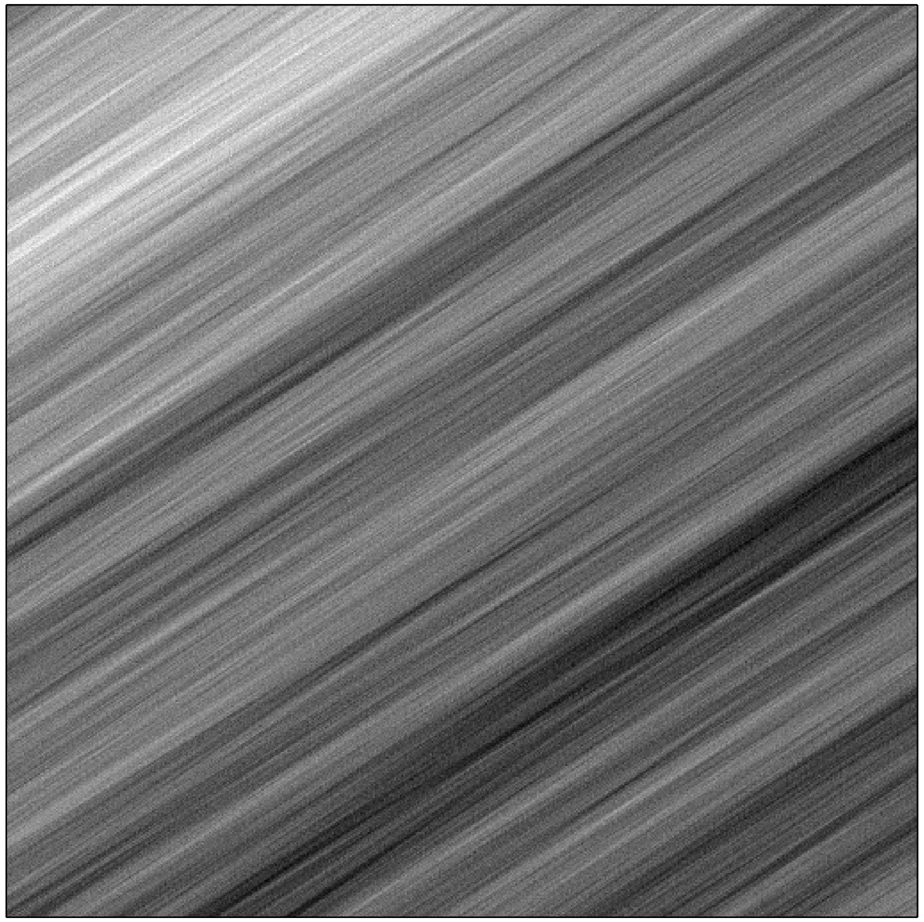


$$\alpha = 0.1$$

Elementary field

$$\alpha_0 = -\frac{\pi}{3}$$





$$\alpha = 0.05$$

Tangent field

$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1)\frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0(\mathbf{x}))}{\|\xi\|^{H+1}} d\widehat{W}(\xi)$$

■ Tangent field.

For a random field X locally asymptotically self-similar of order H,

$$\frac{X(\mathbf{x}_0 + \rho\mathbf{h}) - X(\mathbf{x}_0)}{\rho^H} \xrightarrow[\rho\to 0]{\mathcal{L}} Y_{\mathbf{x}_0}$$

→ $Y_{\mathbf{x}_0}$: tangent field of X at point $\mathbf{x}_0 \in \mathbb{R}^2$

[Benassi,1997]
[Falconer,2002]

Taylor's expansion ⟷ Tangent field

Deterministic case | *Stochastic case*

Tangent field

$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1)\frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0(\mathbf{x}))}{\|\xi\|^{H+1}} d\widehat{W}(\xi)$$

■ **Tangent field.**

For a random field X locally asymptotically self-similar of order H,

$$\frac{X(\mathbf{x}_0 + \rho\mathbf{h}) - X(\mathbf{x}_0)}{\rho^H} \xrightarrow[\rho\to 0]{\mathcal{L}} Y_{\mathbf{x}_0}$$

→ $Y_{\mathbf{x}_0}$: tangent field of X at point $\mathbf{x}_0 \in \mathbb{R}^2$

[Benassi,1997]
[Falconer,2002]

Taylor's expansion ⟷ Tangent field

Deterministic case — *Stochastic case*

Tangent field

$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) \frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0(\mathbf{x}))}{\|\xi\|^{H+1}} d\widehat{W}(\xi)$$

- **Theorem.** The LAFBF $B^{H}_{\alpha_0,\alpha}$ admits for tangent field $Y_{\mathbf{x}_0}$:

$$Y_{\mathbf{x}_0}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) \frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0(\mathbf{x}_0))}{\|\xi\|^{H+1}} d\widehat{W}(\xi)$$

$\longrightarrow$ $Y_{\mathbf{x}_0}$ *elementary field* with global orientation $\alpha_0(\mathbf{x}_0)$

Tangent field

$$B^H_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\boldsymbol{\xi}} - 1) \frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg\boldsymbol{\xi} - \alpha_0(\mathbf{x}))}{\|\boldsymbol{\xi}\|^{H+1}} d\widehat{W}(\boldsymbol{\xi})$$

- **Theorem.** The LAFBF $B^H_{\alpha_0,\alpha}$ admits for tangent field $Y_{\mathbf{x}_0}$:

$$Y_{\mathbf{x}_0}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\boldsymbol{\xi}} - 1) \frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg\boldsymbol{\xi} - \underbrace{\alpha_0(\mathbf{x}_0)}_{\text{constant}})}{\|\boldsymbol{\xi}\|^{H+1}} d\widehat{W}(\boldsymbol{\xi})$$

→ $Y_{\mathbf{x}_0}$ *elementary field* with global orientation $\alpha_0(\mathbf{x}_0)$

Tangent field

$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1)\frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0(\mathbf{x}))}{\|\xi\|^{H+1}}\, d\widehat{W}(\xi)$$

- **Theorem.** The LAFBF $B^{H}_{\alpha_0,\alpha}$ admits for tangent field $Y_{\mathbf{x}_0}$:

$$Y_{\mathbf{x}_0}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1)\frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \underbrace{\alpha_0(\mathbf{x}_0)}_{\text{constant}})}{\|\xi\|^{H+1}}\, d\widehat{W}(\xi)$$

→ $Y_{\mathbf{x}_0}$ *elementary field* with global orientation $\alpha_0(\mathbf{x}_0)$

⇒ $B^{H}_{\alpha_0,\alpha}(\mathbf{x}_0) \approx Y_{\mathbf{x}_0}(x = \mathbf{x}_0)$

Simulation of tangent fields

- **Continuous formulation.** Variogram of $Y_{\mathbf{x}_0}$: [Bierme,Richard,Moisan,2012]

$$
\begin{aligned}
v_{Y_{\mathbf{x}_0}}(\mathbf{x}) &= \frac{1}{2}\int_{\mathbb{R}^2} |e^{i\mathbf{x}\cdot\xi} - 1|^2 f(\mathbf{x}_0,\xi)d\xi \\
&= \frac{1}{2}\gamma(H)\int_{-\pi/2}^{\pi/2} c_{\alpha_0,\alpha}(\mathbf{x}_0,\theta)\ |\mathbf{x}\cdot\mathbf{u}(\theta)|^{2H}d\theta \\
&= \int_{-\pi/2}^{\pi/2} \tilde{v}_\theta(\mathbf{x}\cdot\mathbf{u}(\theta))d\theta
\end{aligned}
$$

Simulation of tangent fields

- **Continuous formulation.** Variogram of $Y_{\mathbf{x}_0}$: [Bierme,Richard,Moisan,2012]

$$\begin{aligned} v_{Y_{\mathbf{x}_0}}(\mathbf{x}) &= \frac{1}{2}\int_{\mathbb{R}^2} |e^{i\mathbf{x}\cdot\xi} - 1|^2 f(\mathbf{x}_0,\xi)d\xi \\ &= \frac{1}{2}\gamma(H)\int_{-\pi/2}^{\pi/2} c_{\alpha_0,\alpha}(\mathbf{x}_0,\theta)\ |\mathbf{x}\cdot\mathbf{u}(\theta)|^{2H}d\theta \\ &= \int_{-\pi/2}^{\pi/2} \tilde{v}_\theta(\mathbf{x}\cdot\mathbf{u}(\theta))d\theta \end{aligned}$$

in polar coordinates

Simulation of tangent fields

- **Continuous formulation.** Variogram of $Y_{\mathbf{x}_0}$: [Bierme,Richard,Moisan,2012]

$$
\begin{aligned}
v_{Y_{\mathbf{x}_0}}(\mathbf{x}) &= \frac{1}{2}\int_{\mathbb{R}^2} |e^{i\mathbf{x}\cdot\xi} - 1|^2 f(\mathbf{x}_0,\xi)d\xi \\
&= \frac{1}{2}\gamma(H)\int_{-\pi/2}^{\pi/2} c_{\alpha_0,\alpha}(\mathbf{x}_0,\theta)\ |\mathbf{x}\cdot\mathbf{u}(\theta)|^{2H}d\theta \\
&= \int_{-\pi/2}^{\pi/2} \tilde{v}_\theta(\mathbf{x}\cdot\mathbf{u}(\theta))d\theta
\end{aligned}
$$

in polar coordinates

$$
\begin{aligned}
\tilde{v}_\theta &= \frac{1}{2}\gamma(H)c_{\alpha_0,\alpha}(\mathbf{x}_0,\theta)|\cdot|^{2H} \\
\mathbf{u}(\theta) &= (\cos\theta,\sin\theta) \\
\gamma(H) &= \frac{\pi}{H\Gamma(2H)\sin(H\pi)}
\end{aligned}
$$

Simulation of tangent fields

Continuous formulation. Variogram of $Y_{\mathbf{x}_0}$: [Bierme,Richard,Moisan,2012]

$$\begin{aligned} v_{Y_{\mathbf{x}_0}}(\mathbf{x}) &= \frac{1}{2}\int_{\mathbb{R}^2} |e^{i\mathbf{x}\cdot\xi}-1|^2 f(\mathbf{x}_0,\xi)d\xi \\ &= \frac{1}{2}\gamma(H)\int_{-\pi/2}^{\pi/2} c_{\alpha_0,\alpha}(\mathbf{x}_0,\theta)\ |\mathbf{x}\cdot\mathbf{u}(\theta)|^{2H}d\theta \\ &= \int_{-\pi/2}^{\pi/2} \tilde{v}_\theta(\mathbf{x}\cdot\mathbf{u}(\theta))d\theta \end{aligned}$$

in polar coordinates

variogram of a fractional brownian motion (FBM) of order H

$$\tilde{v}_\theta = \frac{1}{2}\gamma(H)c_{\alpha_0,\alpha}(\mathbf{x}_0,\theta)|\cdot|^{2H}$$

$$\mathbf{u}(\theta) = (\cos\theta,\sin\theta)$$

$$\gamma(H) = \frac{\pi}{H\Gamma(2H)\sin(H\pi)}$$

Simulation of tangent fields

- **Continuous formulation.** Variogram of $Y_{\mathbf{x}_0}$: [Bierme,Richard,Moisan,2012]

$$
\begin{aligned}
v_{Y_{\mathbf{x}_0}}(\mathbf{x}) &= \frac{1}{2}\int_{\mathbb{R}^2} |e^{i\mathbf{x}\cdot\boldsymbol{\xi}} - 1|^2 f(\mathbf{x}_0, \boldsymbol{\xi}) d\boldsymbol{\xi} \\
&= \frac{1}{2}\gamma(H) \int_{-\pi/2}^{\pi/2} c_{\alpha_0,\alpha}(\mathbf{x}_0, \theta)\ |\mathbf{x} \cdot \mathbf{u}(\theta)|^{2H} d\theta \\
&= \int_{-\pi/2}^{\pi/2} \tilde{v}_\theta(\mathbf{x} \cdot \mathbf{u}(\theta)) d\theta
\end{aligned}
$$

in polar coordinates

variogram of a fractional brownian motion (FBM) of order H

$$\tilde{v}_\theta = \frac{1}{2}\gamma(H) c_{\alpha_0,\alpha}(\mathbf{x}_0, \theta) | \cdot |^{2H}$$

$$\mathbf{u}(\theta) = (\cos\theta, \sin\theta)$$

$$\gamma(H) = \frac{\pi}{H\Gamma(2H)\sin(H\pi)}$$

$Y_{\mathbf{x}_0}$

=

Infinite sum of independant rotating FBM of order H

Simulation of tangent fields

Discrete formulation.

[Bierme,Richard,Moisan,2012]

- $(\theta_i)_{1 \leqslant i \leqslant n}$ are n bands orientations and $\lambda_i = \theta_{i+1} - \theta_i$
- The **turning band field** is defined as

$$Y_{\mathbf{x}_0}{}^{[n]}(\mathbf{x}) = \gamma(H)^{\frac{1}{2}} \sum_{i=1}^{n} \sqrt{\lambda_i c_{\alpha_0,\alpha}(\mathbf{x}_0, \theta_i)} B_i^H(\mathbf{x} \cdot \mathbf{u}(\theta_i))$$

- B_i^H are n independent FBM of order H
- Good approximation provided $\max_i \lambda_i \leqslant \varepsilon$

Simulation of tangent fields

Discrete formulation.

[Bierme,Richard,Moisan,2012]

- $(\theta_i)_{1\leqslant i\leqslant n}$ are n bands orientations and $\lambda_i = \theta_{i+1} - \theta_i$
- The **turning band field** is defined as

$$Y_{\mathbf{x}_0}{}^{[n]}(\mathbf{x}) = \gamma(H)^{\frac{1}{2}} \sum_{i=1}^{n} \sqrt{\lambda_i c_{\alpha_0,\alpha}(\mathbf{x}_0, \theta_i)}\, B_i^H(\mathbf{x} \cdot \mathbf{u}(\theta_i))$$

- B_i^H are n independent FBM of order H
- Good approximation provided $\max_i \lambda_i \leqslant \varepsilon$

not equispaced ✗

Simulation of tangent fields

Simulation along particular bands.

[Bierme,Richard,Moisan,2012]

- Discrete grid $r^{-1}\mathbb{Z}^2 \cap [0,1]^2$ with $r = 2^k - 1, k \in \mathbb{N}^\star$
- Choose (θ_i) such that $\tan\theta_i = \frac{p_i}{q_i}$ and $\max_i \lambda_i \leqslant \epsilon$
- Then $B_i^H(\mathbf{x}\cdot\mathbf{u}(\theta_i))$ becomes

$$\left\{B_i^H\left(\frac{k_1}{r}\cos\theta_i + \frac{k_2}{r}\sin\theta_i\right);0 \leqslant k_1, k_2 \leqslant r\right\} \overset{\mathcal{L}}{=} \left(\frac{\cos\theta_i}{rq_i}\right)^H \{B_i^H(k_1q_i + k_2p_i); 0 \leqslant k_1, k_2 \leqslant r\}$$

Simulation of tangent fields

- **Simulation along particular bands.** [Bierme,Richard,Moisan,2012]

- Discrete grid $r^{-1}\mathbb{Z}^2 \cap [0,1]^2$ with $r = 2^k - 1$, $k \in \mathbb{N}^\star$
- Choose (θ_i) such that $\tan\theta_i = \frac{p_i}{q_i}$ and $\max_i \lambda_i \leqslant \epsilon$
- Then $B_i^H(\mathbf{x} \cdot \mathbf{u}(\theta_i))$ becomes

Dynamic programming

$$\left\{ B_i^H\left(\frac{k_1}{r}\cos\theta_i + \frac{k_2}{r}\sin\theta_i\right); 0 \leqslant k_1, k_2 \leqslant r \right\} \overset{\mathcal{L}}{=} \left(\frac{\cos\theta_i}{rq_i}\right)^H \{B_i^H(k_1 q_i + k_2 p_i); 0 \leqslant k_1, k_2 \leqslant r\}$$

Simulation of tangent fields

Simulation along particular bands.

[Bierme,Richard,Moisan,2012]

- Discrete grid $r^{-1}\mathbb{Z}^2 \cap [0,1]^2$ with $r = 2^k - 1$, $k \in \mathbb{N}^\star$
- Choose (θ_i) such that $\tan\theta_i = \frac{p_i}{q_i}$ and $\max_i \lambda_i \leqslant \epsilon$
- Then $B_i^H(\mathbf{x}\cdot\mathbf{u}(\theta_i))$ becomes

Dynamic programming

$$\left\{ B_i^H\left(\frac{k_1}{r}\cos\theta_i + \frac{k_2}{r}\sin\theta_i\right); 0 \leqslant k_1, k_2 \leqslant r \right\} \overset{\mathcal{L}}{=}$$

$$\left(\frac{\cos\theta_i}{rq_i}\right)^H \{B_i^H(k_1 q_i + k_2 p_i); 0 \leqslant k_1, k_2 \leqslant r\}$$

self-similarity

$B^H(\lambda\cdot) \overset{\mathcal{L}}{=} \lambda^H B^H(\cdot)$

Simulation of tangent fields

Simulation along particular bands.

[Bierme,Richard,Moisan,2012]

- Discrete grid $r^{-1}\mathbb{Z}^2 \cap [0,1]^2$ with $r = 2^k - 1$, $k \in \mathbb{N}^\star$
- Choose (θ_i) such that $\tan\theta_i = \frac{p_i}{q_i}$ and $\max_i \lambda_i \leqslant \epsilon$
- Then $B_i^H(\mathbf{x}\cdot\mathbf{u}(\theta_i))$ becomes

Dynamic programming

$$\left\{B_i^H\left(\frac{k_1}{r}\cos\theta_i+\frac{k_2}{r}\sin\theta_i\right);0\leqslant k_1,k_2\leqslant r\right\}\overset{\mathcal{L}}{=}$$

$$\left(\frac{\cos\theta_i}{rq_i}\right)^H\{B_i^H(k_1q_i+k_2p_i);0\leqslant k_1,k_2\leqslant r\}$$

self-similarity

$B^H(\lambda\cdot)\overset{\mathcal{L}}{=}\lambda^H B^H(\cdot)$

equispaced ✓

Simulation of LAFBF

using tangent fields

[Polisano et al.,2014]

- **Algorithm.** For each pixel $\mathbf{x}_0 = (k_1, k_2) \in [\![0, r]\!]^2$

$$B^{H}_{\alpha_0,\alpha}((k_1, k_2)) = \gamma(H)^{\frac{1}{2}} \sum_{i=1}^{n} \sqrt{\lambda_i c_{\alpha_0,\alpha}((k_1, k_2), \theta_i)} \left(\frac{\cos\theta_i}{rq_i} \right)^{H} B^{H}_{i}(k_1 q_i + k_2 p_i)$$

Simulation of LAFBF

using tangent fields

[Polisano et al.,2014]

- **Algorithm.** For each pixel $\mathbf{x}_0 = (k_1, k_2) \in [\![0, r]\!]^2$

$$B^H_{\alpha_0,\alpha}(\mathbf{x}_0) \approx Y_{\mathbf{x}_0}(x = \mathbf{x}_0)$$

$$B^H_{\alpha_0,\alpha}((k_1, k_2)) = \gamma(H)^{\frac{1}{2}} \sum_{i=1}^{n} \sqrt{\lambda_i c_{\alpha_0,\alpha}((k_1, k_2), \theta_i)} \left(\frac{\cos\theta_i}{rq_i}\right)^H B_i^H(k_1 q_i + k_2 p_i)$$

Simulation of LAFBF

using tangent fields

[Polisano et al.,2014]

- **Algorithm.** For each pixel $\mathbf{x}_0 = (k_1, k_2) \in [\![0, r]\!]^2$

$$B^{H}_{\alpha_0,\alpha}((k_1, k_2)) = \gamma(H)^{\frac{1}{2}} \sum_{i=1}^{n} \sqrt{\lambda_i c_{\alpha_0,\alpha}((k_1, k_2), \theta_i)} \left(\frac{\cos\theta_i}{rq_i}\right)^{H} B_i^H(k_1 q_i + k_2 p_i)$$

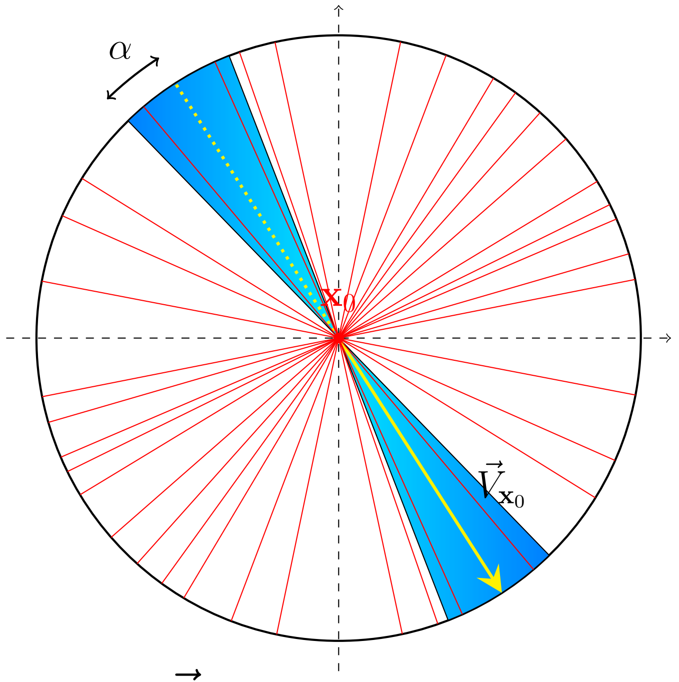
Simulation of LAFBF

using tangent fields

[Polisano et al.,2014]

- **Algorithm.** For each pixel $\mathbf{x}_0 = (k_1, k_2) \in [\![0, r]\!]^2$

$$B^{H}_{\alpha_0,\alpha}((k_1, k_2)) = \gamma(H)^{\frac{1}{2}} \sum_{i=1}^{n} \sqrt{\lambda_i c_{\alpha_0,\alpha}((k_1, k_2), \theta_i)} \left(\frac{\cos\theta_i}{rq_i}\right)^{H} B_i^{H}(k_1 q_i + k_2 p_i)$$



$\vec{V}_{\mathbf{x}_0} = \alpha_0(k_1, k_2)$

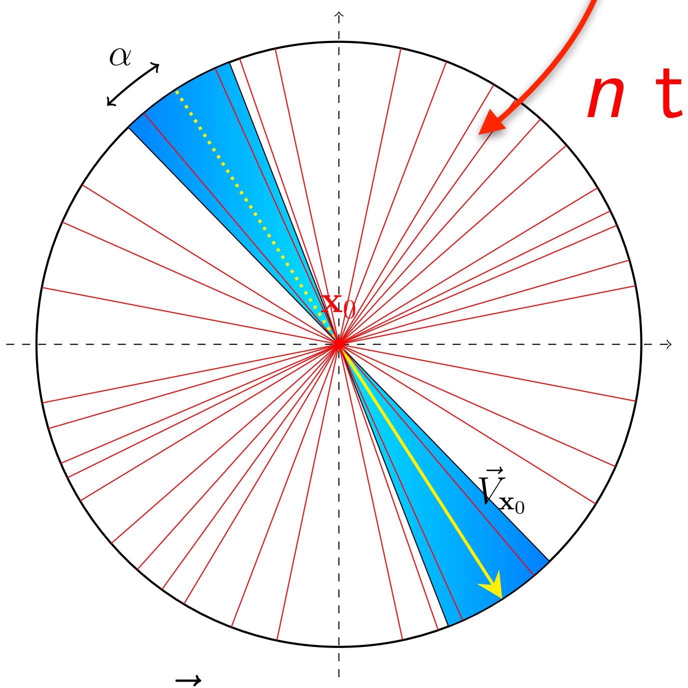
Simulation of LAFBF

using tangent fields

[Polisano et al.,2014]

- **Algorithm.** For each pixel $\mathbf{x}_0 = (k_1, k_2) \in [\![0, r]\!]^2$

$$B^H_{\alpha_0,\alpha}((k_1,k_2)) = \gamma(H)^{\frac{1}{2}} \sum_{i=1}^{n} \sqrt{\lambda_i c_{\alpha_0,\alpha}((k_1,k_2),\theta_i)} \left(\frac{\cos\theta_i}{rq_i}\right)^H B_i^H(k_1 q_i + k_2 p_i)$$



n turning bands θ_i

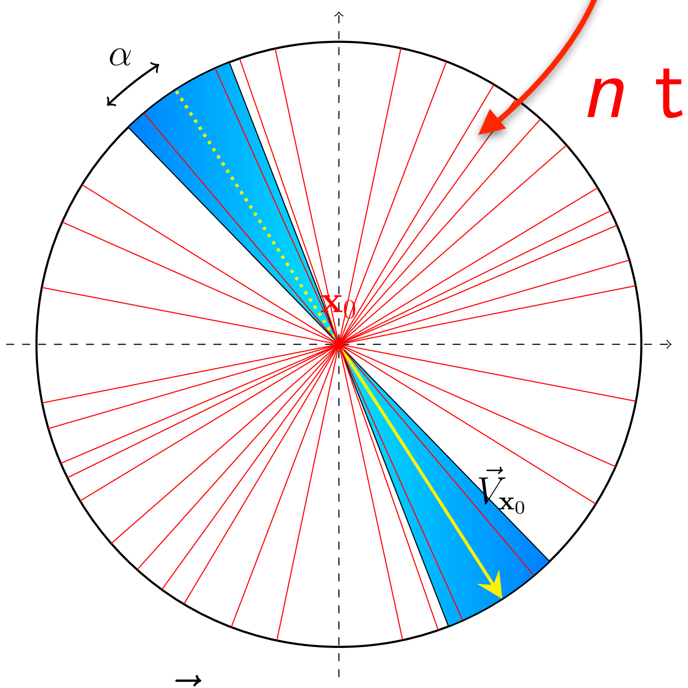
$\vec{V}_{\mathbf{x}_0} = \alpha_0(k_1, k_2)$

Simulation of LAFBF using tangent fields

[Polisano et al.,2014]

- **Algorithm.** For each pixel $\mathbf{x}_0 = (k_1, k_2) \in [\![0, r]\!]^2$

$$B^H_{\alpha_0,\alpha}((k_1,k_2)) = \gamma(H)^{\frac{1}{2}} \sum_{i=1}^{\boxed{n}} \sqrt{\lambda_i c_{\alpha_0,\alpha}((k_1,k_2),\theta_i)} \left(\frac{\cos\theta_i}{rq_i}\right)^H B_i^H(k_1 q_i + k_2 p_i)$$



$\vec{V}_{\mathbf{x}_0} = \alpha_0(k_1, k_2)$

n turning bands θ_i

$$\mathbb{1}_{[-\alpha,\alpha]}(\theta_i - \alpha_0((k_1,k_2))) \neq 0 \\ \Leftrightarrow \\ |\theta_i - \alpha_0((k_1,k_2))| \leqslant \alpha$$

Simulation of LAFBF

using tangent fields

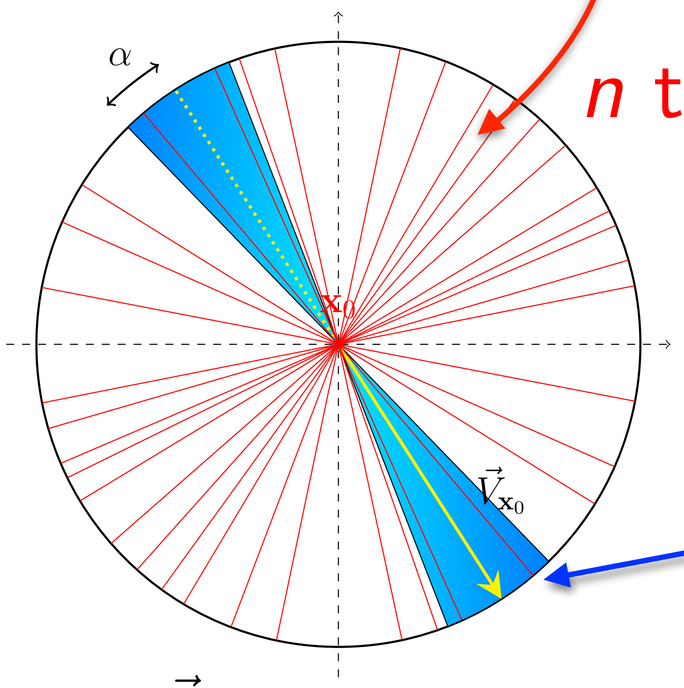
[Polisano et al.,2014]

- **Algorithm.** For each pixel $\mathbf{x}_0 = (k_1, k_2) \in [\![0, r]\!]^2$

$$B^H_{\alpha_0,\alpha}((k_1, k_2)) = \gamma(H)^{\frac{1}{2}} \sum_{i=1}^{\boxed{n}} \sqrt{\lambda_i c_{\alpha_0,\alpha}((k_1, k_2), \theta_i)} \left(\frac{\cos\theta_i}{rq_i}\right)^H B_i^H(k_1 q_i + k_2 p_i)$$

n turning bands θ_i

$$\mathbb{1}_{[-\alpha,\alpha]}(\theta_i - \alpha_0((k_1, k_2))) \neq 0 \Leftrightarrow |\theta_i - \alpha_0((k_1, k_2))| \leqslant \alpha$$



$\vec{V}_{\mathbf{x}_0} = \alpha_0(k_1, k_2)$

Simulation of LAFBF

using tangent fields

[Polisano et al.,2014]

- **Algorithm.** For each pixel $\mathbf{x}_0 = (k_1, k_2) \in [\![0, r]\!]^2$

$$B^H_{\alpha_0,\alpha}((k_1,k_2))$$

$$=$$

$$\gamma(H)^{\frac{1}{2}} \sum_{i=1}^{\boxed{n}} \sqrt{\lambda_i c_{\alpha_0,\alpha}((k_1,k_2),\theta_i)} \left(\frac{\cos\theta_i}{rq_i}\right)^H B_i^H(k_1 q_i + k_2 p_i)$$

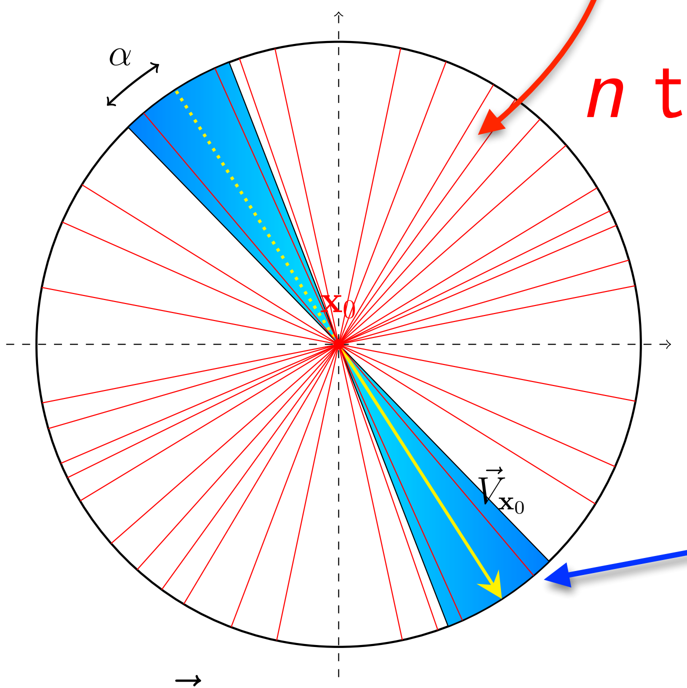
n turning bands θ_i

$$\mathbb{1}_{[-\alpha,\alpha]}(\theta_i - \alpha_0((k_1,k_2))) \neq 0$$

$$\Leftrightarrow$$

$$|\theta_i - \alpha_0((k_1,k_2))| \leqslant \alpha$$

few bands in the cone



$\vec{V}_{\mathbf{x}_0} = \alpha_0(k_1, k_2)$

Simulation of LAFBF

using tangent fields

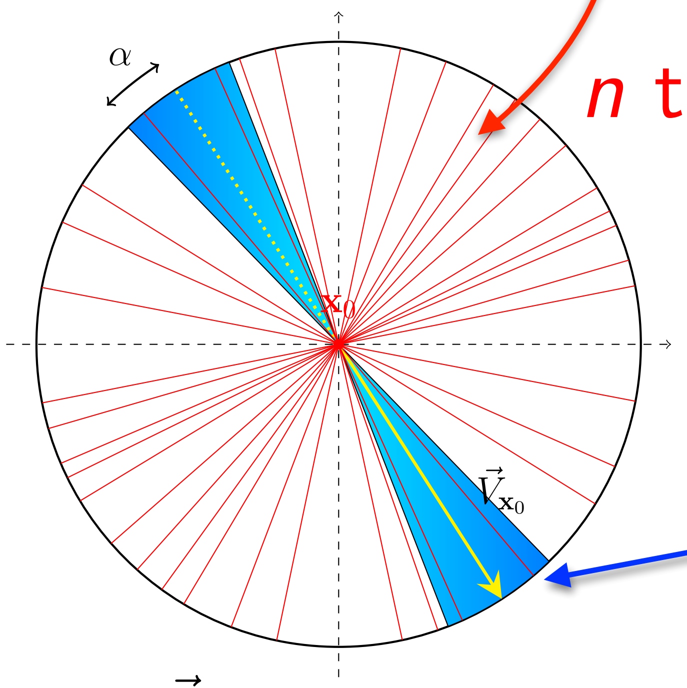
[Polisano et al.,2014]

- **Algorithm.** For each pixel $\mathbf{x}_0 = (k_1, k_2) \in [\![0, r]\!]^2$

$$B^H_{\alpha_0,\alpha}((k_1, k_2))$$

$$=$$

$$\gamma(H)^{\frac{1}{2}} \sum_{i=1}^{\boxed{n}} \sqrt{\lambda_i c_{\alpha_0,\alpha}((k_1, k_2), \theta_i)} \left(\frac{\cos\theta_i}{rq_i}\right)^H B_i^H(k_1 q_i + k_2 p_i)$$



$\vec{V}_{\mathbf{x}_0} = \alpha_0(k_1, k_2)$

n turning bands θ_i

$$\mathbb{1}_{[-\alpha,\alpha]}(\theta_i - \alpha_0((k_1, k_2))) \neq 0$$

$$\Leftrightarrow$$

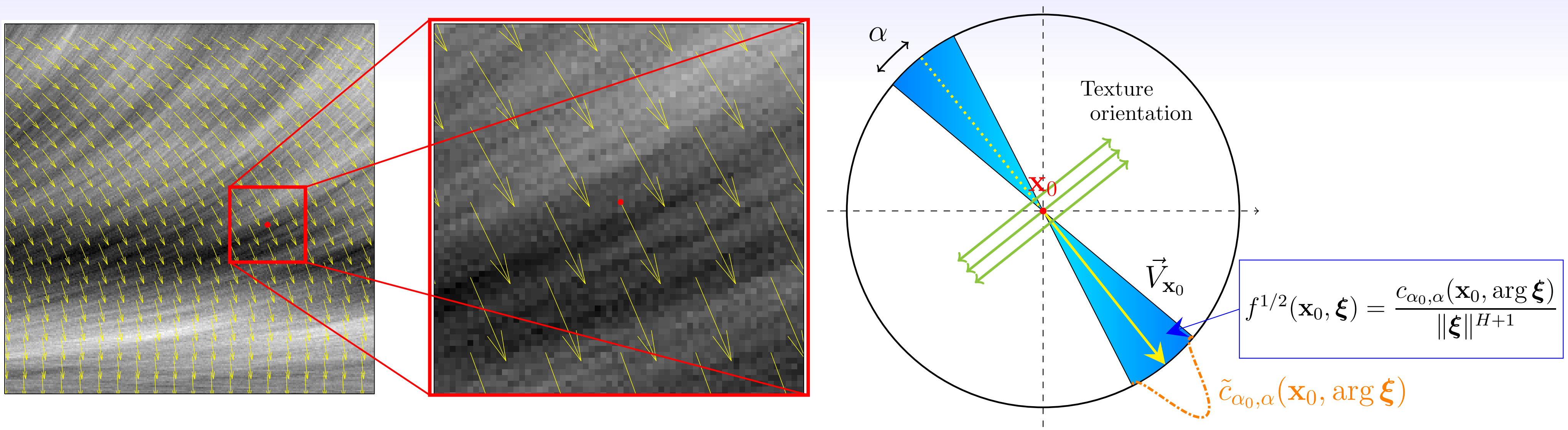
$$|\theta_i - \alpha_0((k_1, k_2))| \leqslant \alpha$$

few bands in the cone

Complexity $O(r^2 \log n)$

Numerical experiments

$$\vec{V}^1_{(x,y)} = (\cos(-\pi/2 + y), \sin(-\pi/2))$$



(a) (b) (c)

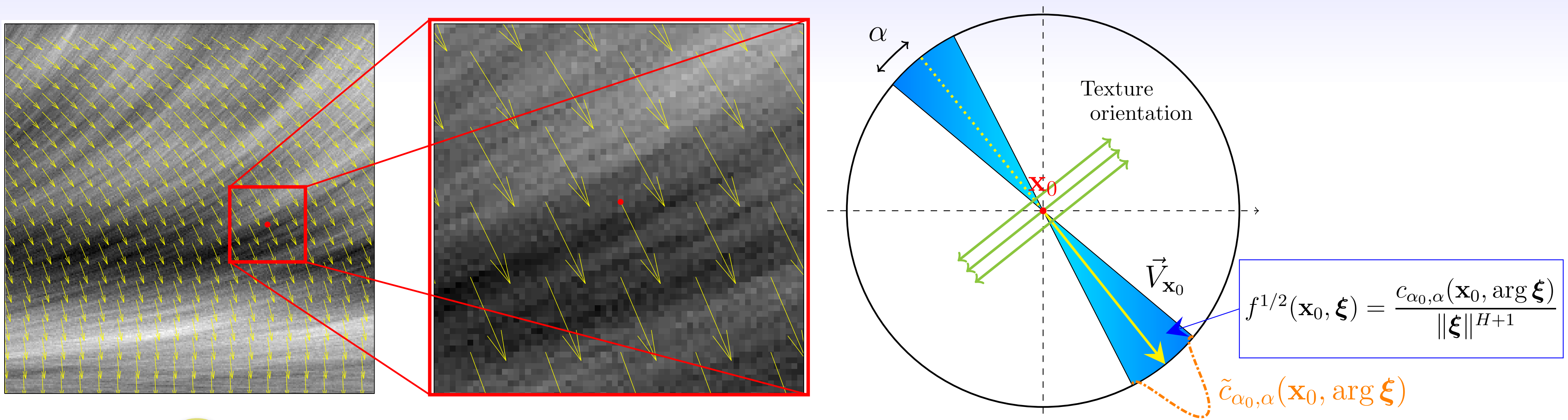
Parameters

$r = 255$ $H = 0.2$

$\alpha = 10^{-1}$ $\epsilon = 10^{-2}$

Numerical experiments

$$\vec{V}^1_{(x,y)} = (\cos(-\pi/2 + y), \sin(-\pi/2))$$



(a) (b) (c)

Texture with prescribed local orientation at each point $\mathbf{x}_0$ given by a vector field

$$\vec{V}_{\mathbf{x}_0} = \mathbf{u}(\alpha_0(\mathbf{x}_0))$$

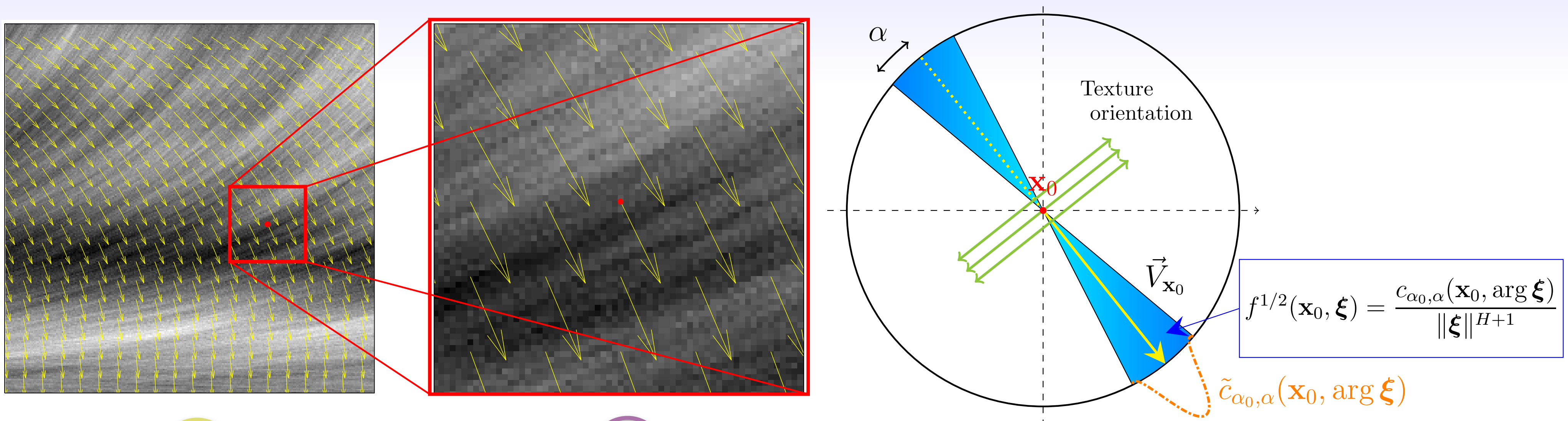
Parameters

$r = 255 \quad H = 0.2$

$\alpha = 10^{-1} \quad \epsilon = 10^{-2}$

Numerical experiments

$$\vec{V}^1_{(x,y)} = (\cos(-\pi/2 + y), \sin(-\pi/2))$$



(a) (b) (c)

Texture with prescribed local orientation at each point $\mathbf{x}_0$ given by a vector field

$$\vec{V}_{\mathbf{x}_0} = \mathbf{u}(\alpha_0(\mathbf{x}_0))$$

A zoom around a point $\mathbf{x}_0$ shows that locally a LAFBF behaves as an elementary field

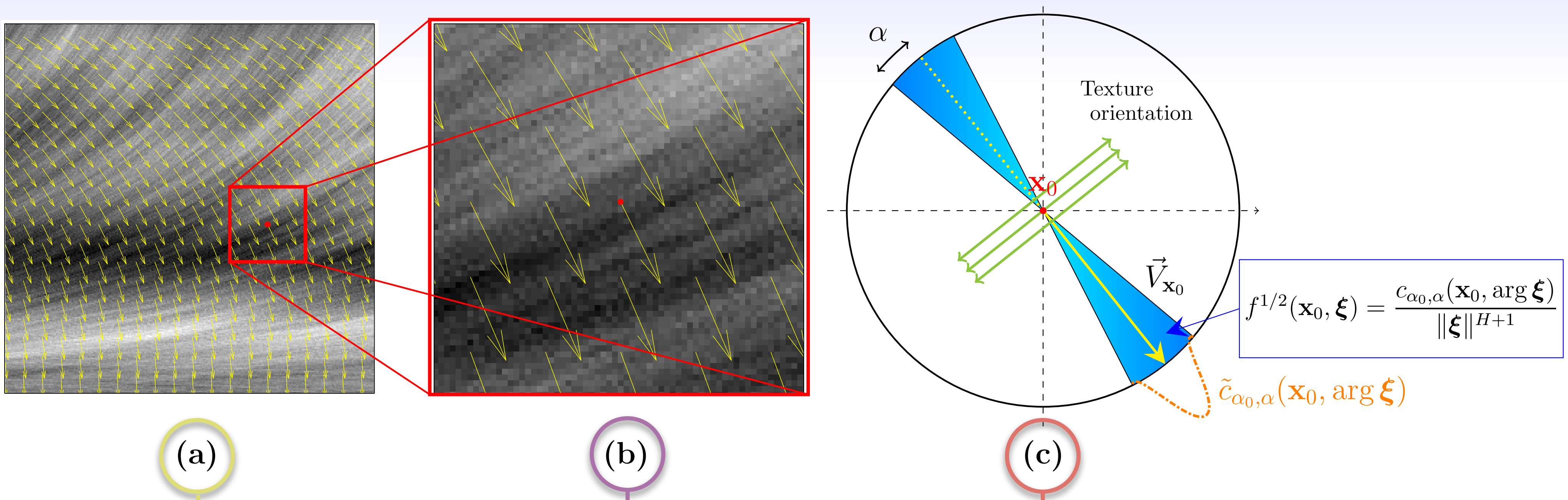
Parameters

$r = 255 \quad H = 0.2$

$\alpha = 10^{-1} \quad \epsilon = 10^{-2}$

Numerical experiments

$$\vec{V}^1_{(x,y)} = (\cos(-\pi/2 + y), \sin(-\pi/2))$$



Texture with prescribed local orientation at each point $\mathbf{x}_0$ given by a vector field

$$\vec{V}_{\mathbf{x}_0} = \mathbf{u}(\alpha_0(\mathbf{x}_0))$$

Regularized version of the anisotropy function

A zoom around a point $\mathbf{x}_0$ shows that locally a LAFBF behaves as an elementary field

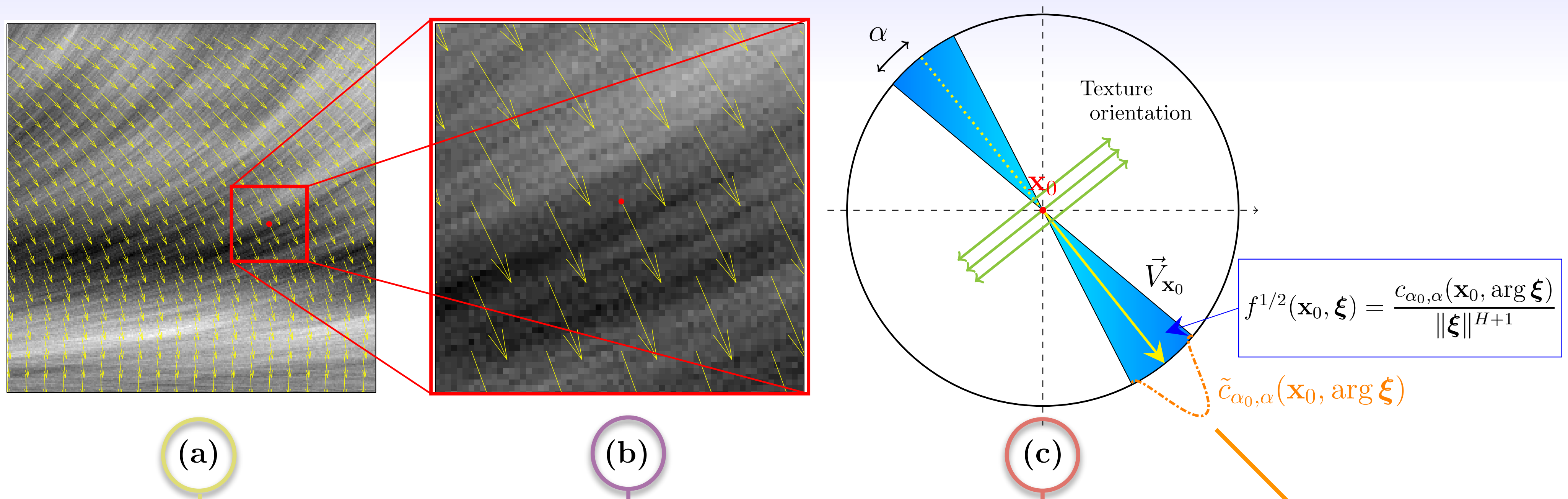
Parameters

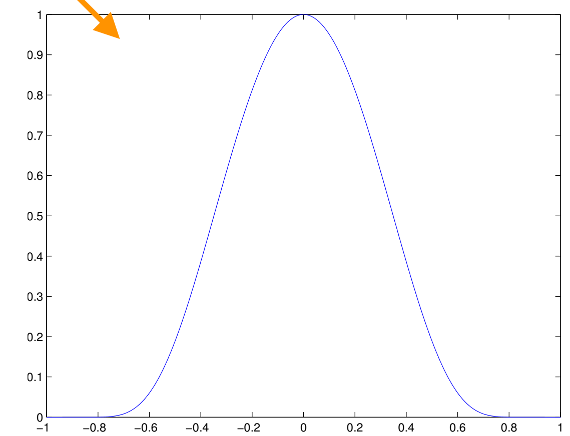
$r = 255 \quad H = 0.2$

$\alpha = 10^{-1} \quad \epsilon = 10^{-2}$

Numerical experiments

$$\vec{V}^1_{(x,y)} = (\cos(-\pi/2 + y), \sin(-\pi/2))$$





Texture with prescribed local orientation at each point $\mathbf{x}_0$ given by a vector field

$$\vec{V}_{\mathbf{x}_0} = \mathbf{u}(\alpha_0(\mathbf{x}_0))$$

Regularized version of the anisotropy function

A zoom around a point $\mathbf{x}_0$ shows that locally a LAFBF behaves as an elementary field

Parameters

$r = 255 \quad H = 0.2$

$\alpha = 10^{-1} \quad \epsilon = 10^{-2}$

Numerical experiments

$\vec{\mathbf{V}}^2_{(x,y)} = (\cos(\cos(36xy)), \sin(\cos(36xy)))$

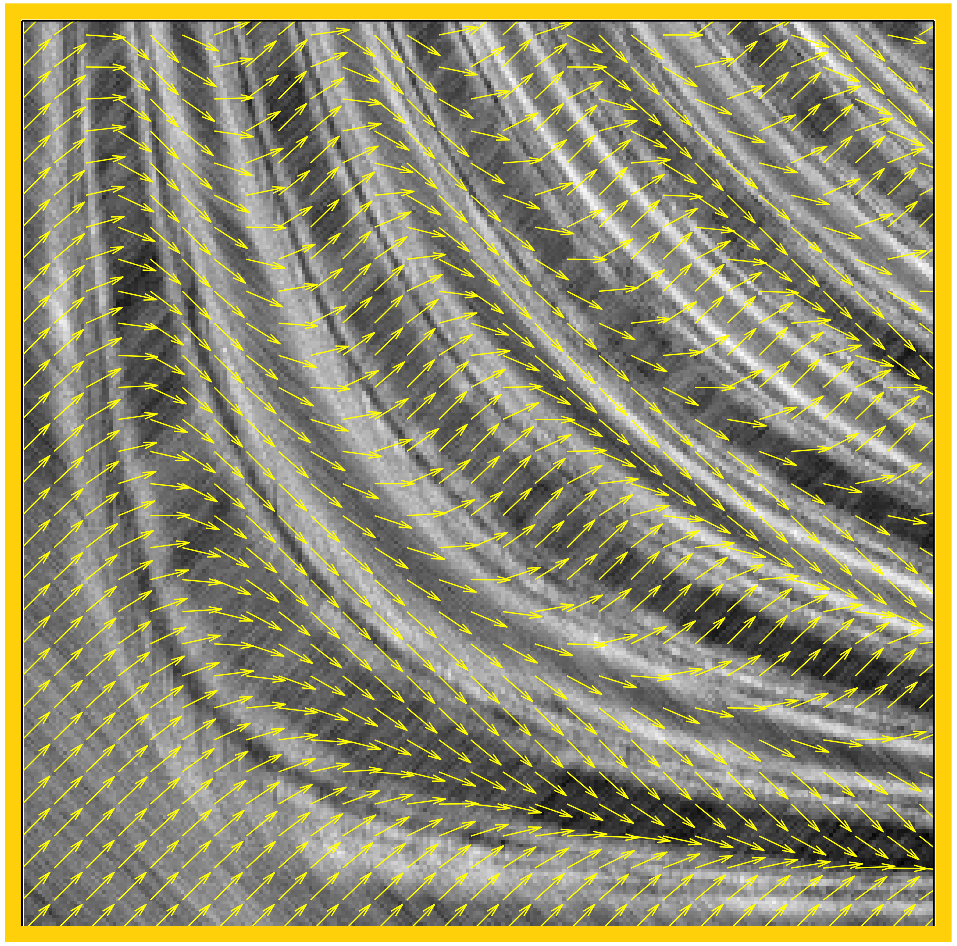
$\vec{\mathbf{V}}^3_{(x,y)} = \nabla F(x,y)$

$F(x,y) = (4x-2)e^{-(4x-2)^2-(4y-2)^2}$

Numerical experiments

$$\vec{\mathbf{V}}^2_{(x,y)} = (\cos(\cos(36xy)), \sin(\cos(36xy)))$$

$$\vec{\mathbf{V}}^3_{(x,y)} = \nabla F(x,y)$$

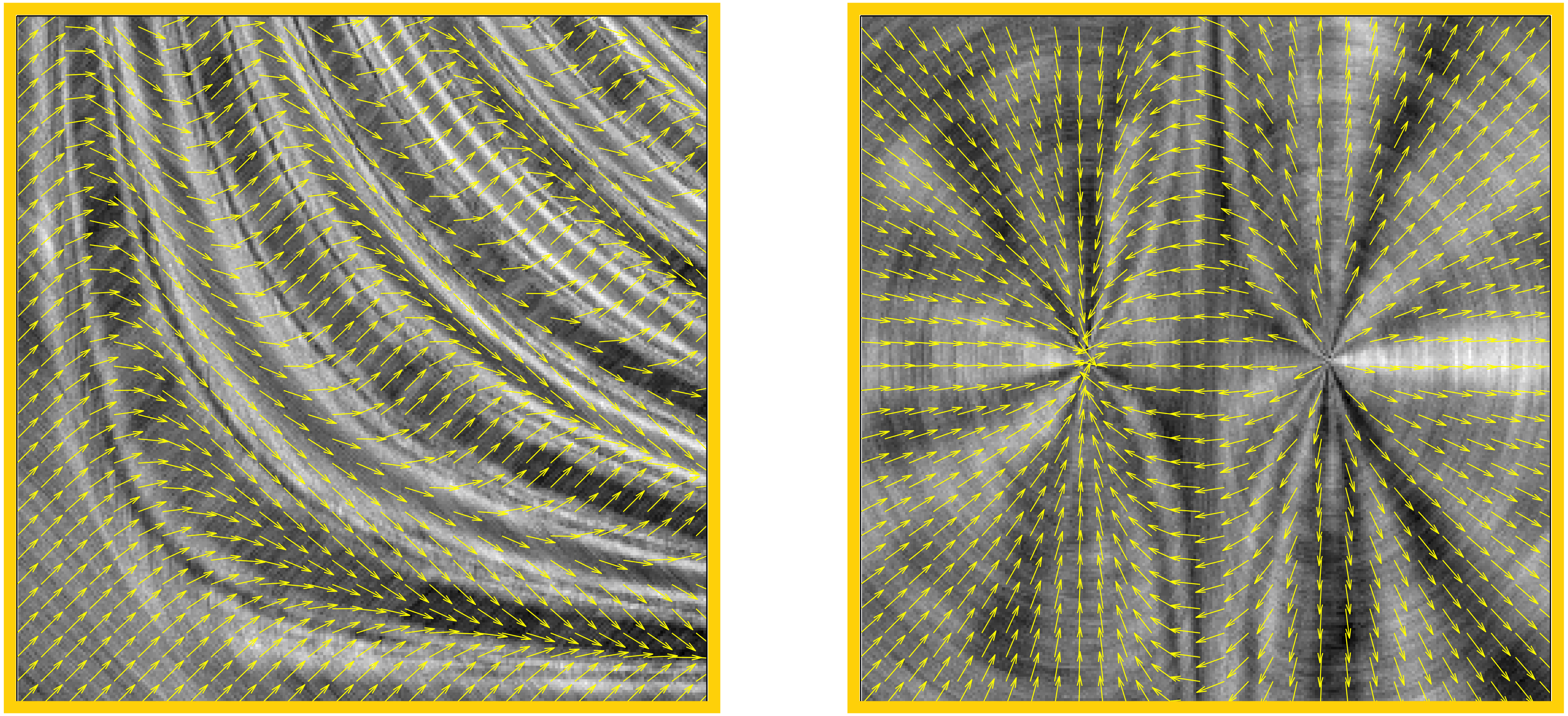


$$F(x,y) = (4x-2)e^{-(4x-2)^2-(4y-2)^2}$$

Numerical experiments

$$\vec{\mathbf{V}}^2_{(x,y)} = (\cos(\cos(36xy)), \sin(\cos(36xy)))$$

$$\vec{\mathbf{V}}^3_{(x,y)} = \nabla F(x, y)$$



$$F(x, y) = (4x - 2)e^{-(4x-2)^2-(4y-2)^2}$$

Numerical experiments

H=0.2

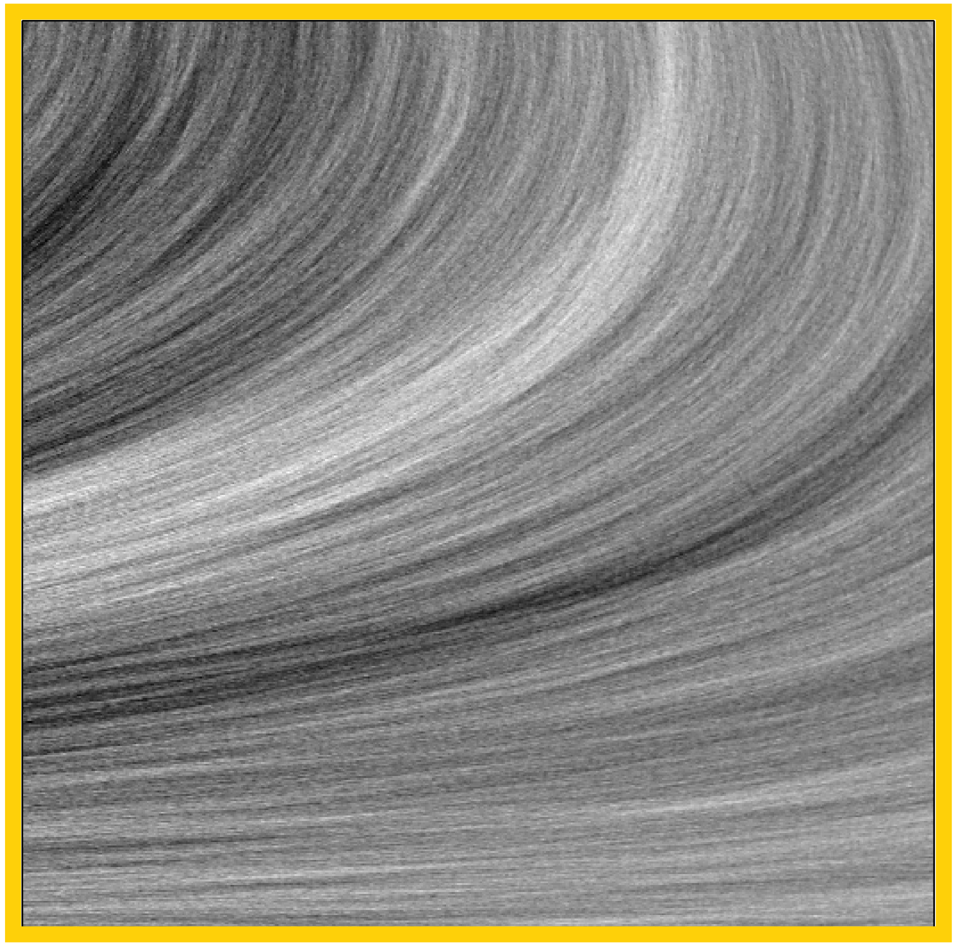
H=0.5

$$\vec{V}^{1}_{(x,y)} = (\cos(-\pi/2 + y), \sin(-\pi/2))$$

Numerical experiments

H=0.2

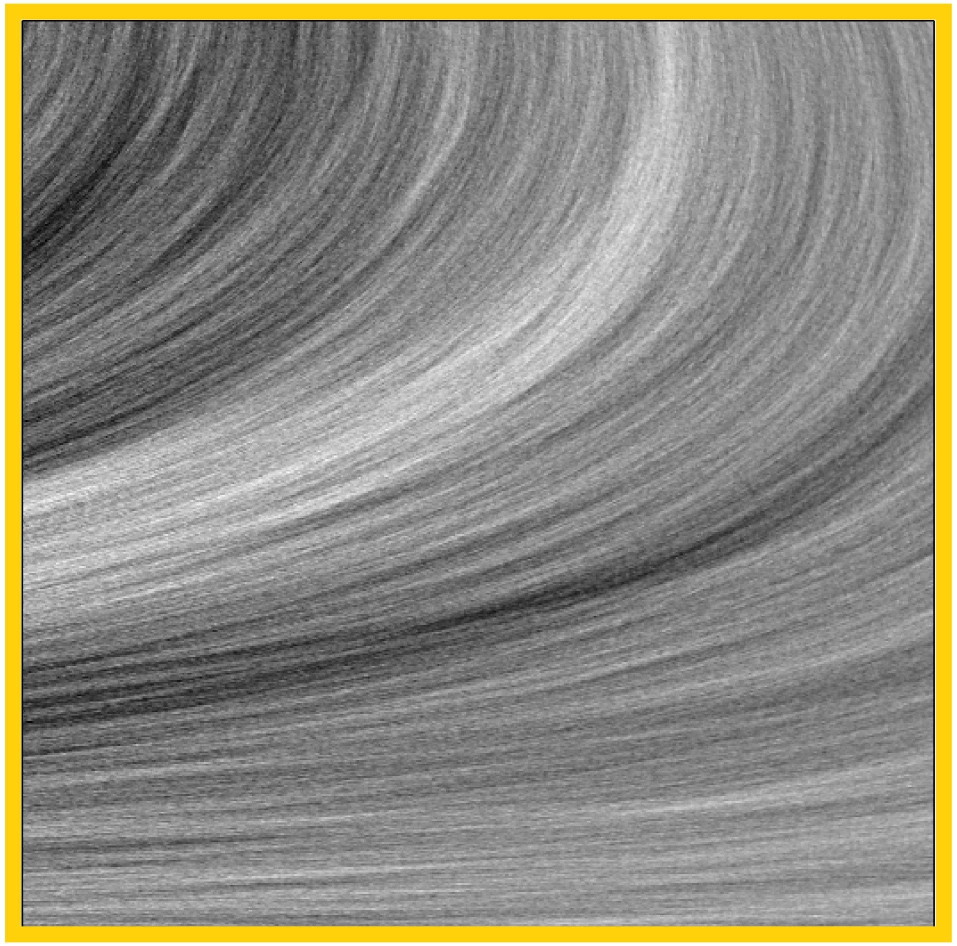
H=0.5



$$\vec{V}^1_{(x,y)} = (\cos(-\pi/2 + y), \sin(-\pi/2))$$

Numerical experiments

H=0.2



H=0.5

$$\vec{V}^1_{(x,y)} = (\cos(-\pi/2 + y), \sin(-\pi/2))$$

Conclusion and future work

- **Conclusion**

Conclusion and future work

- **Conclusion**
  - Introduce a **new stochastic model** defined as a local version of an AFBF.

Conclusion and future work

- **Conclusion**
  - Introduce a **new stochastic model** defined as a local version of an AFBF.
  - Simulations based on **tangent field formulation** and the **turning bands method** produce textures with prescribed local orientations.

Conclusion and future work

Conclusion

- Introduce a **new stochastic model** defined as a local version of an AFBF.
- Simulations based on **tangent field formulation** and the **turning bands method** produce textures with prescribed local orientations.

Conclusion and future work

Conclusion

- Introduce a **new stochastic model** defined as a local version of an AFBF.
- Simulations based on **tangent field formulation** and the **turning bands method** produce textures with prescribed local orientations.

Future work

- Extensions of our model include Hurst index may **vary spatially**.

Conclusion and future work

🟥 Conclusion

- Introduce a **new stochastic model** defined as a local version of an AFBF.
- Simulations based on **tangent field formulation** and the **turning bands method** produce textures with prescribed local orientations.

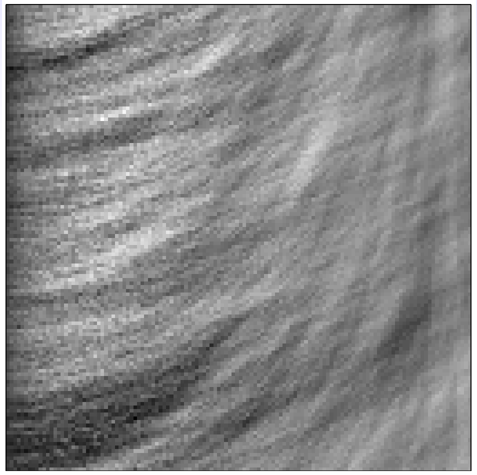
🟩 Future work

- Extensions of our model include Hurst index may **vary spatially**.

Conclusion and future work

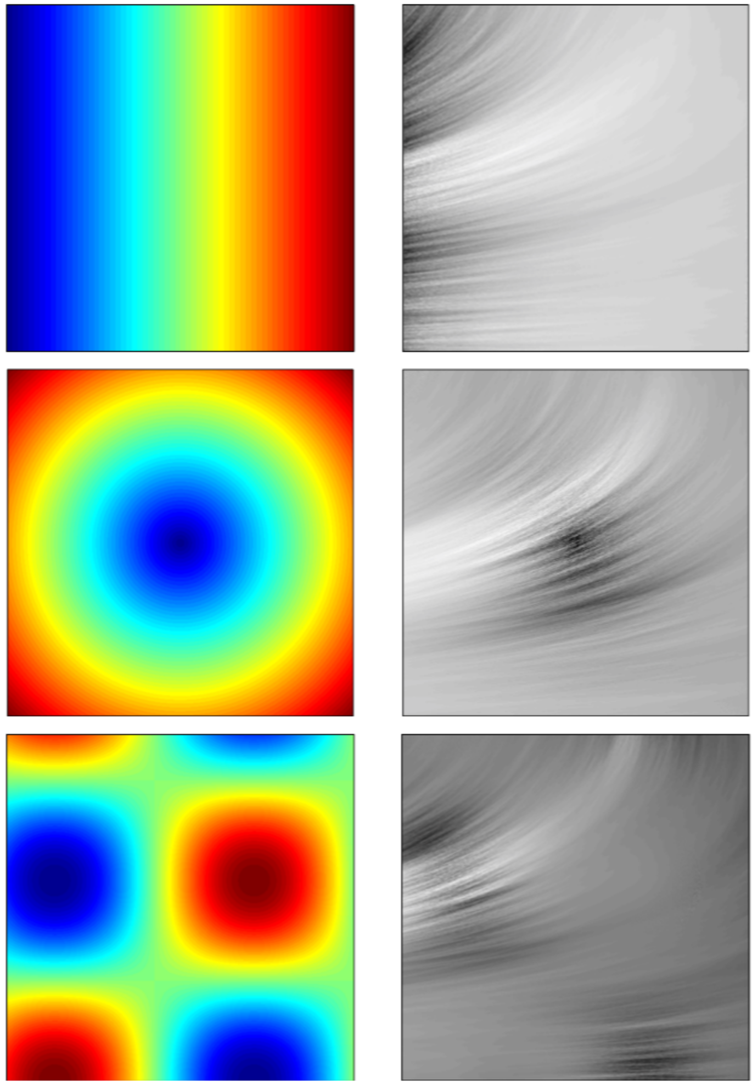
🟥 Conclusion

- Introduce a **new stochastic model** defined as a local version of an AFBF.
- Simulations based on **tangent field formulation** and the **turning bands method** produce textures with prescribed local orientations.



🟩 Future work

- Extensions of our model include Hurst index may **vary spatially**.



Bibliography

Selected papers

- K. Polisano, M. Clausel, V. Perrier and L. Condat, ”Texture modeling by Gaussian fields with prescribed local orientation”, *IEEE ICIP*, 2014.
- A. Bonami and A. Estrade, ”Anisotropic analysis of some Gaussian models”, *Journal of Fourier Analysis and Applications*, vol. 9, no. 3, pp. 215–236, 2003.
- H. Bierme, L. Moisan, and F. Richard, “A turning-band method for the simulation of anisotropic fractional Brownian fields,” *preprint MAP5 No. 2012-312012*, 2012.
- K.J. Falconer, “Tangent fields and the local structure of random fields,” *Journal of Theoretical Probability*, vol. 15, no. 3, pp. 731–750, 2002.

Questions ?

Questions ?

Thank you for your attention.

- **Dynamic programming.** The choice of the bands orientation $(\theta_i)_{1 \leqslant i \leqslant n}$ is governed by the computational cost of the B_i^H 's within dynamic programming.
  - Let the error ϵ fixed. Taking $N = \lceil \frac{1}{\tan \epsilon} \rceil$ consider the following set:

$$\mathcal{V}_N = \left\{ (p,q) \in \mathbb{Z}^2 / -N \leqslant p \leqslant N, 1 \leqslant q \leqslant N, p \wedge q = 1, -\frac{\pi}{2} < \arctan\left(\frac{p}{q}\right) < \frac{\pi}{2} \right\}$$

  - The aim is to find n pairs in the set $\mathcal{V}_N$ which minimize the following global cost:

$$C(\Theta) = \sum_{k=1}^{s} C(r(|p_{i_k}| + q_{i_k}))$$

  where $C(\ell)$ is the cost of one FBM B_i^H in $O(n \log n)$, under the constraint $\max_i (\theta_{i+1} - \theta_i) \leqslant \epsilon$