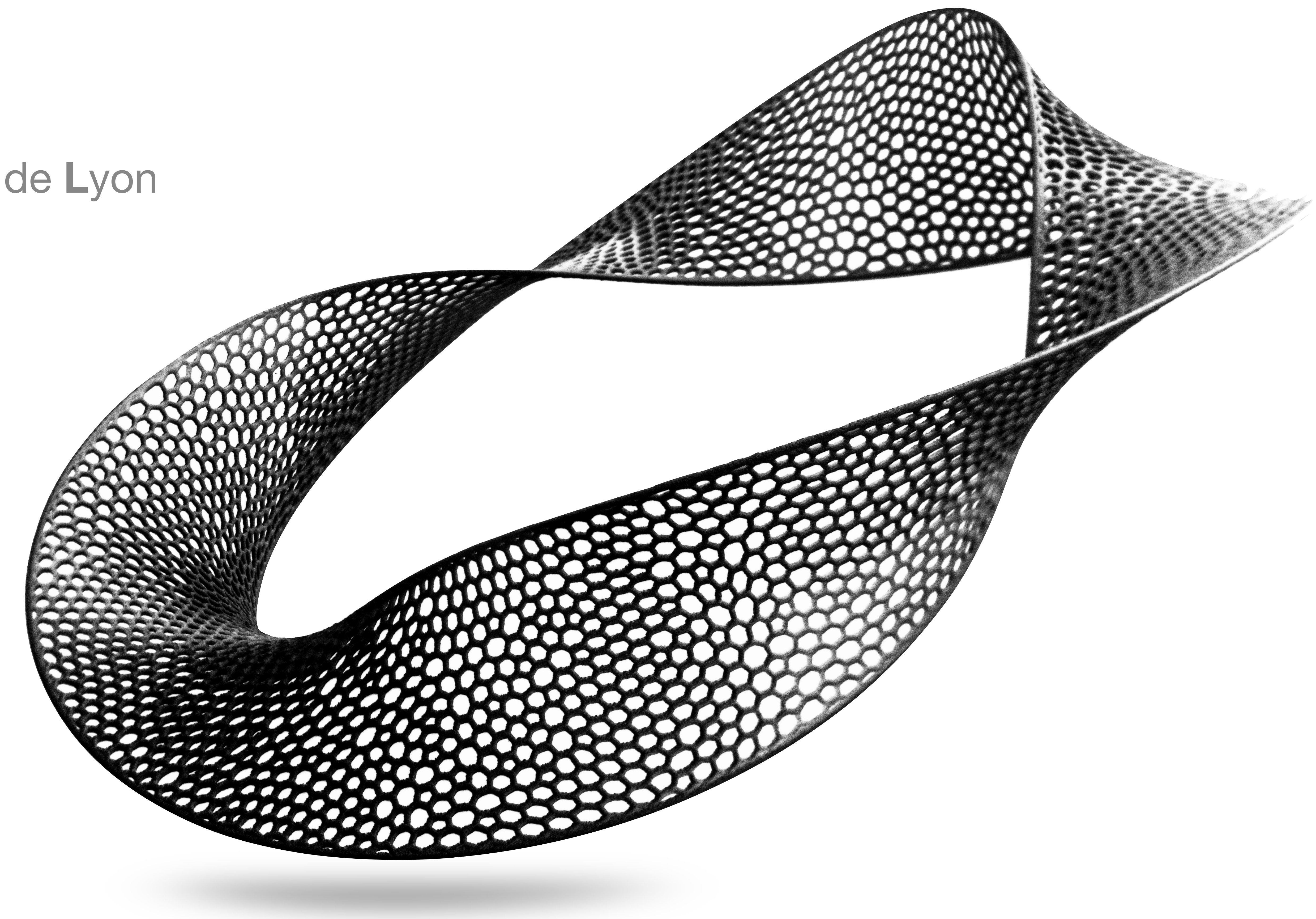
Twisted Matter



David Carpentier
CNRS and **E**cole **N**ormale **S**upérieure de **L**yon

Properties of Matter

mechanical properties

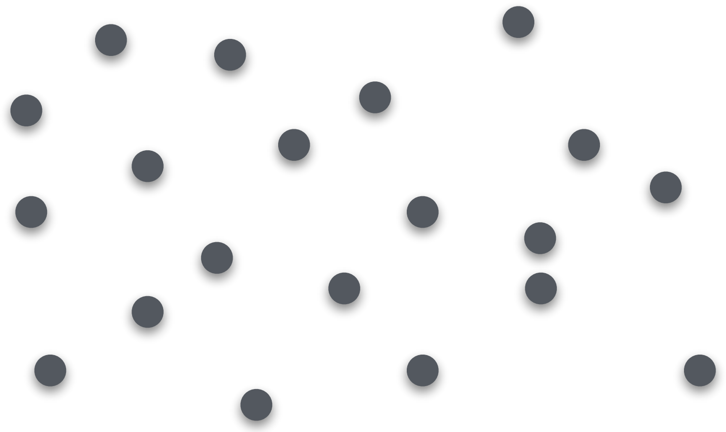
Liquid

Solid

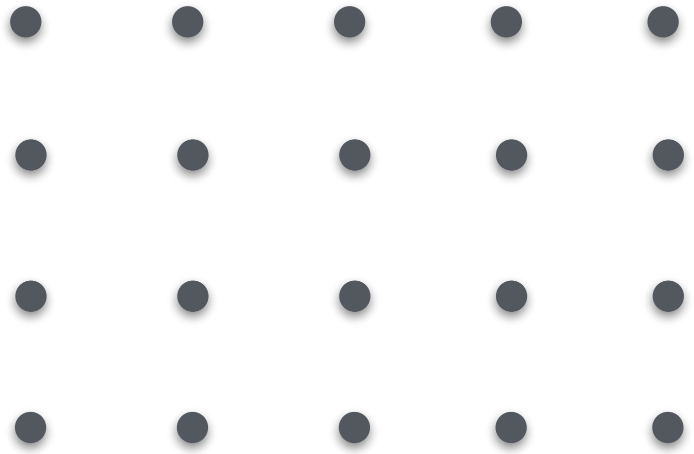
Properties of Matter

mechanical properties

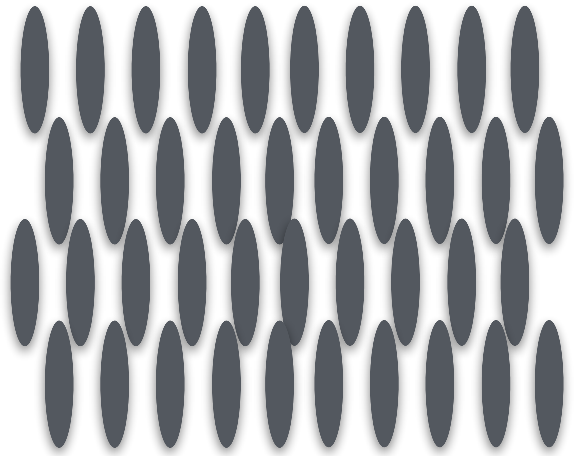
Liquid



Solid



nematic



Properties of Matter

mechanical properties

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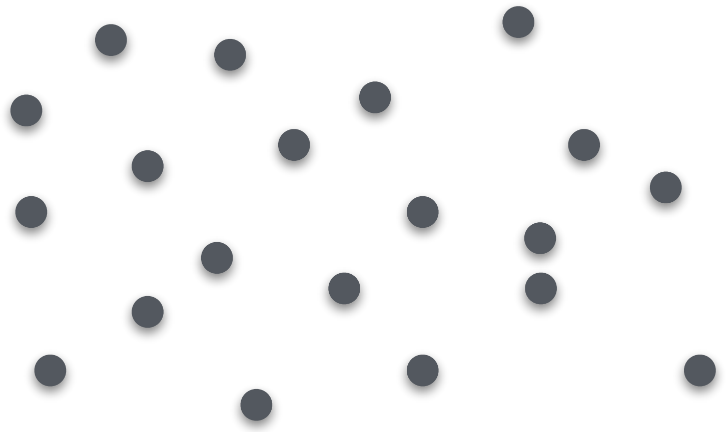

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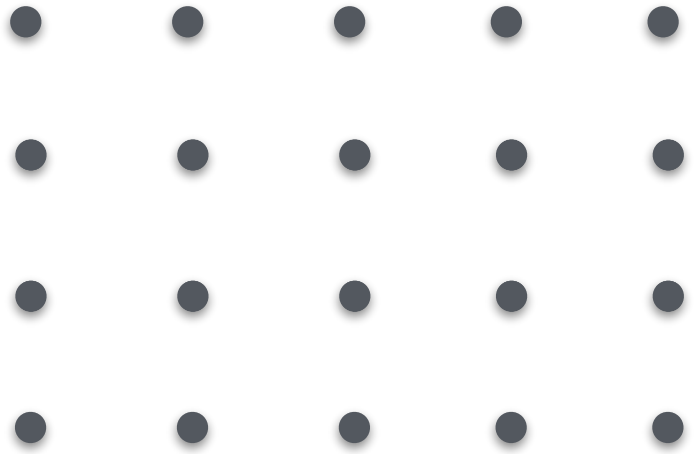

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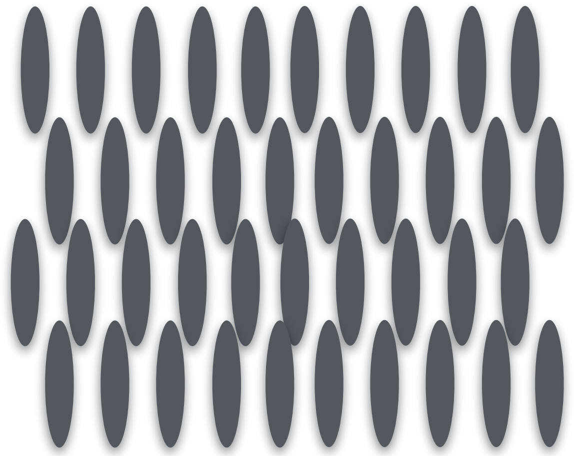
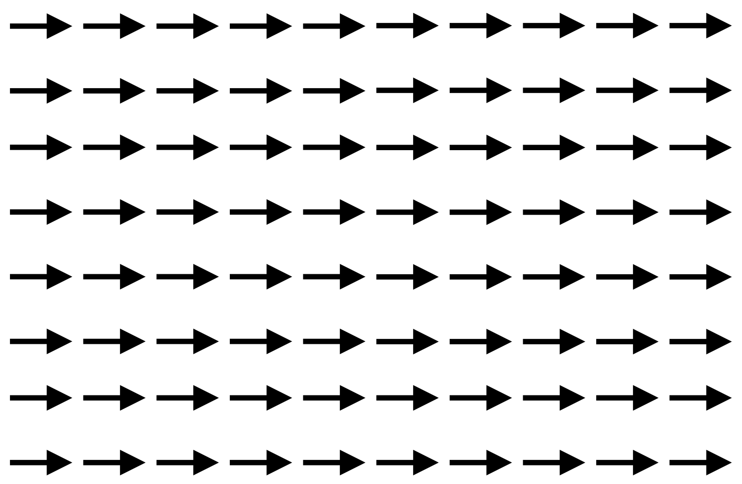


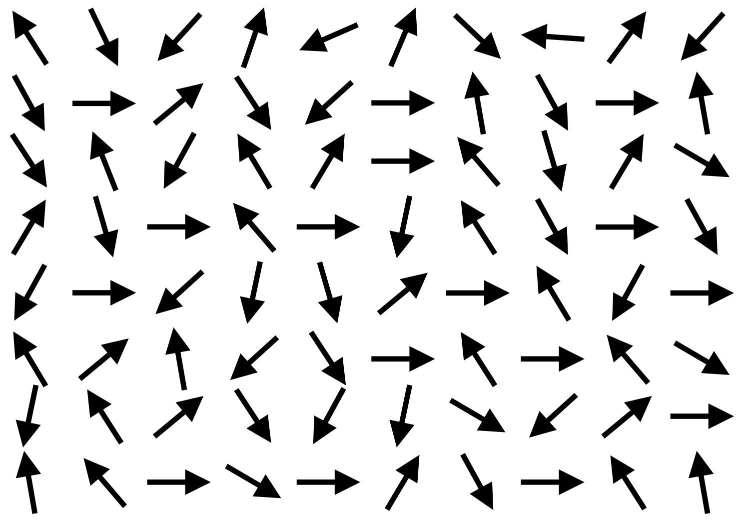
magnetic properties

electronic properties

Ferromagnet



Paramagnet



Metals

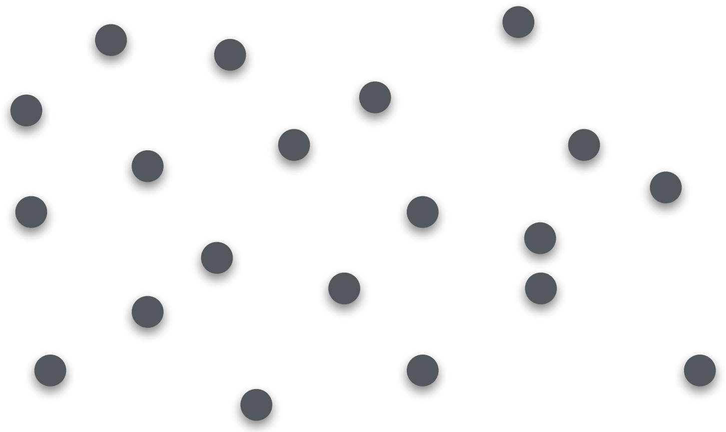
Insulators

Properties of Matter

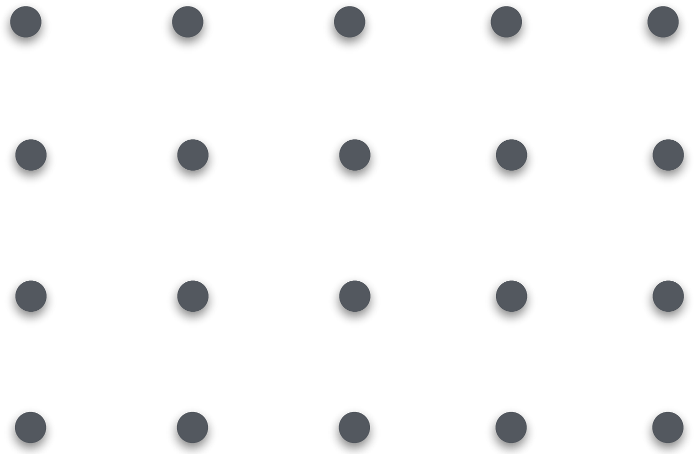
mechanical properties

Symmetry : leaves the phase invariant

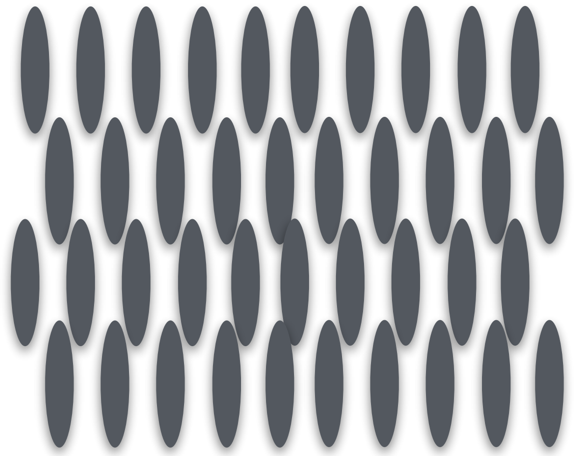
Liquid



Solid



nematic



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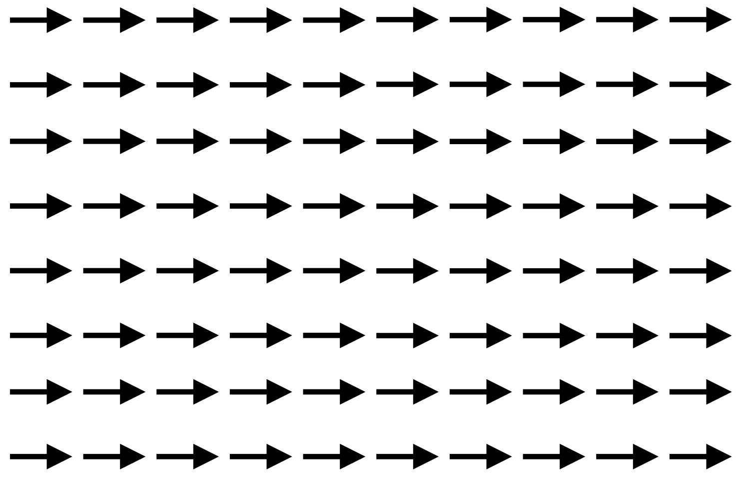
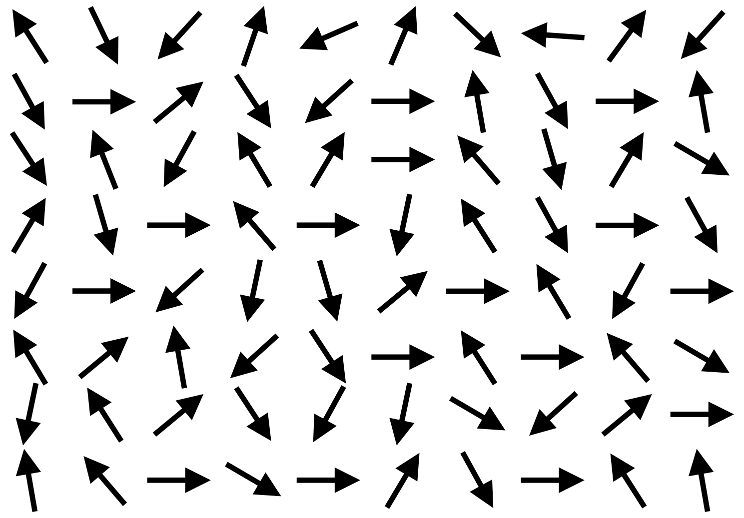


Paramagnet



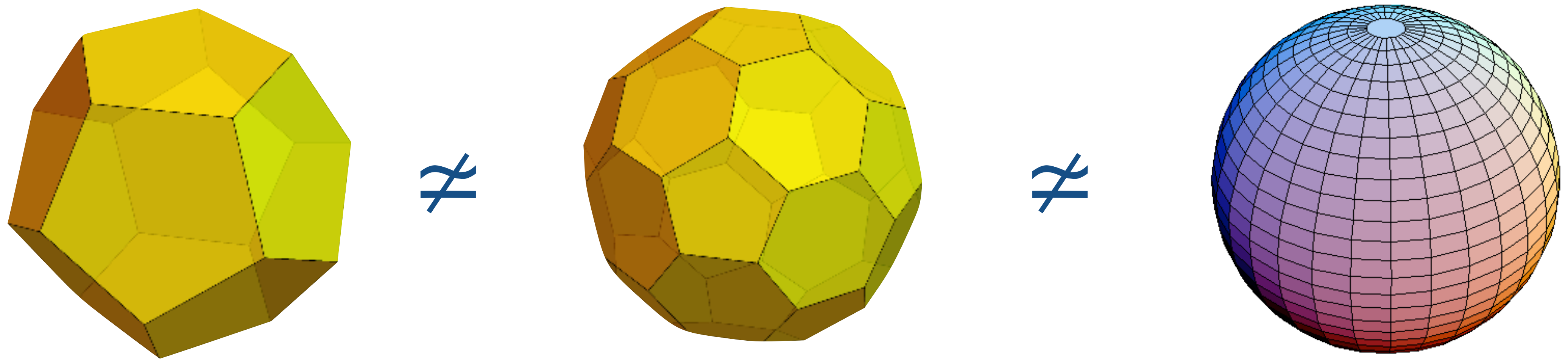
electronic properties

Metals

Insulators

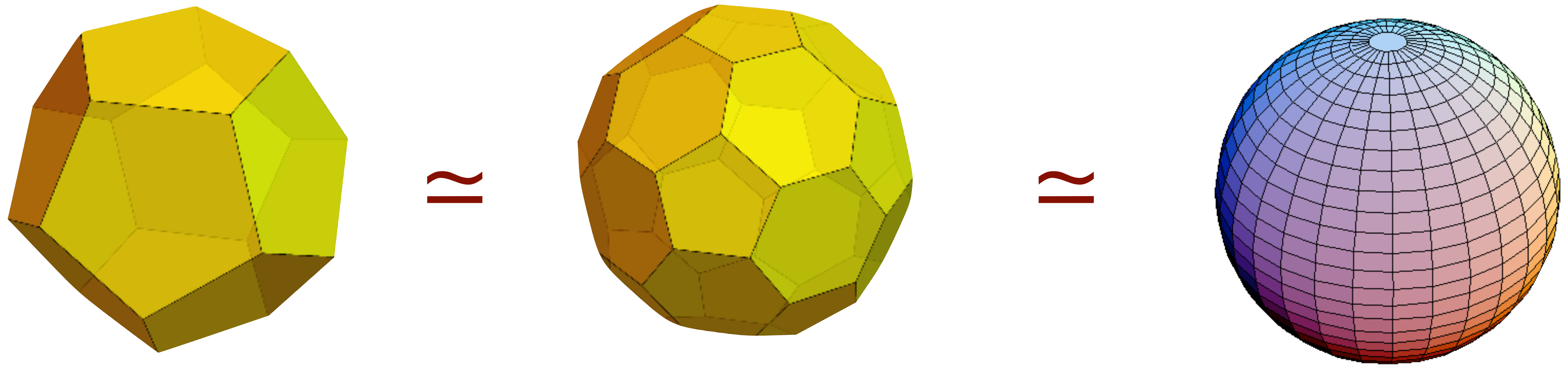
Topology versus Geometry

Symmetry : leaves the object invariant



Topology : global shape

the study of properties unaffected by the continuous change of shape or size



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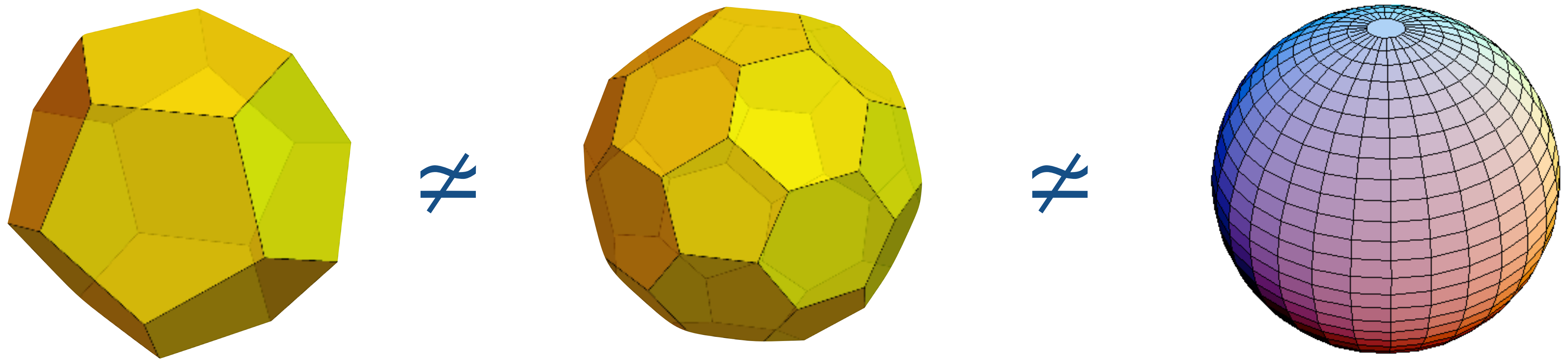

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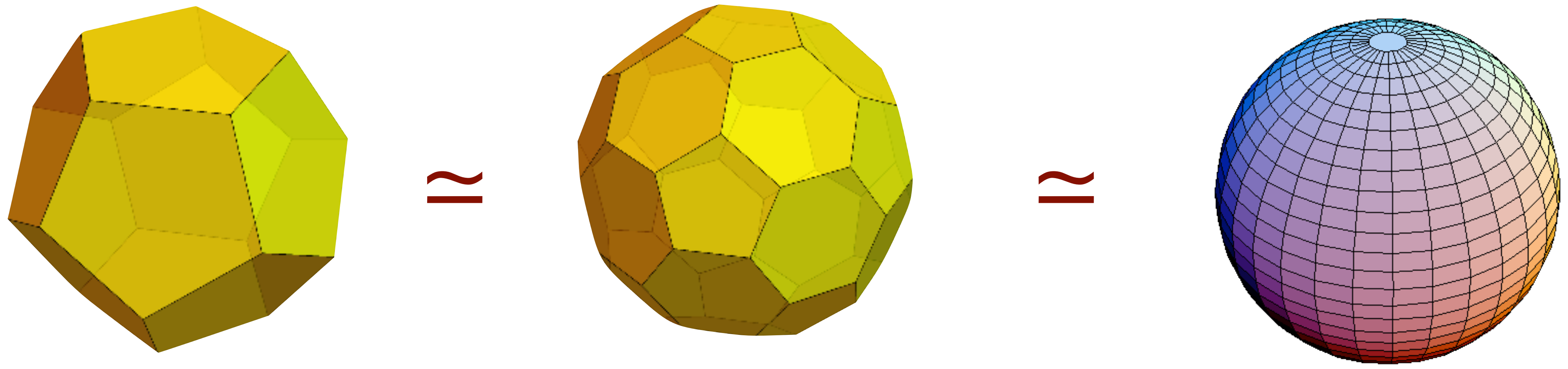
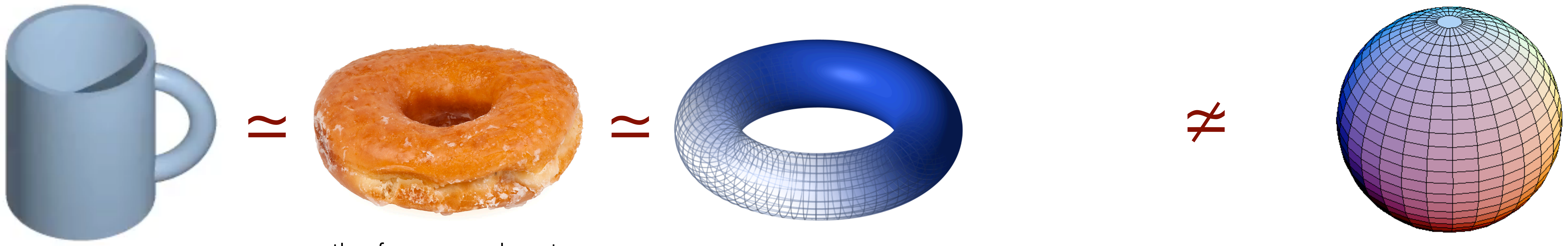




the famous «donut »

Twisted Matter : characterization via topology

Electronic Properties of Quantum Matter

Topological Insulators

Metals   Insulators

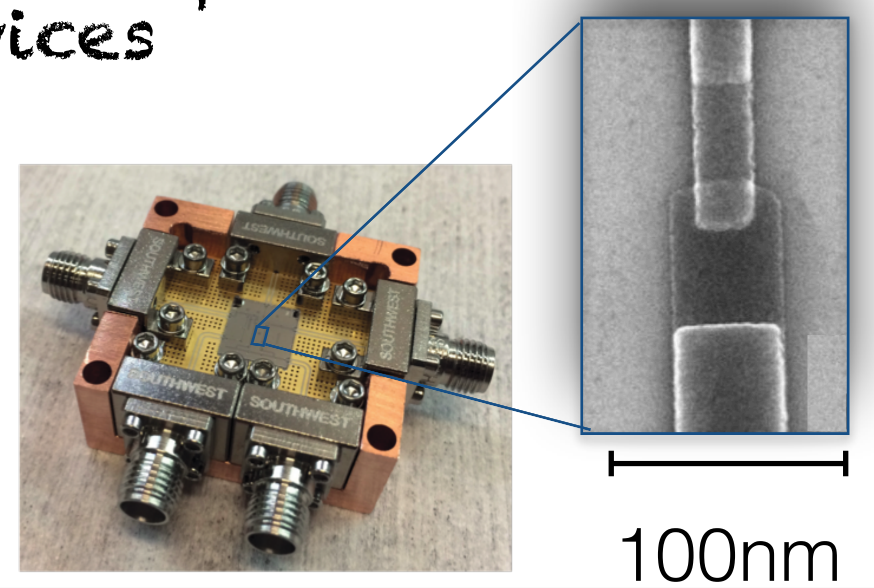
Topology : global shape

the study of properties unaffected by the continuous change of shape or size

Twisted Matter : characterization via topology

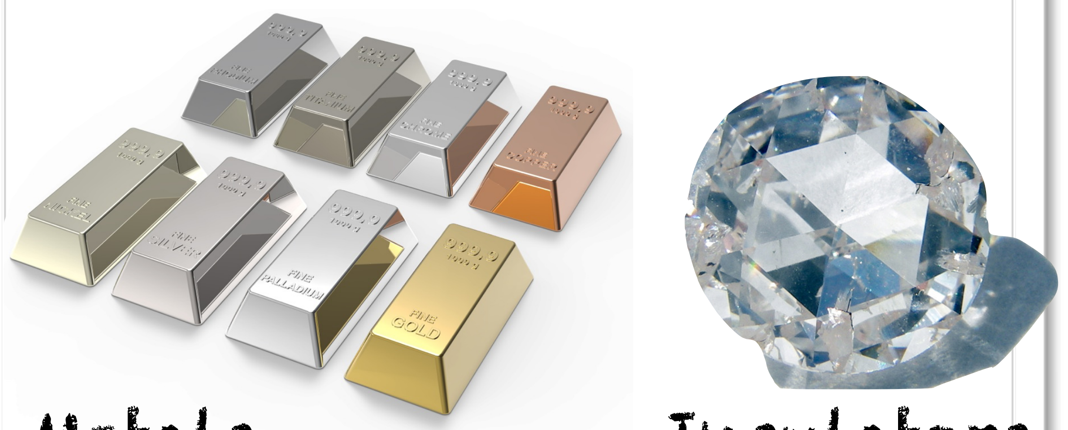
Quantum Technologies

Robust quantum devices



Electronic Properties of Quantum Matter

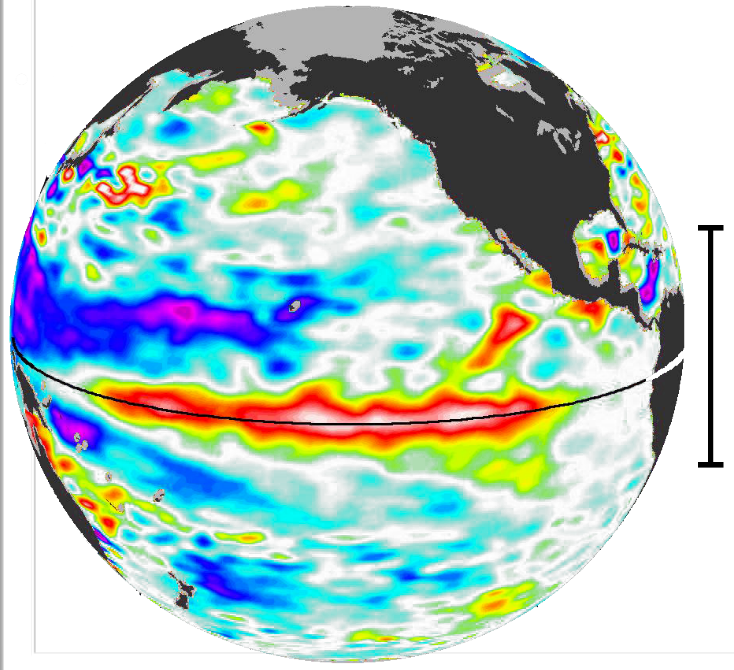
Topological Insulators



Topology
global shape

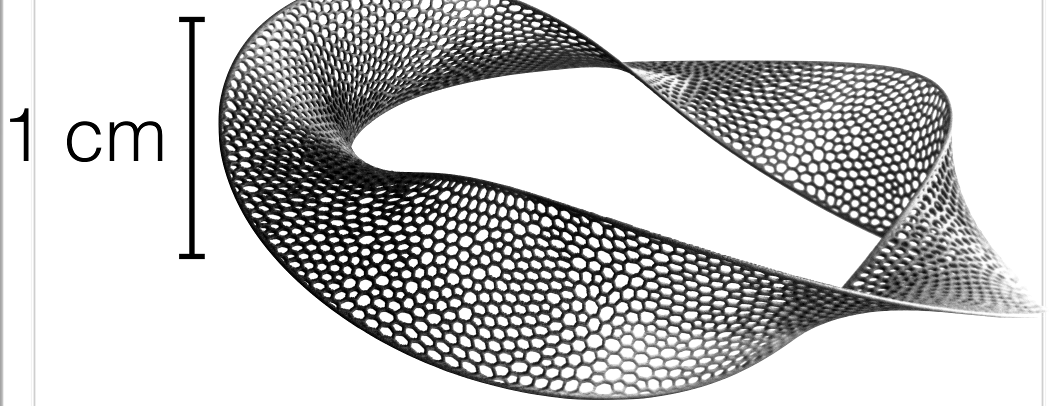
Geofluids

equatorial waves of topological origin



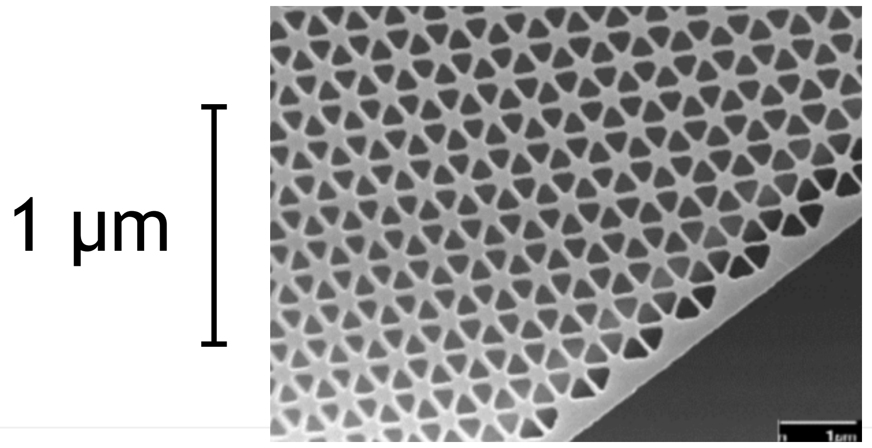
Mechanics / Metamaterials

deformations constrained by topology



Optics / Metamaterials

optical modes constrained by topology



Topological Matter in Lyon

Quantum Technologies

D. Carpentier,
P. Delplace

B. Huard

Electronic and Magnetic Properties of Matter

D. Carpentier,
P. Delplace

A. Fedorenko

P. Holdsworth

T. Rolscilde

L. Savary

E. Orignac

Topology
global shape

K. Gawedzki

J. Kellendonk

J.-M. Stephan

(Institute Camille Jourdan)

Geofluids

P. Delplace

A. Venaille

Mechanics / Metamaterials

D. Carpentier

D. Bartolo

Optics / Metamaterials

D. Carpentier, P. Delplace

L. Ferrier

H.-S. Nguyen

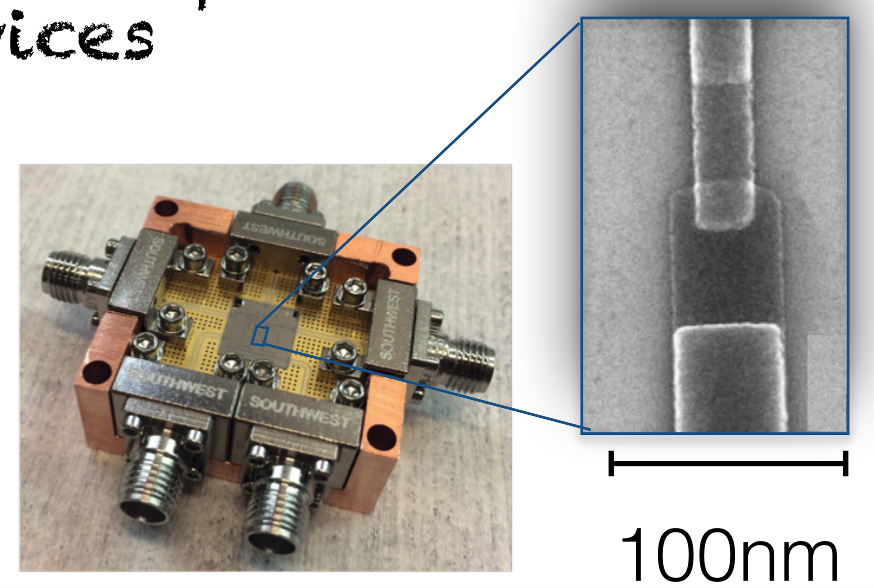
(Lyon Institute of Nanotechnology)

ToRe Breakthrough Project

IDEX LYON
UNIVERSITÉ DE LYON

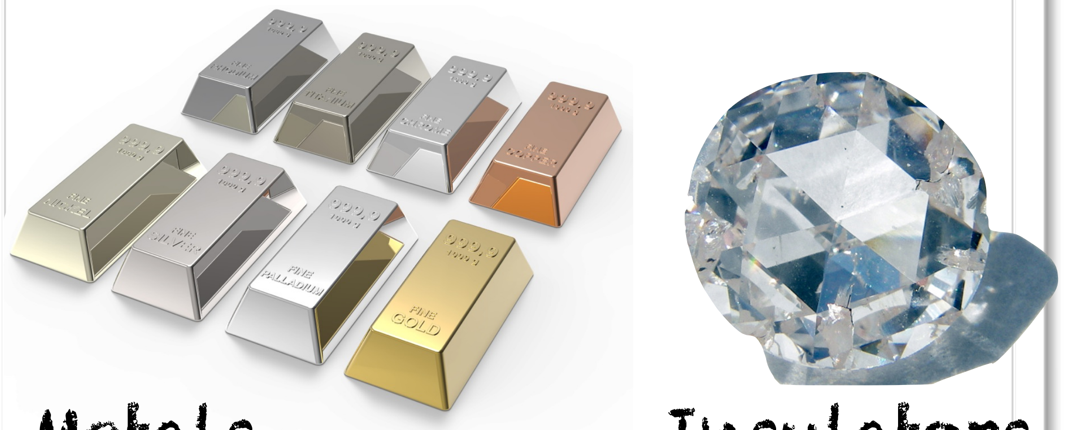
Quantum Technologies

Robust quantum devices



Electronic Properties of Quantum Matter

Topological Insulators



Topology
global shape

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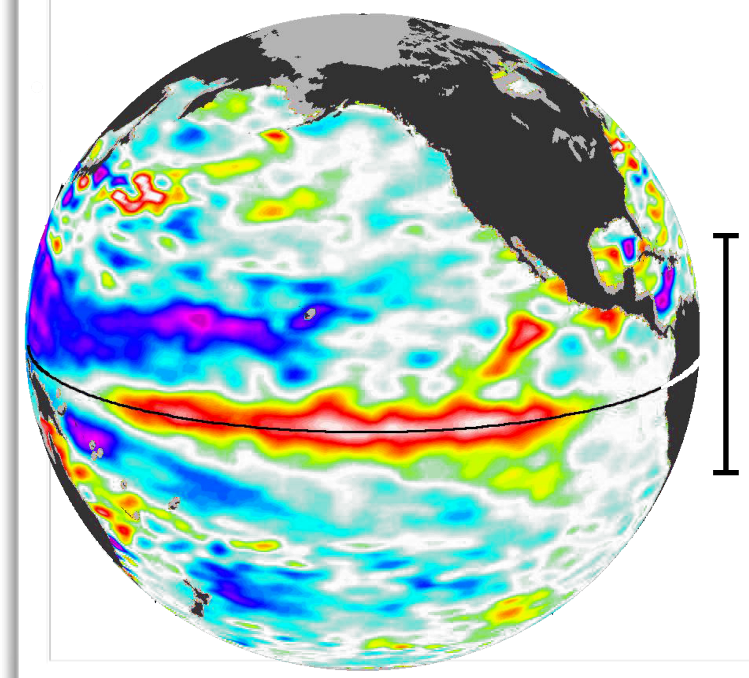
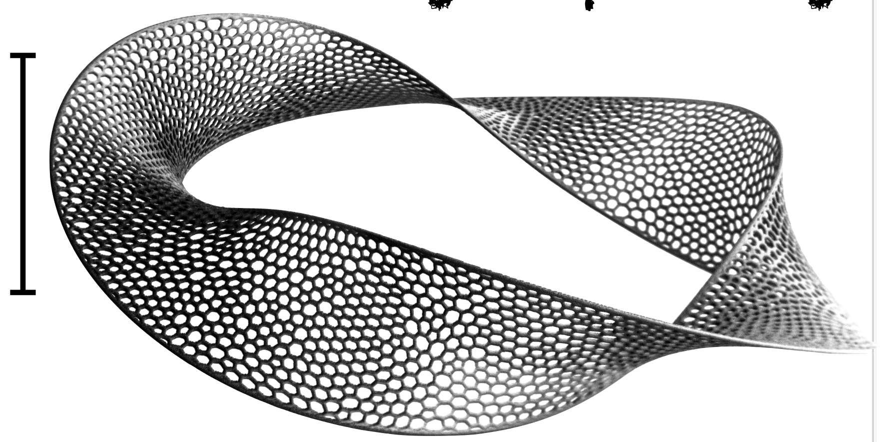


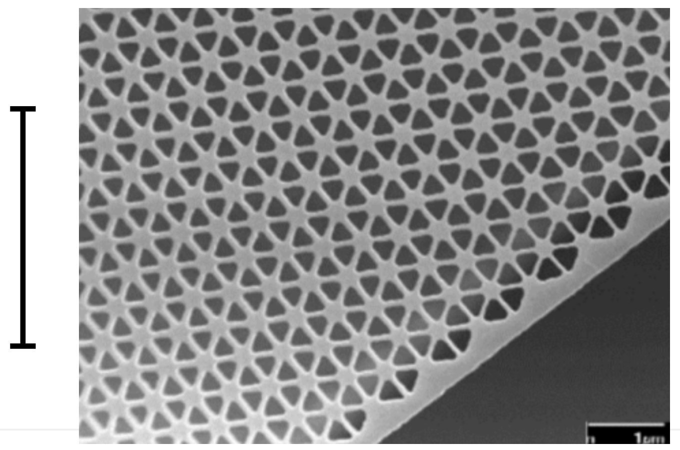
Mechanics / Metamaterials

deformations constrained by topology



Optics / Metamaterials

optical modes constrained by topology



Outline

1. Electronic Properties of Quantum Matter

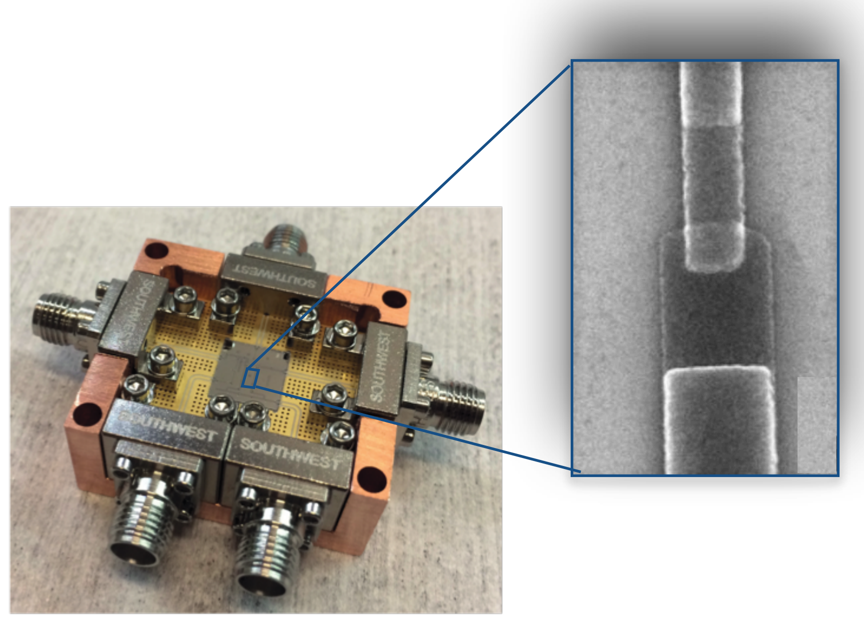
   Topological Insulators

Metals

Insulators

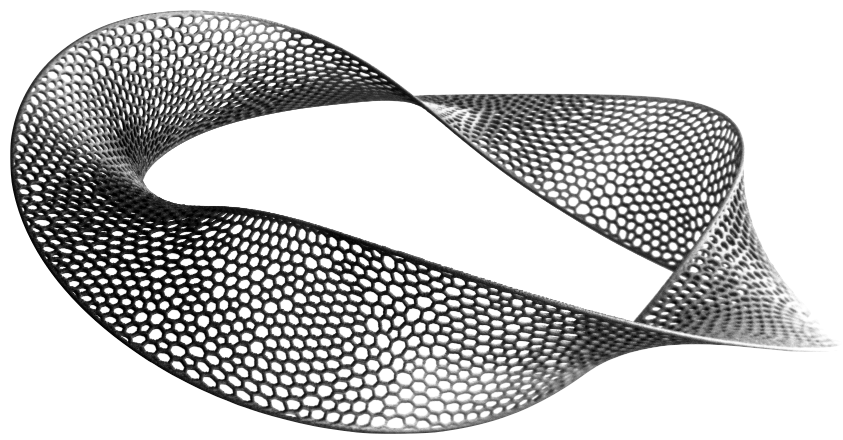
2. Quantum Technologies

   Robust quantum devices



3. Mechanics / Metamaterials

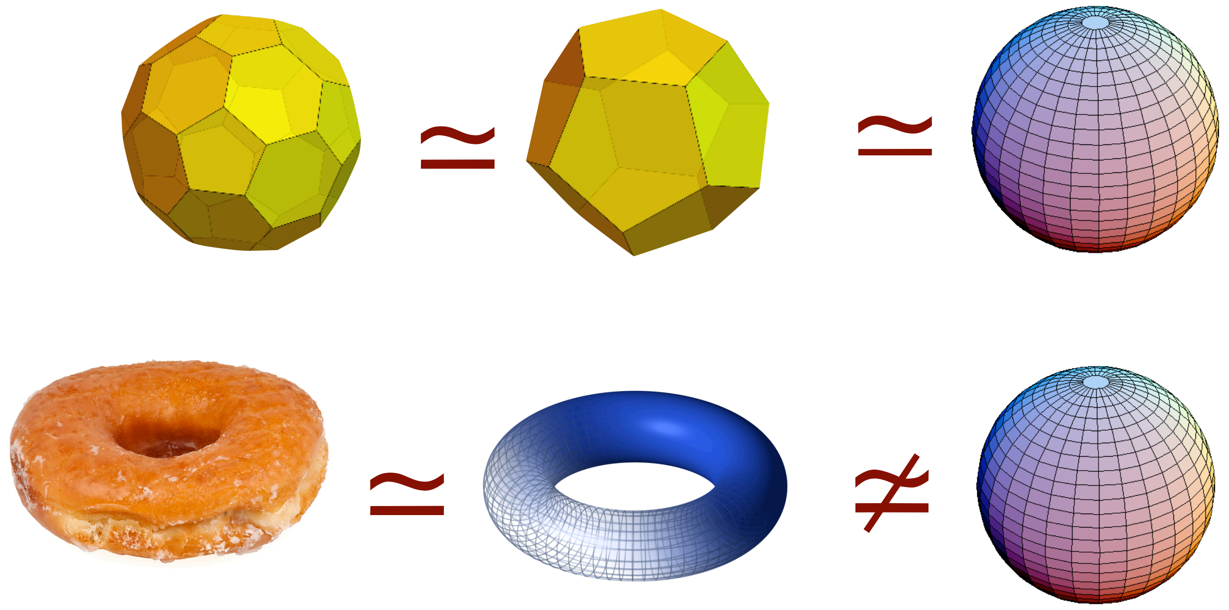
   deformations constrained by topology



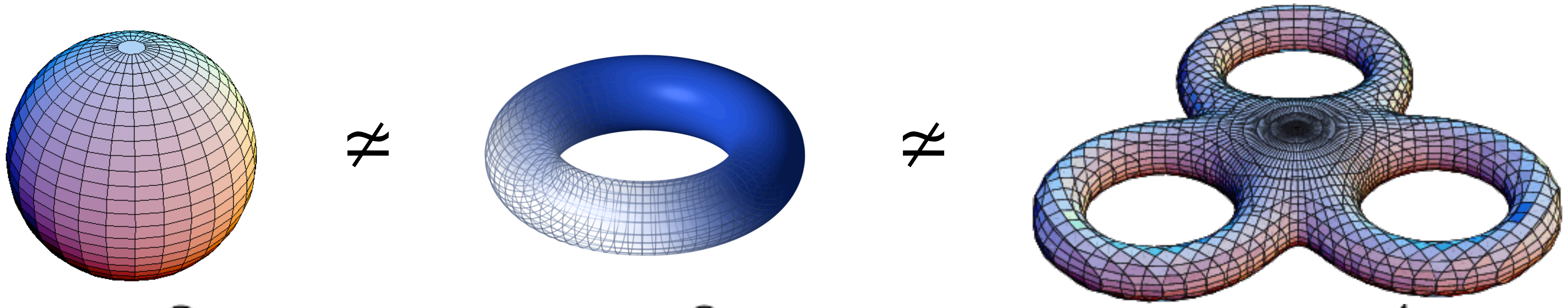
Topology

Topology: aims at classifying objects

- identifies properties of objects that are preserved under continuous deformations
- uses **integer number** to distinguish classes of objects



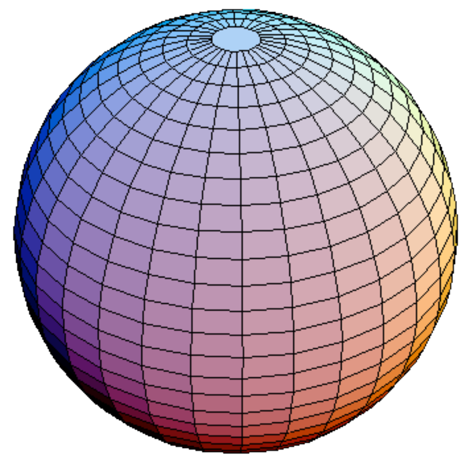
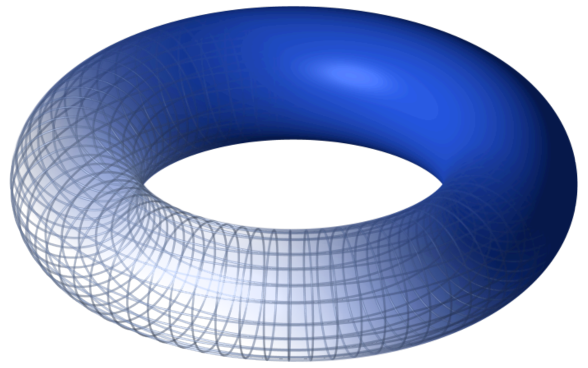
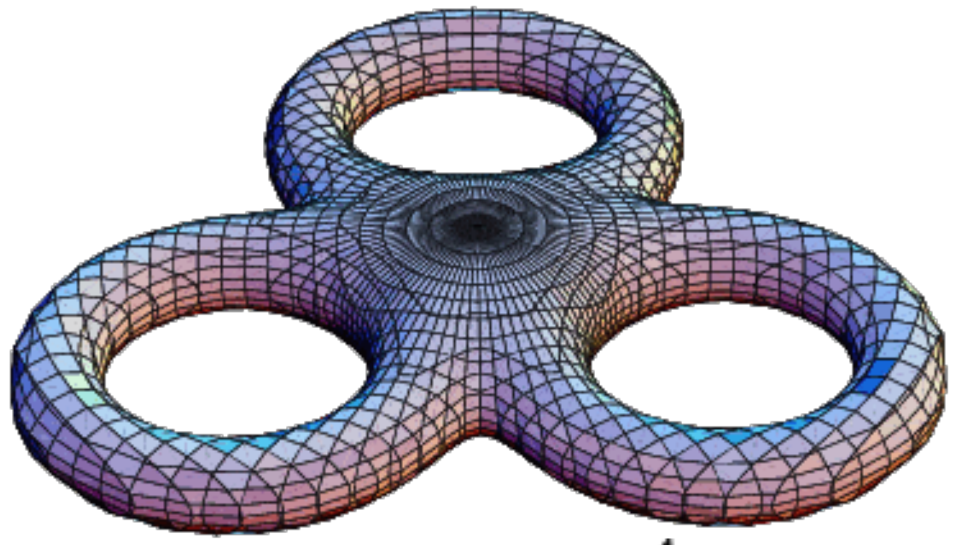
Example of 2d surfaces :



$\chi = 2$ $\qquad$ $\chi = 0$ $\qquad$ $\chi = -4$

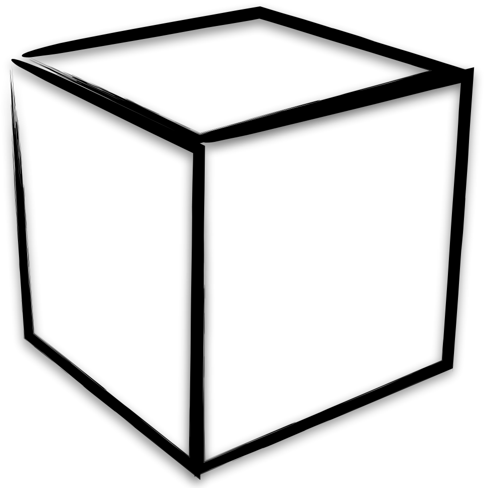
(Euler characteristic)

Topology

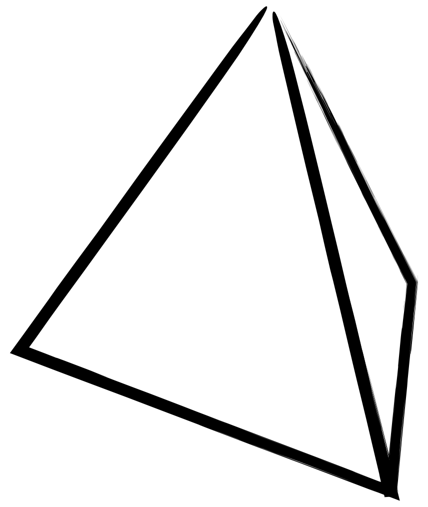
 $\neq$  $\neq$ 

$\chi = 2$ $\chi = 0$ $\chi = -4$

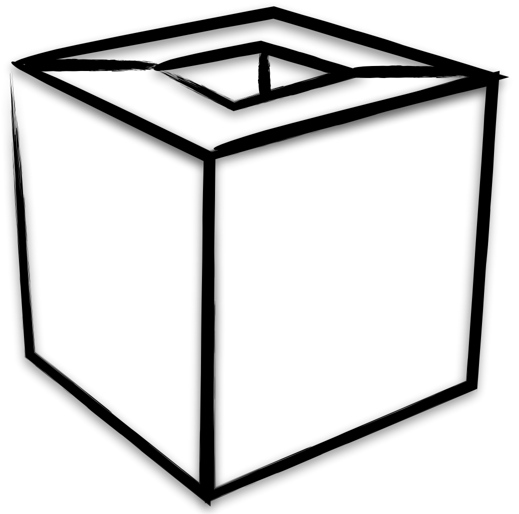
- For polygons: **Euler characteristic** χ = #vertices - # edges + # faces



$$\chi = 8 - 12 + 6 = +2$$

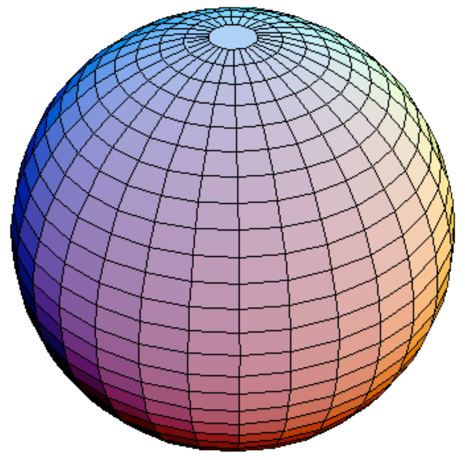
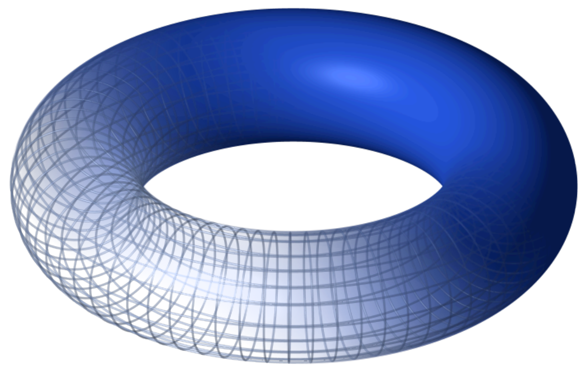
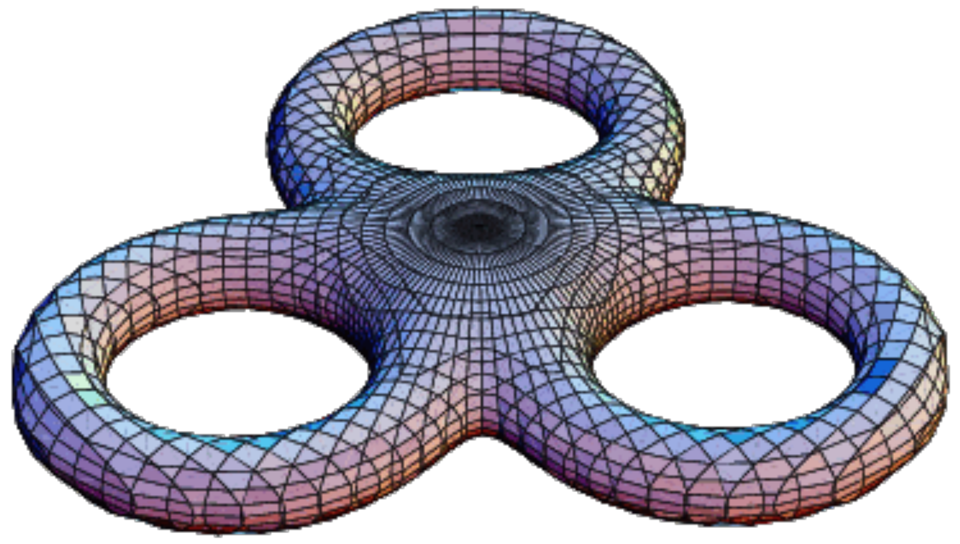


$$\chi = 4 - 6 + 4 = +2$$



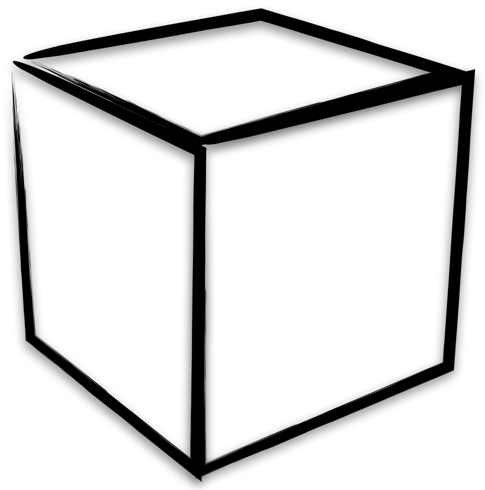
$$\chi = 16 - 28 + 12 = 0$$

Topology

 $\neq$  $\neq$ 

$\chi = 2, g = 0$ $\qquad$ $\chi = 0, g = 1$ $\qquad$ $\chi = -4, g = 3$

- For polygons: **Euler characteristic** χ = #vertices - # edges + # faces



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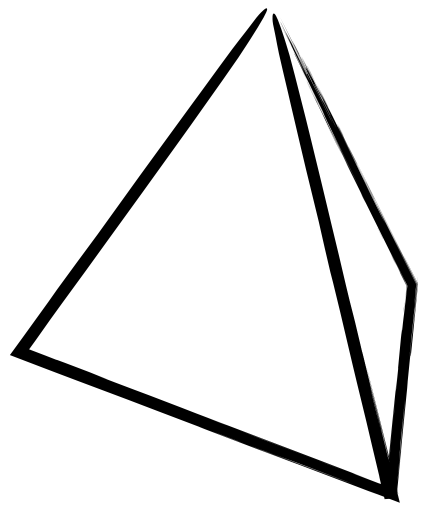
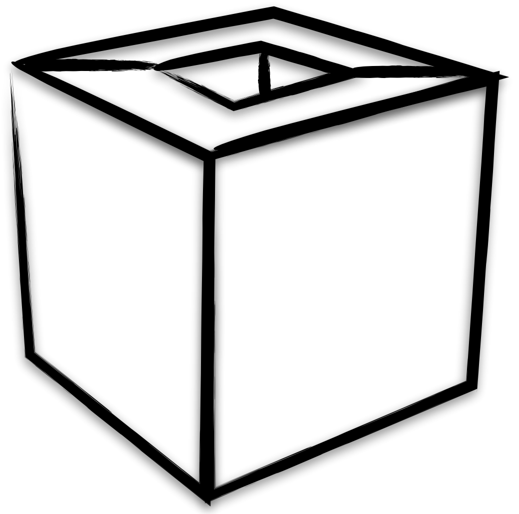


$\chi = 4 - 6 + 4 = +2$



$\chi = 16 - 28 + 12 = 0$

- **Euler characteristic** ↔ genus g : $\chi = 2 - 2g$

- Gauss-Bonnet theorem $\chi = \int dS\ \kappa$

Gaussian curvature : $\kappa = 1/(R_1 R_2)$

– curvature : depends on «local properties»
– Integral of curvature : «global property» (topology)

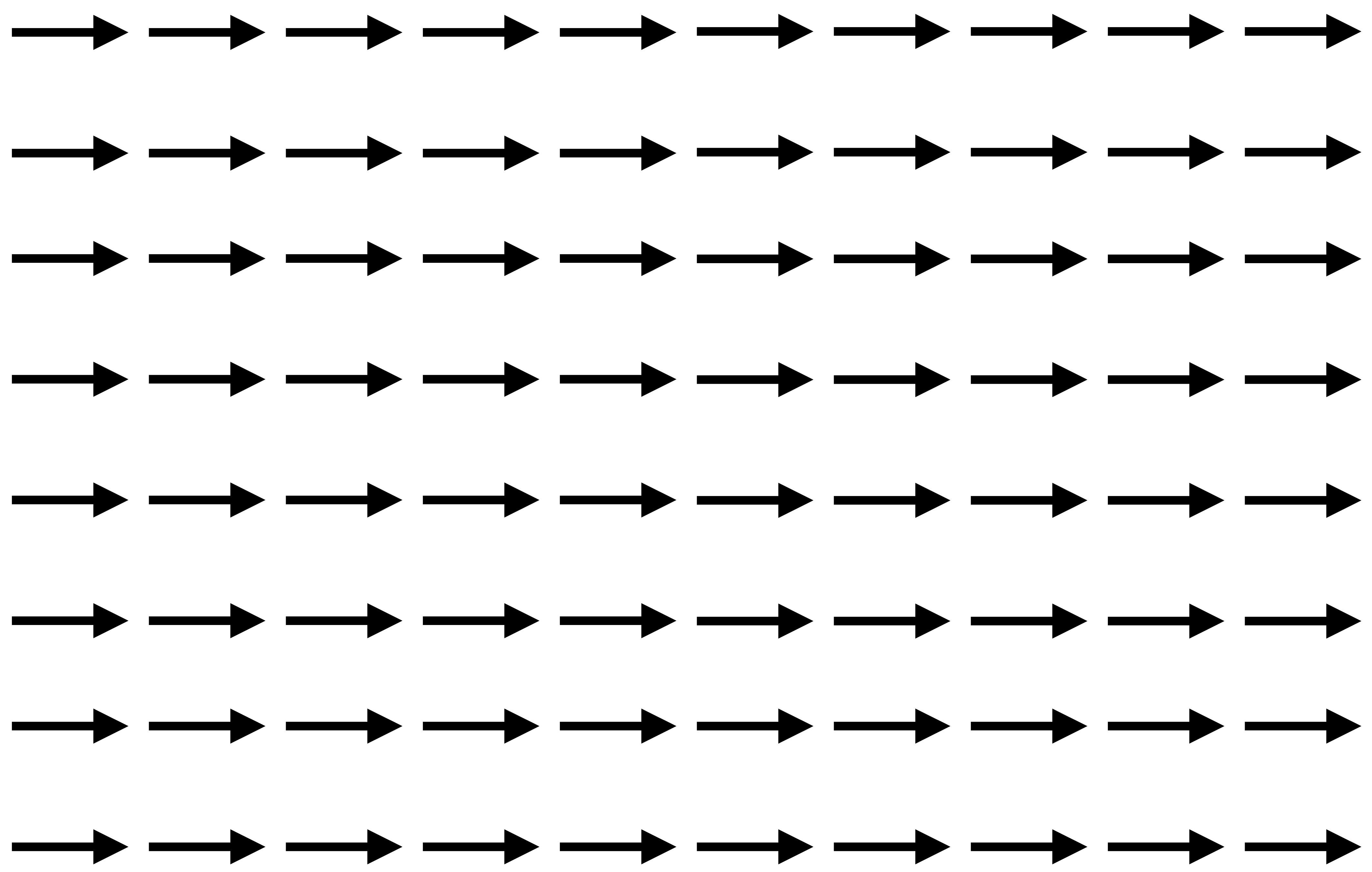
Topology

In condensed matter : **topological defects** of ordered phase / topological **textures**

- vortices (superfluid, superconductor, XY spins),
- dislocations and disclinations (solids, liquid crystals),
- hedgehog / skyrmions (SU(2) spins), etc.

Ordered phase :

- order parameter $\psi(x) \in \mathbb{C}$
- spatial order



d=2, complex order parameter

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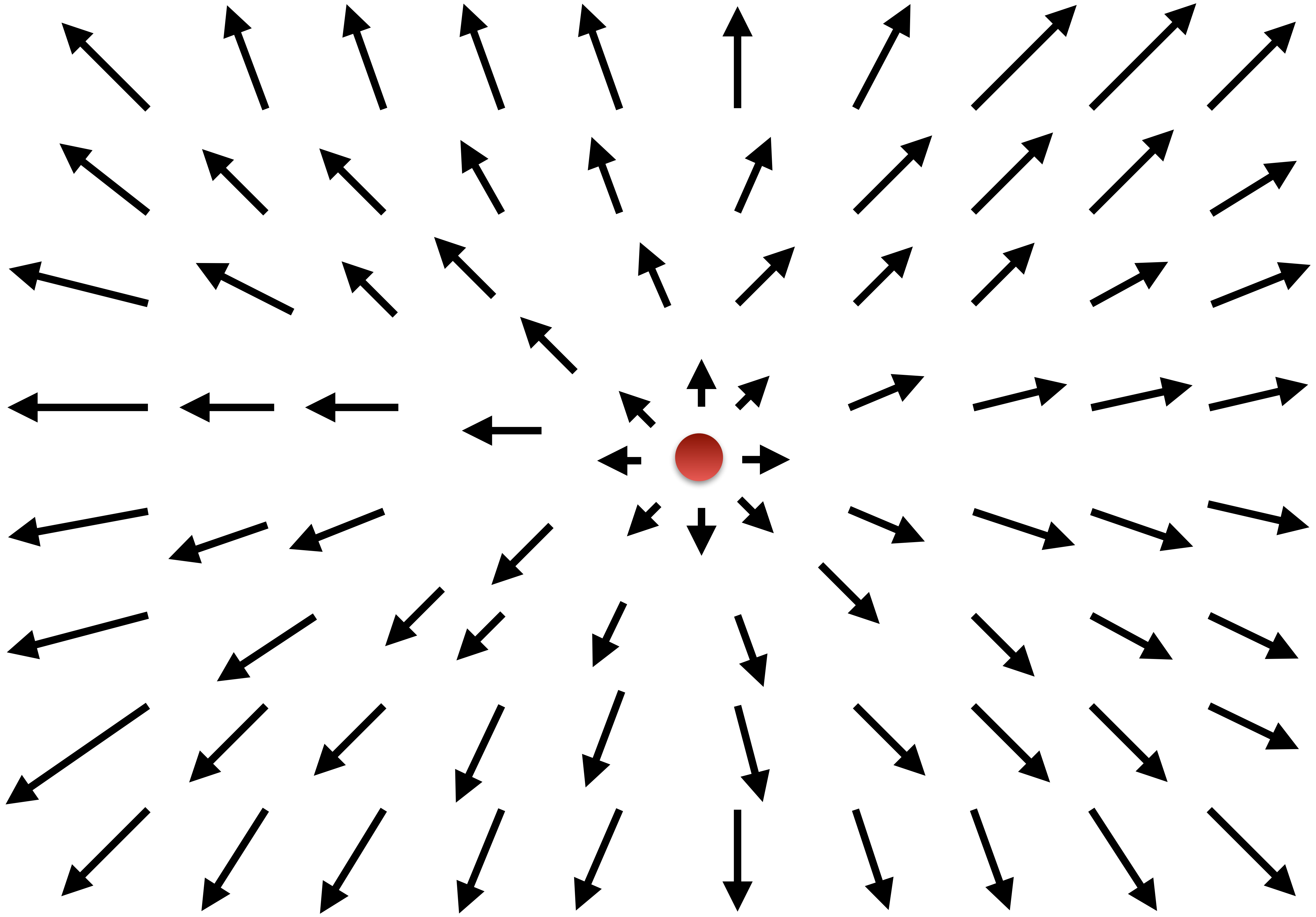
Ordered phase :

- order parameter
- spatial order

Associated Defect

- singularity of order field



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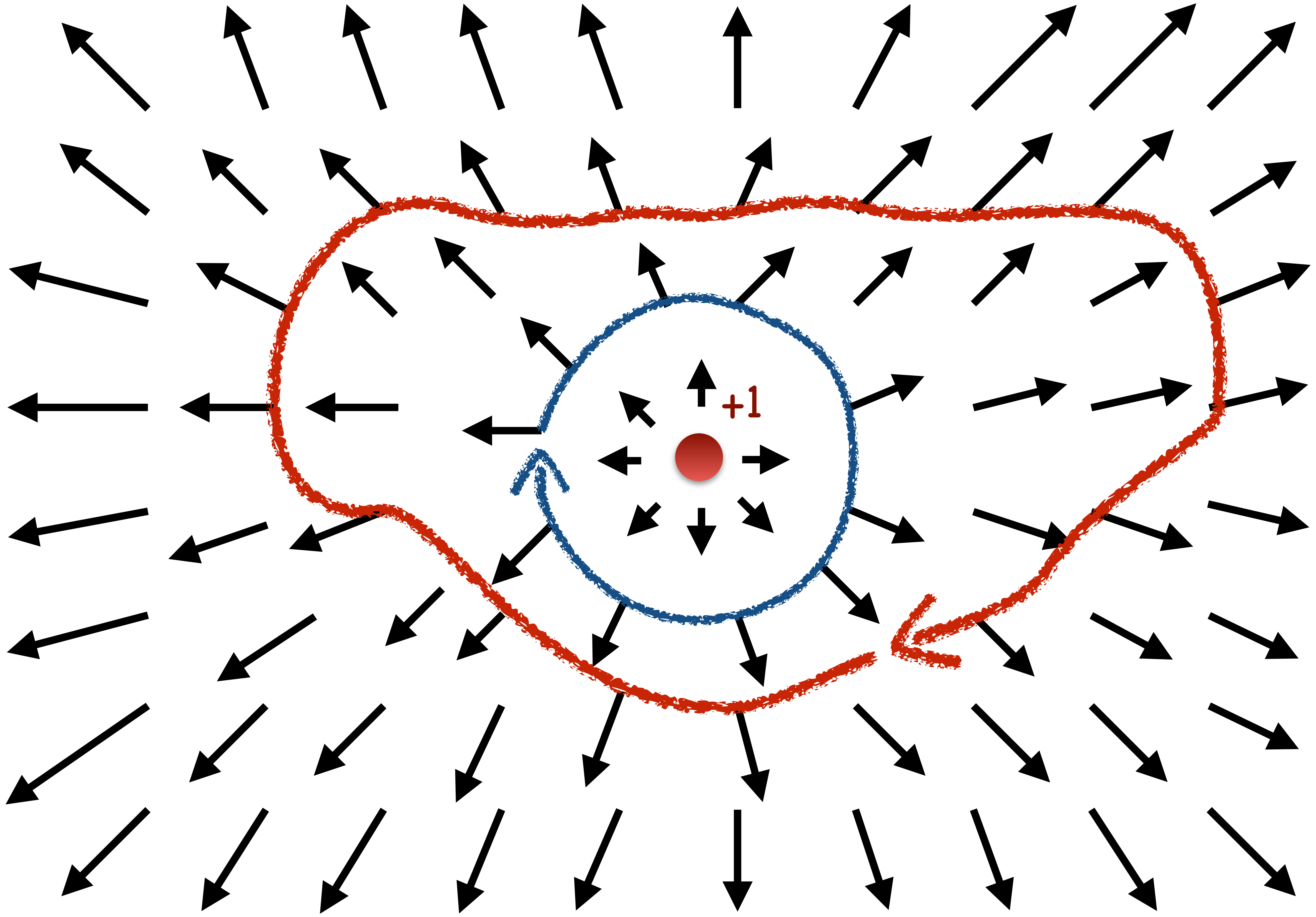
Ordered phase :

- order parameter
- spatial order

Associated Defect

- singularity of order field
- winding of order parameter : topological number

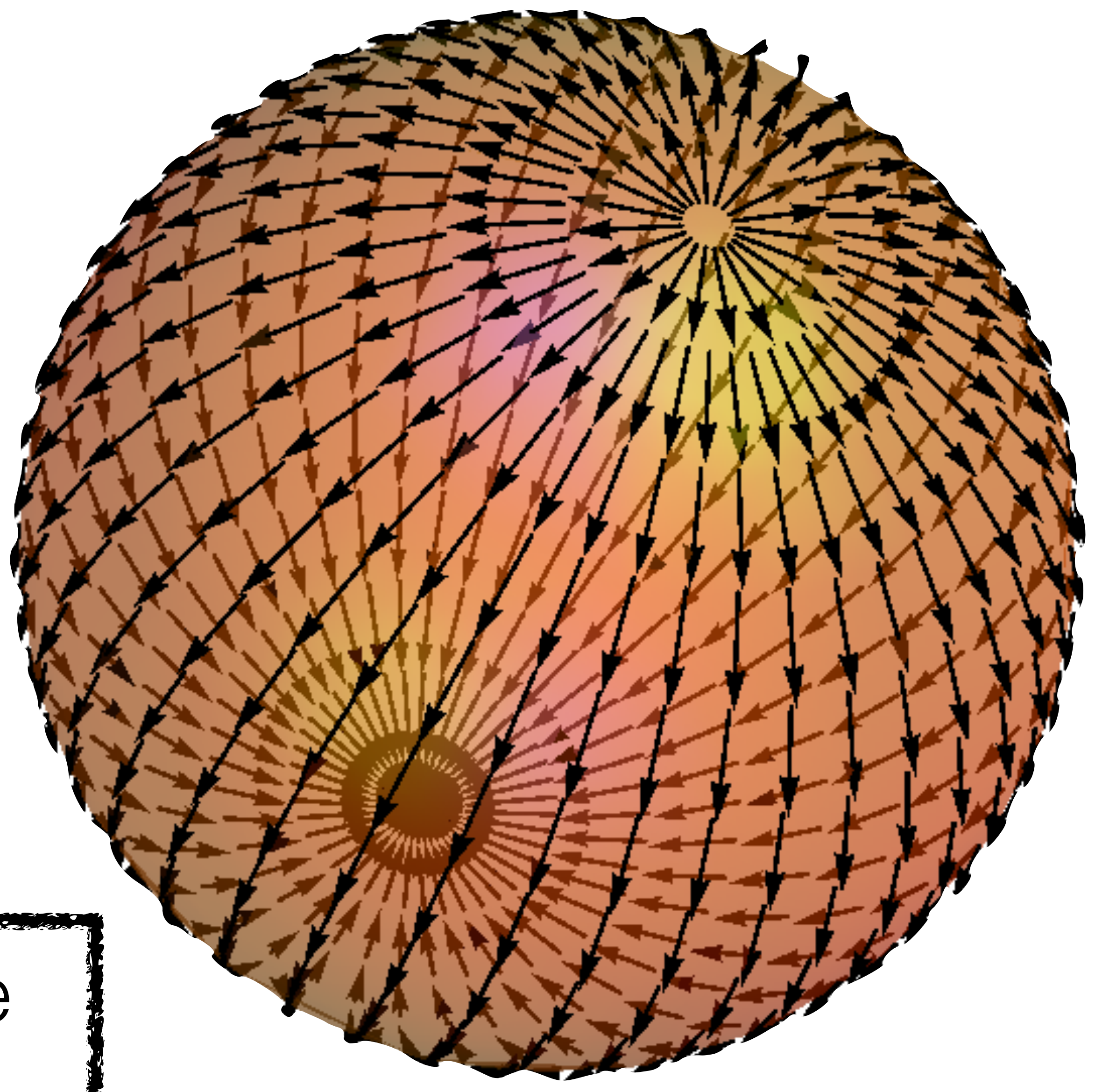


d=2, complex order parameter

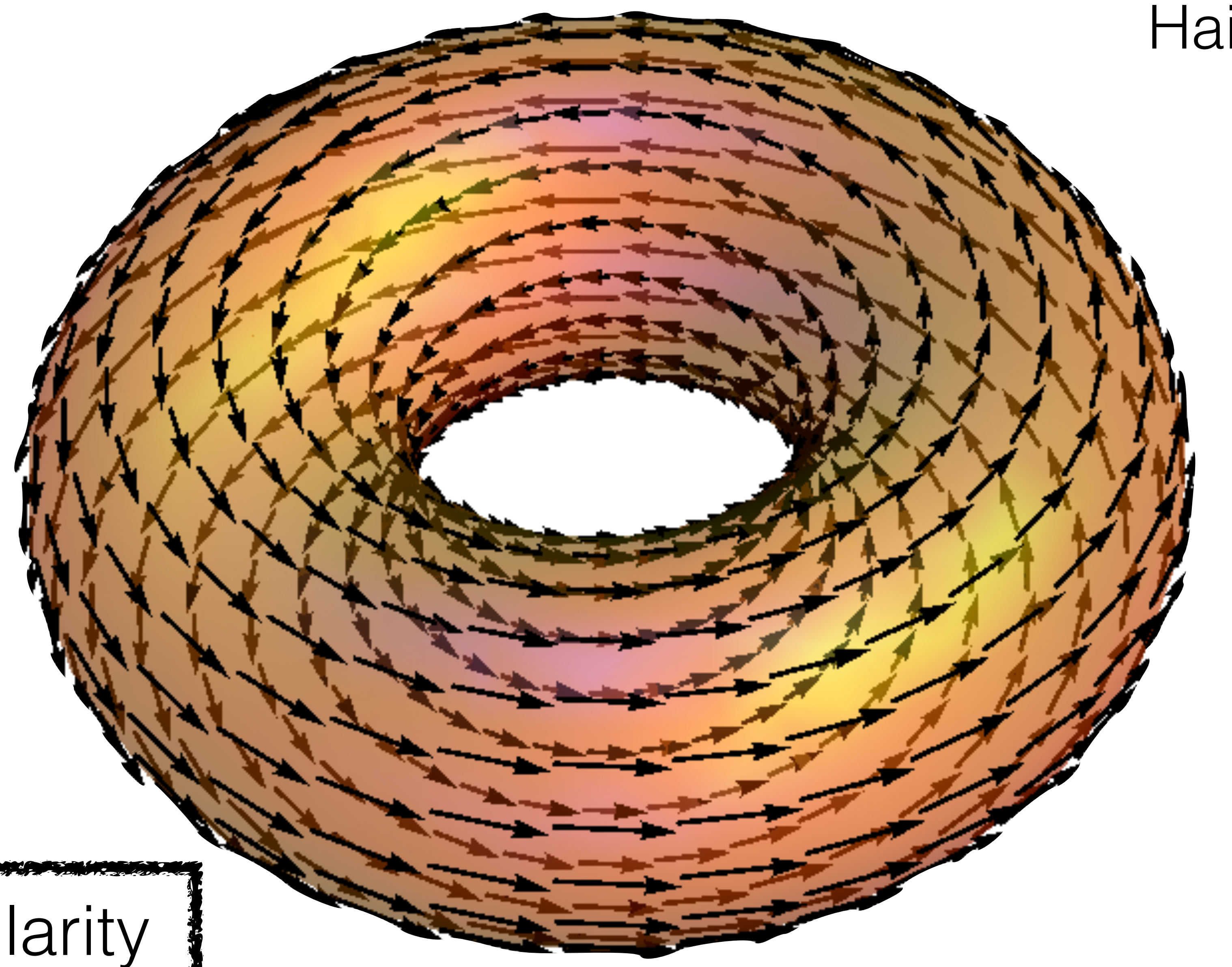
Topology

If vector field defined on a manifold : **non singular vector fields** do not necessarily exist

Hairy ball theorem



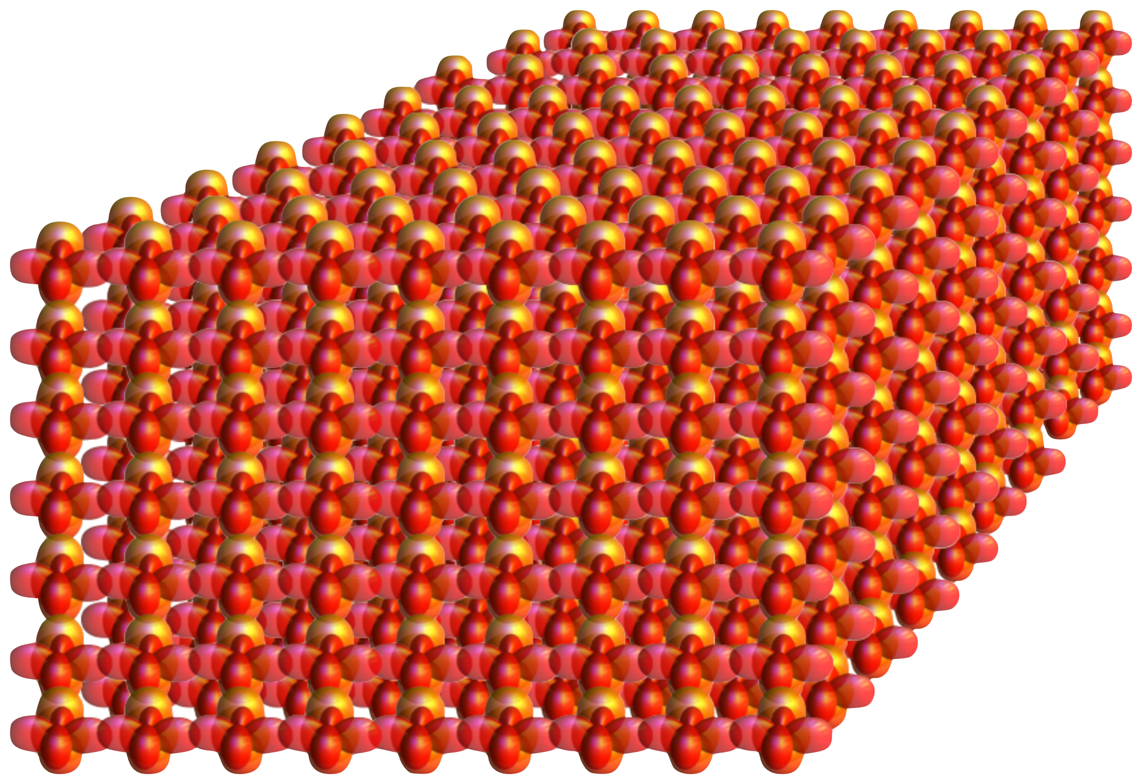
at least one singularity

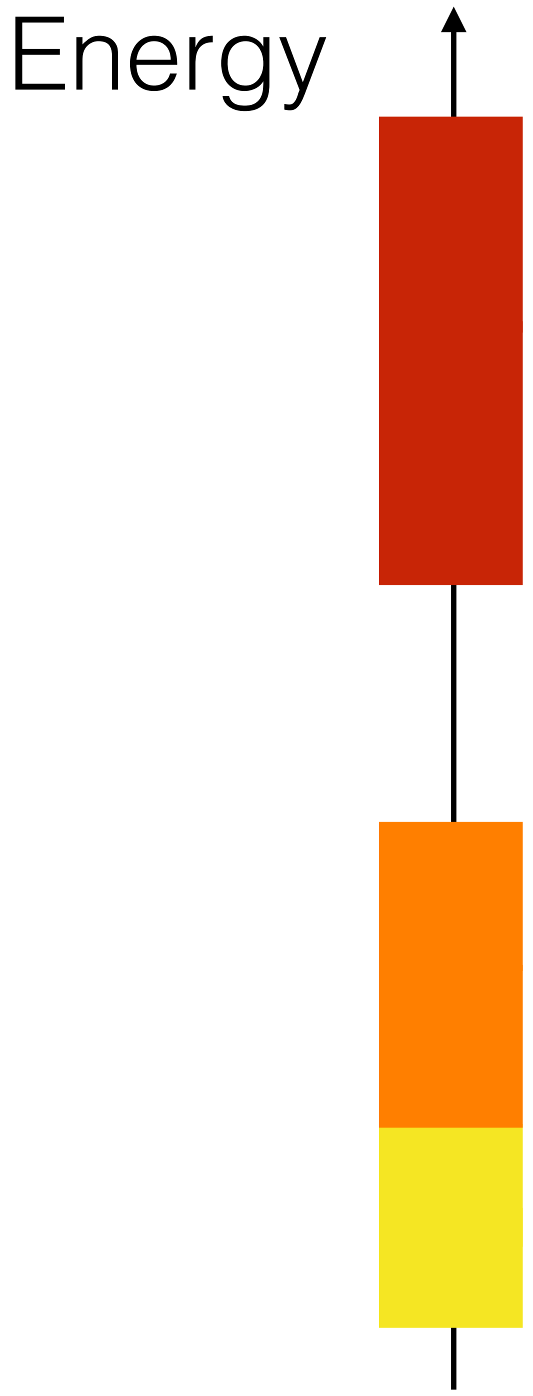


no singularity

- defines a **vector bundle** (manifold + vector space above each point)
- all vector fields singular ↔ **non trivial vector bundle**
- **topological property**, associated with a « topological Chern number »

Band Theory of Electrons in Solids





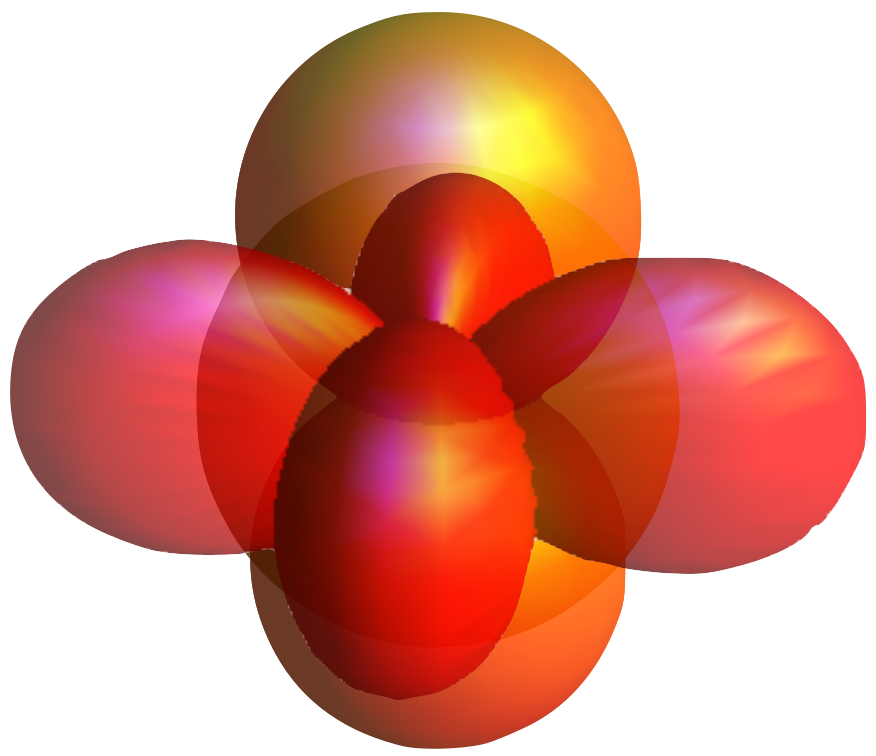
Band Theory of Electrons in Solids

Electronic Orbitals

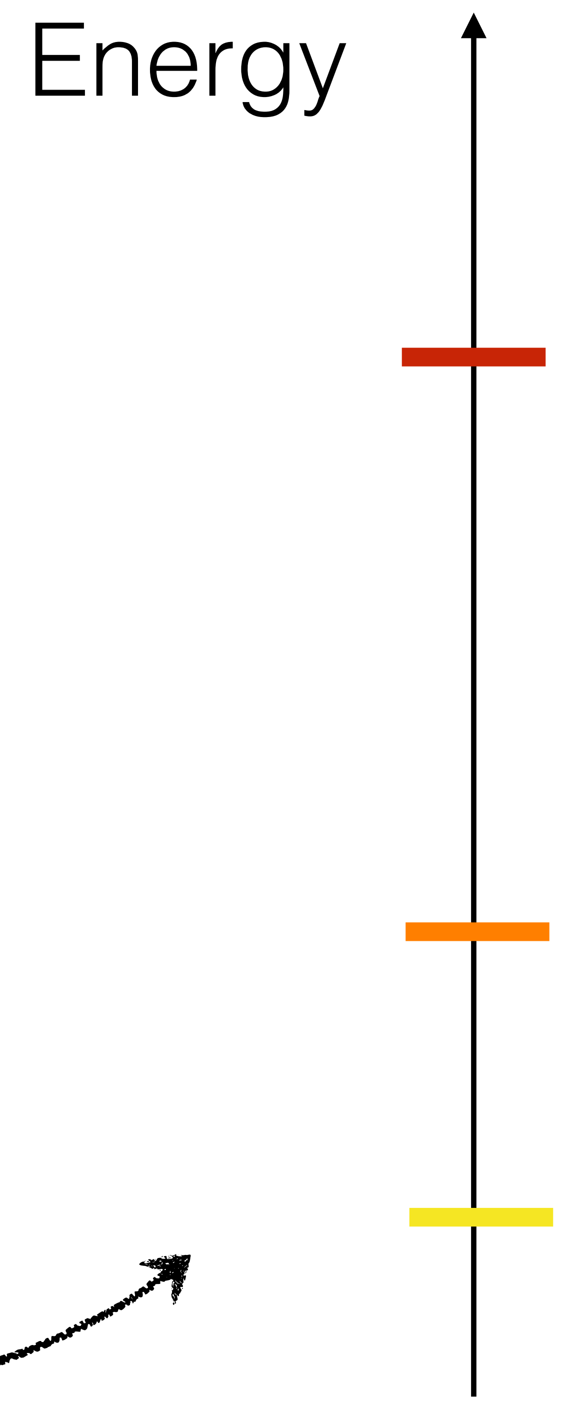
1s

2p

3d



Single atom



Discrete energy levels

Schrödinger equation :

$$H|\psi\rangle = E\ |\psi\rangle$$

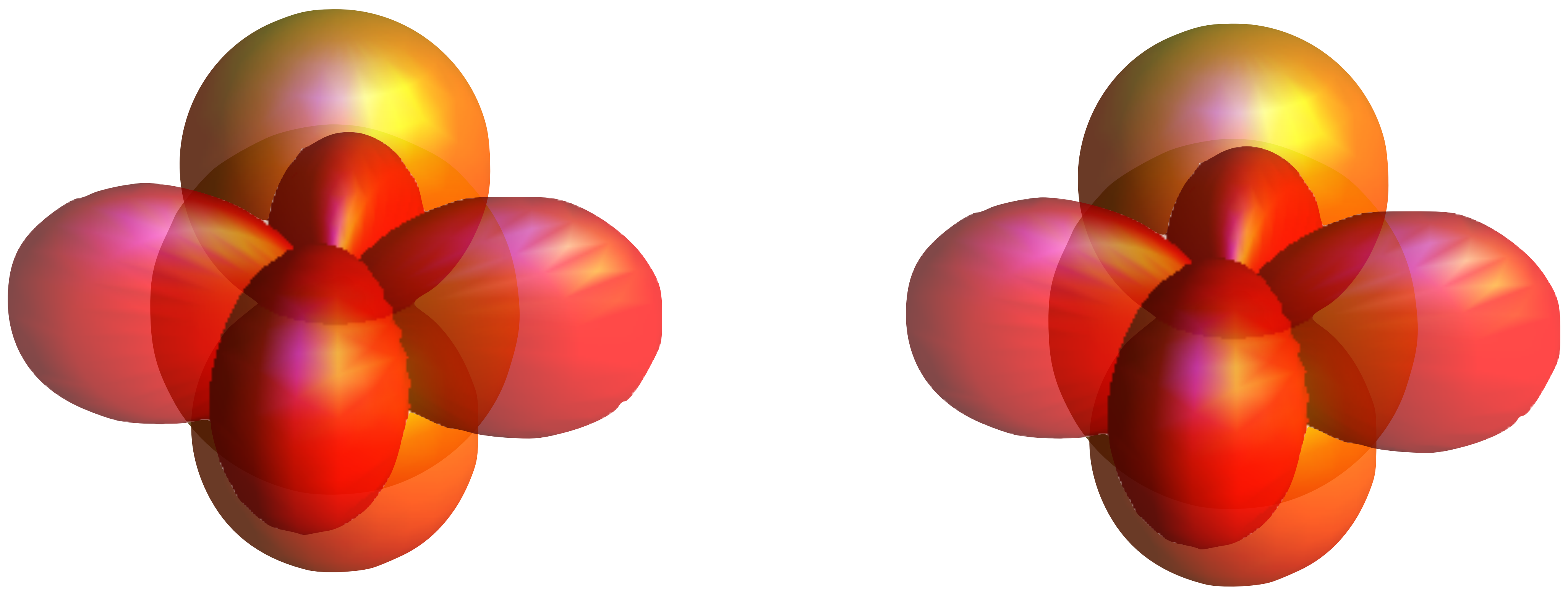
Band Theory of Electrons in Solids

Electronic Orbitals

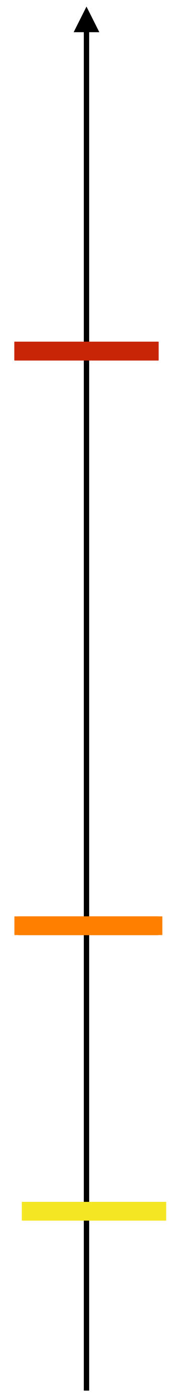
1s

2p

3d



Energy



Discrete energy levels

Two atoms

Schrödinger equation :

$$H|\psi\rangle = E\ |\psi\rangle$$

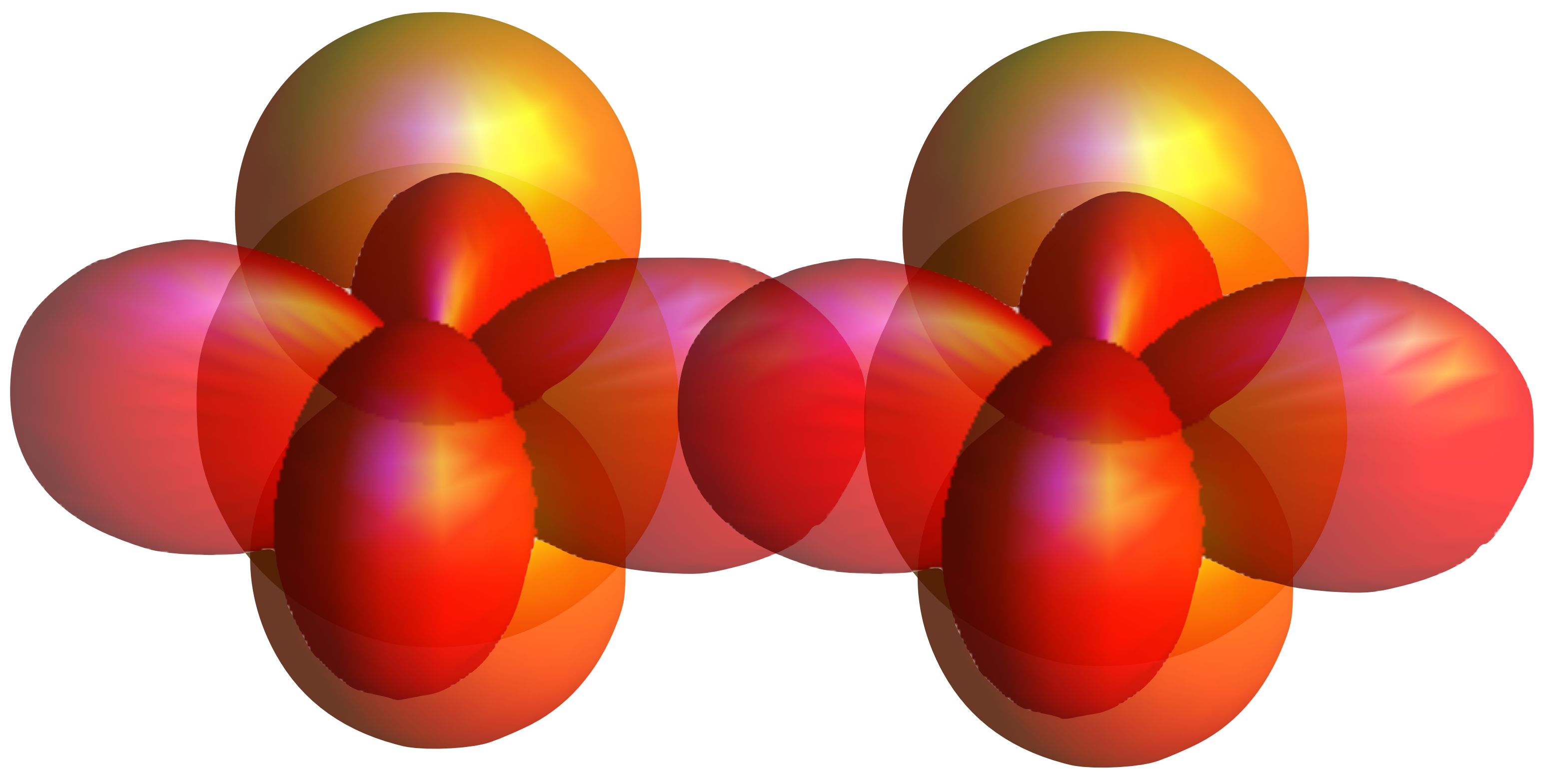
Band Theory of Electrons in Solids

Electronic Orbitals

1s

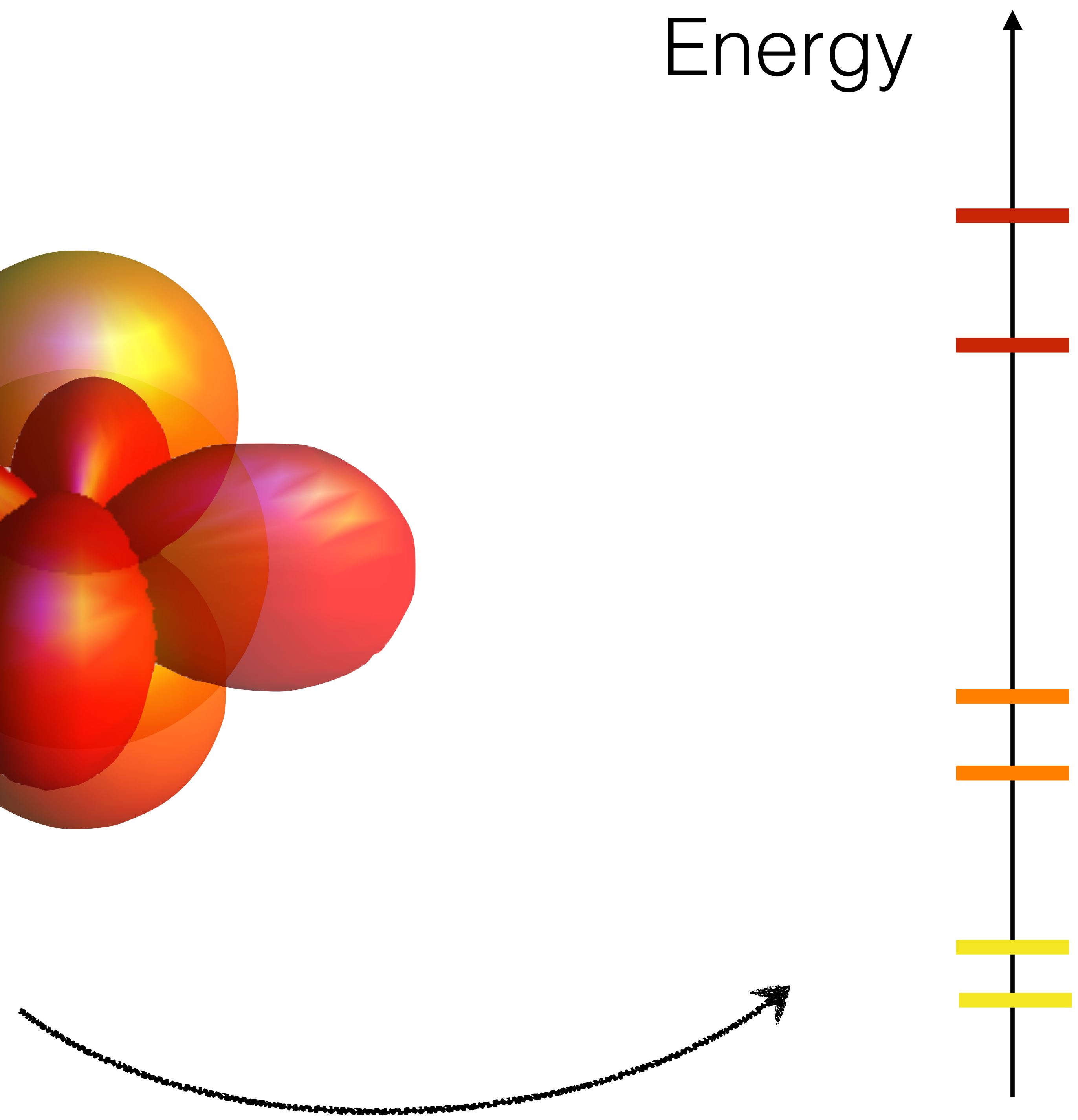
2p

3d



Two atoms

Energy



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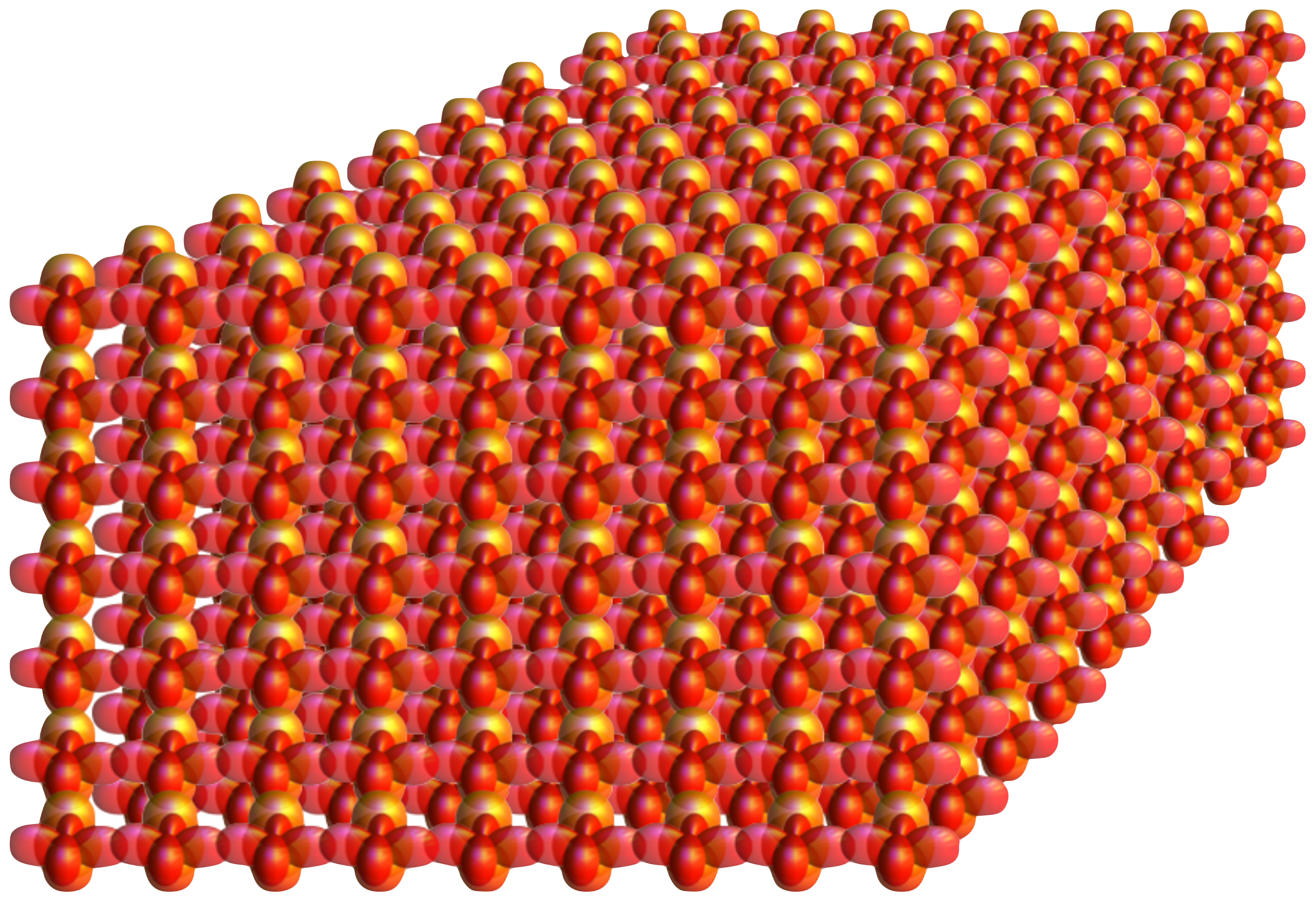
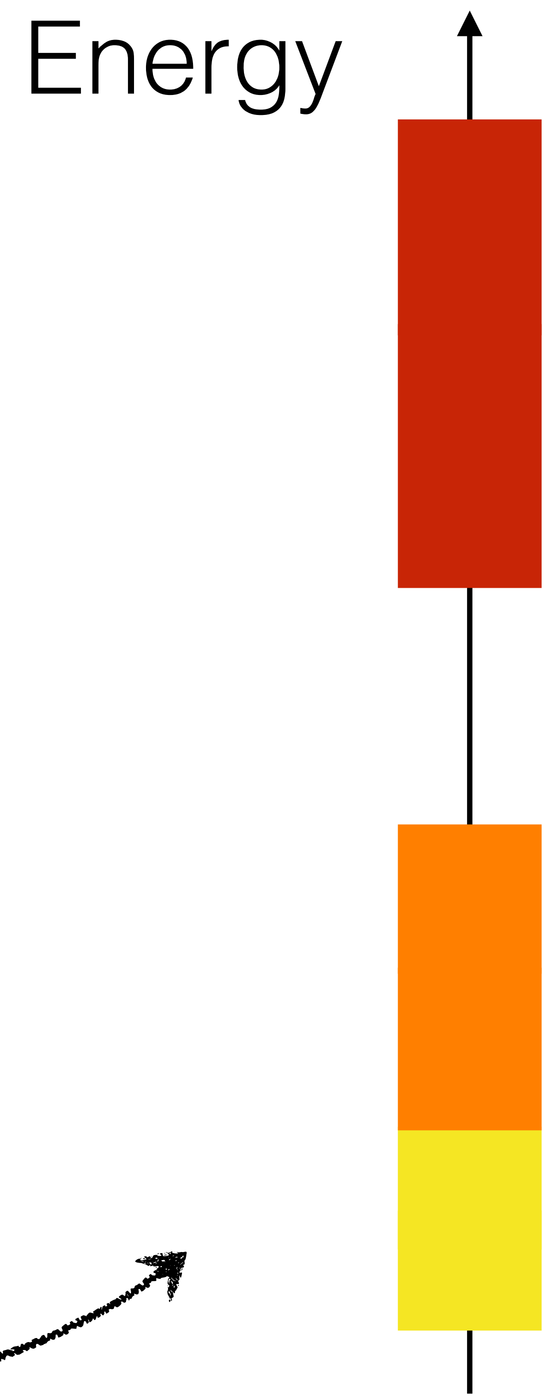
Band Theory of Electrons in Solids

Electronic Orbitals



Solids : ~10^{23} atoms



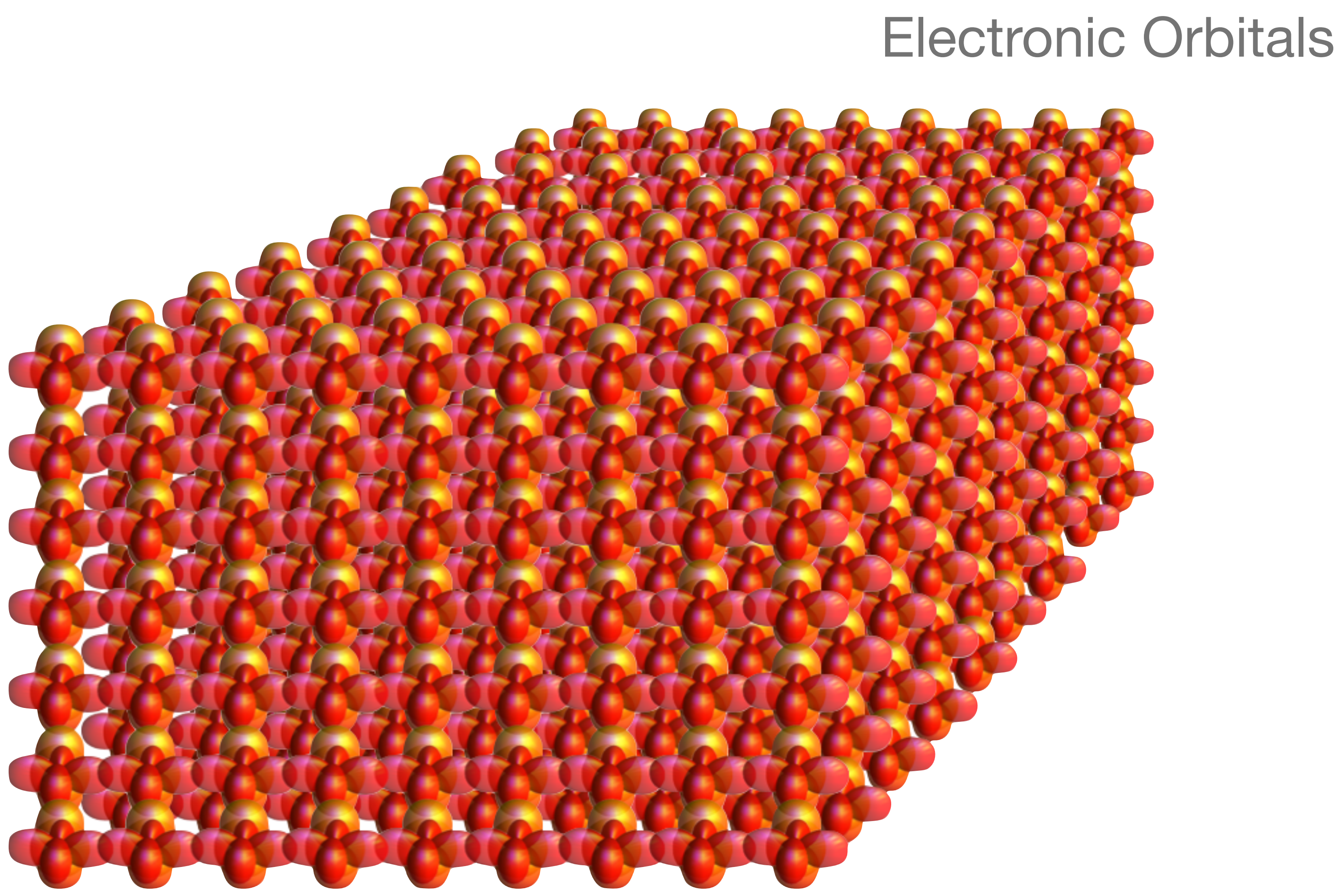
Discrete energy levels

Continuous energy bands

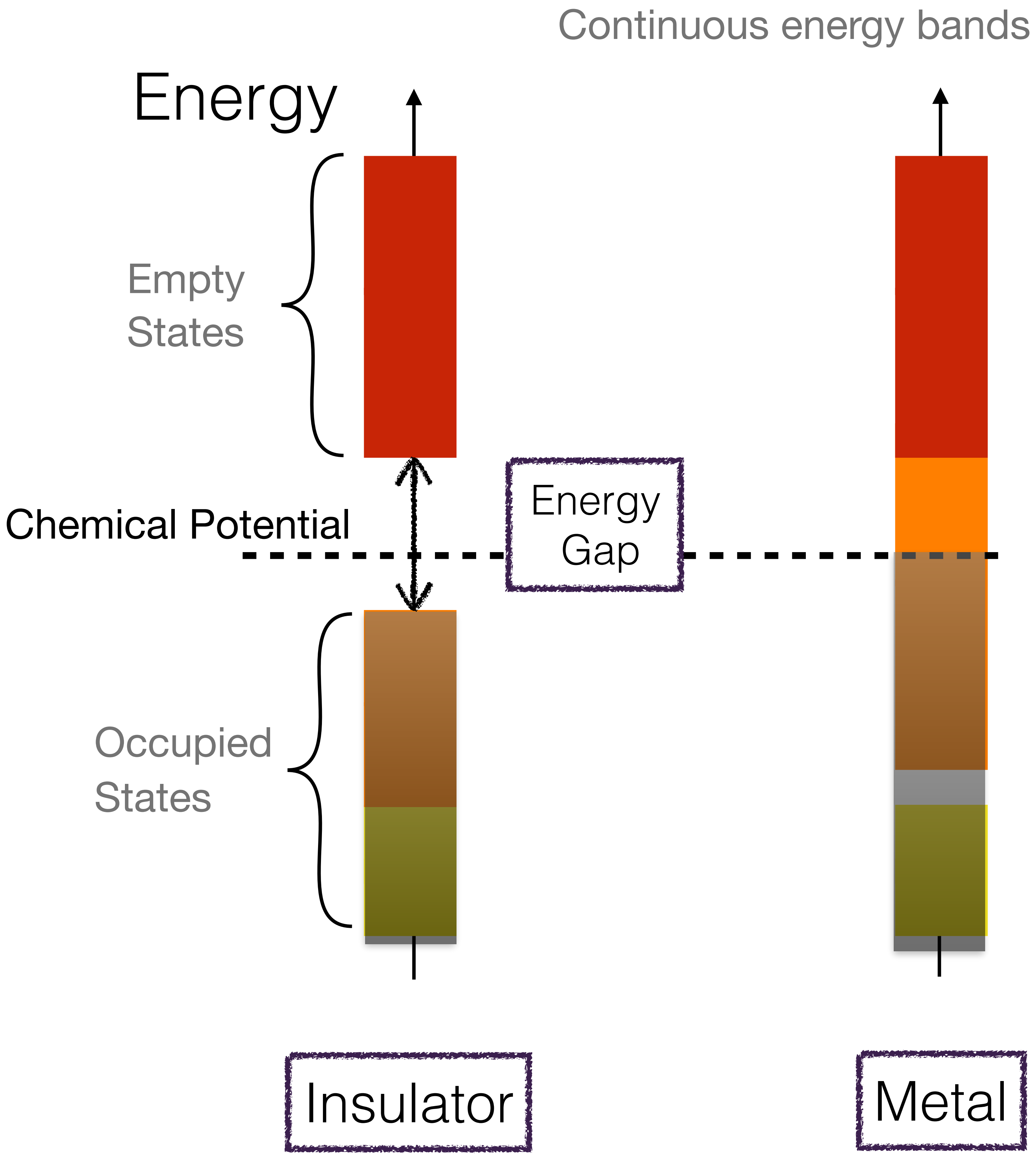
Schrödinger equation :

$$H|\psi\rangle = E\;|\psi\rangle$$

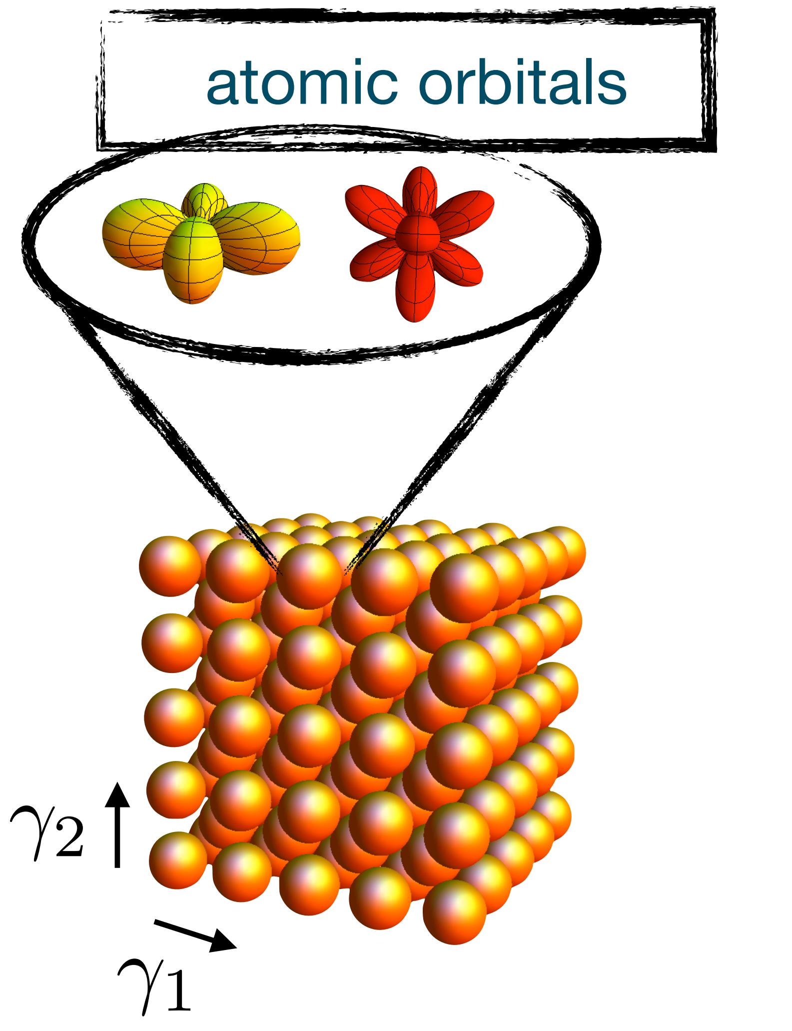
Band Theory of Electrons in Solids



Solids : ~10^{23} atoms



Band Theory of Solids



- Band theory : single particle description of electronic states
  - Diagonalisation of a lattice Hamiltonian : $H_0|\psi\rangle = |\psi\rangle$
- Periodicity of lattice **(symmetry !)** :
  - Bravais lattice : translations T_γ that leave physical lattice invariant
  - if $|\psi\rangle$ eigenstate, then $T_\gamma|\psi\rangle$ also with same energy $T_\gamma\psi(x) = \psi(x-\gamma)$
  - diagonalize simultaneously H_0 and T_γ
- Bloch wavefunctions :
  - Eigenstates of translations : $T_\gamma\psi(x) = \psi(x-\gamma) = e^{ik.x}\psi(x)$
  - labelled by quasi-momentum k
  - k and $k+G$ label the same eigenvector $|\psi_k\rangle$ if

    $G.\gamma = n\ 2\pi,\ \ n \in \mathbb{Z}$ for all γ

Band Theory of Solids

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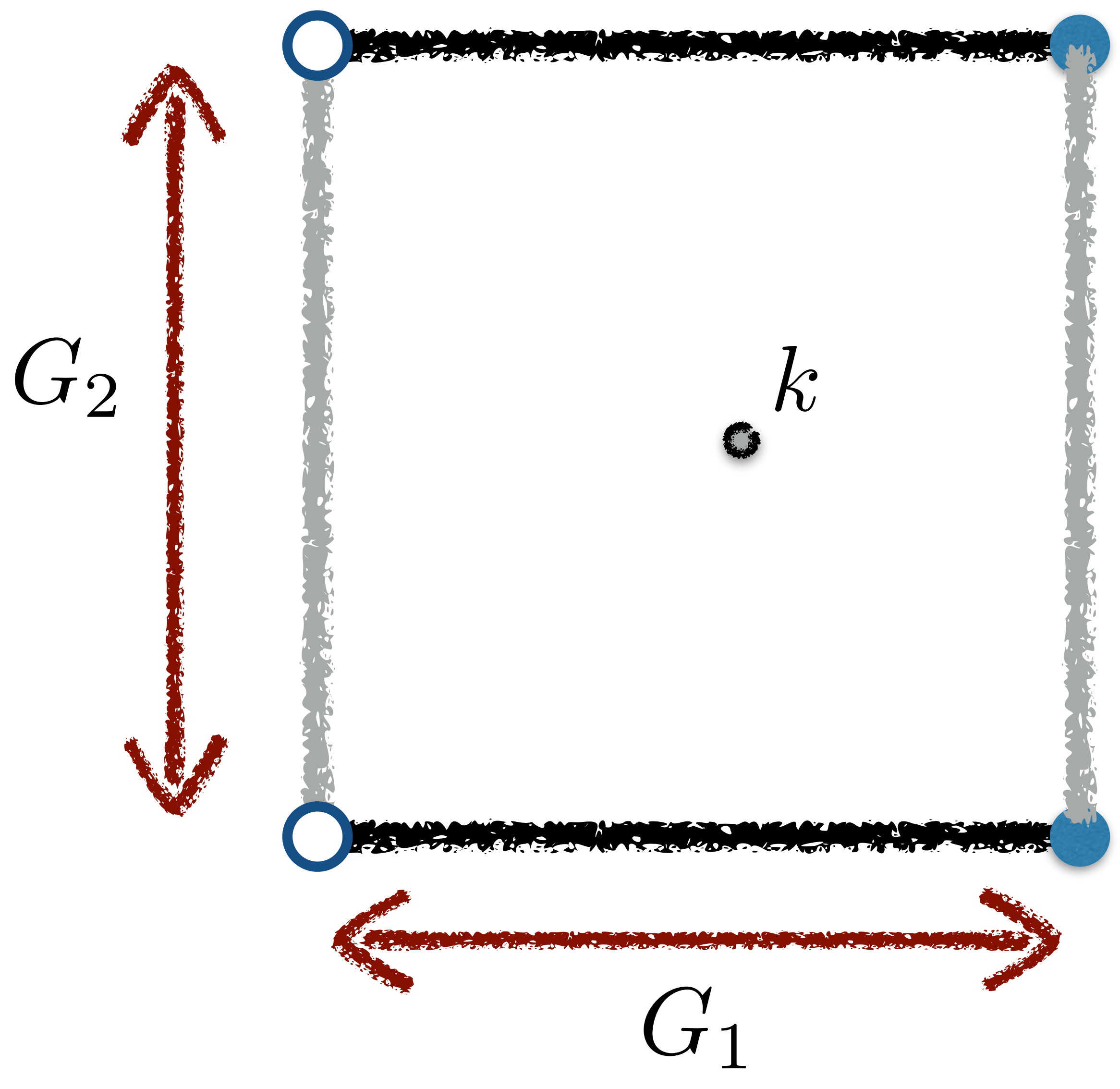
  - k lies in **Brillouin Zone**



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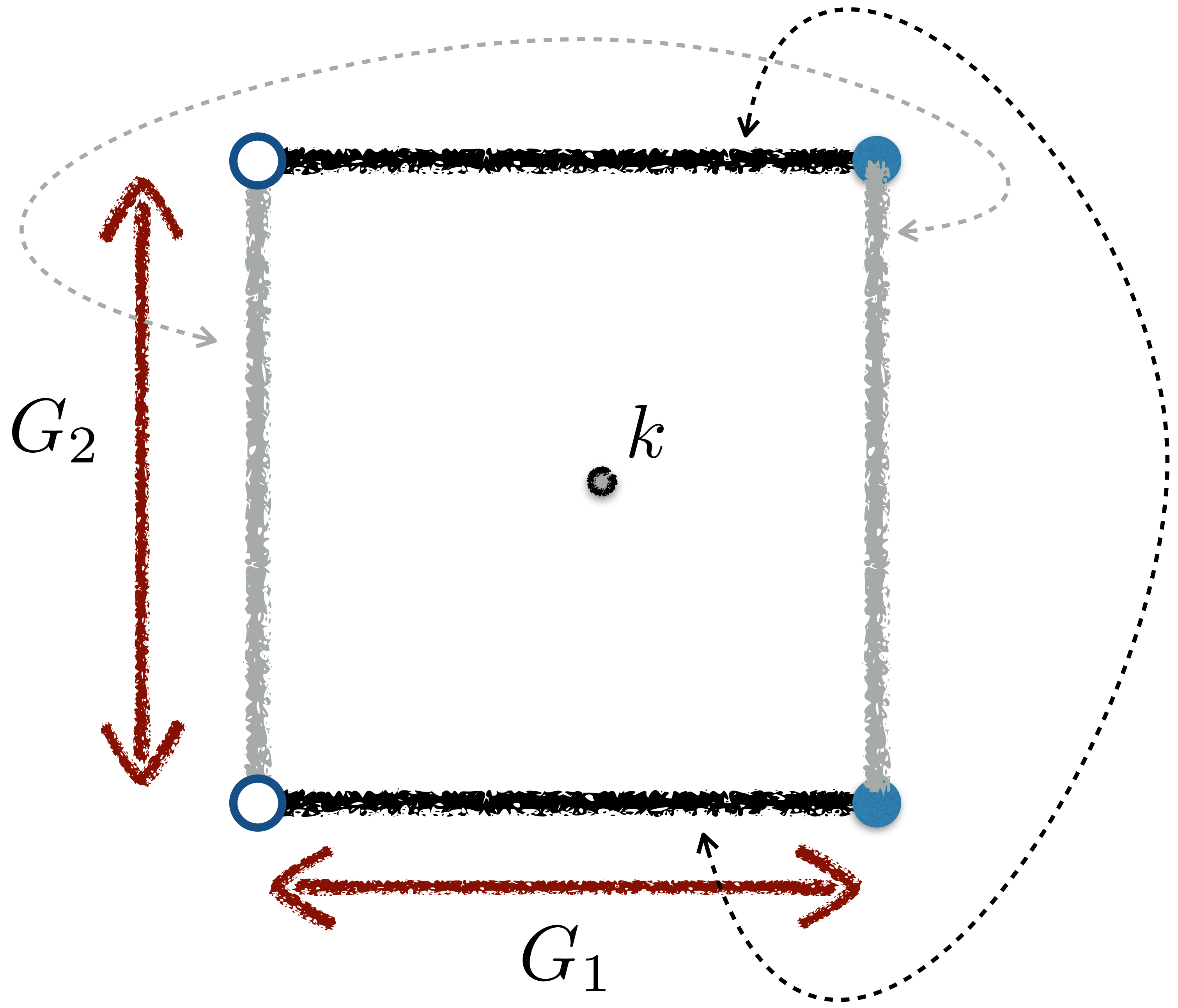

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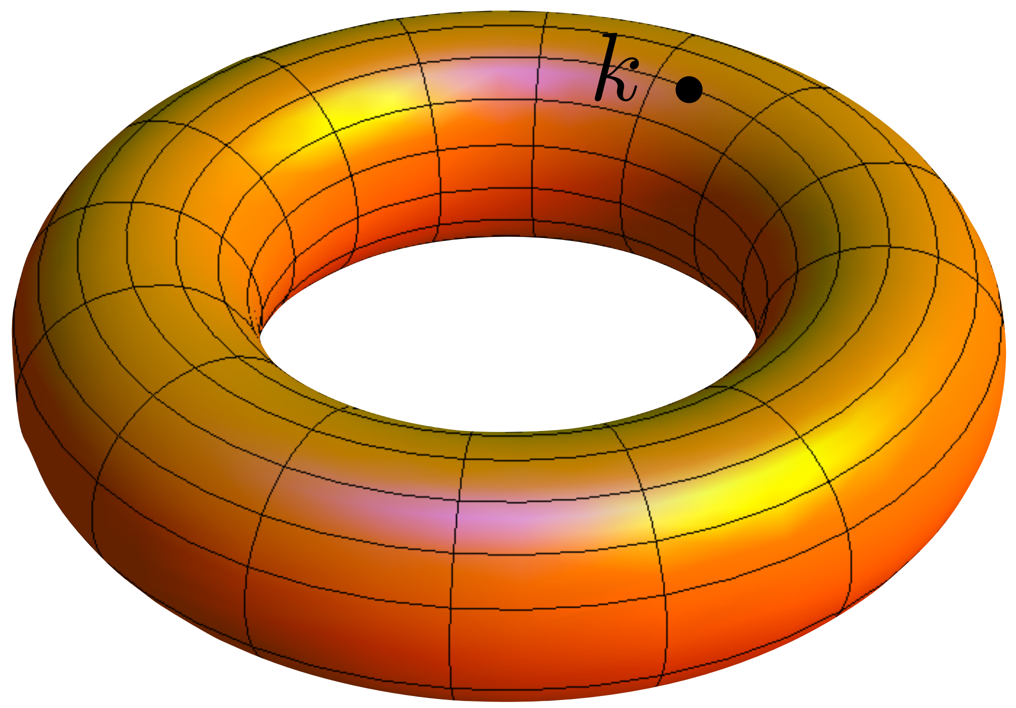
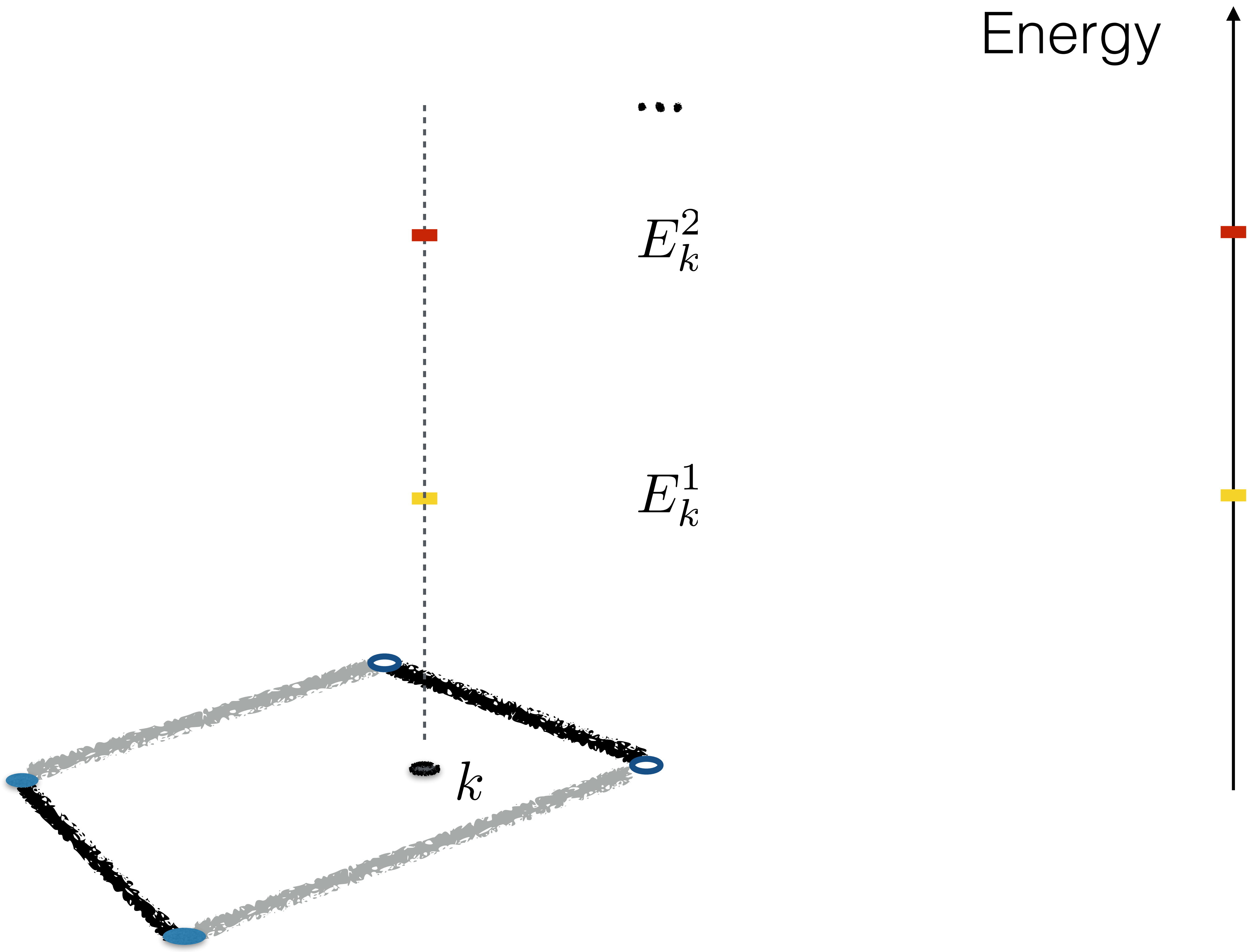


Band Theory of Solids

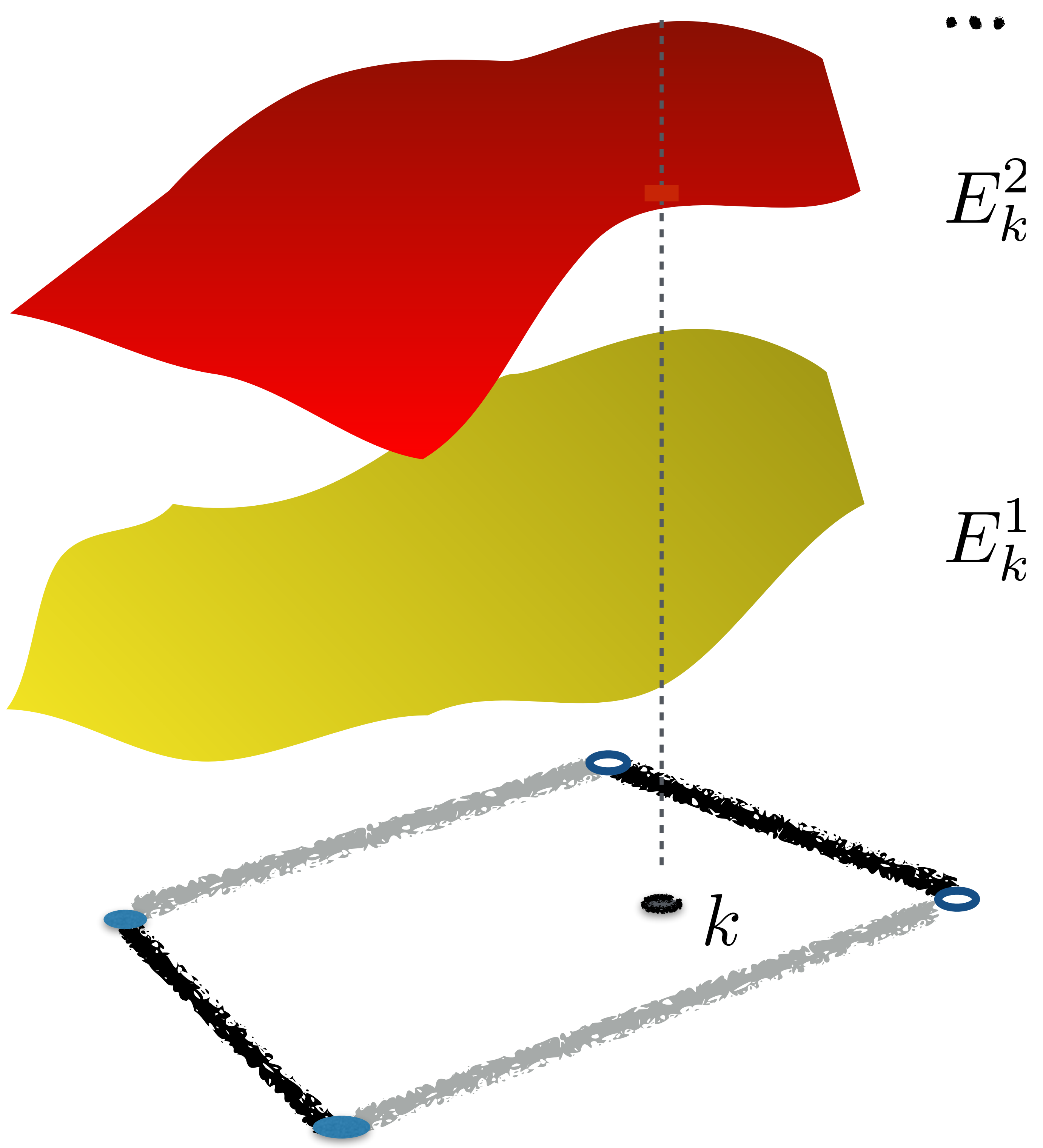
Diagonalization : $H|\psi_k^\alpha\rangle = E_k^\alpha|\psi_k^\alpha\rangle$

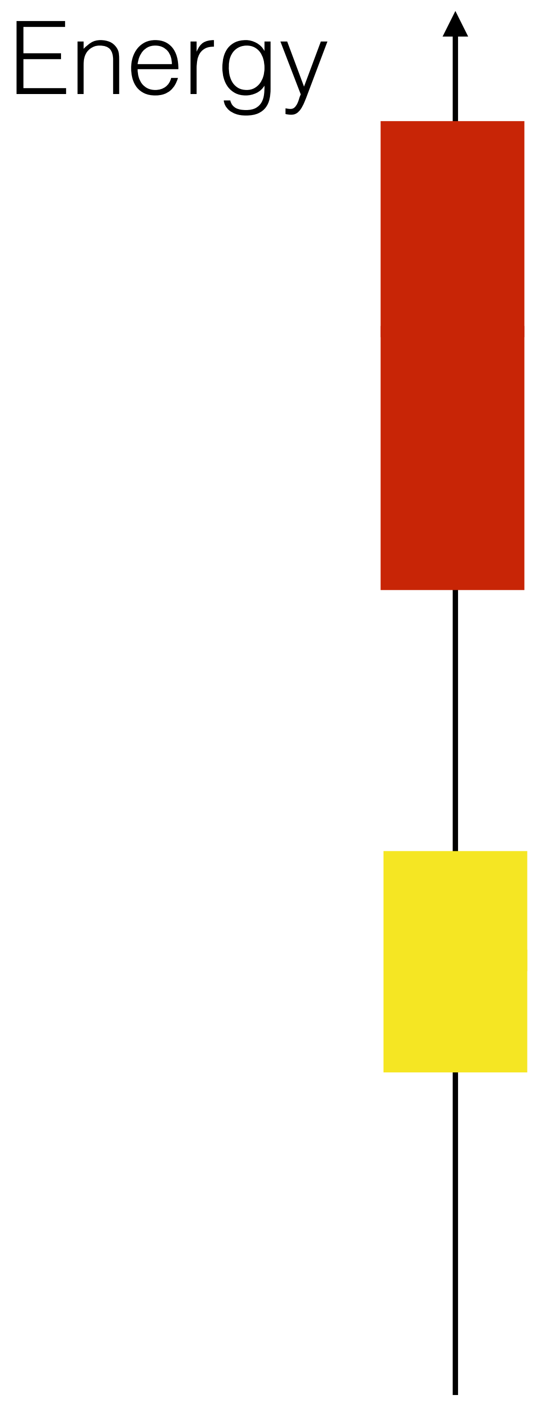


Band Theory of Solids

Diagonalization : $H|\psi_k^\alpha\rangle = E_k^\alpha|\psi_k^\alpha\rangle$

➡ Energy bands : surfaces E_k^α over the Brillouin torus

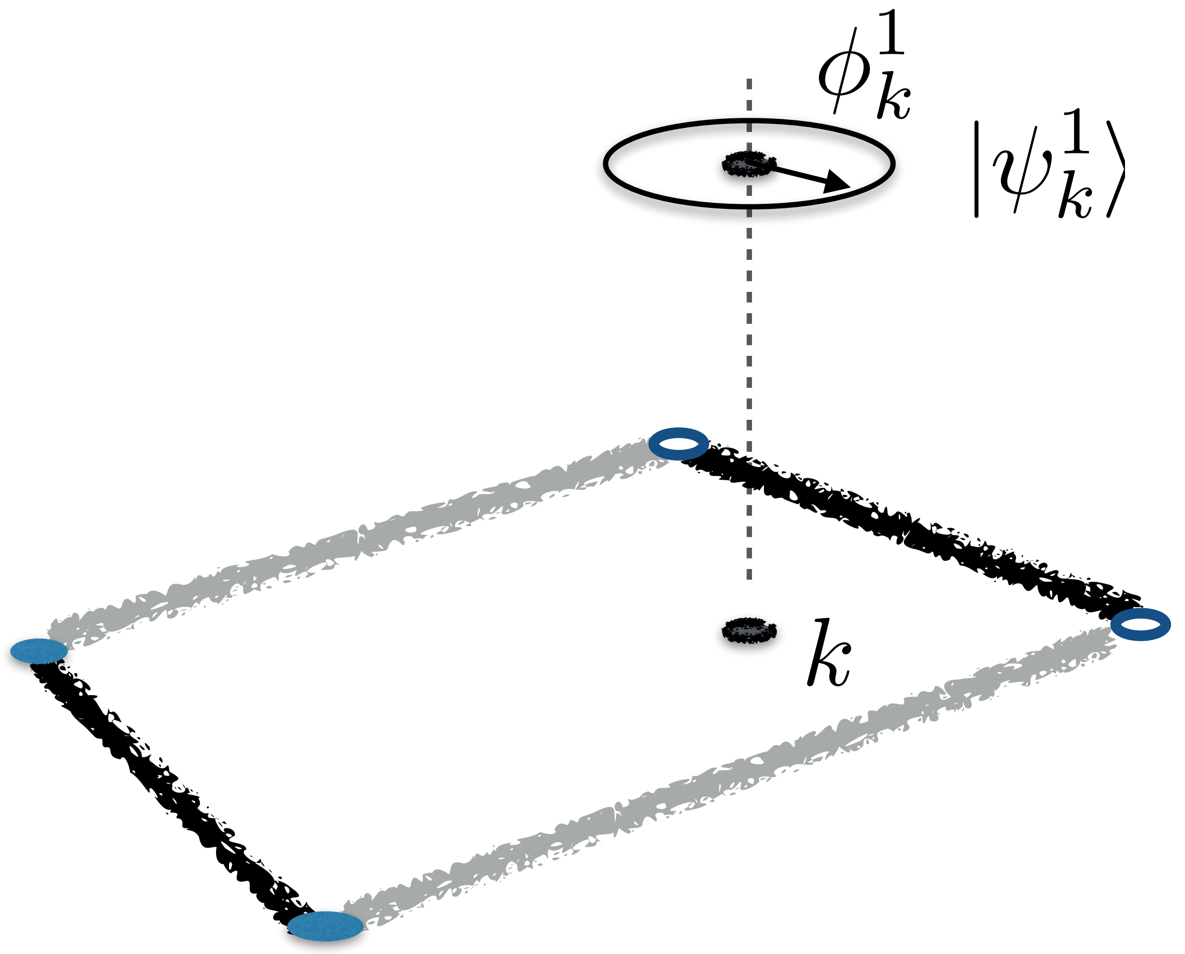




Band Theory of Solids

Diagonalization : $H|\psi_k^\alpha\rangle = E_k^\alpha|\psi_k^\alpha\rangle$

- ➡ Each band : **Vector bundle** of vectors $|\psi_k^\alpha\rangle$ over the Brillouin zone
- ➡ eigenvectors $|\psi_k^\alpha\rangle$ defined up to a phase $|\psi_k^\alpha\rangle \to e^{i\phi_k^\alpha}|\psi_k^\alpha\rangle$



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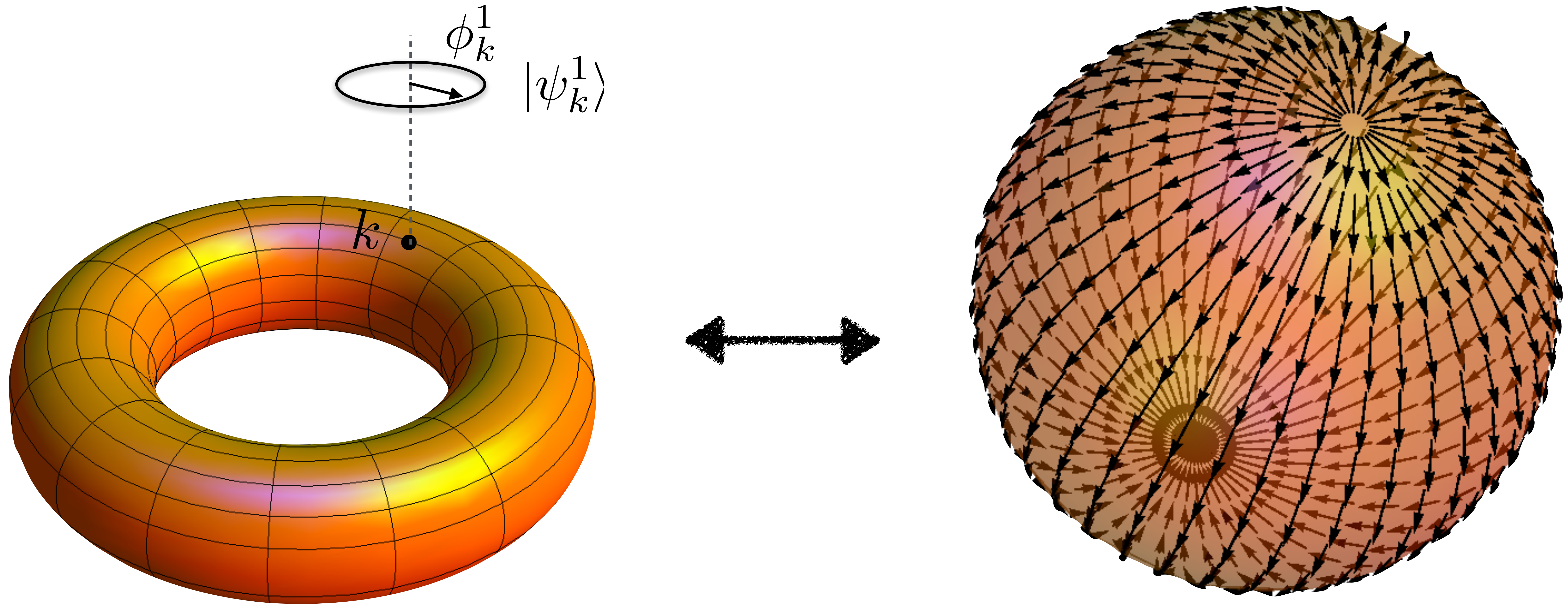

Band Theory of Solids

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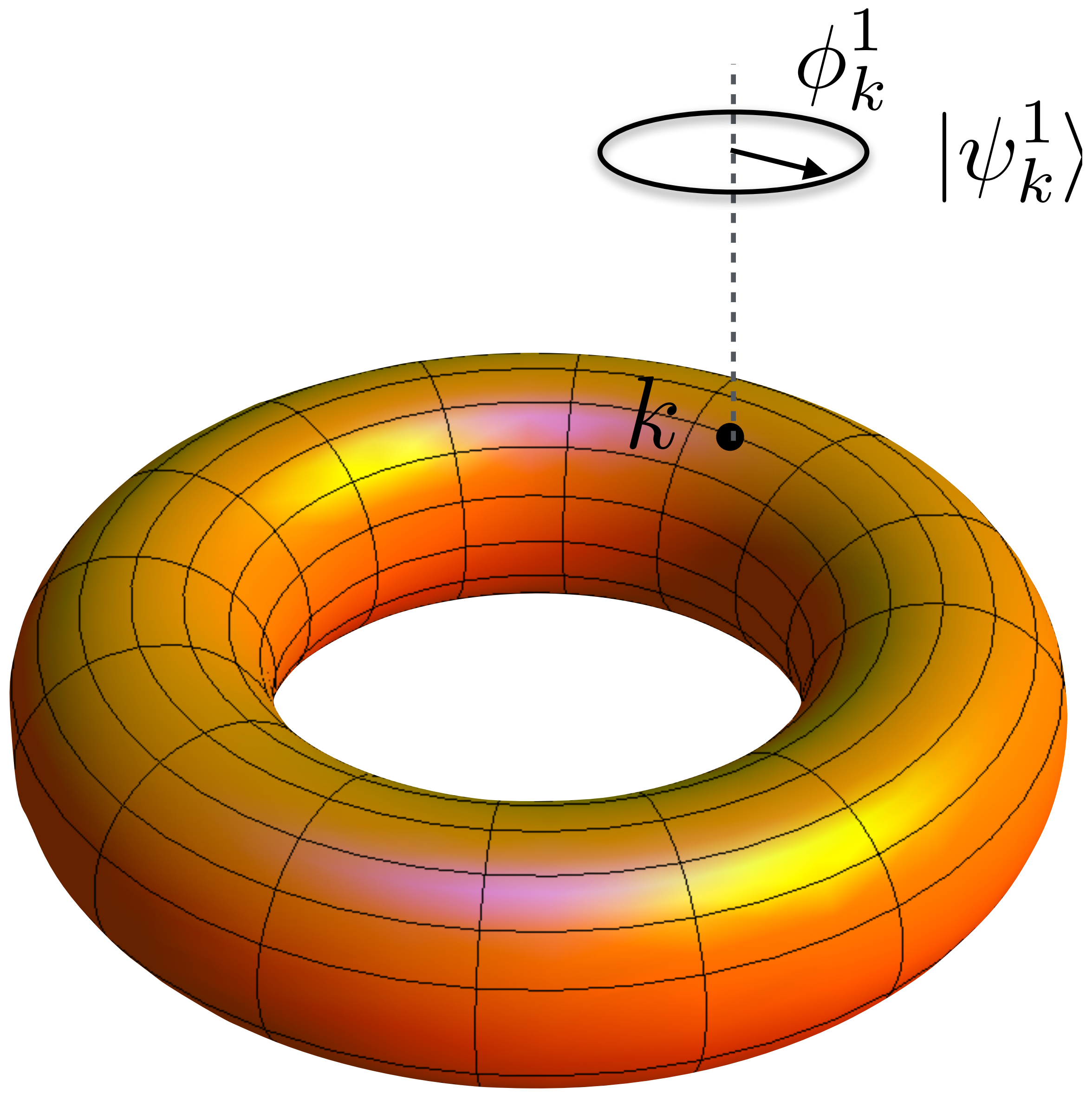
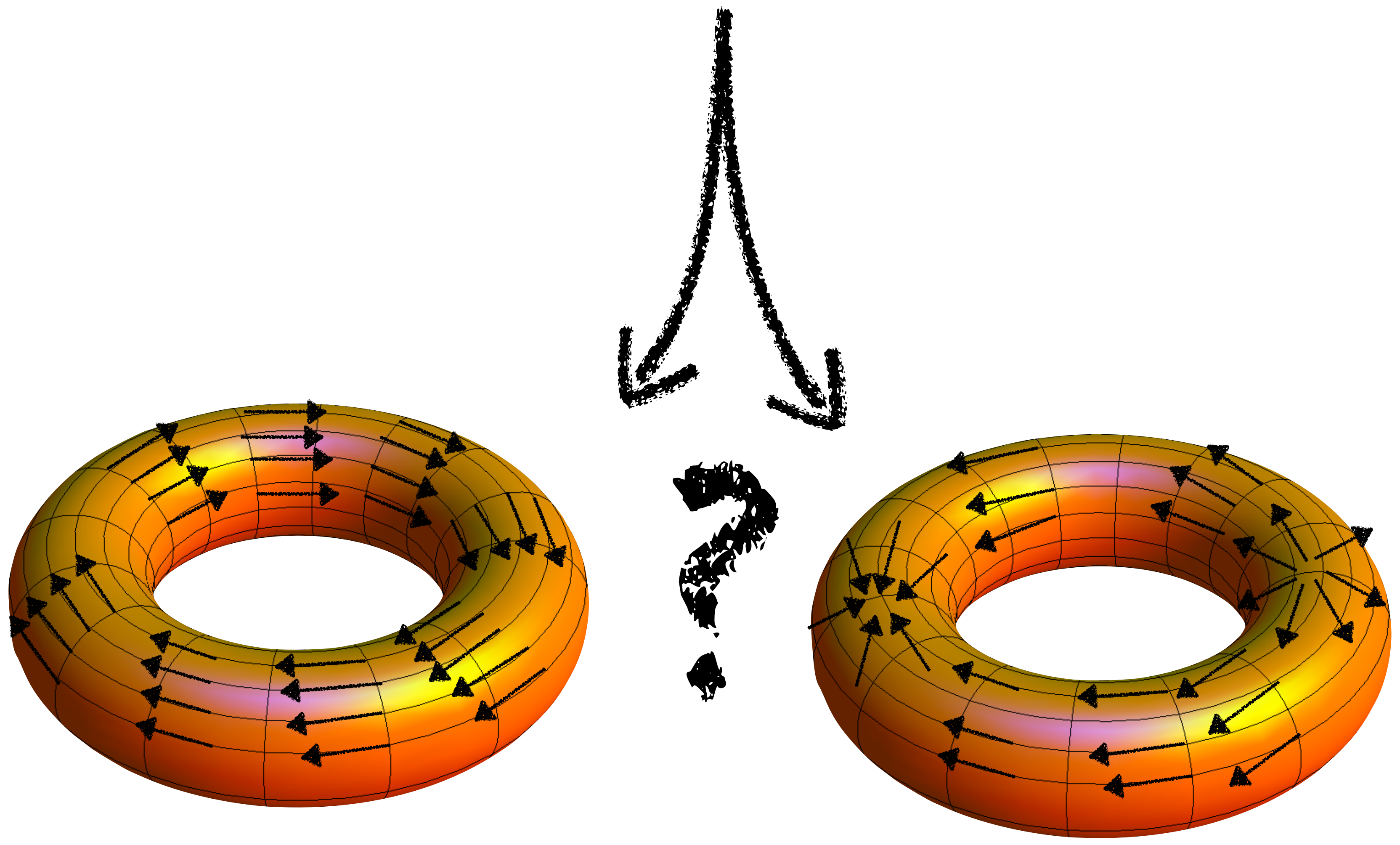
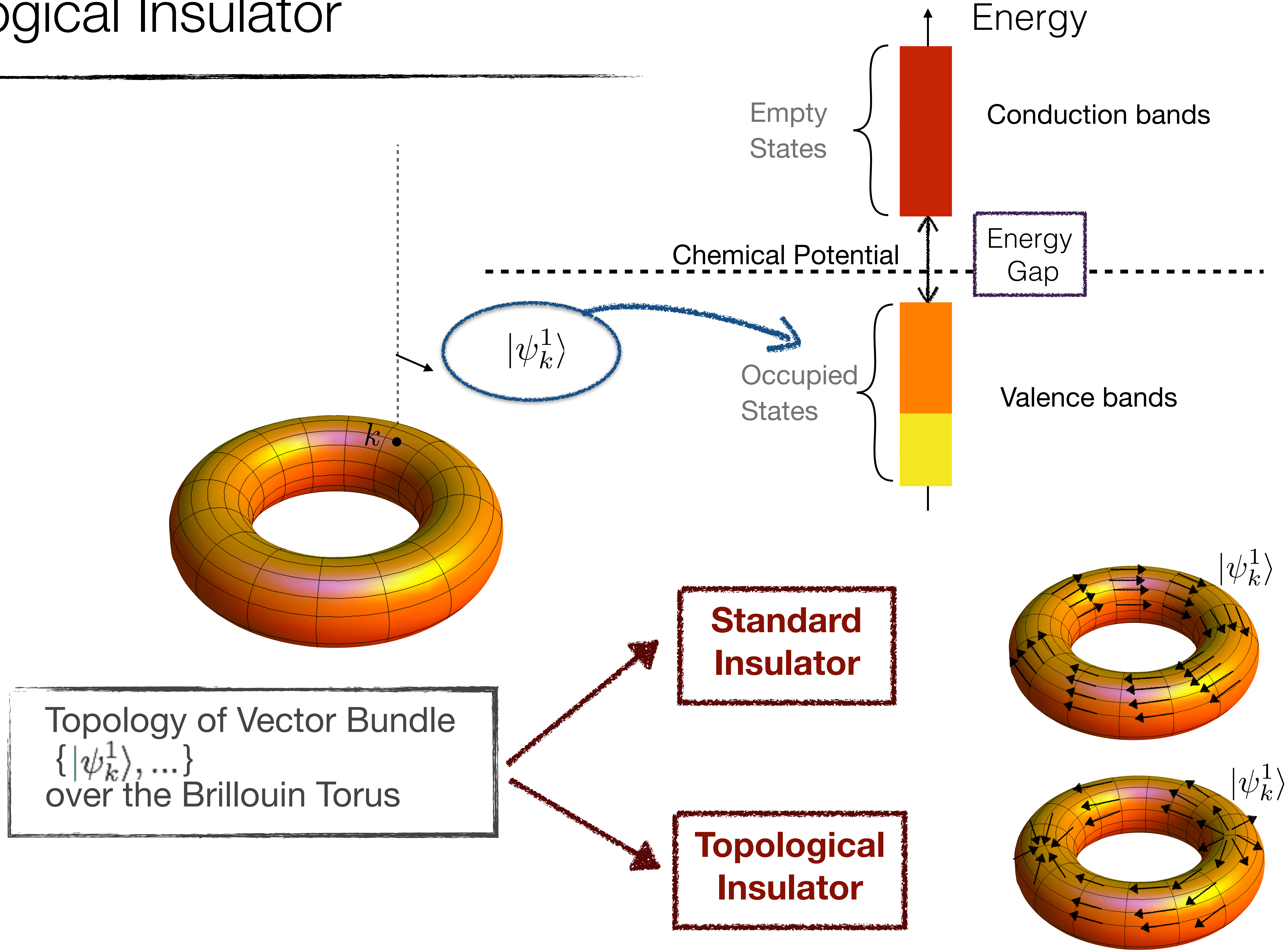
Is it always topologically trivial ?
(*i.e.* continuous vector field exists)



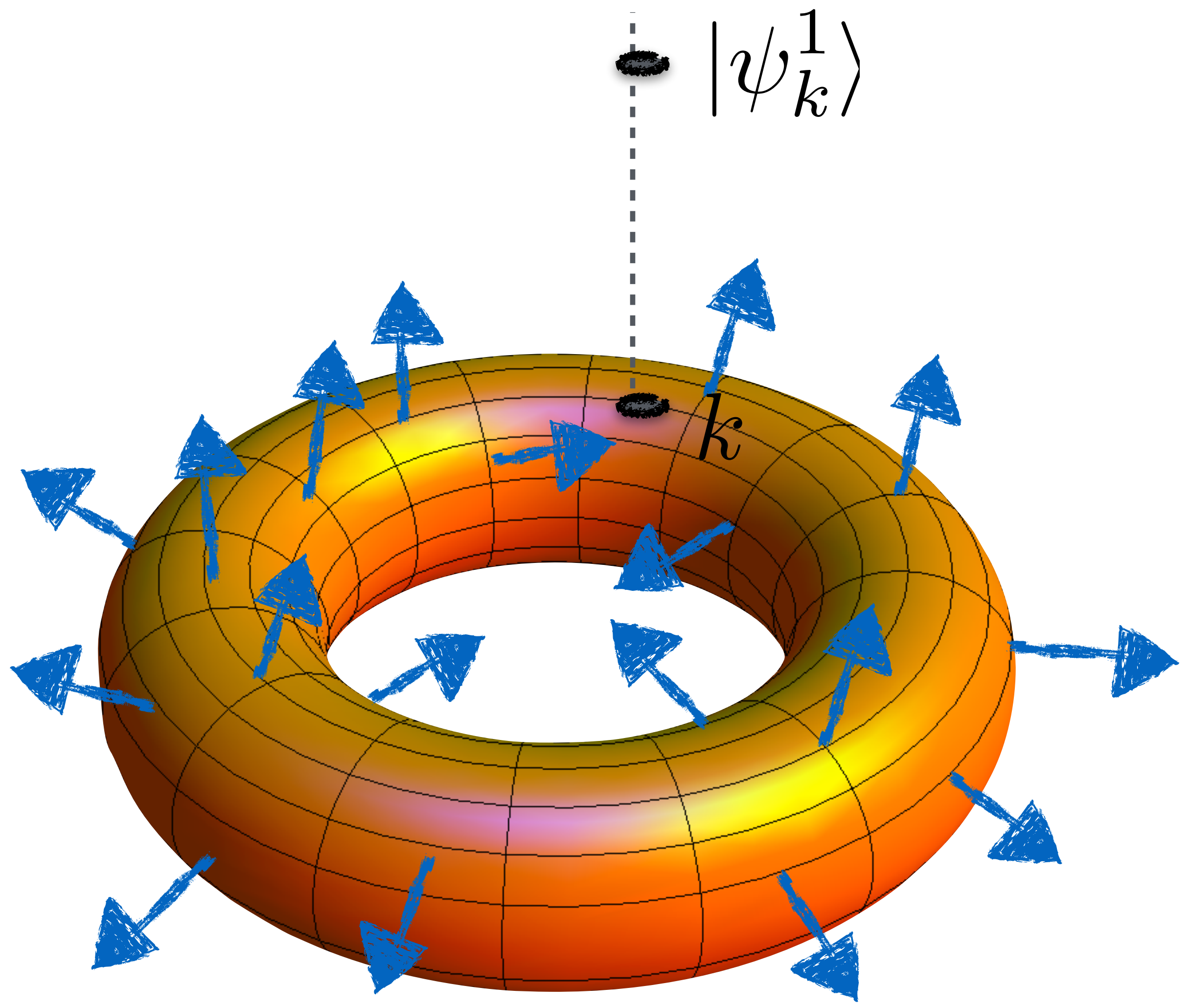


Topological Insulator



Topological index for bands

Thouless *et al.*, (1982)
Berry (1984)
see also Fruchart *et al.*, (2014)



- **Berry Connexion** form (analogous to electr. potential)

$$A_k = \frac{1}{i}\langle u_k^1|\nabla_k|u_k^1\rangle \qquad |\psi_k^1\rangle = e^{ik.\hat{r}}|u_k^1\rangle$$

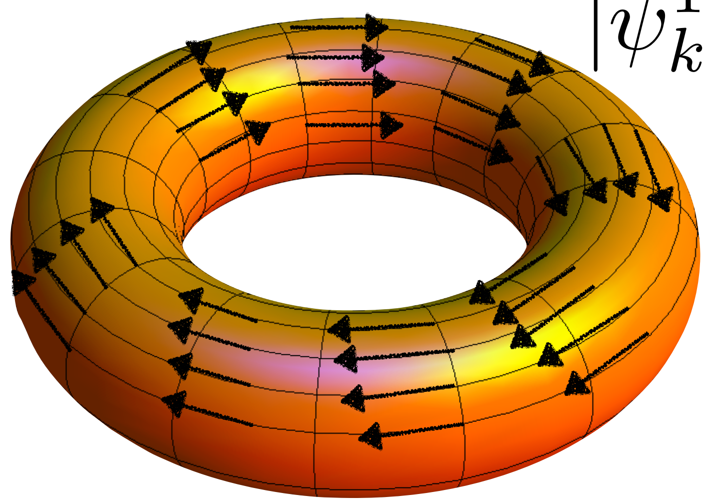
- **Berry curvature** (analogous to Flux) :

$$F_k = \nabla_k \times A_k$$

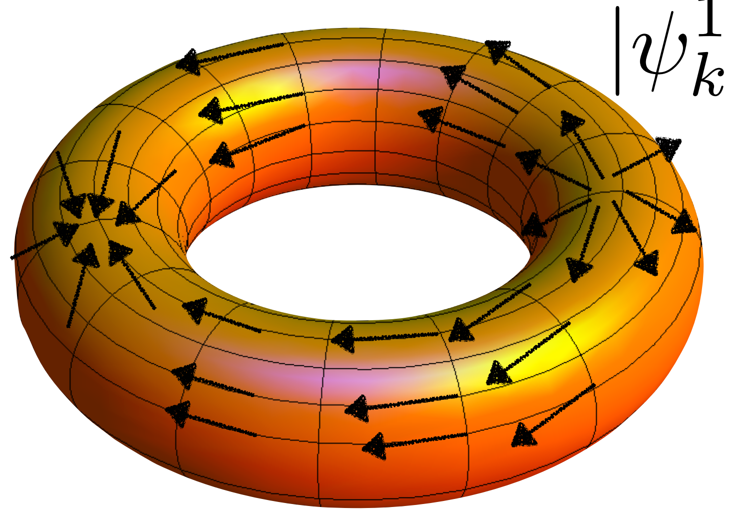
- **Topological number** : Chern number

$$C_1 = \frac{1}{2\pi}\int_{\mathrm{BZ}} F$$

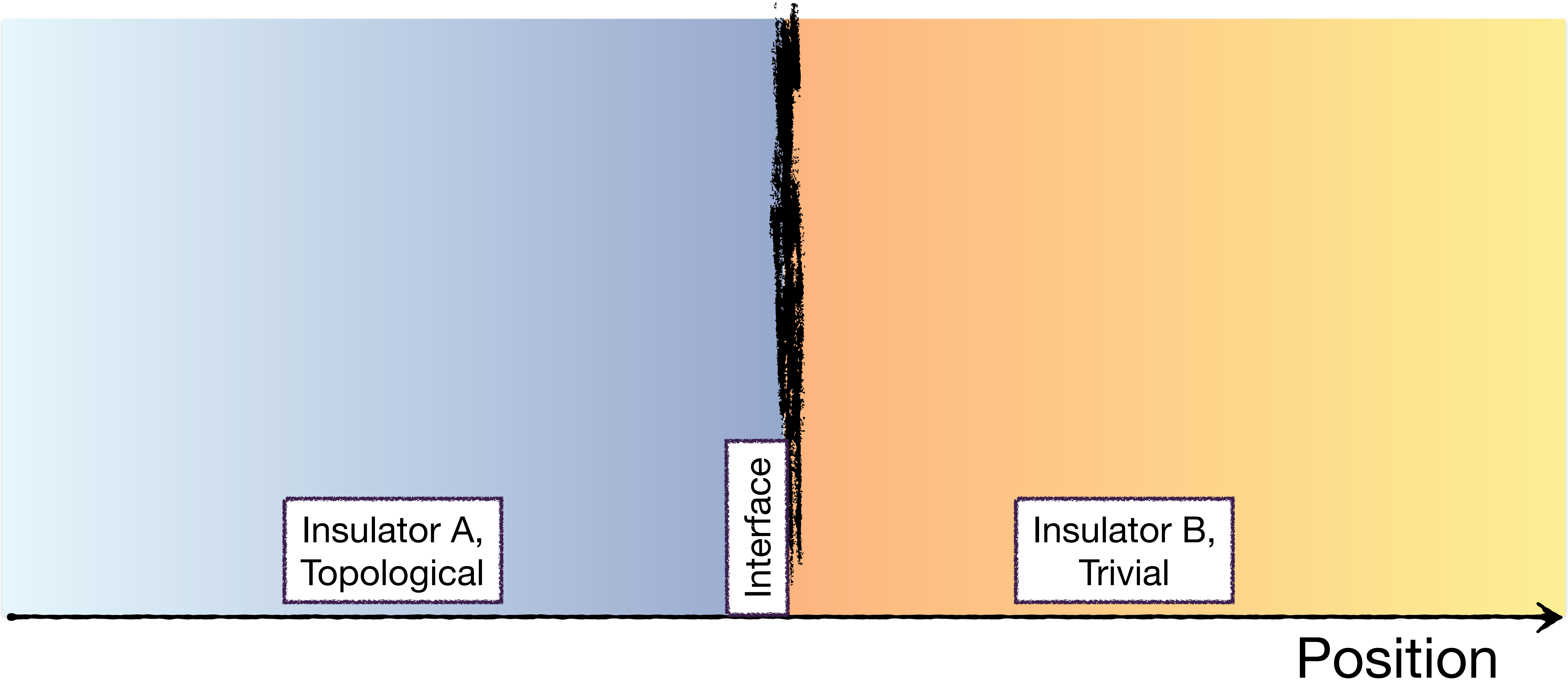
Chern number C = 0



Chern number C ≠ 0



Interface between Insulators



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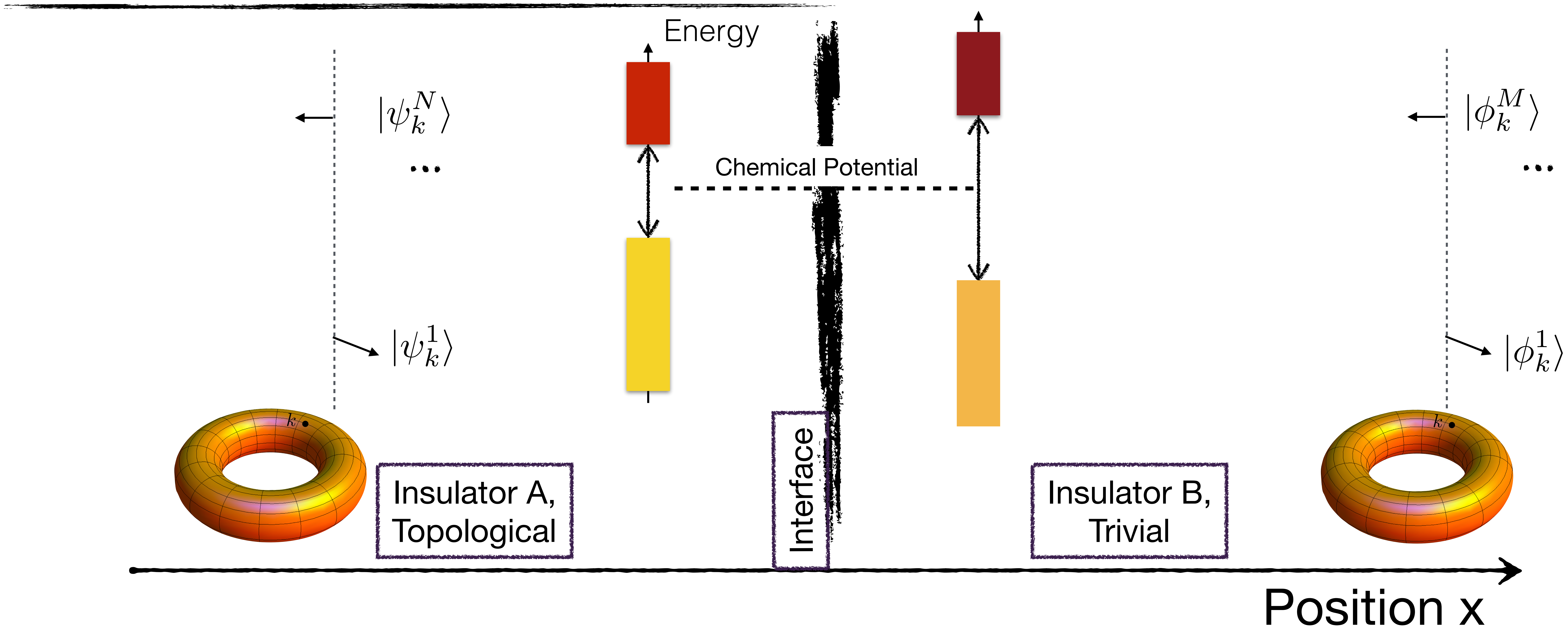
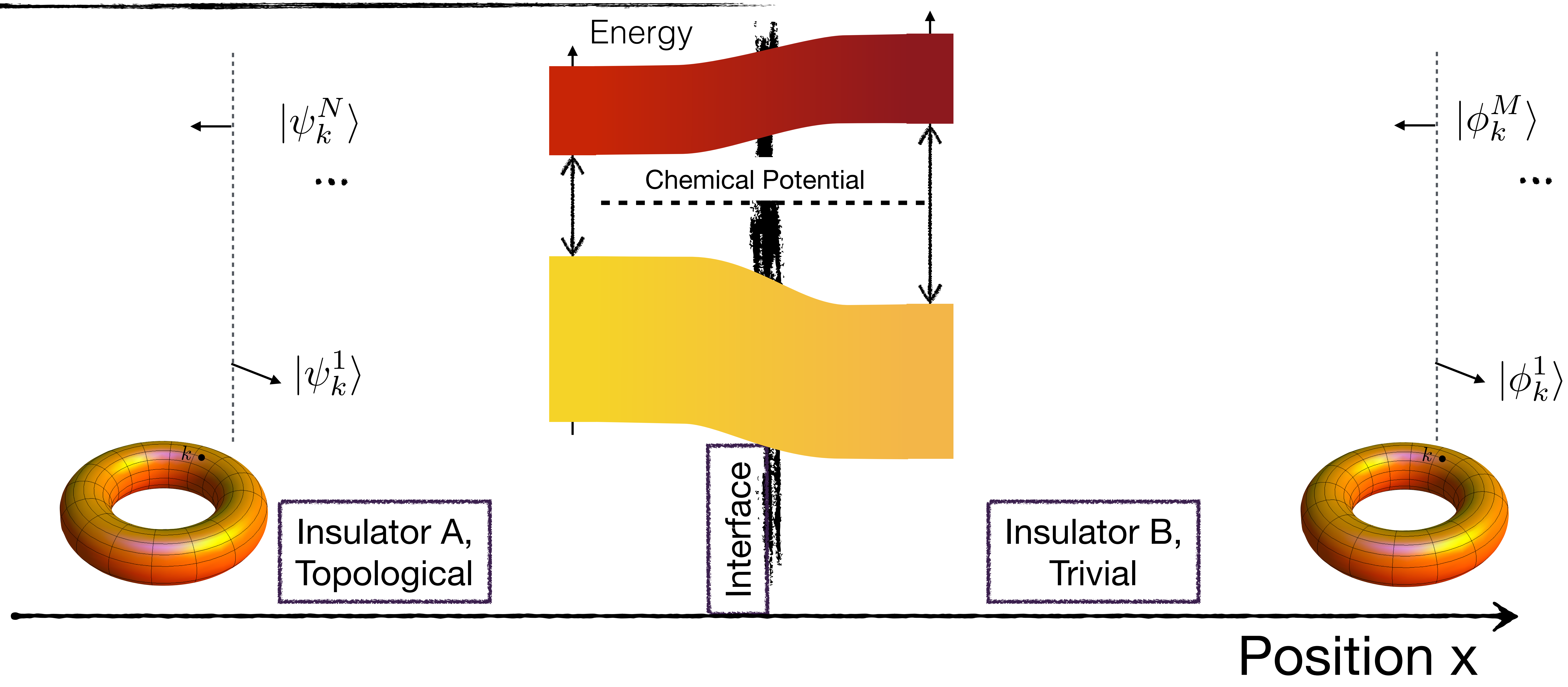


Description of the interface :

- consider eigenstates $\{|\psi_k^\alpha\rangle\}$ and $\{|\phi_k^\beta\rangle\}$ of materials on both sides
- extrapolate (position dependent Hamiltonian H(x))

Interface between Insulators



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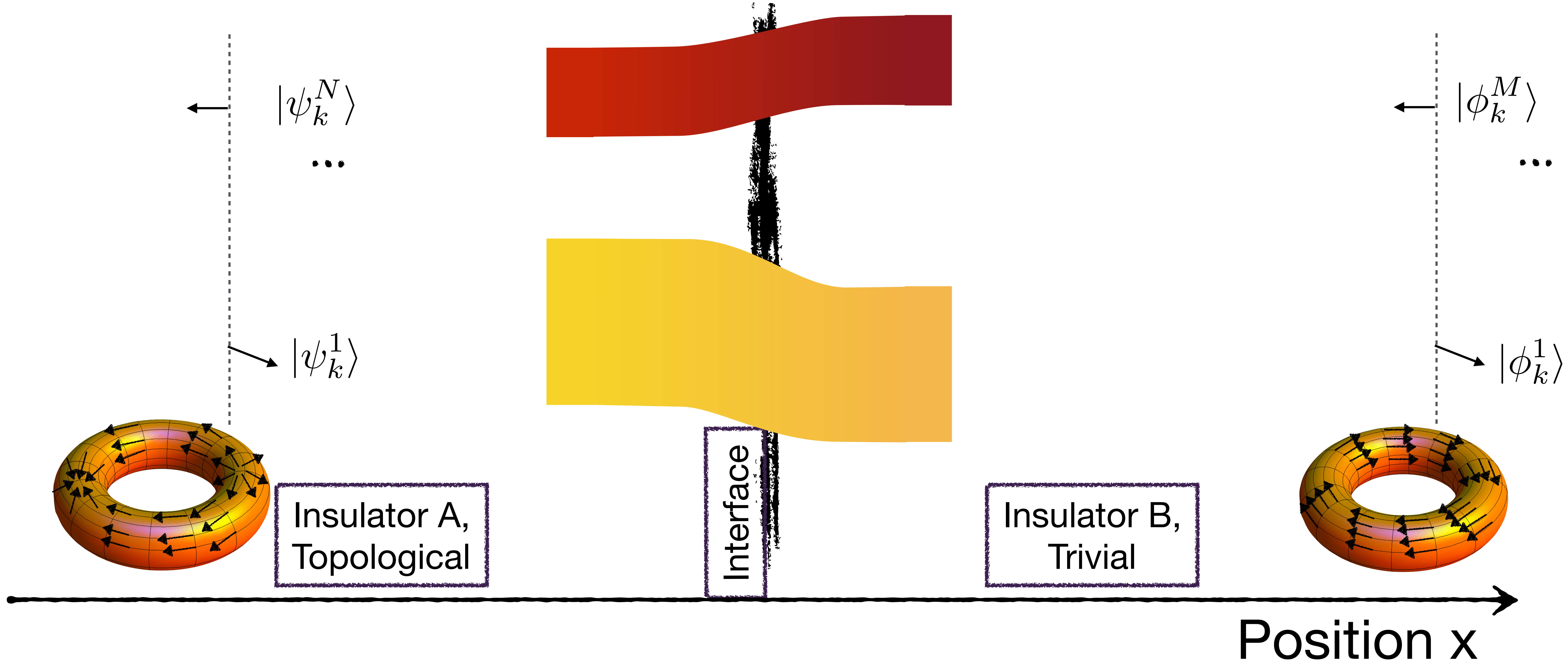

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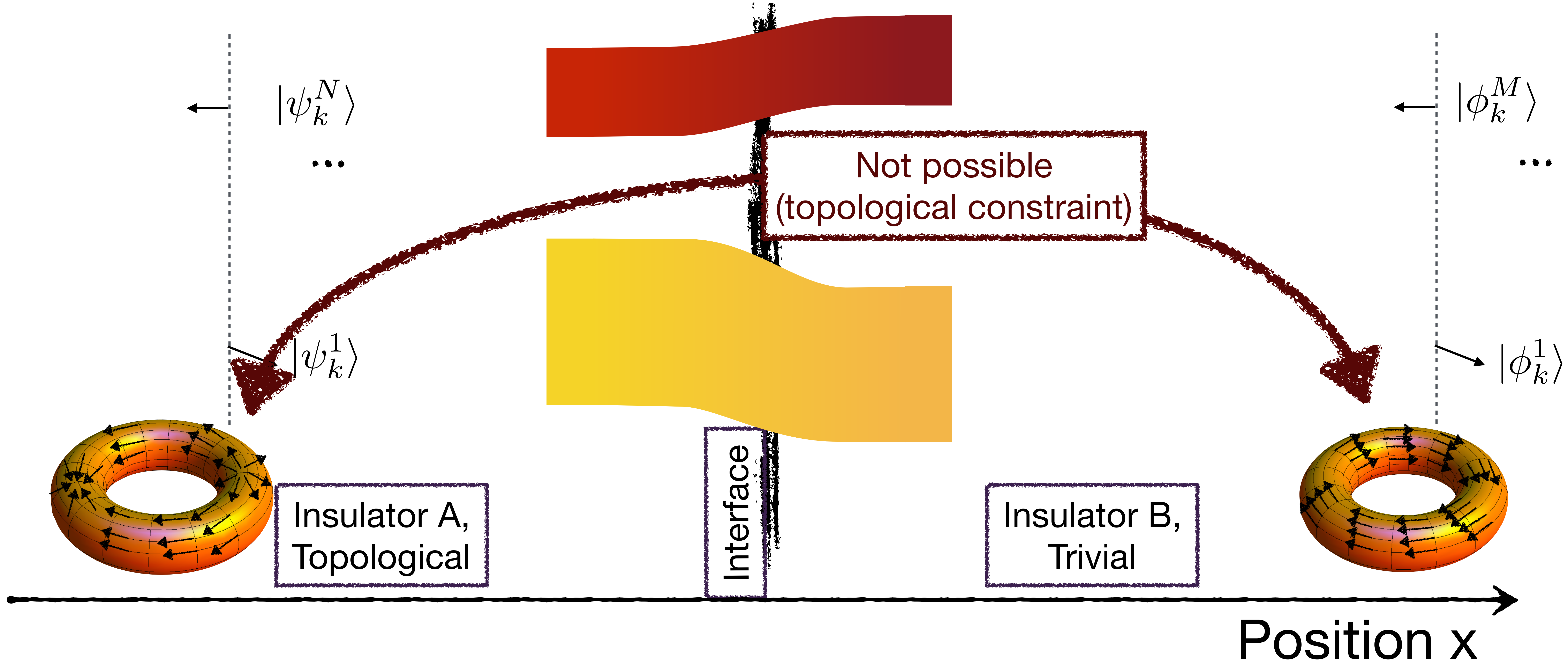

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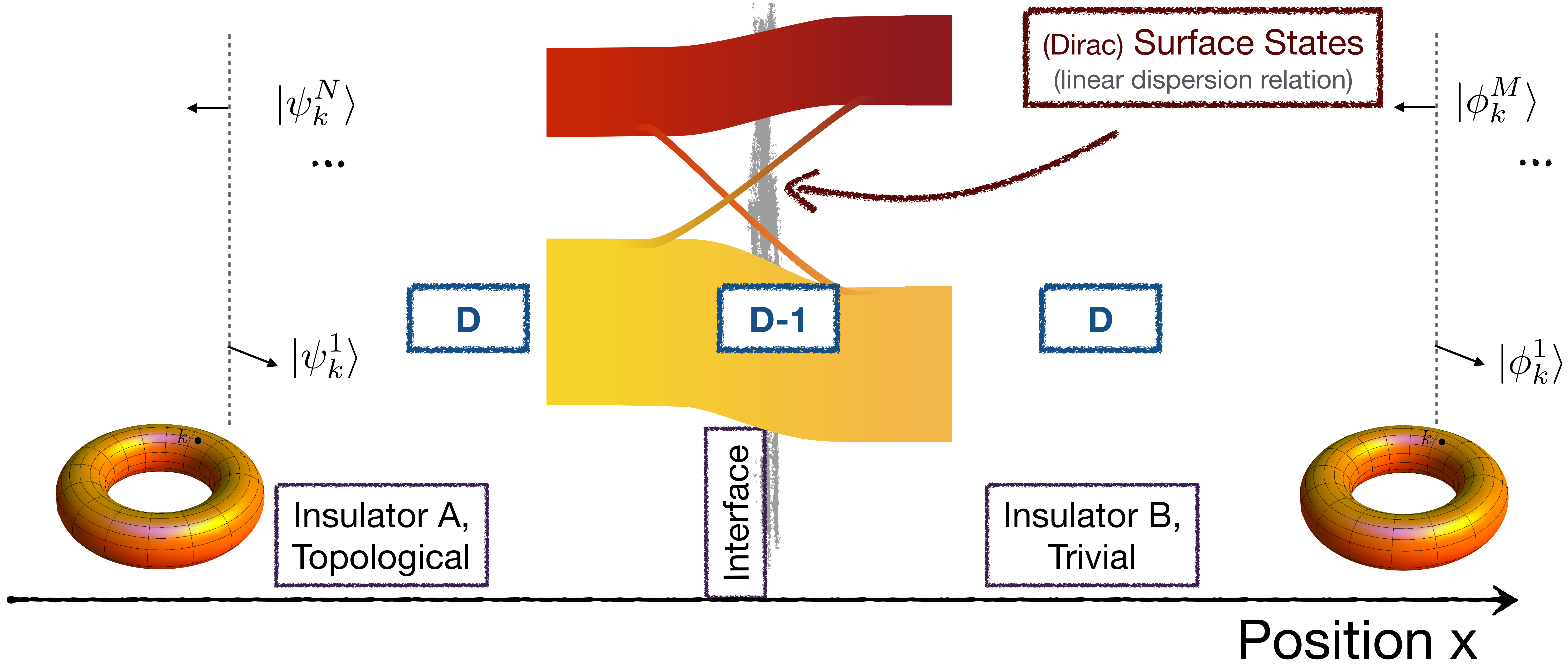

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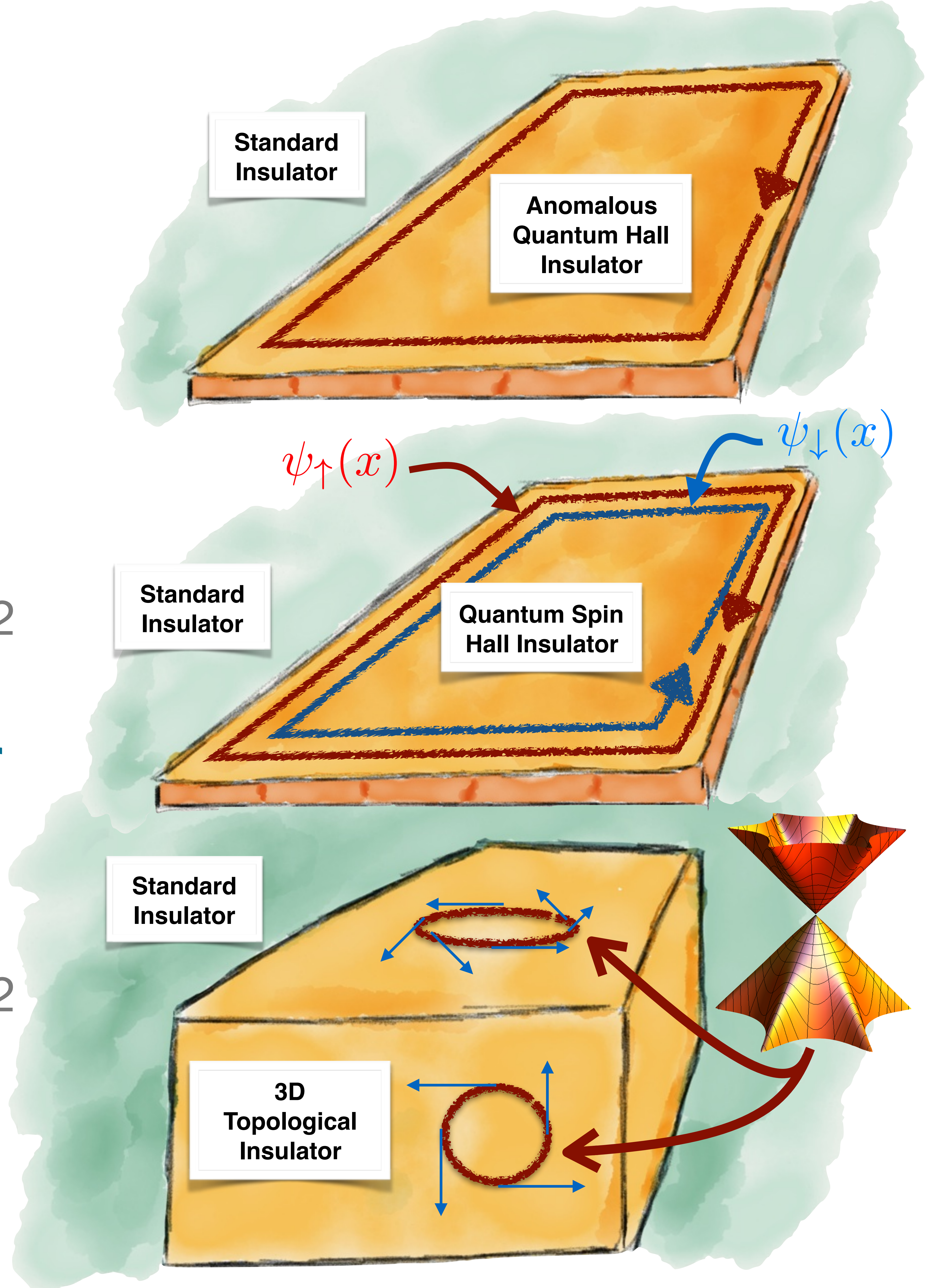
- extrapolate (position dependent Hamiltonian H(x))

Topological Surface States

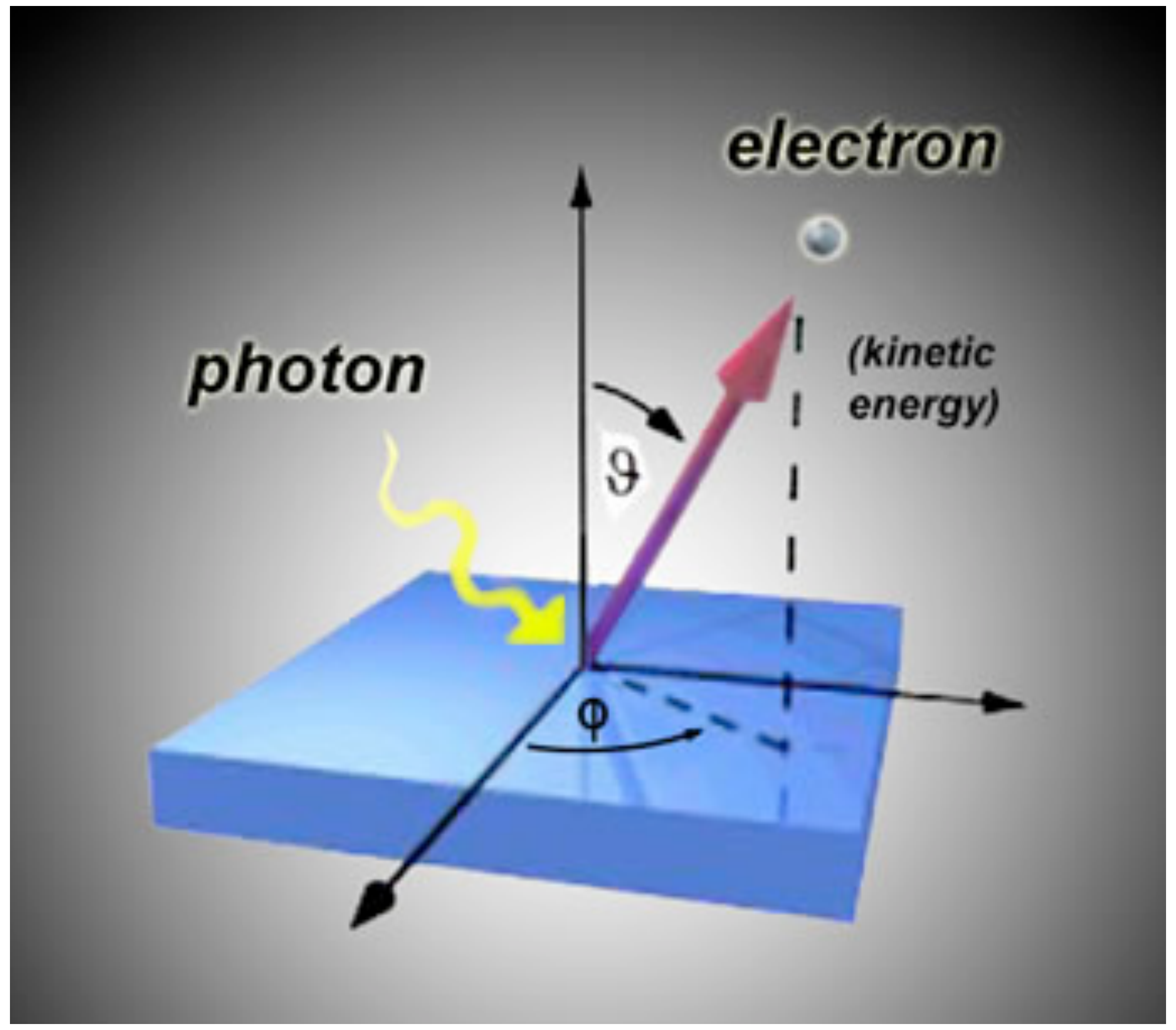
- Quantum Hall Insulator
  - Two dimensions
  - breaks Time-Reversal Symmetry
  - Chern index
  - ➡ **Chiral edge states**
- Quantum Spin Hall Insulator
  - Two dimensions
  - Time-Reversal Symmetry + spins 1/2
  - Kane-Mele Z_2 index
  - ➡ **Helical edge states : Kramers pair**
- 3D Topological Insulators
  - Three dimensions
  - Time-Reversal Symmetry + spins 1/2
  - Kane-Mele Z_2 index
  - ➡ (odd number of) **Dirac cone**

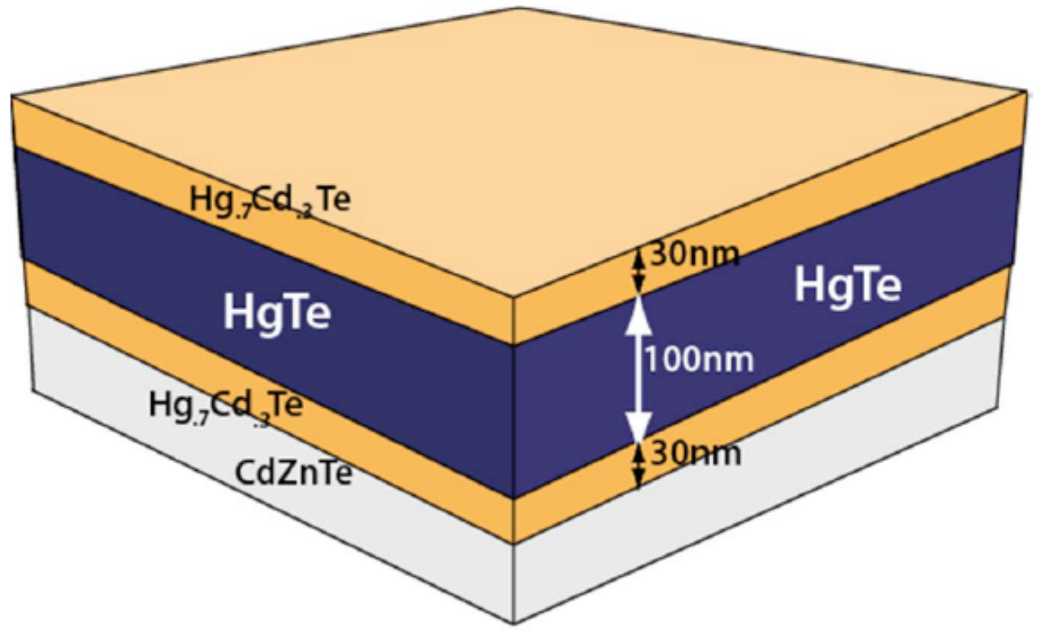


Probing Topological Insulator : Surface States

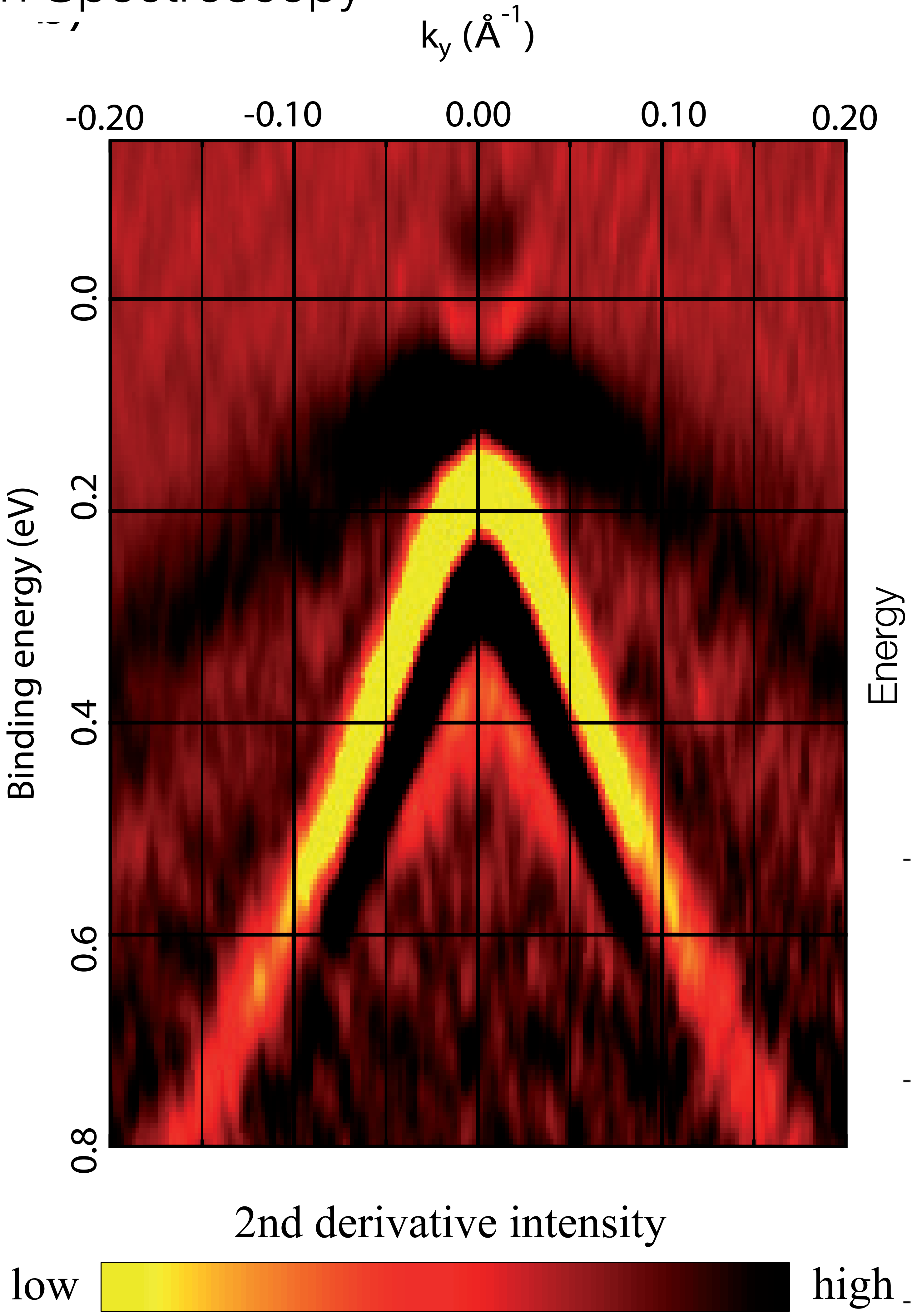
Coll. Inst. Néel / CEA Leti / ENS-Lyon
O. Crauste, *et al.*, arXiv:1307.2008

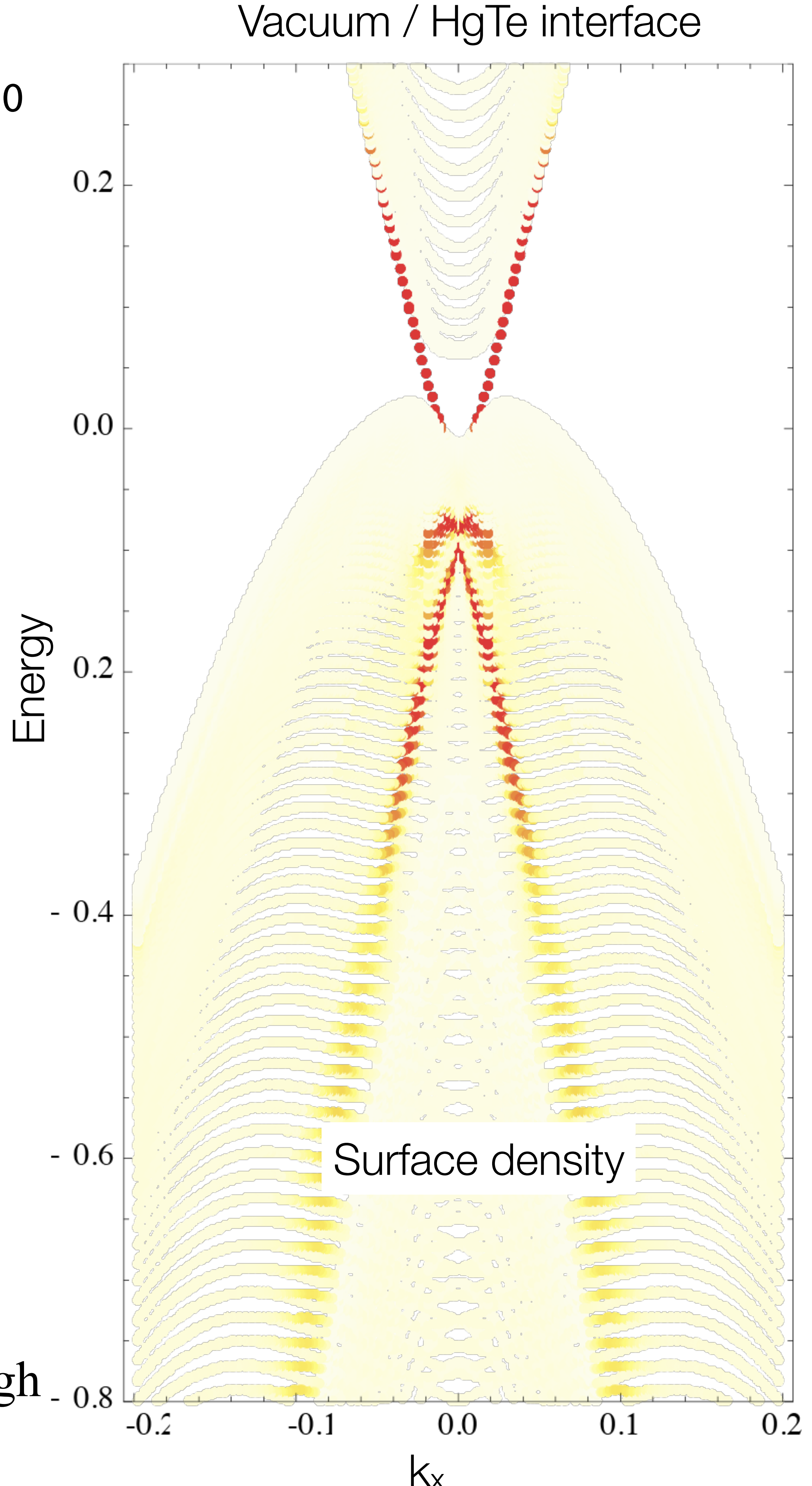
Angle **R**esolved **P**hoto**E**mission **S**pectroscopy





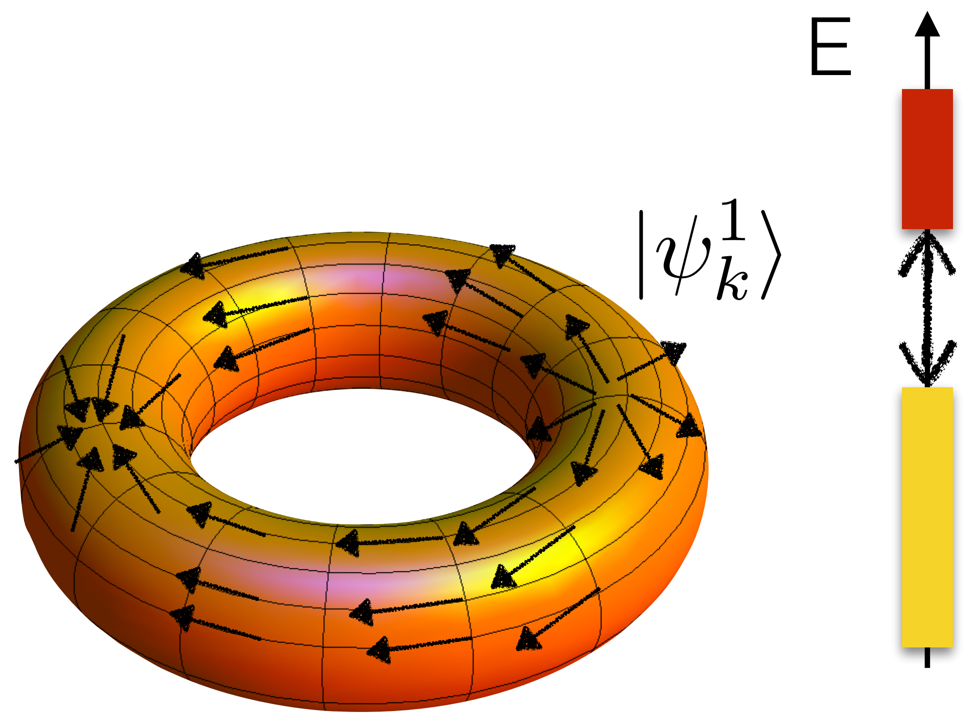
3D strained HgTe





Summary

What is a topological gapped phase ?



▷ Bulk topological property :

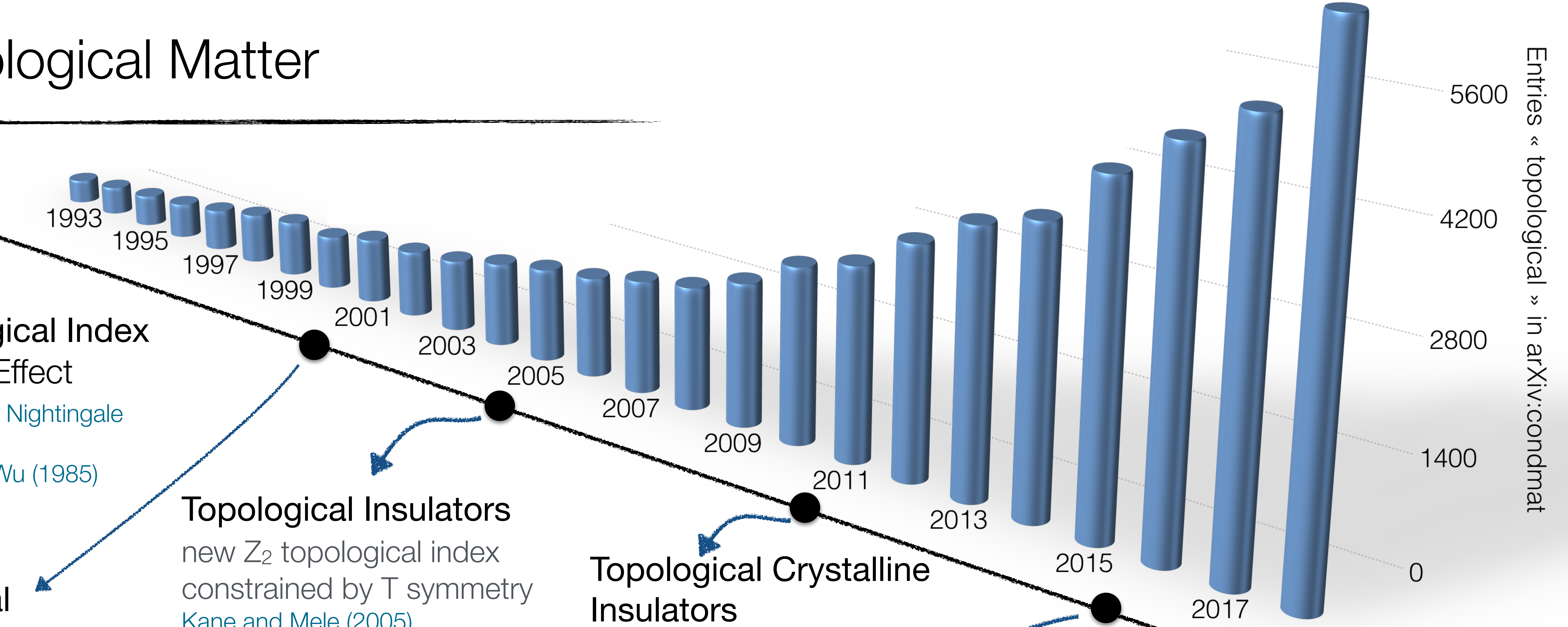
- property of states below the gap
- no continuous Bloch states over Brillouin zone

Topological number : Chern number $$C_1 = \frac{1}{2\pi}\int_{\mathrm{BZ}} F$$

▷ Surface / edge states (inside gap)

- robust (related to topology)
- unique metals (not conventional)

Topological Matter



Chern Topological Index
Quantum Hall Effect

Thouless, Kohmoto, Nightingale and den Nijs (1982)
Niu, Thouless, and Wu (1985)
Haldane (1985)

Topological Superconductor

formal 1D model
Kitaev (2001)

Topological Insulators

new Z_2 topological index
constrained by T symmetry
Kane and Mele (2005)
Bernevig, Hughes, and Zhang (2006)
Fu, Kane et Mele (2007)
Moore and Balents (2007)
Roy (2009)

Topological Crystalline Insulators

constrained by crystal symmetries
Fu (2011)

Topological Dirac & Weyl metals

no gap

Nobel Prize in Physics (2016)
D.J. Thouless,
F.D. Haldane and J.M. Kosterlitz
« for theoretical discoveries of topological phase transitions and topological phases of matter »

Topological Quantum Chemistry

New band theory of materials (crystal symmetres & topology)

Topological Quantum Chemistry

... combining symmetry representations and topology

Catalogue of Topological Electronic Materials

Tiantian Zhang, Yi Jiang, Zhida Song, He Huang, Yuqing He, Zhong Fang, Hongming Weng, Chen Fang
Nature, 566, 475 (2019)

Towards ideal topological materials: Comprehensive database searches using symmetry indicators

Feng Tang, Hoi Chun Po, Ashvin Vishwanath, Xiangang Wan
Nature 566, 486 (2019)

The (High Quality) Topological Materials In The World

M. G. Vergniory, L. Elcoro, C. Felser, B. A. Bernevig, Z. Wang
Nature 566, 480 (2019)

Abstract: « Topological Quantum Chemistry (TQC) links the chemical and symmetry structure of a given material with its topological properties.

Out of **26938** stoichiometric materials in our filtered ICSD database, we find **2861** topological insulators (TI) and **2936** topological semimetals.

Remarkably, our exhaustive results show that a large proportion **(~ 24% !)** of all materials in nature are topological

We added an open-source code and end-user button on the Bilbao Crystallographic Server (BCS)

Outline

1. Electronic Properties of Quantum Matter

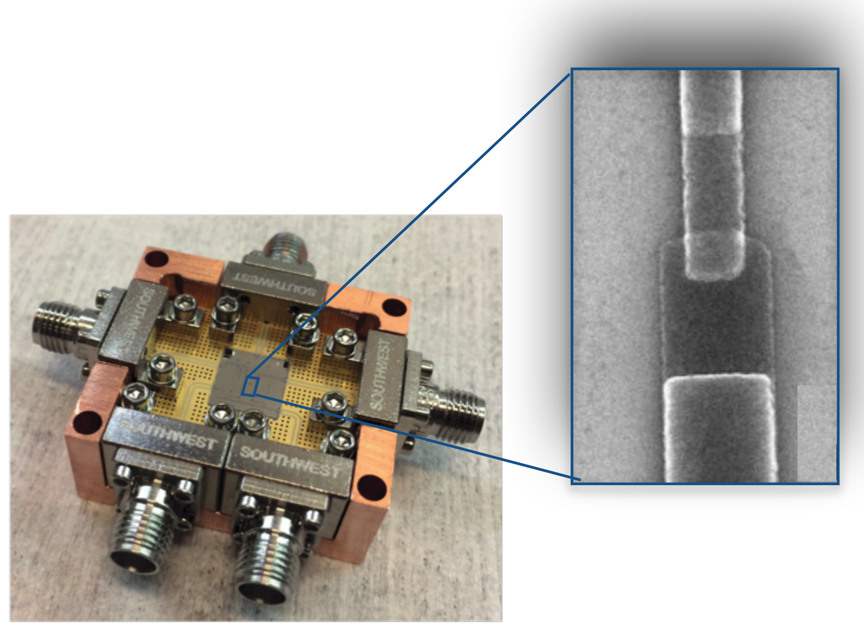
   Topological Insulators

Metals

Insulators

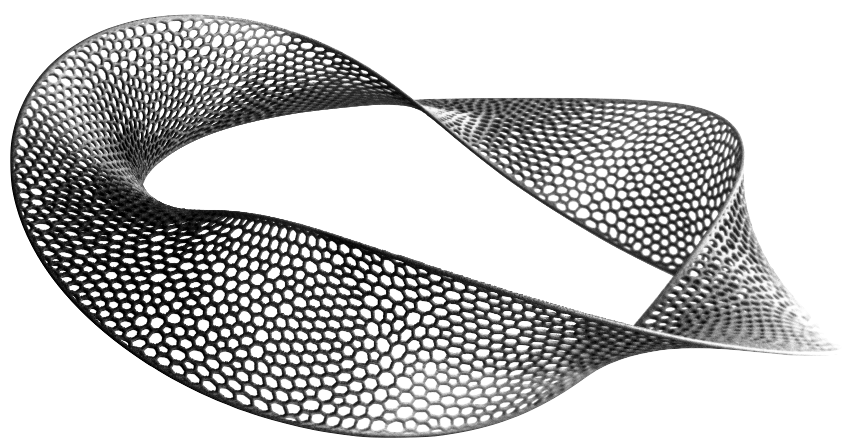
2. Quantum Technologies

   Robust quantum devices

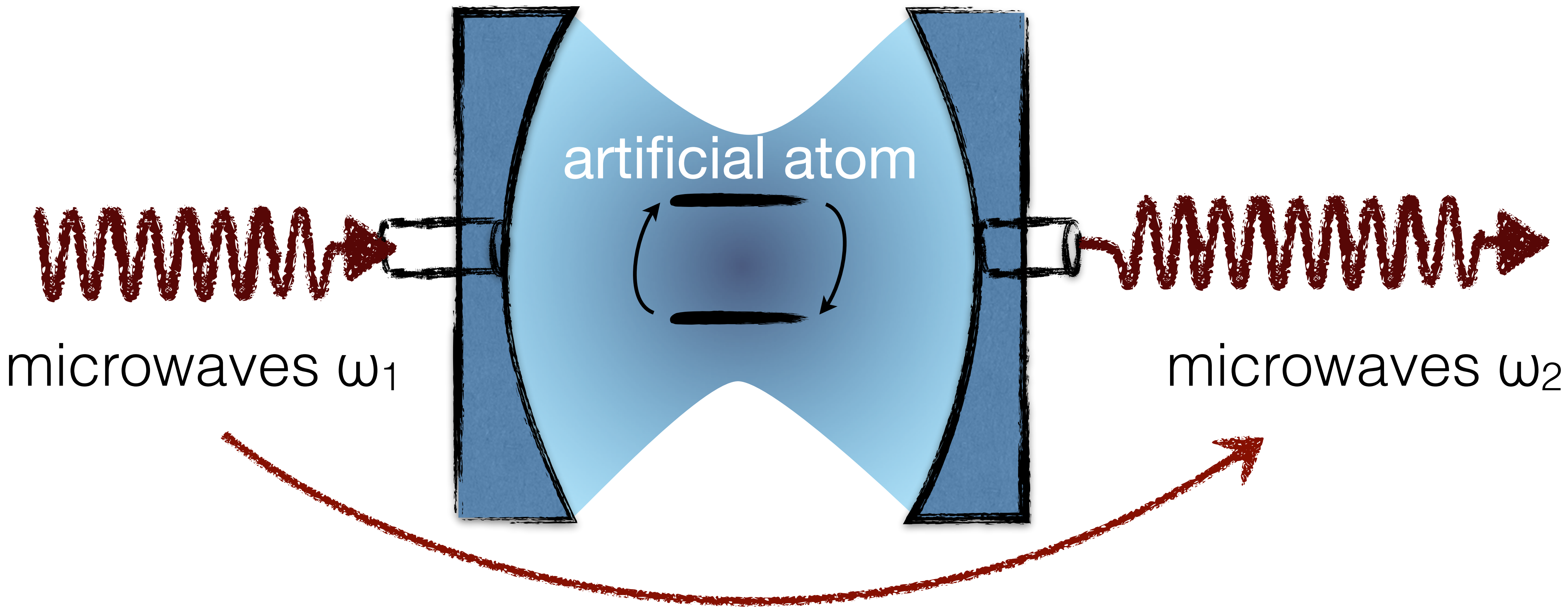


3. Mechanics / Metamaterials

   deformations constrained by topology



A Topological Energy Pump

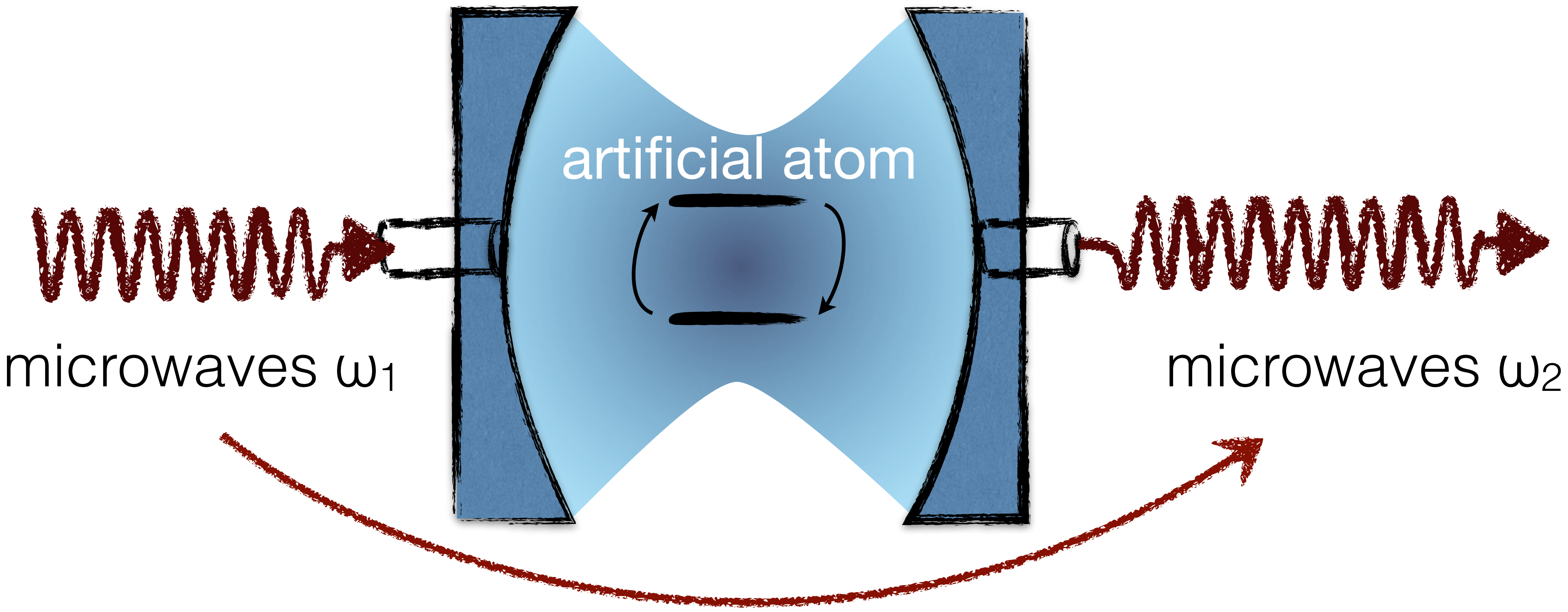


Transfer of energy, quantized power imposed by topology

Clément Dutreix (Univ. Bordeaux) Quentin Ficheux (Univ. Maryland) Pierre Delplace (ENS Lyon) Benjamin Huard (ENS Lyon)

C. Dutreix et al., in preparation (2019)

A Topological Energy Pump



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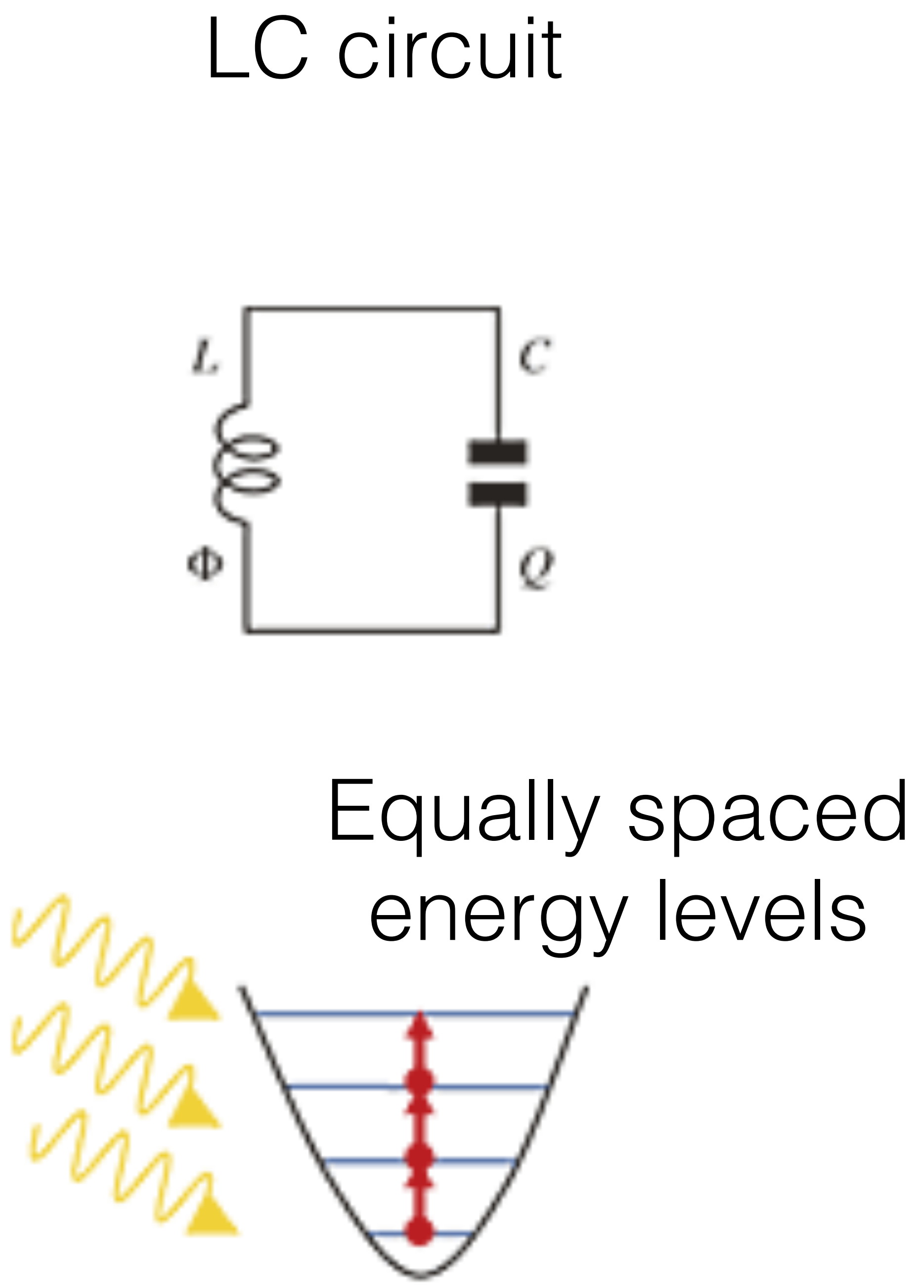

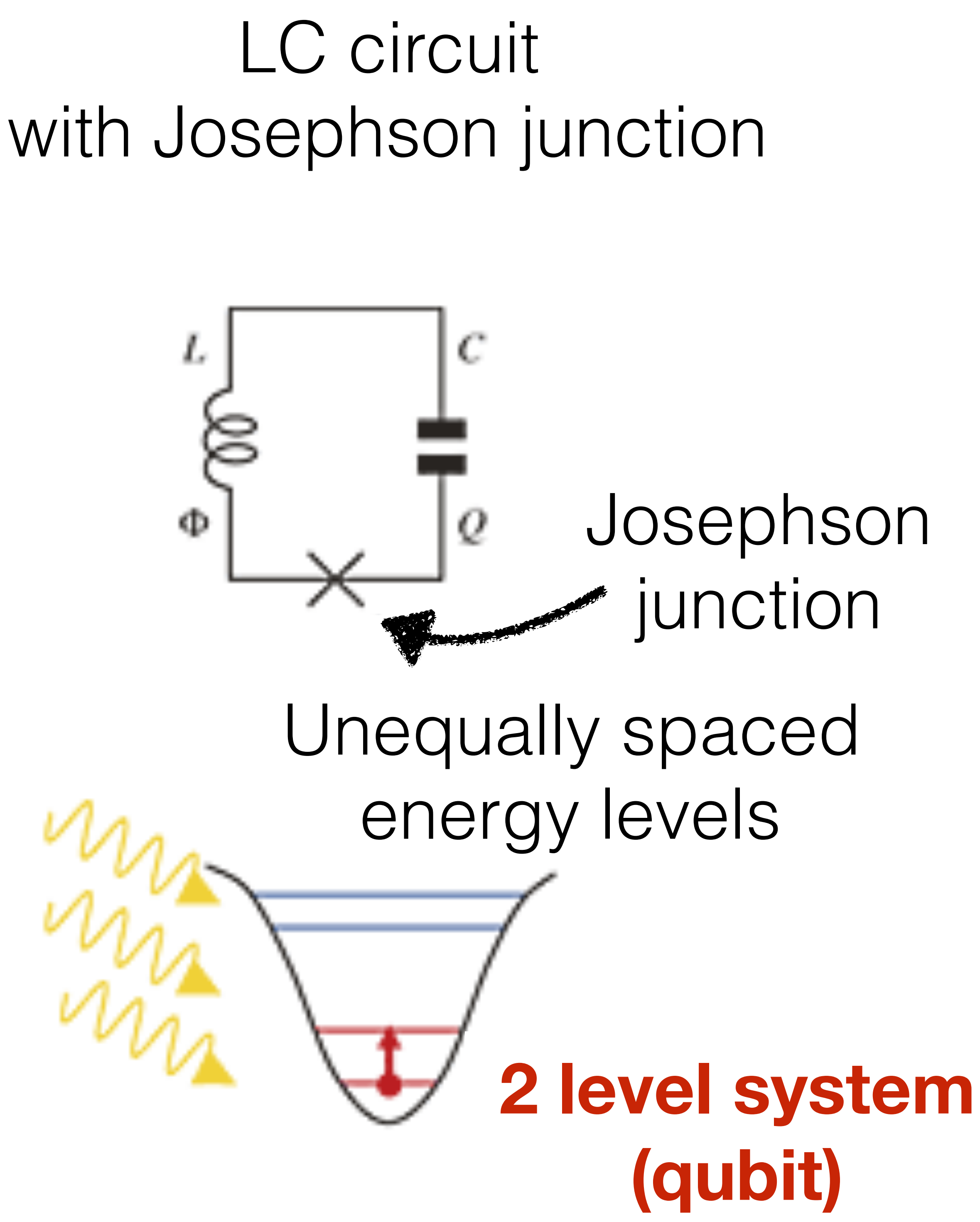

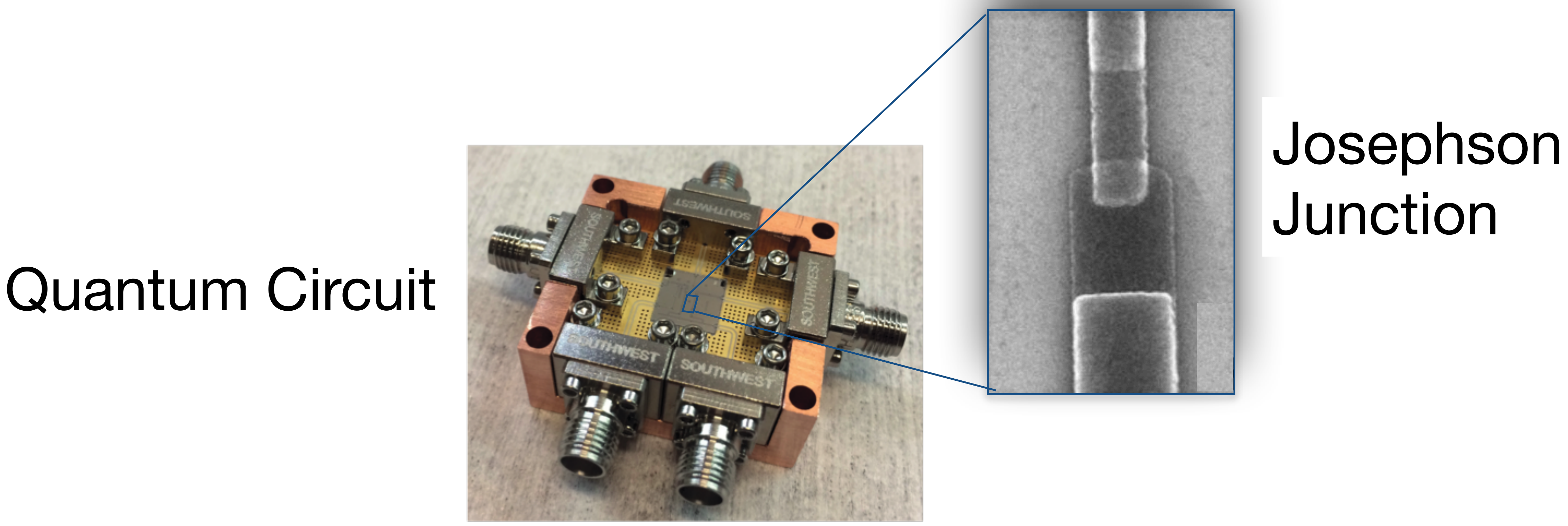

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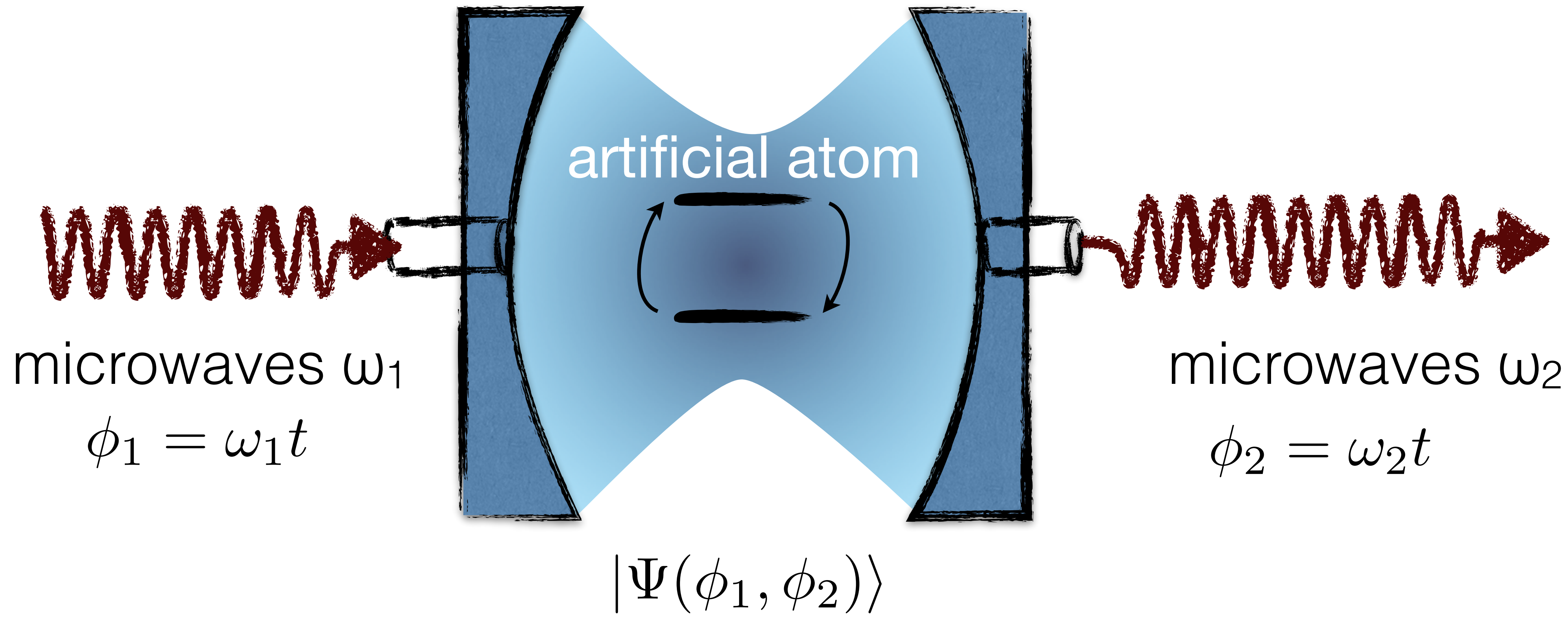
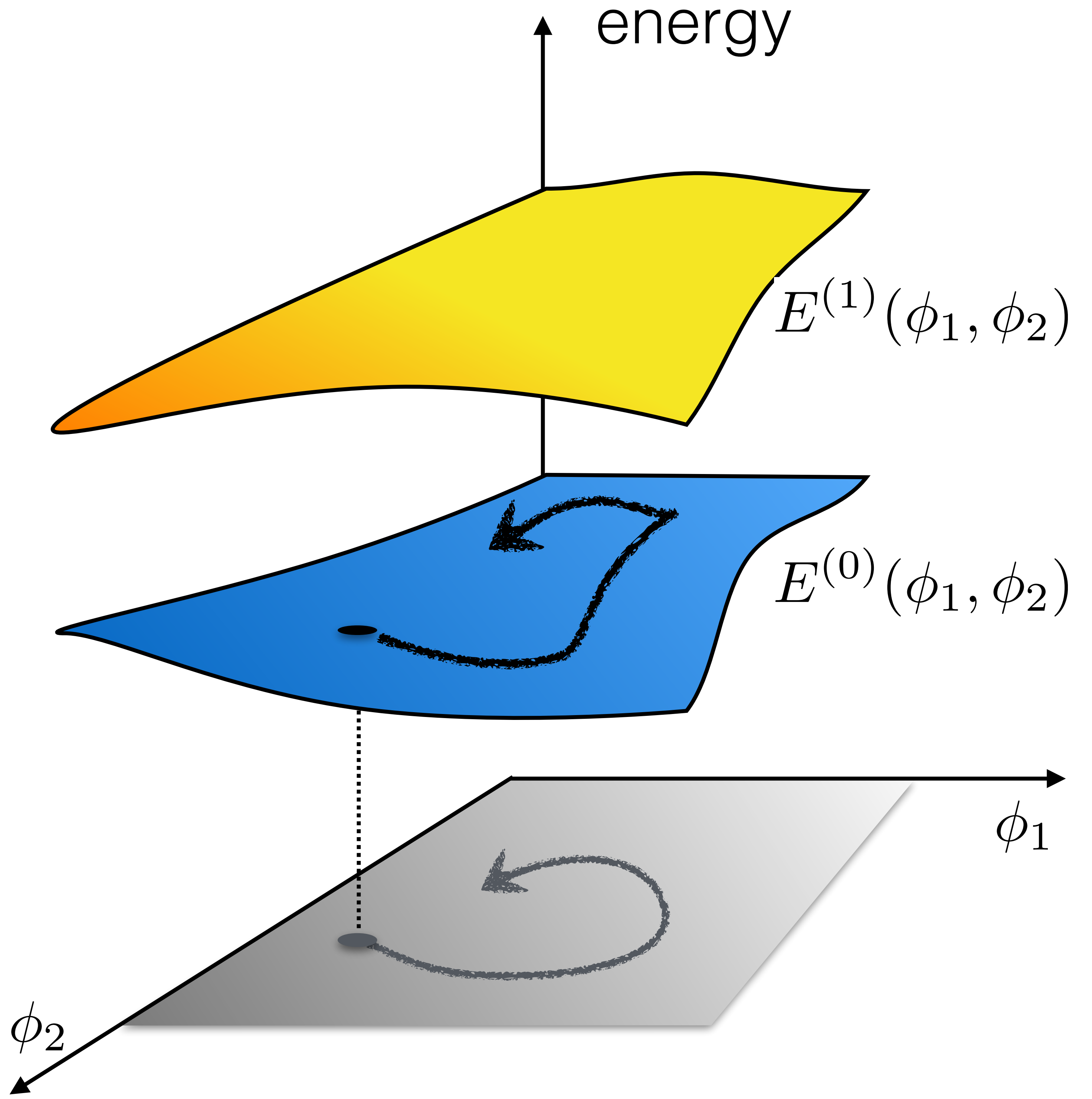


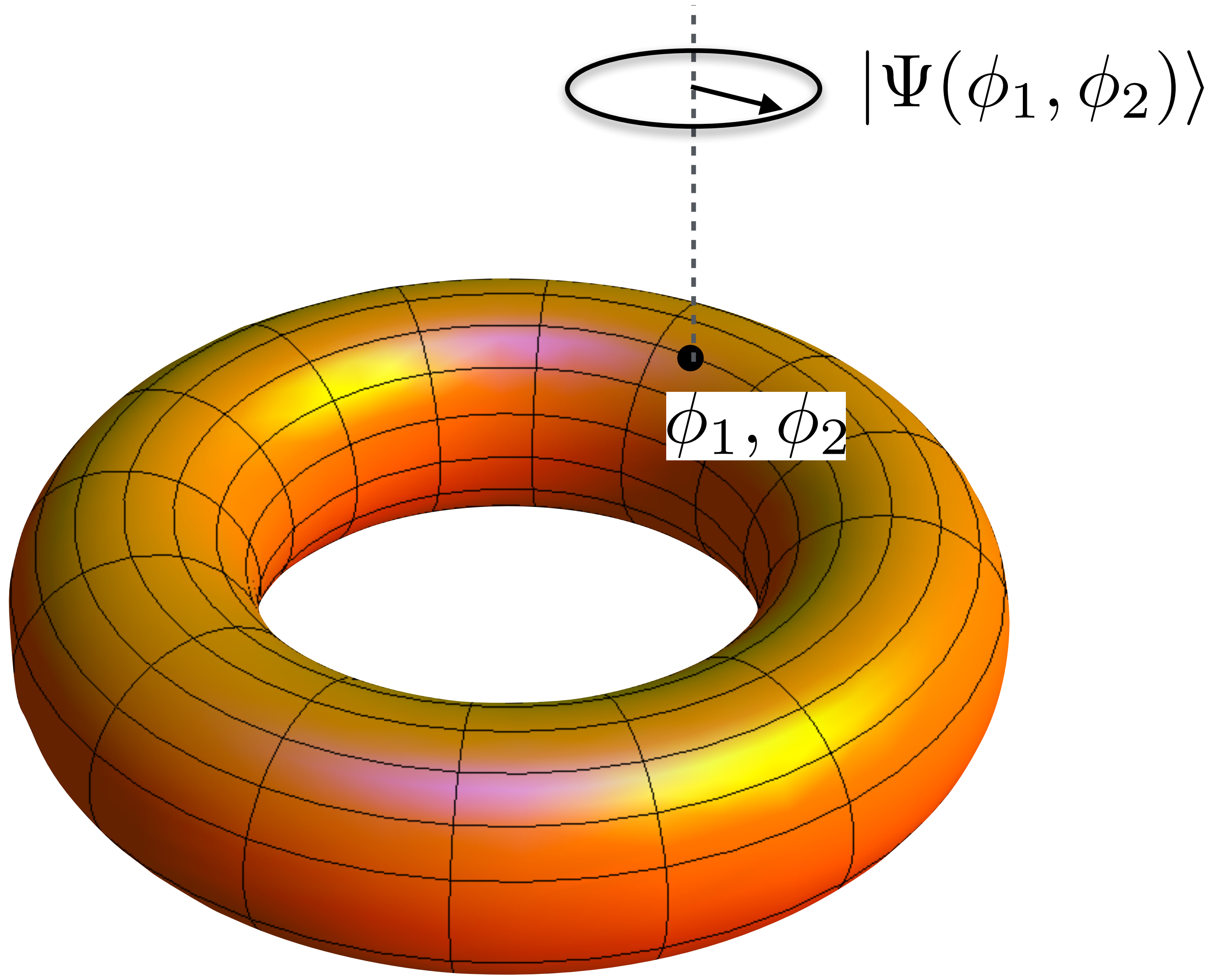


Periodicity

- Hamiltonian $H(\phi_1, \phi_2)$ is 2π periodic in ϕ_1, ϕ_2

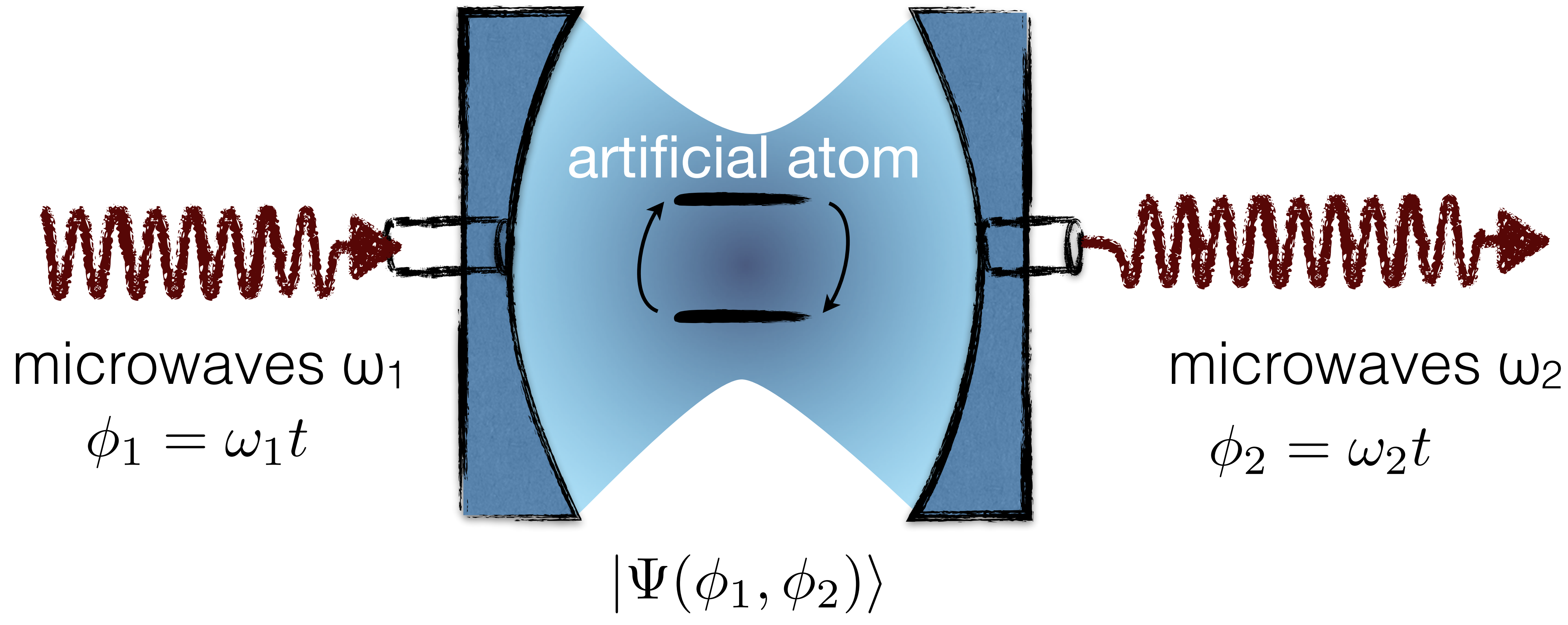
Adiabatic dynamics

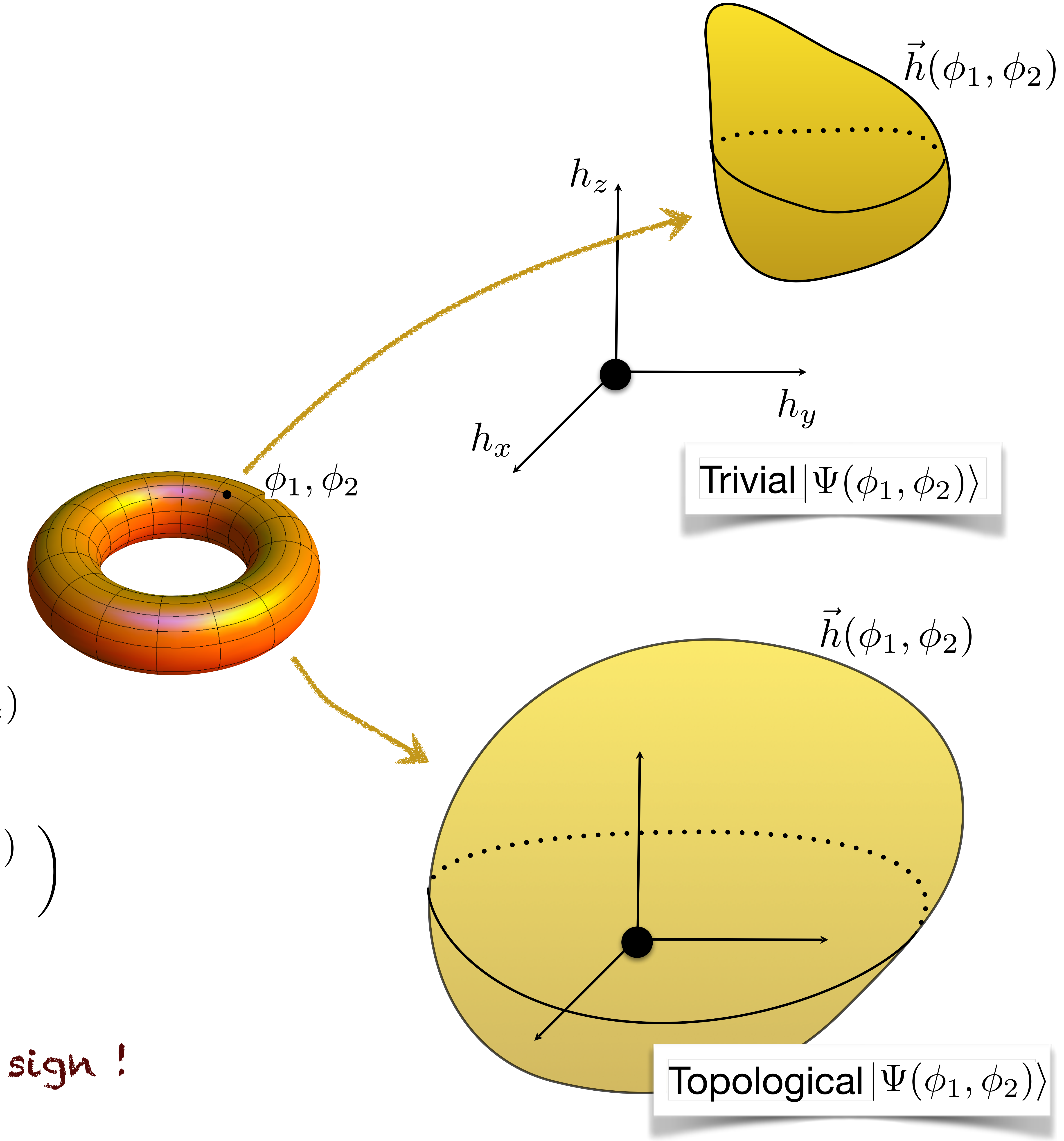
- gap in energy for all ϕ_1, ϕ_2
- quantum system remains in 1 (ground) state



➡ topological (driven) quantum state ?

A Topological Energy Pump





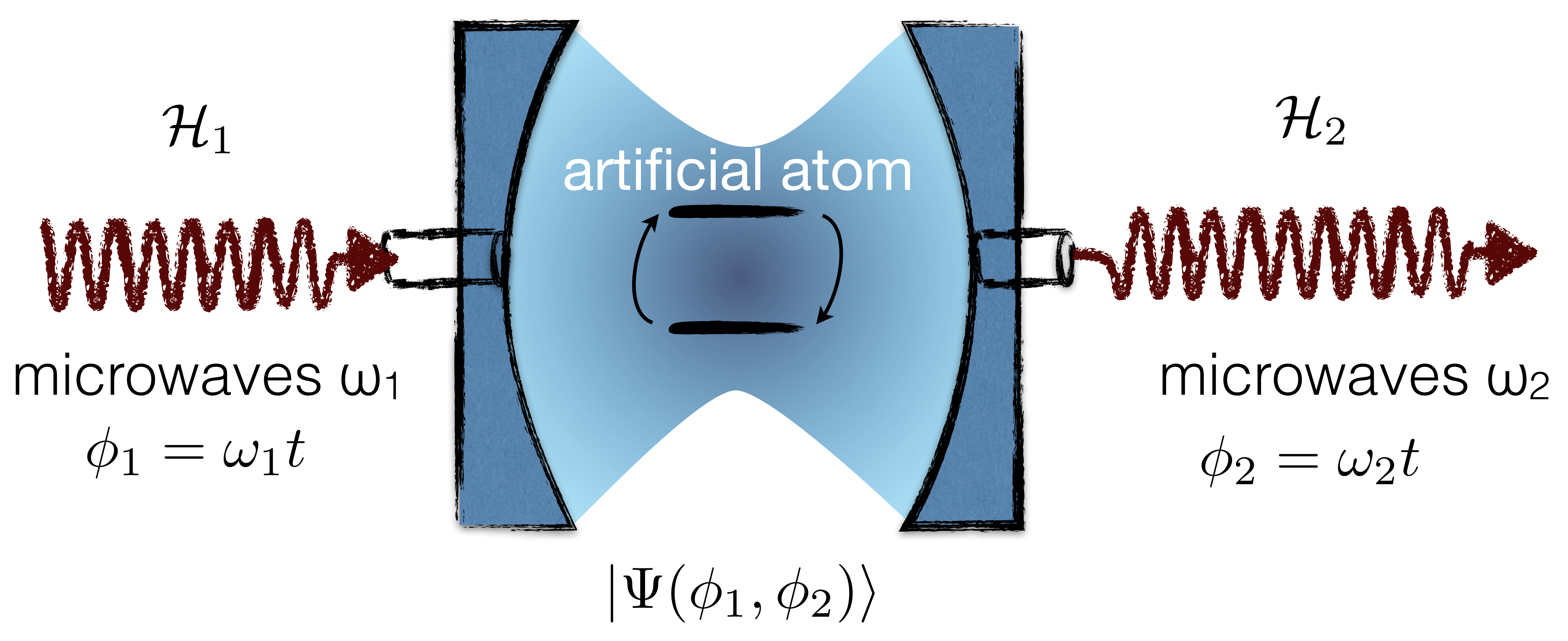
2-level system : recipe for topology

- general parametrization in terms of vector $\vec{h} = (h_x, h_y, h_z)$
- driven 2-level system : map $\vec{h}(\phi_1, \phi_2)$

$$H(\phi_1, \phi_2) = \hbar \begin{pmatrix} h_z(\phi_1, \phi_2) & h_x(\phi_1, \phi_2) - ih_y(\phi_1, \phi_2) \\ h_x(\phi_1, \phi_2) + ih_y(\phi_1, \phi_2) & -h_z(\phi_1, \phi_2) \end{pmatrix}$$

necessary condition : all couplings must change sign !

A Topological Energy Pump



Chern number (topological index)

$$c_{12}^{(\Psi)} = \frac{1}{2\pi} \int d\phi_1 d\phi_2 \; F_{1,2}^{(\Psi)}$$

➡ Average Berry curvature

$$\overline{F_{1,2}^{(\Psi)}} = \frac{1}{2\pi} c_{12}^{(\Psi)}$$

Berry curvature of the eigenstate $|\Psi(\phi_1, \phi_2)\rangle$

General Formalism for topological pumping

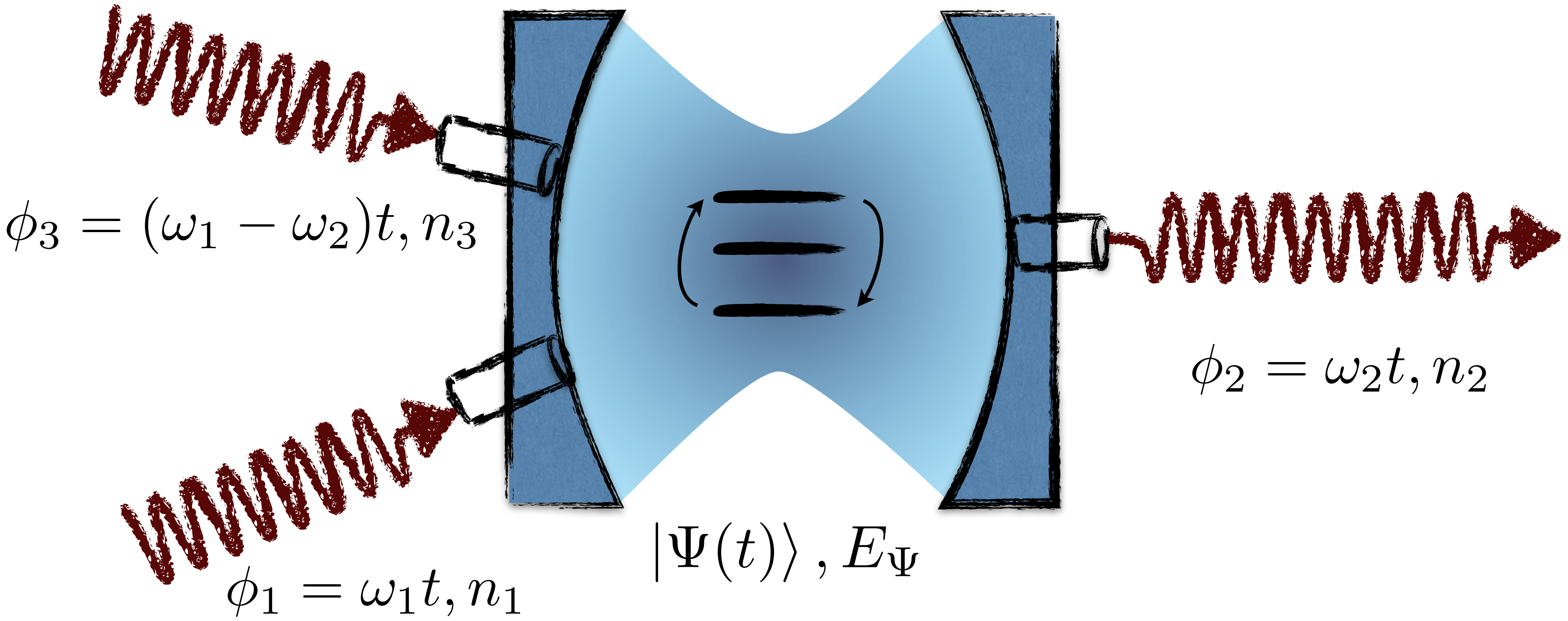
- each bath: conjugate classical variables n_α, ϕ_α , Hamiltonian $\mathcal{H}_\alpha$
- equation of motion:

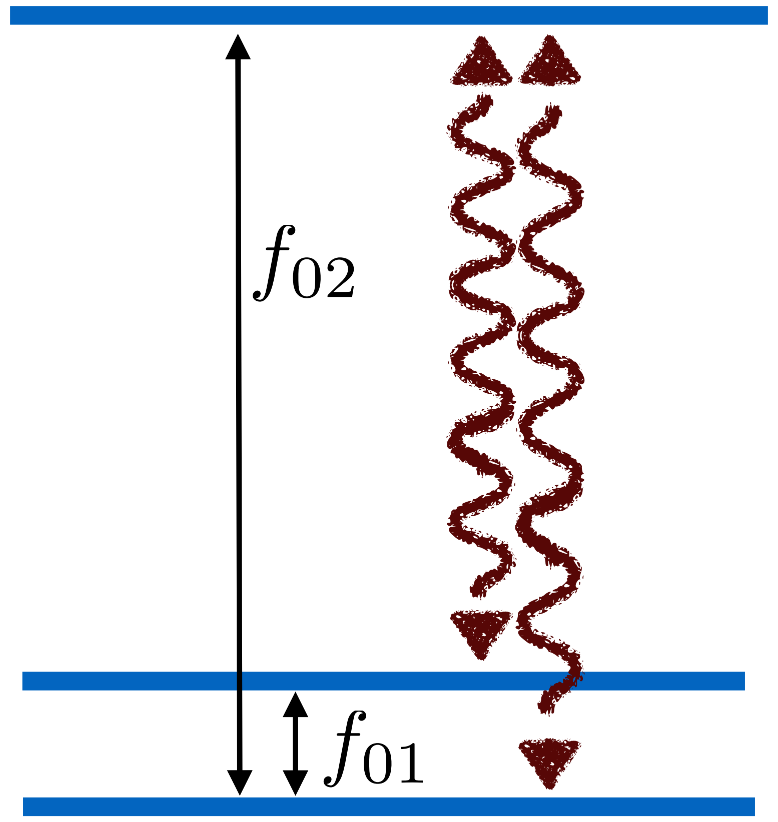
$$\dot{n}_\alpha = -\frac{\partial \mathcal{H}_\alpha}{\partial \phi_\alpha} - \langle \Psi(t) | \frac{\partial H}{\partial \phi_\alpha} | \Psi(t) \rangle = -\frac{\partial \mathcal{H}_\alpha}{\partial \phi_\alpha} - \frac{\partial E_\Psi}{\partial \phi_\alpha} - \hbar \sum_\beta F_{\alpha,\beta}^{(\Psi)} \; \dot{\phi}_\beta$$

$$\dot{\phi}_\alpha = \frac{\partial \mathcal{H}_\alpha}{\partial n_\alpha} = \omega_\alpha$$

- power transfer $\Delta \mathcal{E}_1 = \dot{n}_1 \omega_1 = \frac{\hbar}{2\pi} c_{12} \; \omega_1 \omega_2$

A Topological Energy Pump : effective 2-level system (qutrit)





ω_{01}	$2\pi \times 200$ MHz
ω_{02}	$2\pi \times 5$ GHz
Ω_{01}	$2\pi \times 100$ MHz
Ω_{02}	$2\pi \times 1$ GHz
Ω_{12}	$2\pi \times 1.2$ GHz
$\dot{\phi}_a = 2\dot{\phi}_b$	$2\pi \times 20$MHz

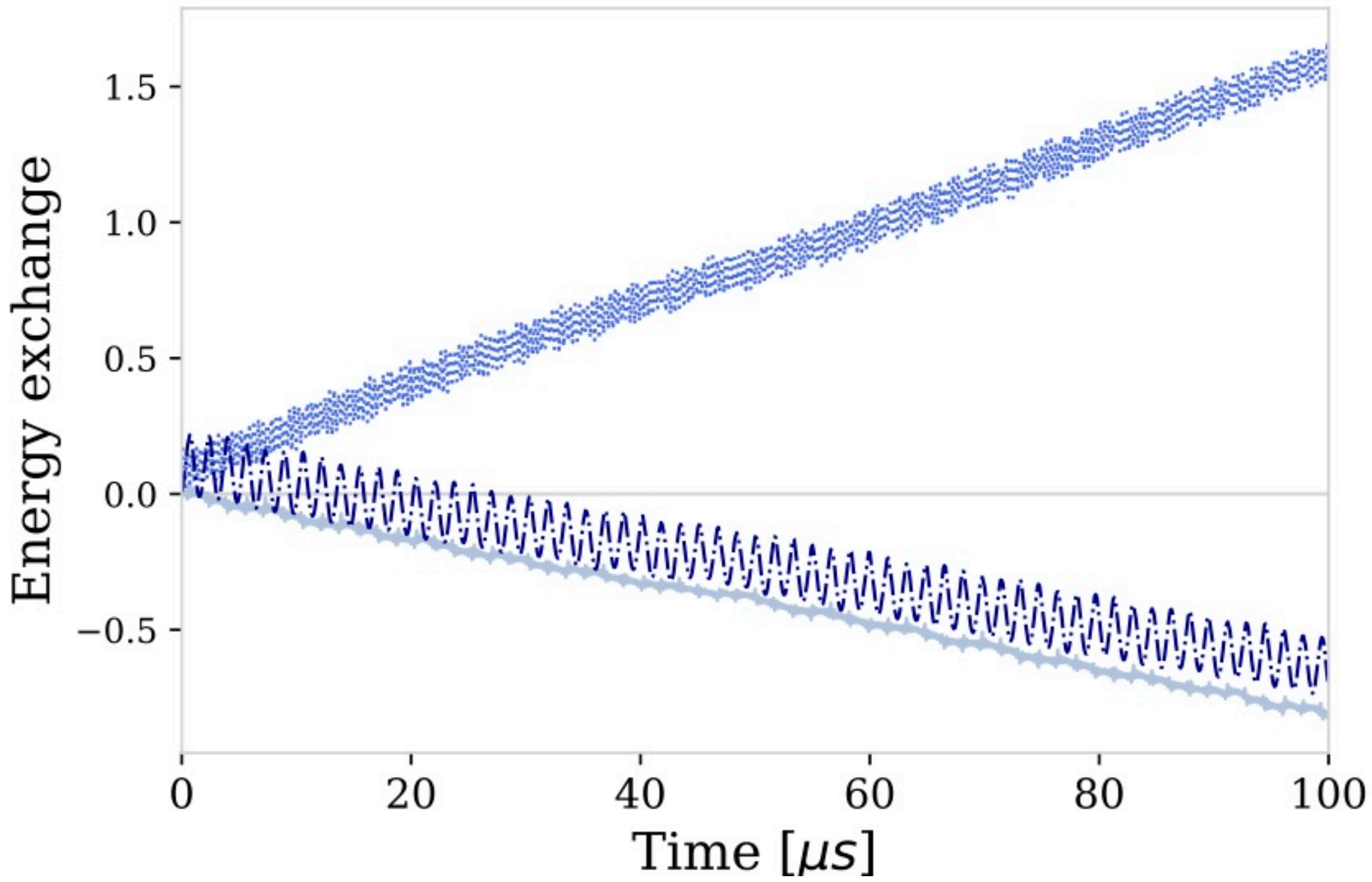
Topological transfer :

$$\dot{n}_I = \dot{n}_1 + \dot{n}_3 = \hbar F^{12}\omega_2$$

$$\dot{n}_{II} = \dot{n}_2 - \dot{n}_3 = -\hbar F^{12}\omega_1$$

Energy exchange : $\Delta\mathcal{E}_i = \dot{n}_i \omega_i$ ➠ $\Delta\mathcal{E}_1 = -\Delta\mathcal{E}_2$

Topological rate : $$\frac{\Delta\mathcal{E}_1}{\omega_1} + \frac{\Delta\mathcal{E}_3}{\omega_1 - \omega_2} = \frac{\hbar}{2\pi} c_{12}\ \omega_2$$



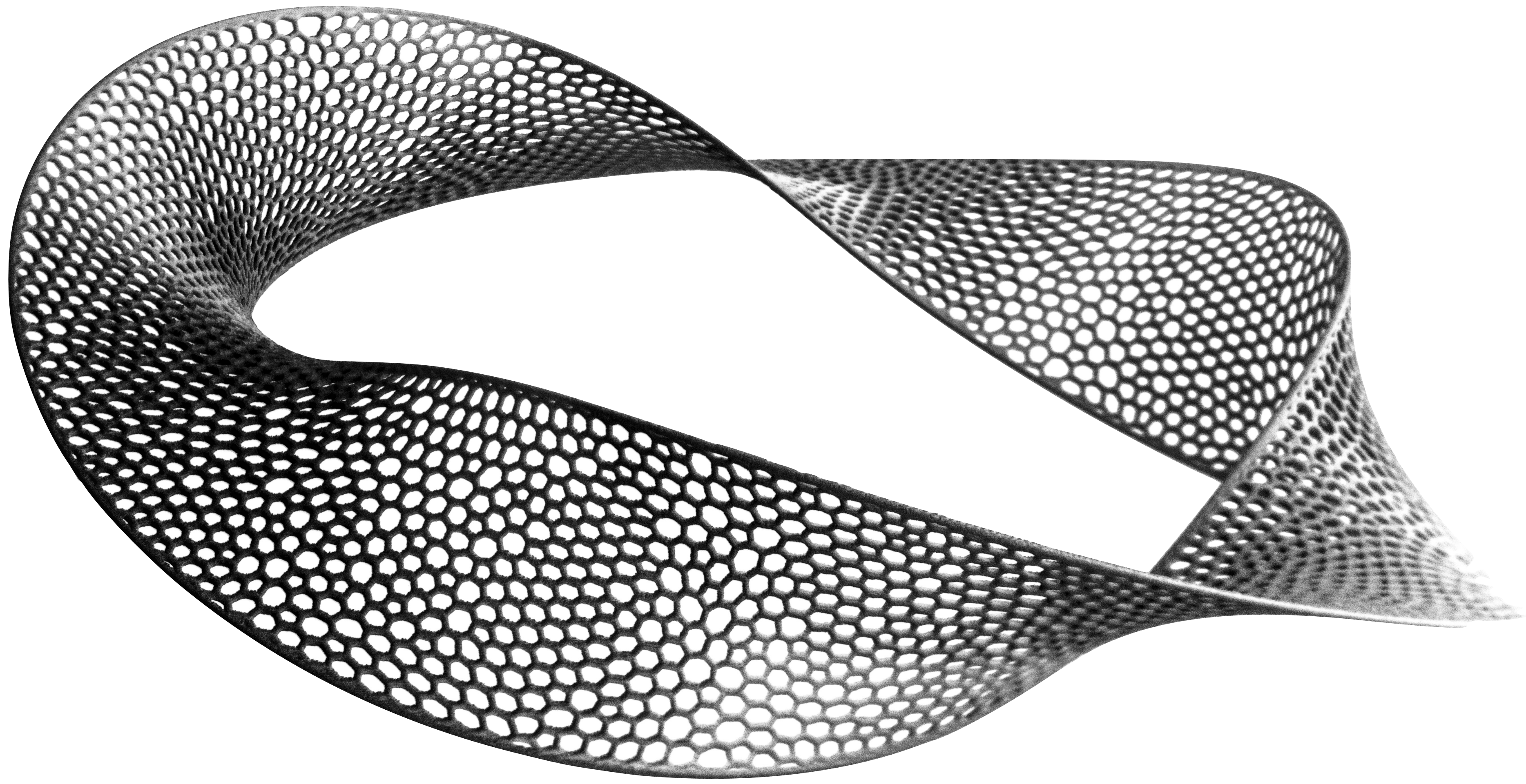
Elasticity of non-orientable objects

Denis Bartolo (ENS Lyon)

Marcelo Guzman (ENS Lyon)

Non-orientable object

- cannot define normal vector to surface continuously
- topological property
- simplest example : Möbius strip (3D printed)

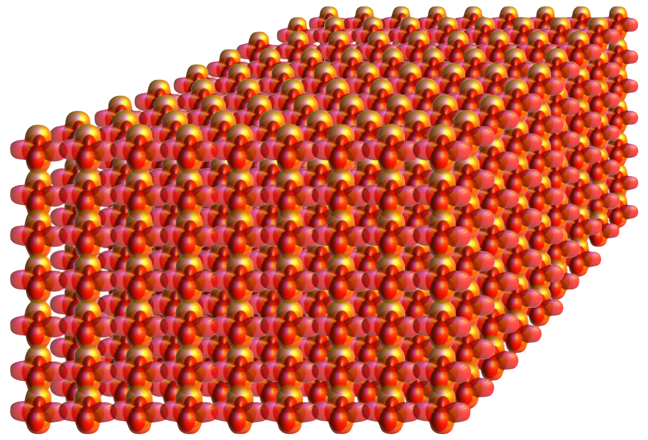


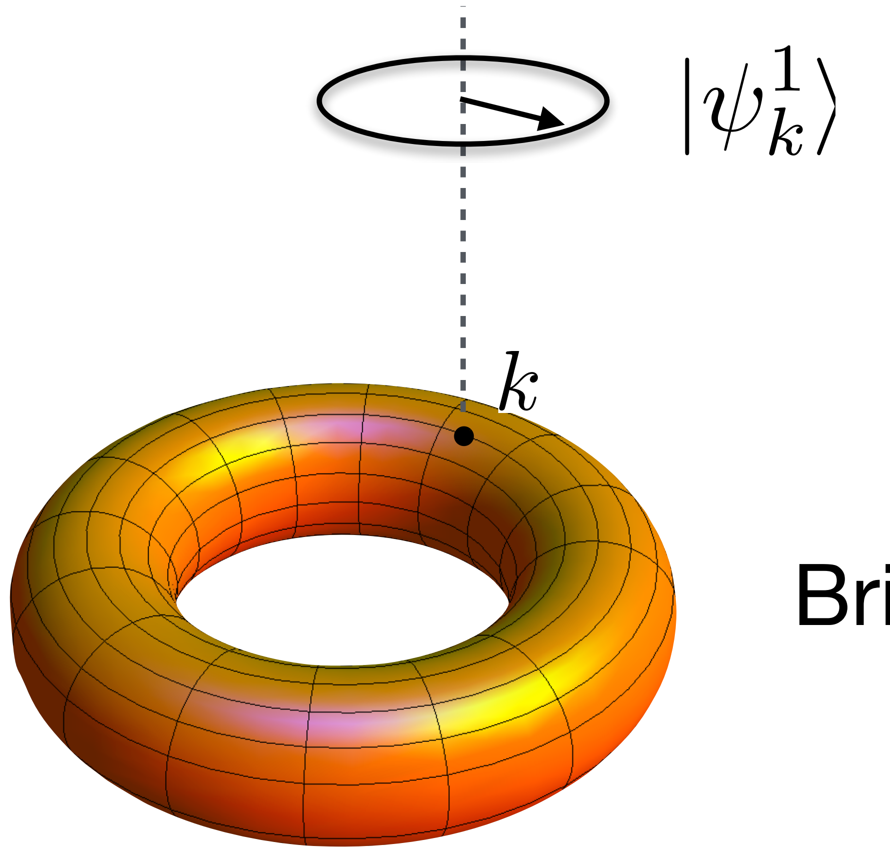
Does the non-orientability (topology) of a Möbius strip manifest itself in its mechanical response ?

Elasticity of non-orientable objects

1. Electronic Properties of Quantum Matter

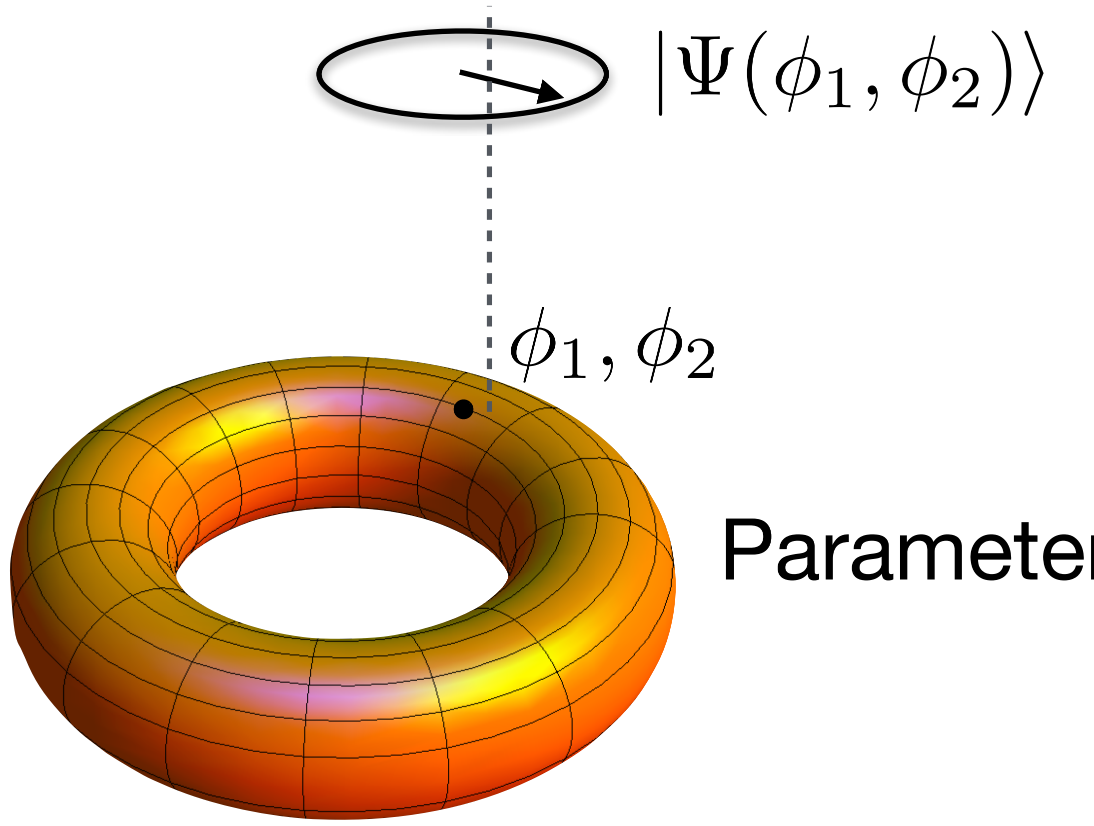
Topological Insulators





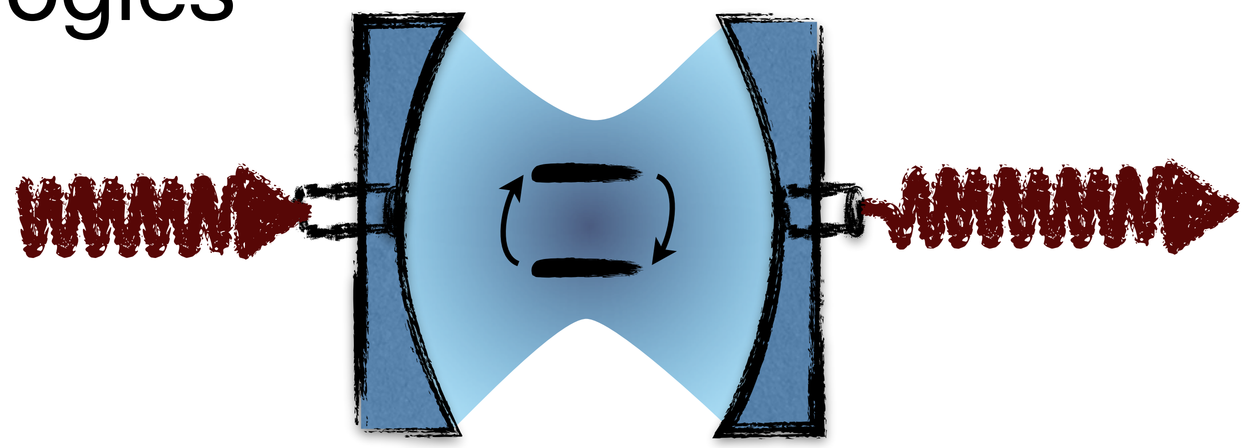
Brillouin zone

topology by « engineering » the vectors



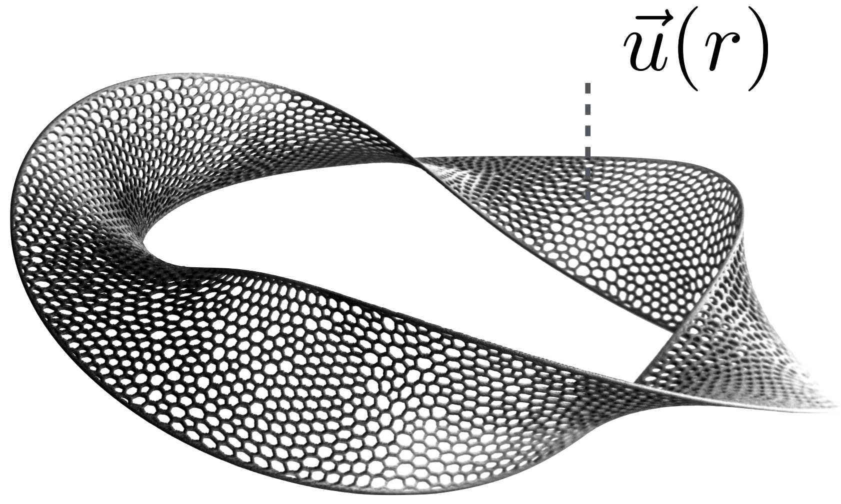
2. Quantum Technologies

Topological pump



Parameter space

3. Mechanics / Metamaterials

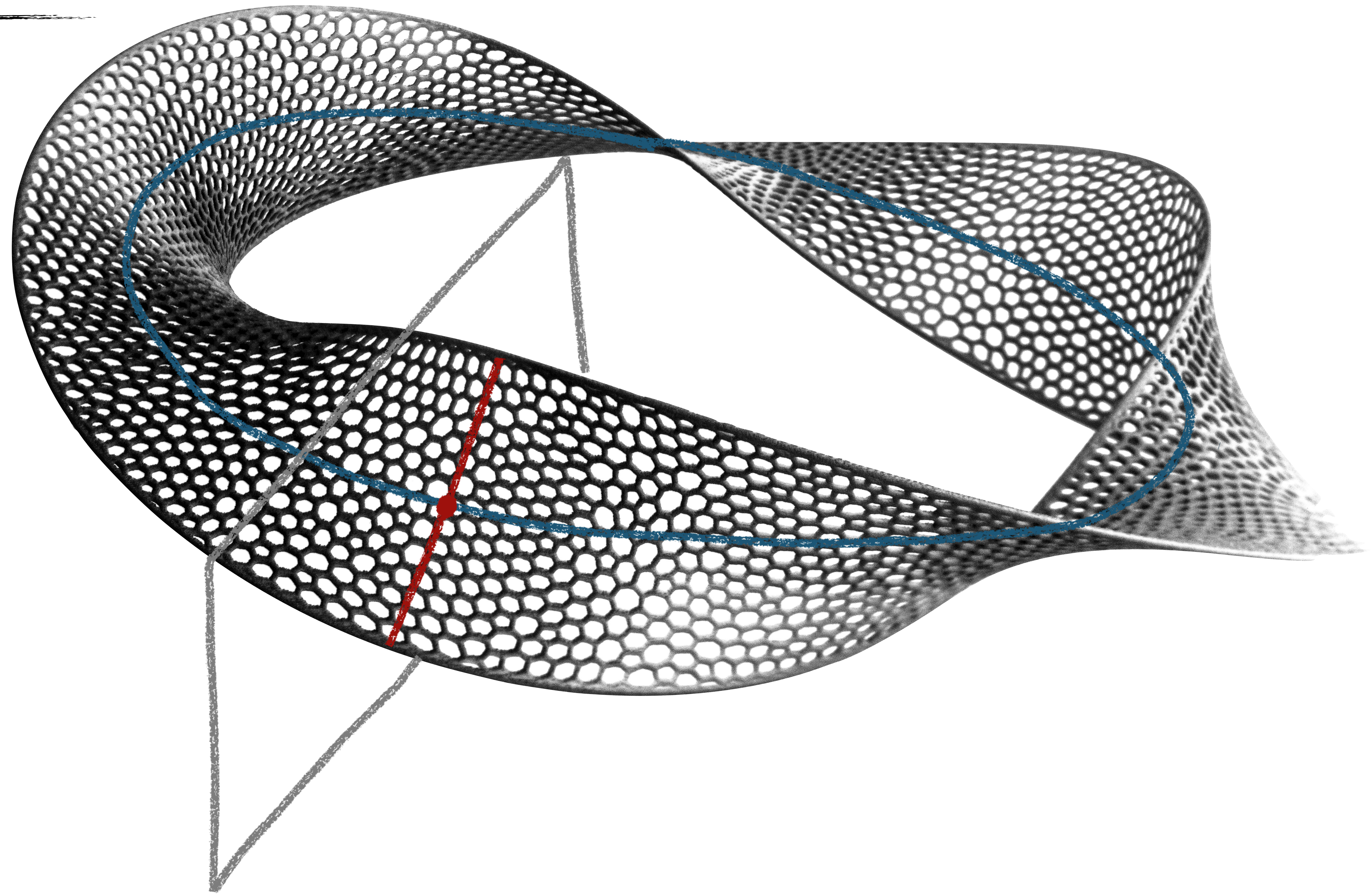


topology by « engineering » the base-space

Elasticity of a Möbius strip

Definition of the shape of the strip

- discrete version of the strip



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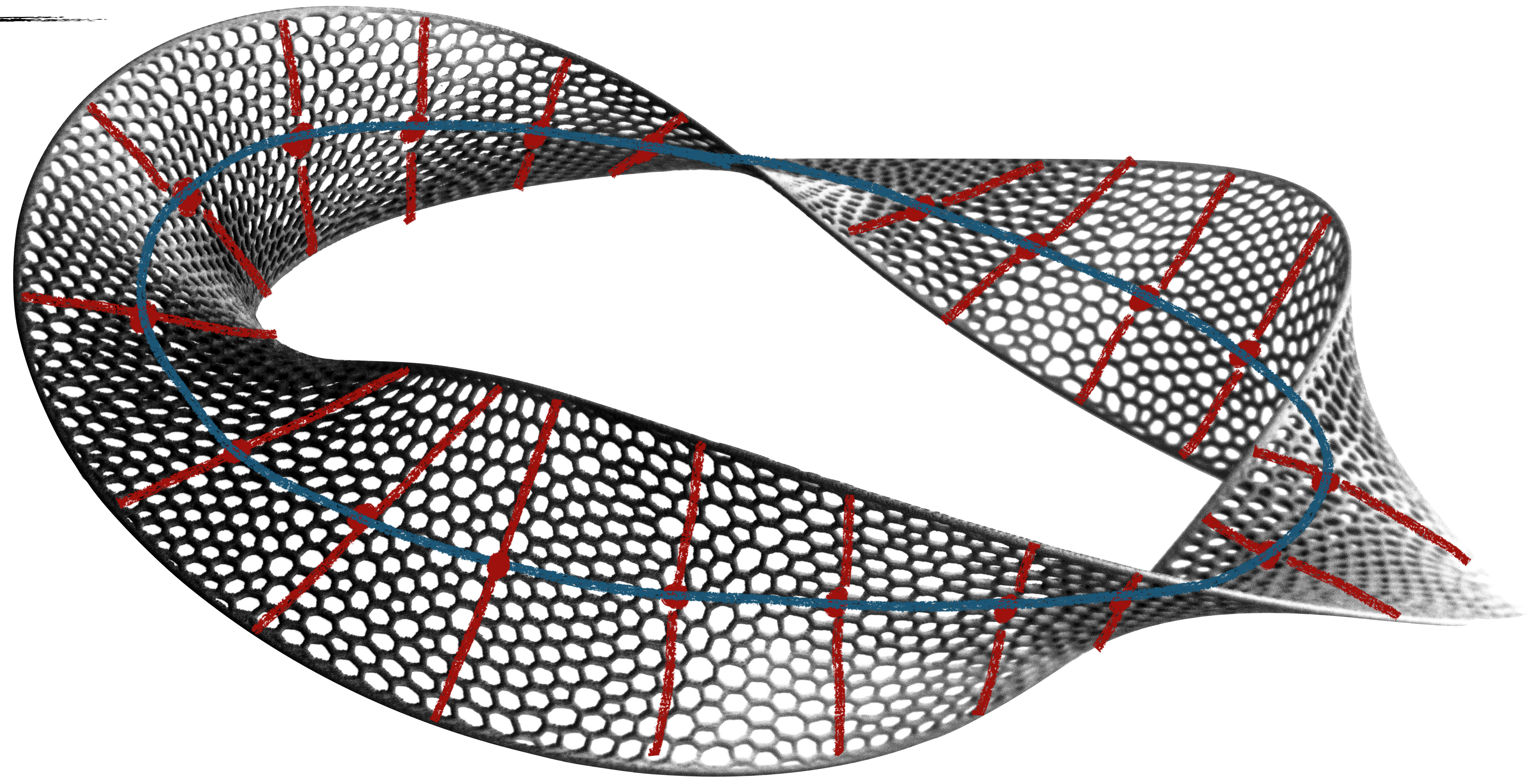
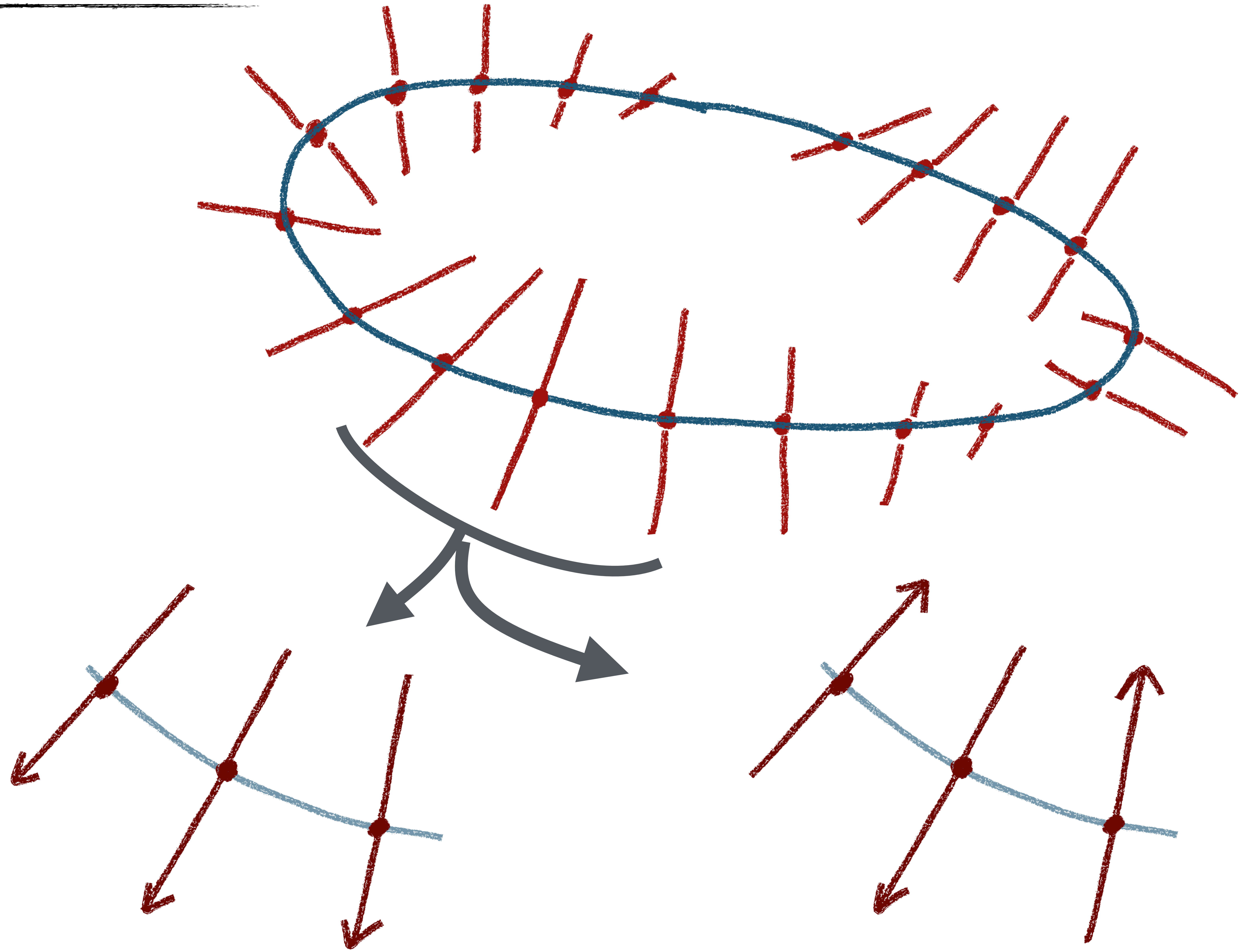


Elasticity of a Möbius strip

Definition of the shape of the strip

- discrete version of the strip
- lattice of « directions »
- but elasticity requires vectors !

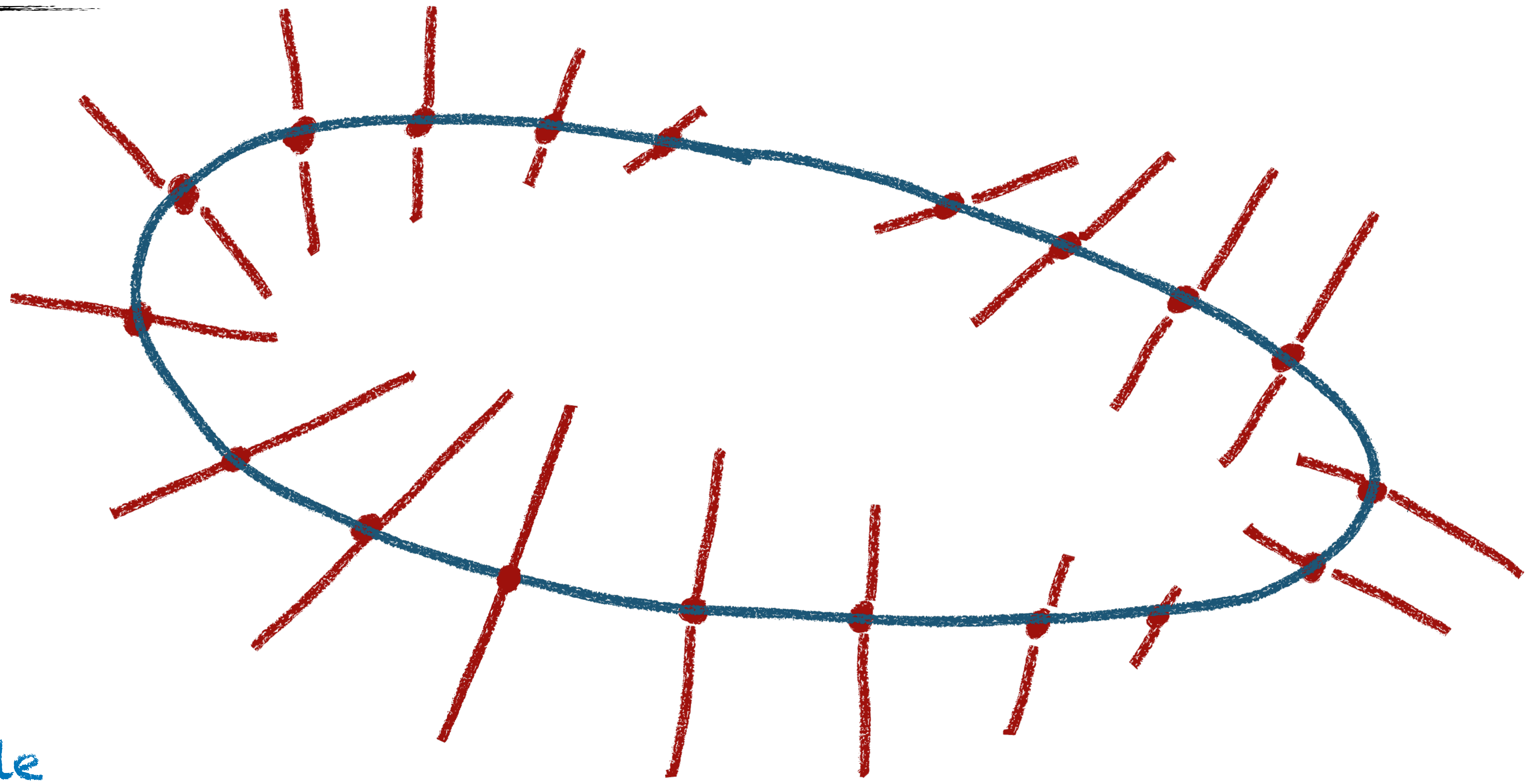


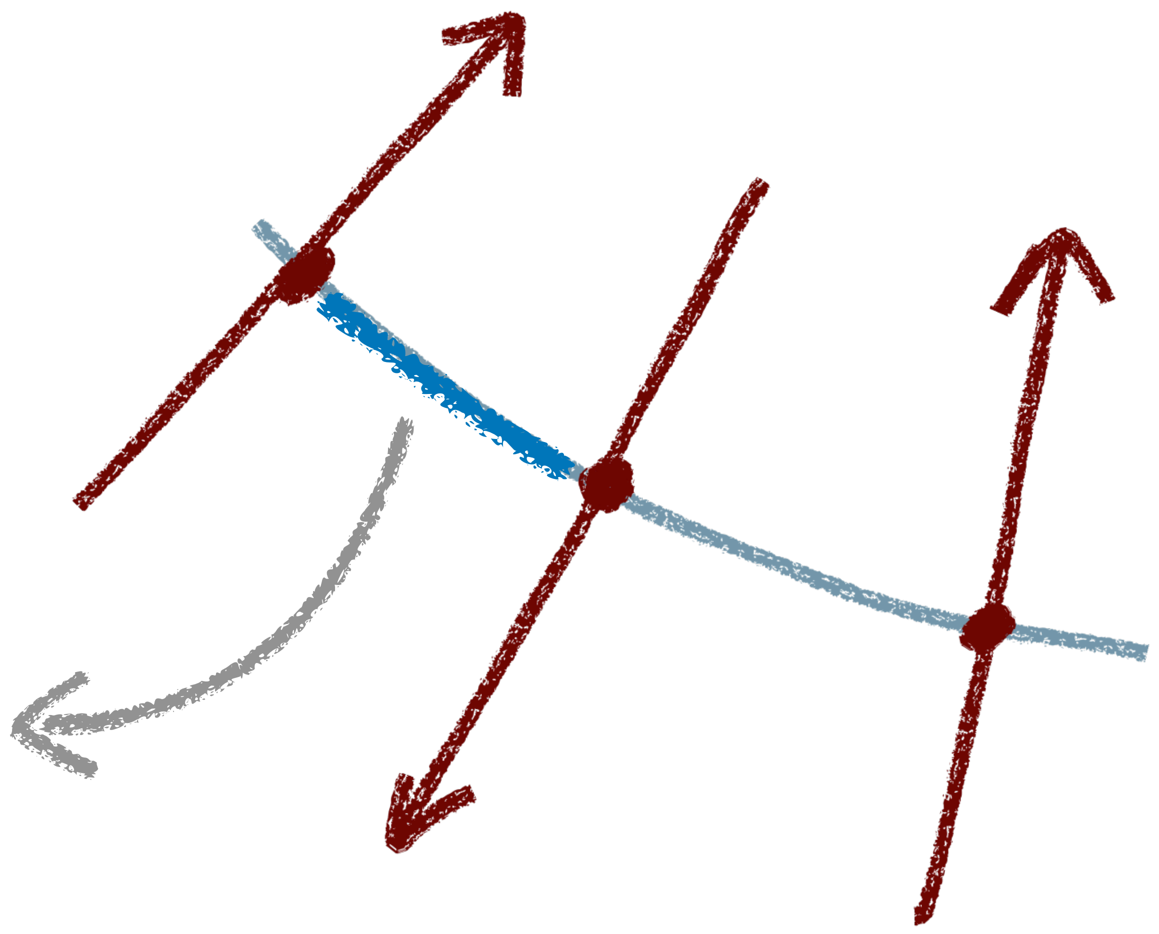
Elasticity of a Möbius strip

Definition of the shape of the strip

- discrete version of the strip
- lattice of « directions »
- but elasticity requires vectors !
- link variable $\eta_{i,i+1}$: coherence of site orientations
- orientability:

$$\mathcal{O} = \prod_{i=1}^{N} \eta_{i,i+1} = \begin{cases} +1 & \text{orientable} \\ -1 & \text{non-orientable} \end{cases}$$





$$\eta_{i,i+1} = \mathrm{sgn}(\mathbf{b}_i \cdot \mathbf{b}_{i+1})$$

Elasticity of a Möbius strip

Definition of the shape of the strip

- discrete version of the strip
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- but elasticity requires vectors !
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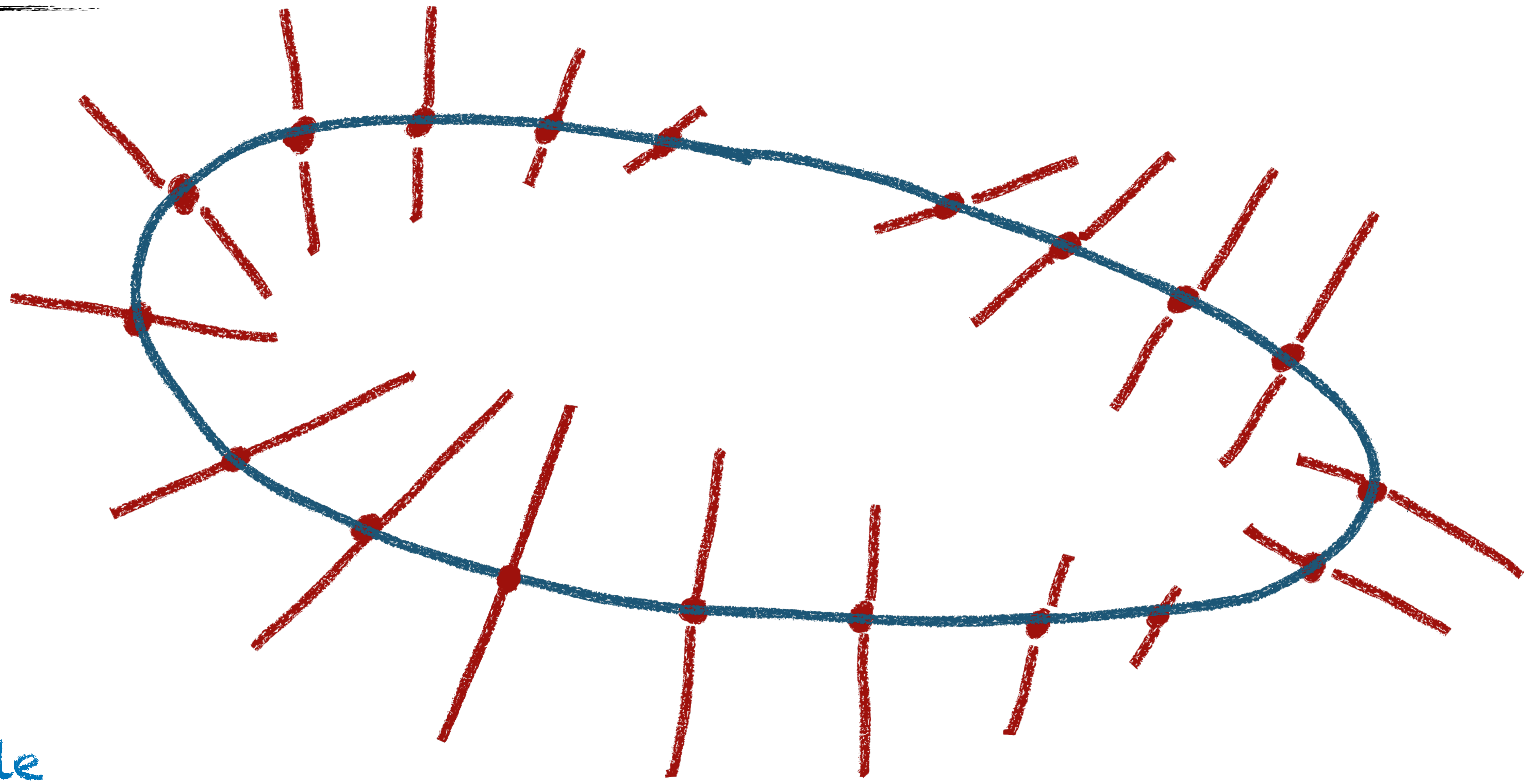
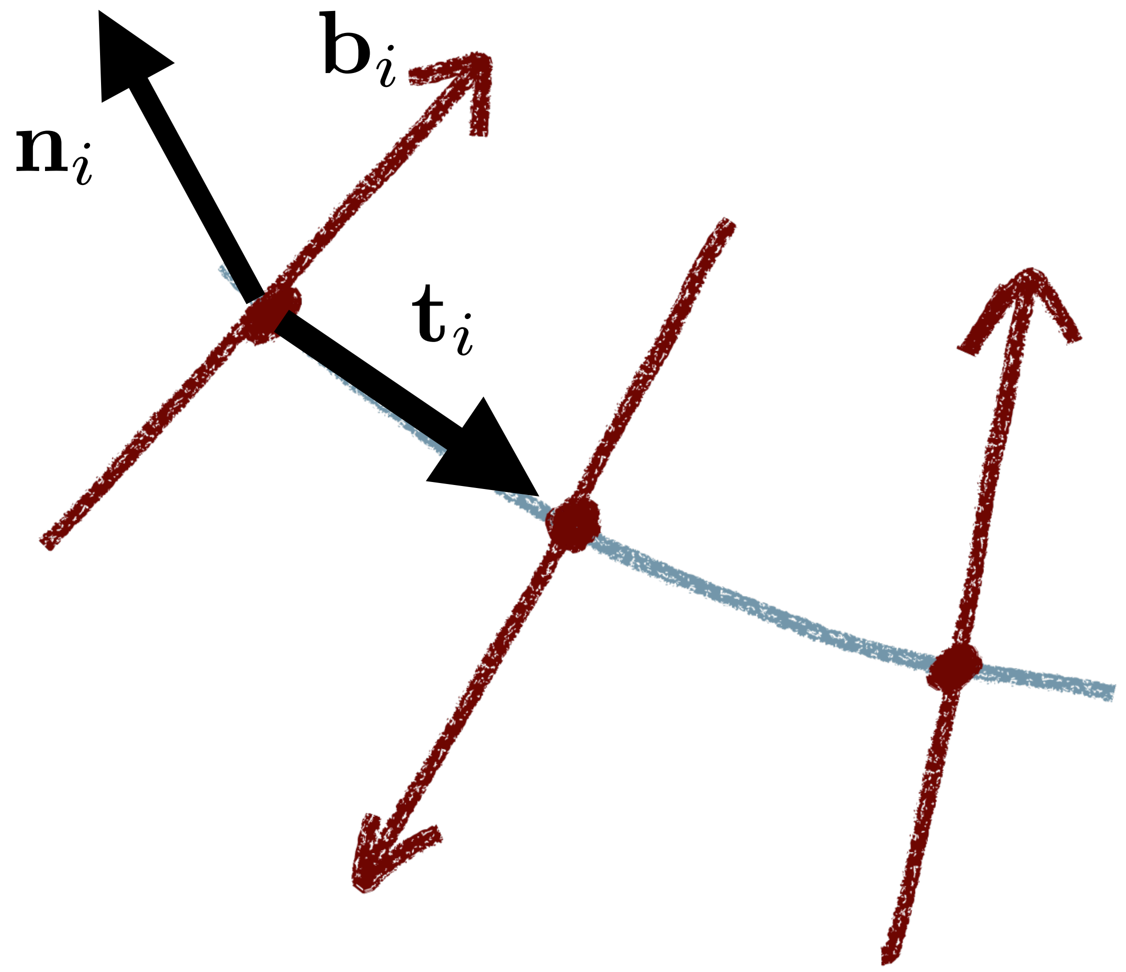
- local basis $\mathbf{t}_i, (\epsilon_i)\mathbf{b}_i, (\epsilon_i)\mathbf{n}_i$





Elasticity of a Möbius strip

Shear deformations

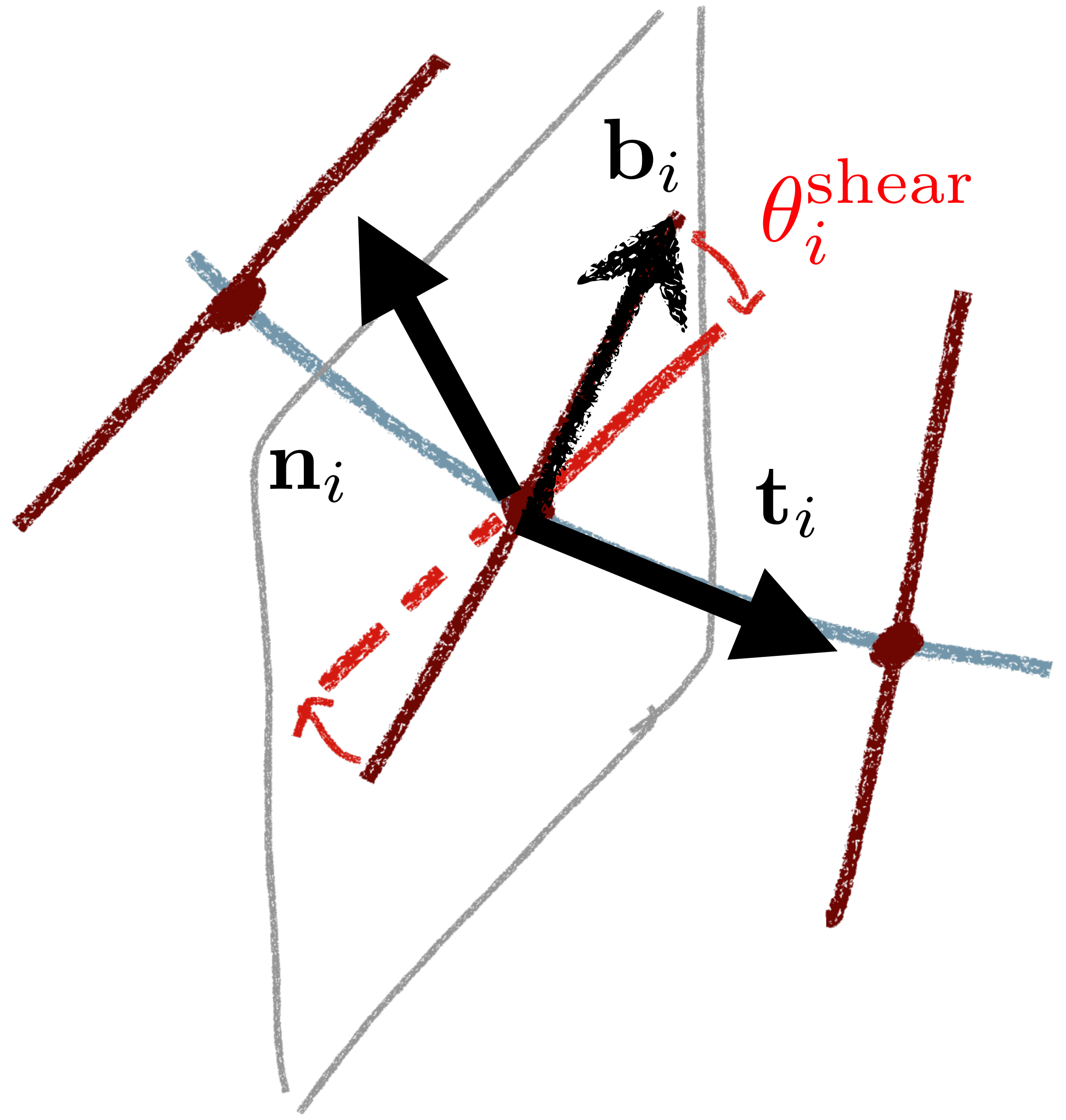
- deformation field: $\epsilon_i \mathbf{u}_i = \epsilon_i \theta_i^{\text{shear}} \ \mathbf{t}_i^0$.
- elastic energy: $\frac{E}{N} = \sum_i \frac{K_i^{\text{s}}}{2} \left[\theta_{i+1}^{\text{shear}} - \eta_{i,i+1} \theta_i^{\text{shear}}\right]^2$

Z_2 gauge theory

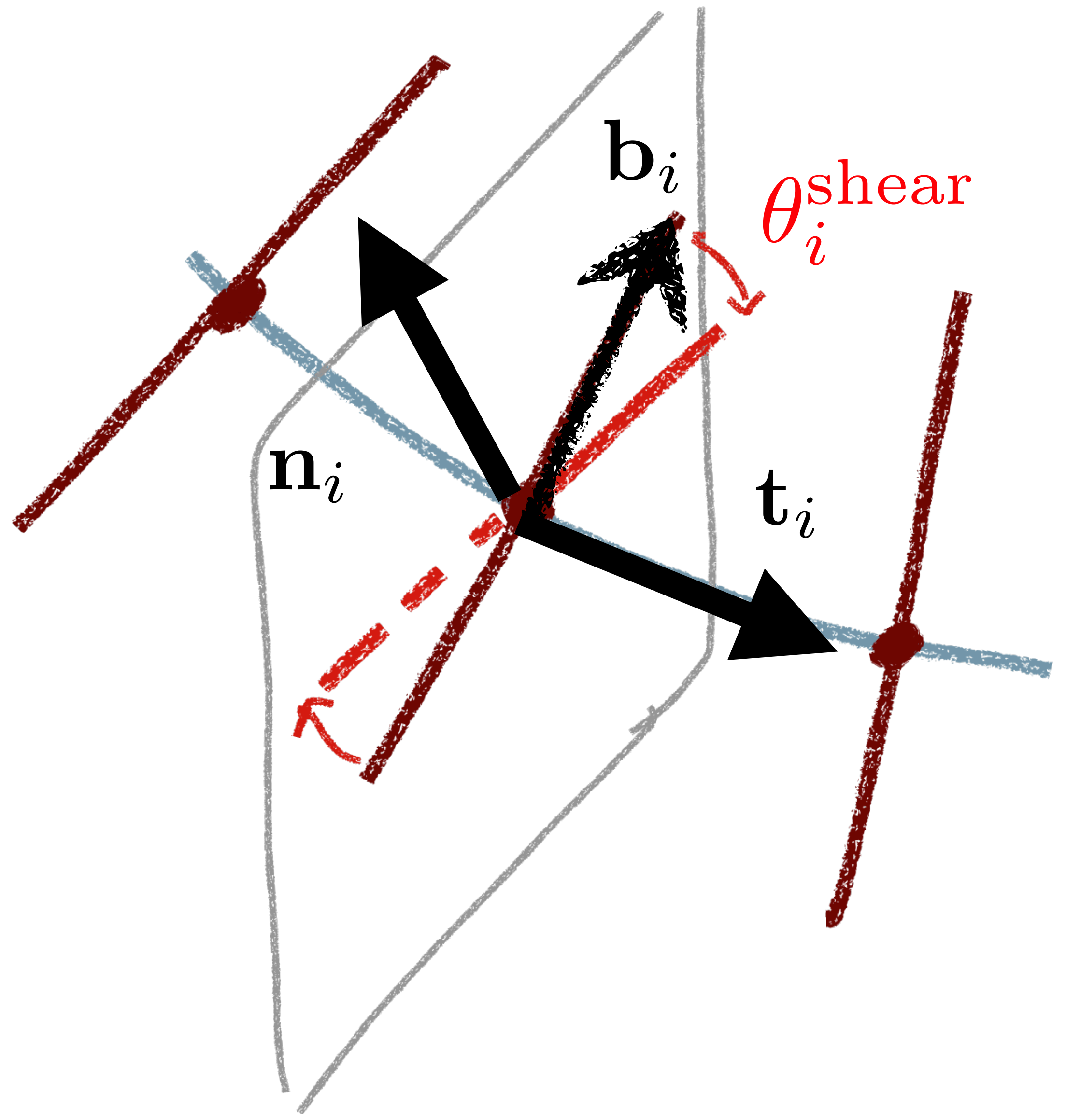
- continuous elasticity: $E = \int \frac{K_{\text{s}}}{2} \left(\partial_s \theta^{\text{shear}}\right)^2$.

additional condition: $\theta^{\text{shear}}(s_0) = 0$ if $\mathcal{O} = -1$

with s_0 free (degree of freedom)



Elasticity of a Möbius strip



Shear deformations

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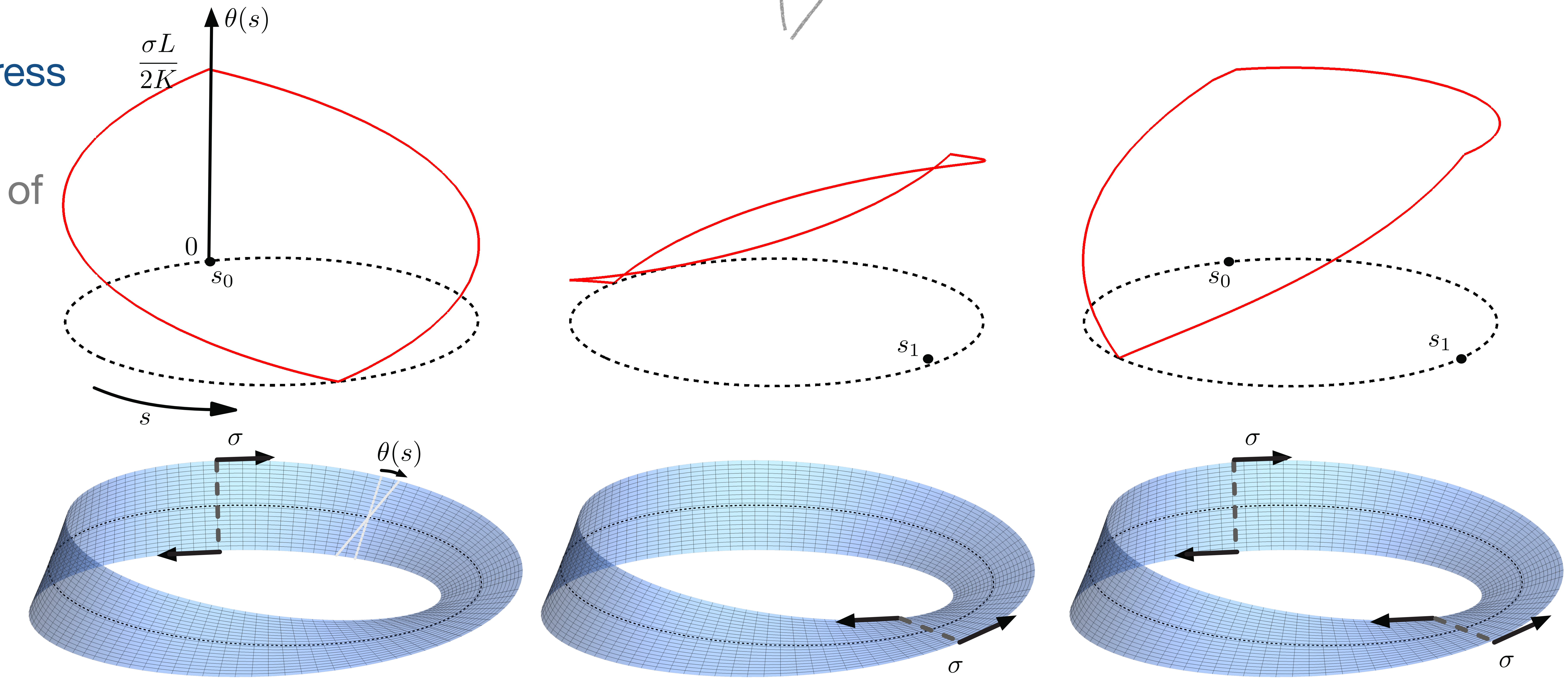
  additional condition: $\theta^{\text{shear}}(s_0) = 0 \text{ if } \mathcal{O} = -1$

  with s_0 free (degree of freedom)

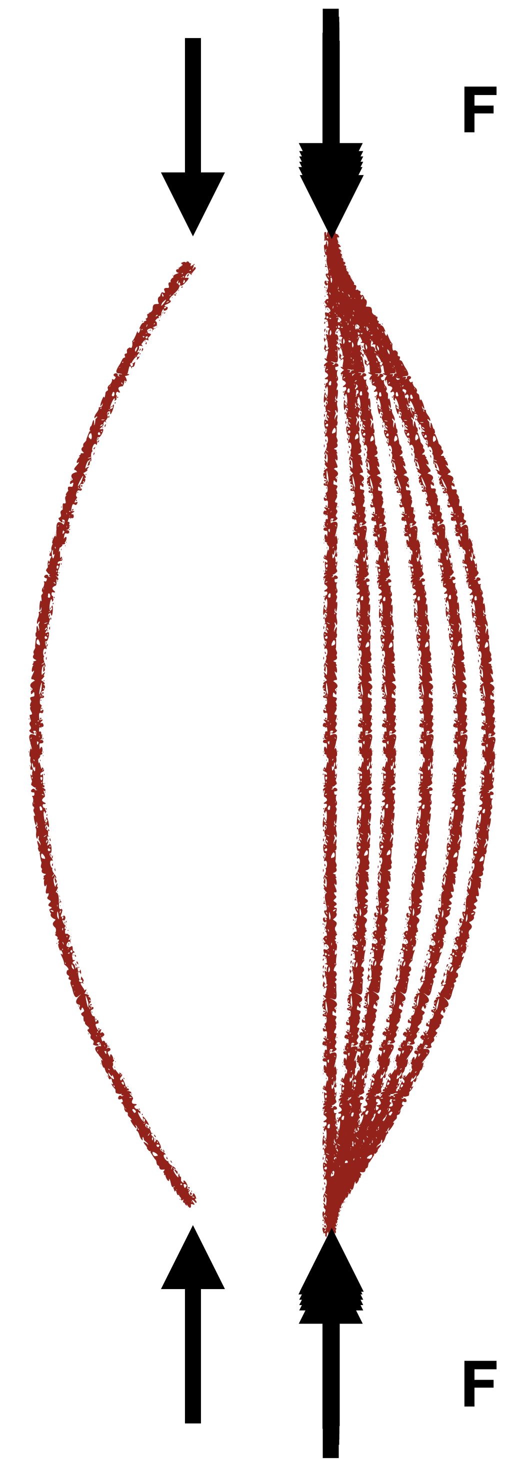
Response to local shear stress

- non linear response
- depends on « history » of constraints



Buckling of a Möbius strip

- What is buckling ?
  - nonlinear relation F ↔ deformation
  - can be applied homogeneously on the Möbius strip



Buckling of a Möbius strip

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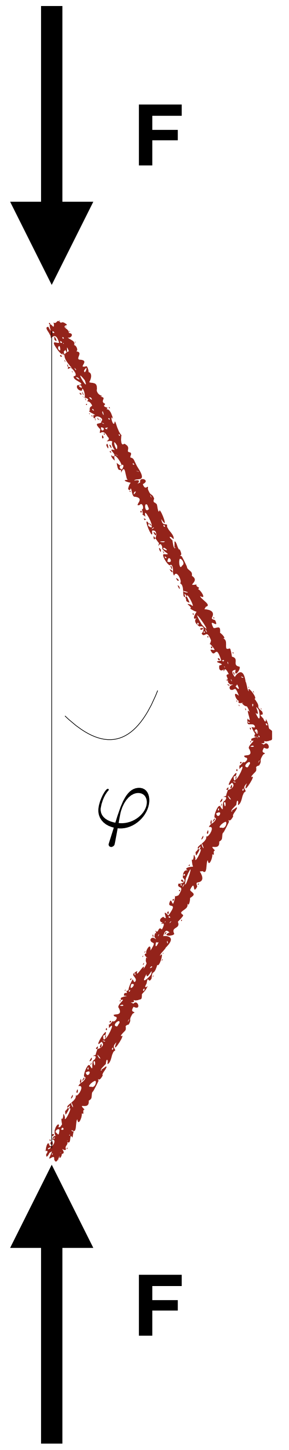
- Buckling of the Möbius strip
  - continuous elasticity

$$\mathcal{E}_{\mathrm{B}} = \frac{1}{2}\int \left[K'_{\mathrm{B}}\,(\partial_s\varphi)^2 + K_{\mathrm{B}}\varphi^2 + \Sigma\ell\cos\varphi\right]\mathrm{d}s \quad \text{with} \quad \varphi(s_0)=0$$



Work of F

$$E = \frac{1}{2}K_B\varphi^2 - \underbrace{FL(1-\cos\varphi)}$$

buckling if $FL > K_B$

Buckling of a Möbius strip

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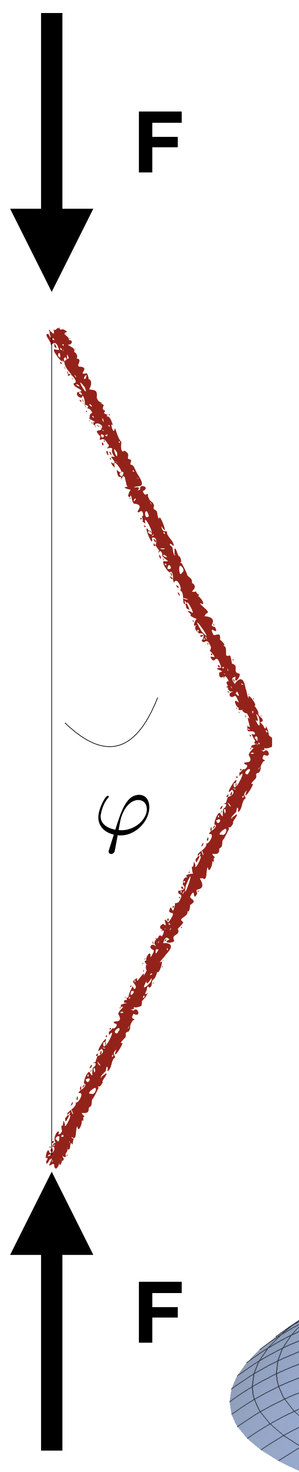

Work of F

$$E = \frac{1}{2}K_B\varphi^2 - \overbrace{FL(1-\cos\varphi)}^{\text{Work of F}}$$

buckling if $FL > K_B$

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  - continuous elasticity

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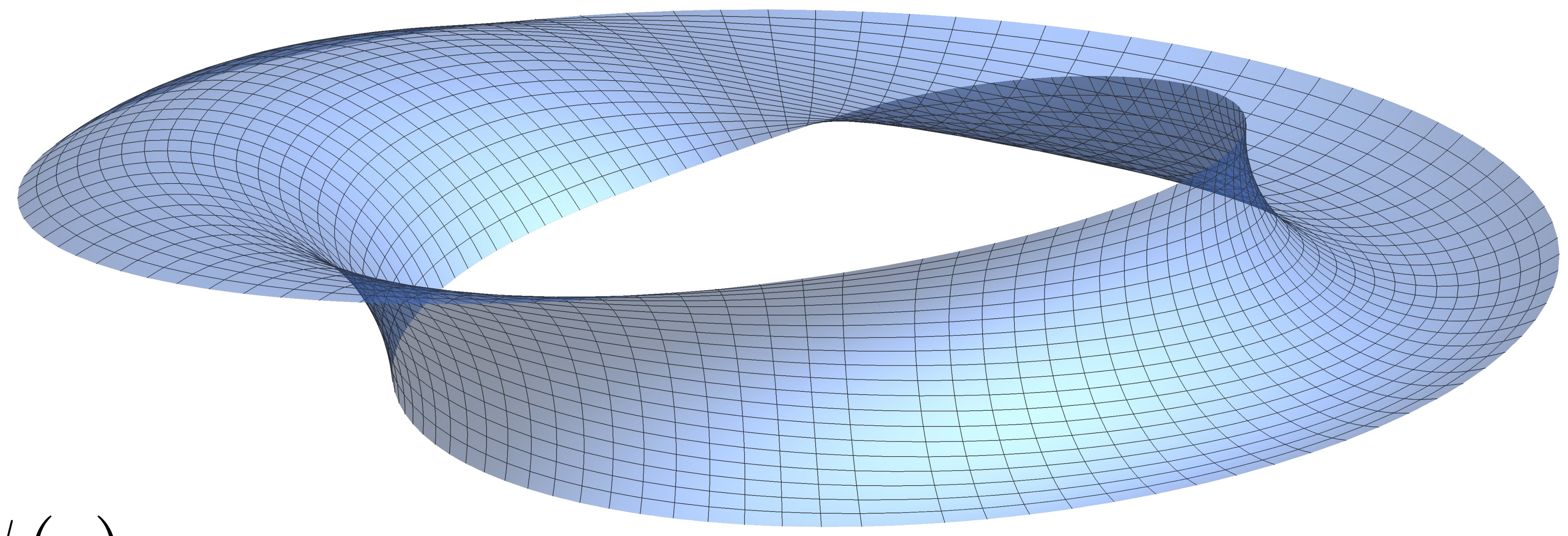
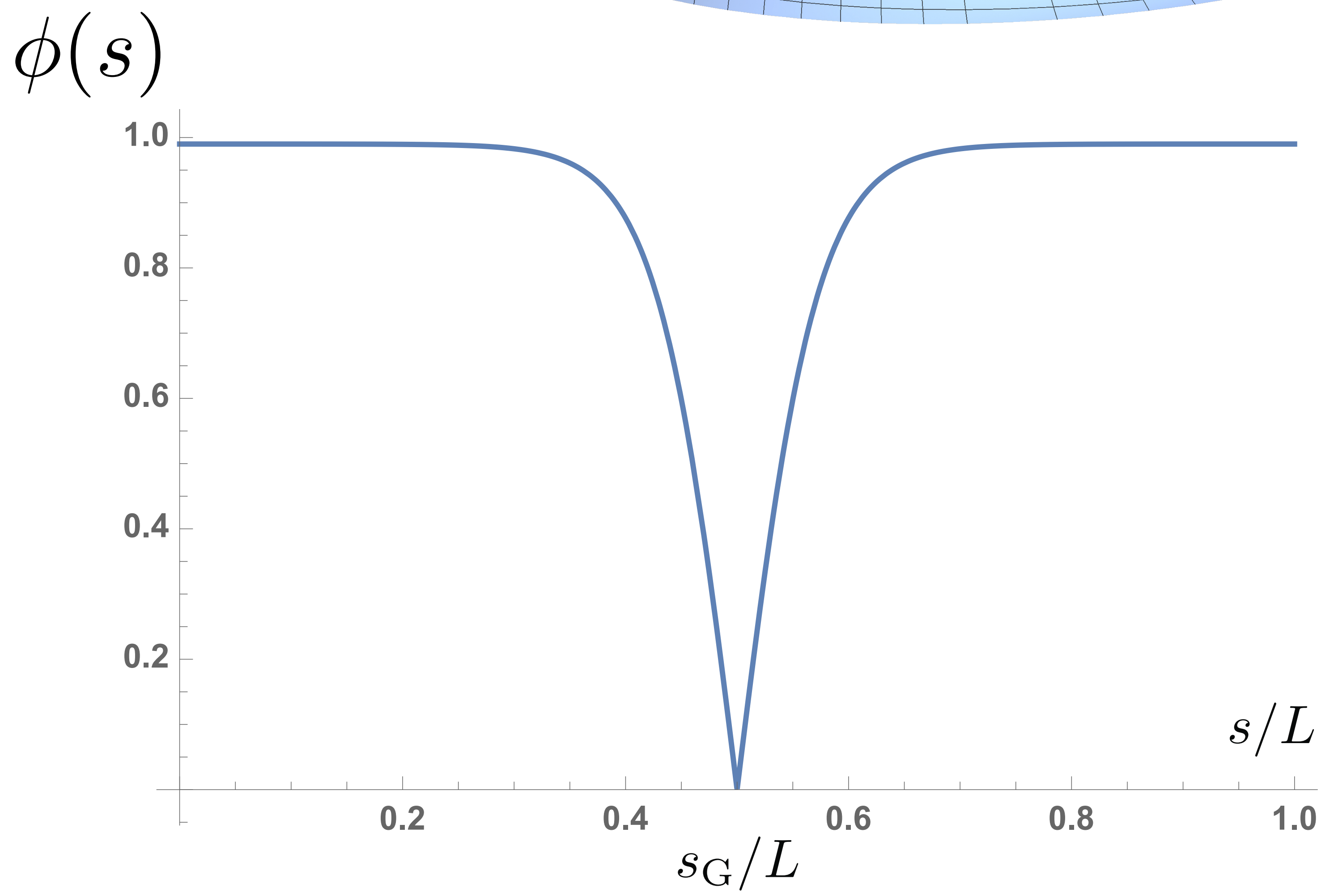
  - **1 point** along the strip remains **undistorted** !
  - topologically protected **solitary wave** of distortion along the strip
  - **topological Z_2 charge** (Witten-Olive 1978)





Buckling of a Möbius strip

Marcelo Guzman

- Buckling of the Möbius strip
  - continuous elasticity

$$\mathcal{E}_{\mathrm{B}} = \frac{1}{2}\int \left[K'_{\mathrm{B}} \left(\partial_s \varphi\right)^2 + K_{\mathrm{B}}\varphi^2 + \Sigma\ell\cos\varphi \right] \mathrm{d}s \quad \text{with} \quad \varphi(s_0) = 0$$

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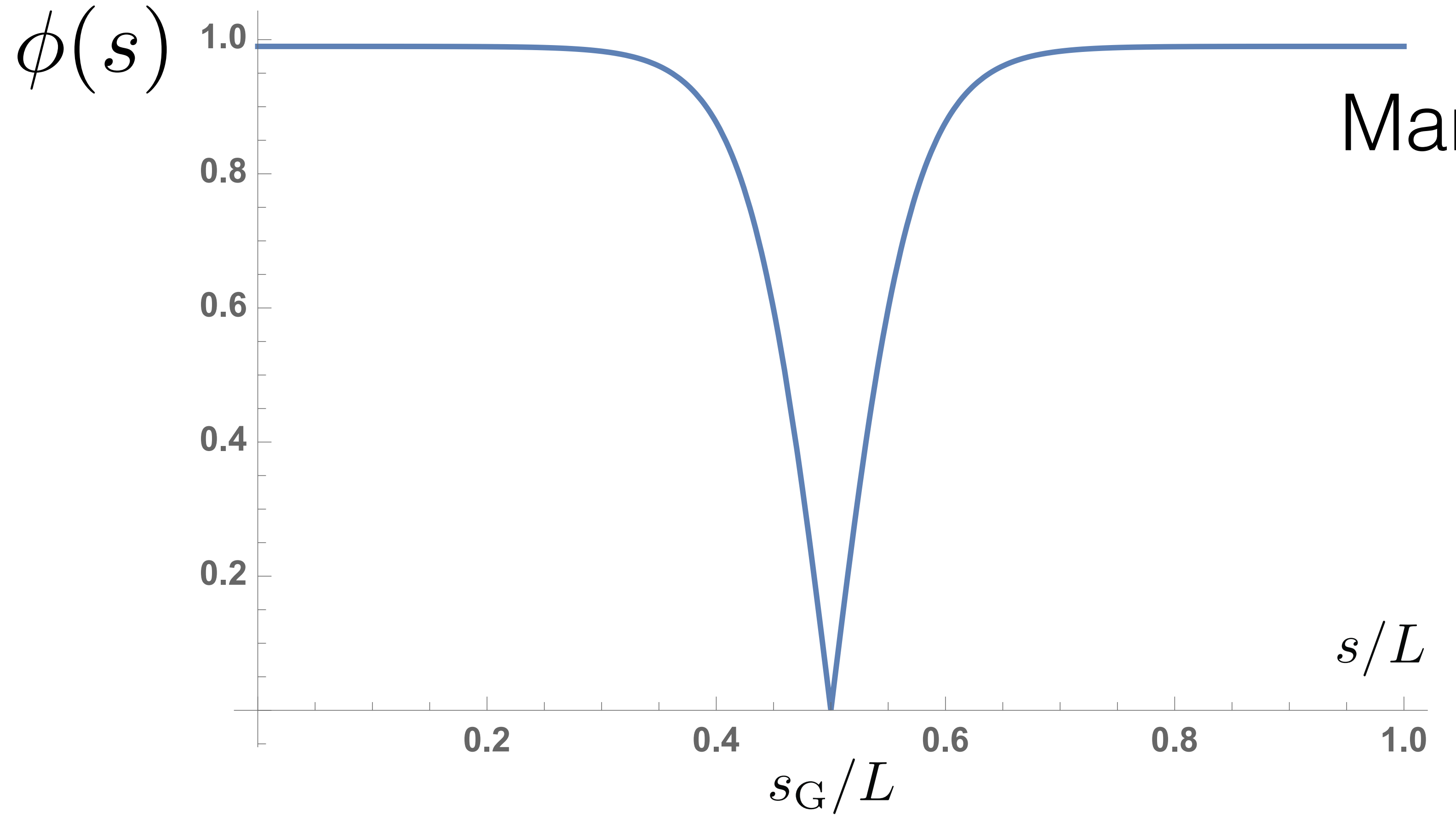
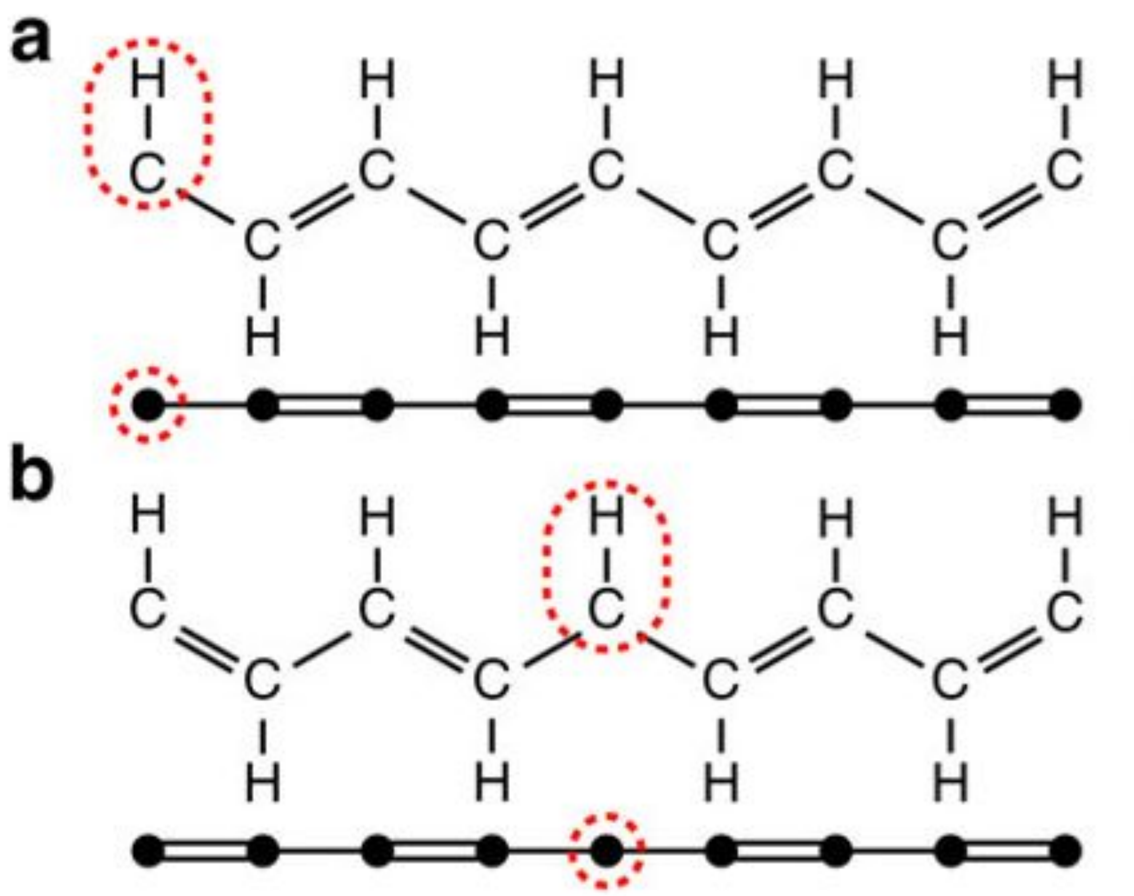


- Su–Schrieffer–Heeger model of polyacetylene
  - 2 configurations topologically distincts
  - boundary state between them = soliton !
  - same **topological Z_2 charge**

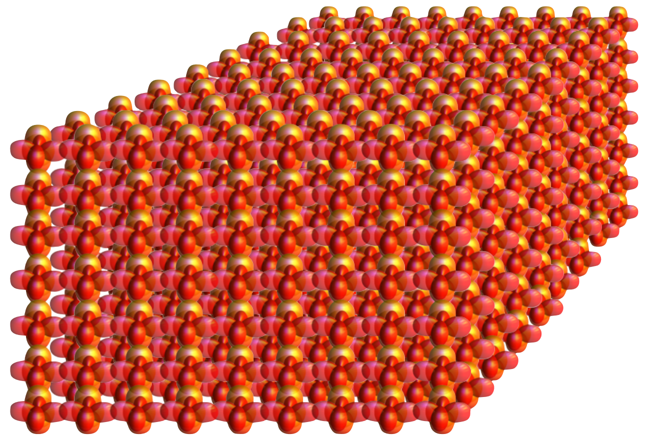


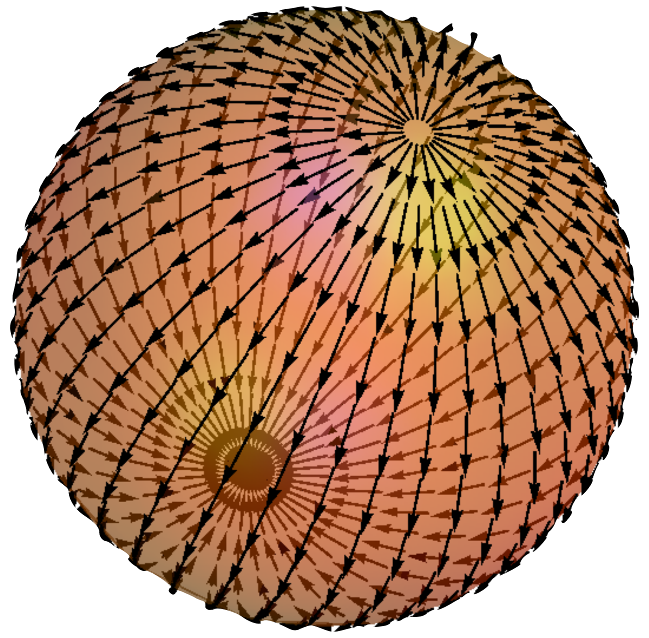
Möbius soliton : topological edge state without an edge !

Summary

1. Electronic Properties of Quantum Matter

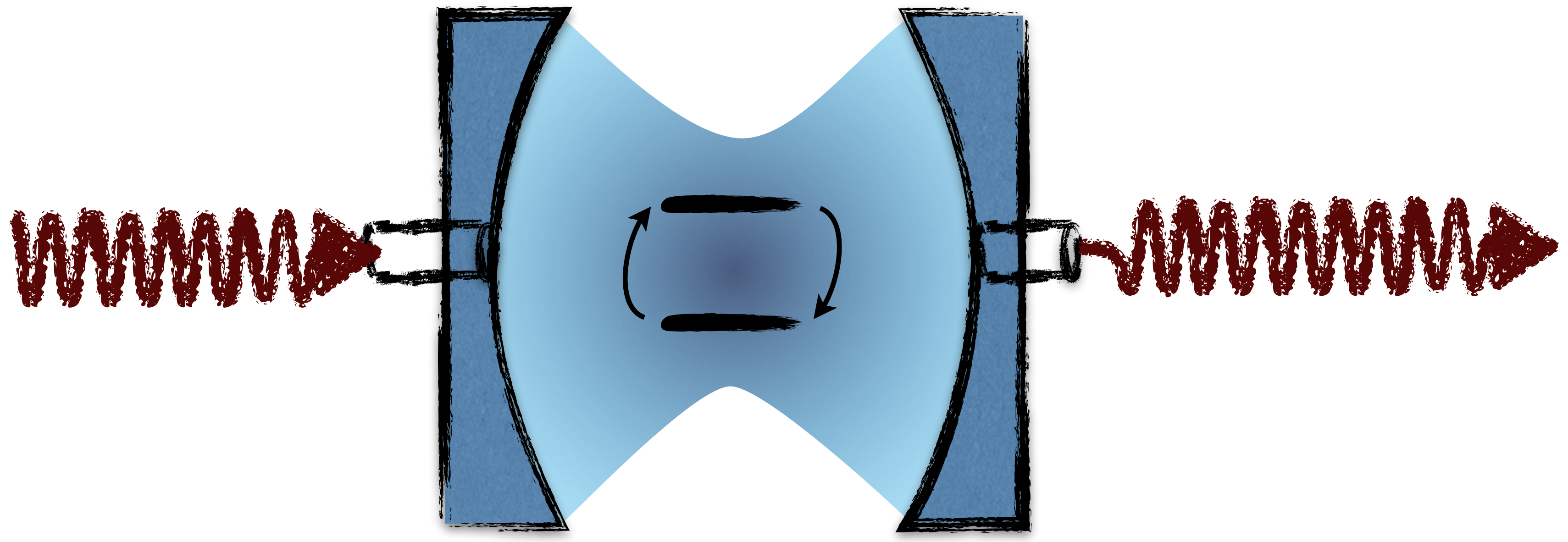
   Topological Insulators



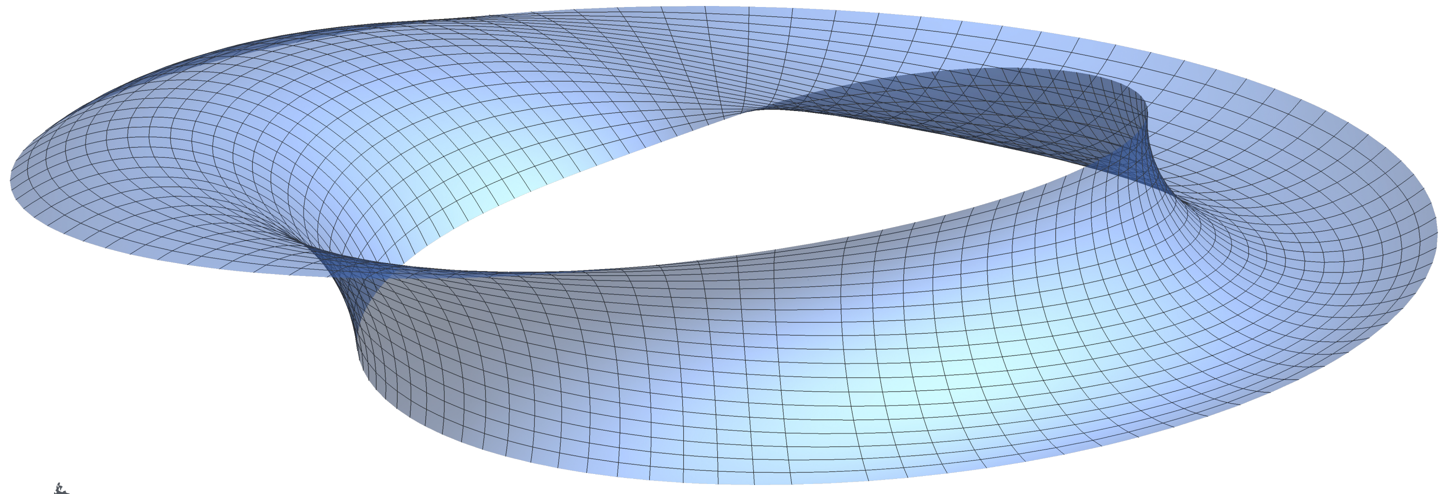


2. Quantum Technologies

   Topological pump



3. Mechanics / Metamaterials



What's next ? (any suggestion ?)

Topology and Symmetry (I)

Kitaev (2010)
Schnyder, Ryu, Furusaki, Ludwig (2008)

	Cartan Nomenclature	TRS	PHS	SLS	$d=1$	$d=2$	$d=3$
standard (Wigner-Dyson)	A (unitary)	0	0	0	-	$\mathbb{Z}$	-
	AI (orthogonal)	+1	0	0	-	-	-
	AII (symplectic)	−1	0	0	-	$\mathbb{Z}_2$	$\mathbb{Z}_2$
chiral (sublattice)	AIII (chiral unitary)	0	0	1	$\mathbb{Z}$	-	$\mathbb{Z}$
	BDI (chiral orthogonal)	+1	+1	1	$\mathbb{Z}$	-	-
	CII (chiral symplectic)	−1	−1	1	$\mathbb{Z}$	-	$\mathbb{Z}_2$
BdG	D	0	+1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	-
	C	0	−1	0	-	$\mathbb{Z}$	-
Superconductors	DIII	−1	+1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
	CI	+1	−1	1	-	-	$\mathbb{Z}$

Time Reversal Symmetry :

- anti unitary ($\Theta = UK$)
- states at same E, $[H, \Theta] = 0$
- $\Theta^2 = \pm\mathbb{I}$

Particle Hole Symmetry :

- anti unitary ($P = VK$)
- states at opposite E, $\{H, P\} = 0$
- $P^2 = \pm\mathbb{I}$

Chiral / Sublattice Symmetry :

- unitary
- states at opposite E, $\{H, C\} = 0$
- $C = \Theta.P$

Symmetries and topology: classification

Kitaev (2010)
Schnyder, Ryu, Furusaki, Ludwig (2008)

Here: only general (ubiquitous) symmetries
Other symmetries: see T. Neupert lectures

(superconductors)

Time Reversal Symmetry :

- anti unitary ($\Theta = UK$)
- states at same E, $[H, \Theta] = 0$
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- anti unitary ($P = VK$)
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Symmetries and topology : classification

Particle Hole (superconductors)

Sub-Lattice (Chiral)

Time Reversal

dimension

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Symmetry Operator : U

0 : no symmetry

+1 : symmetry with $U^2 = \mathbb{I}$

-1 : symmetry with $U^2 = -\mathbb{I}$

10 classes :

TRS : **x3** (0,+1,-1)

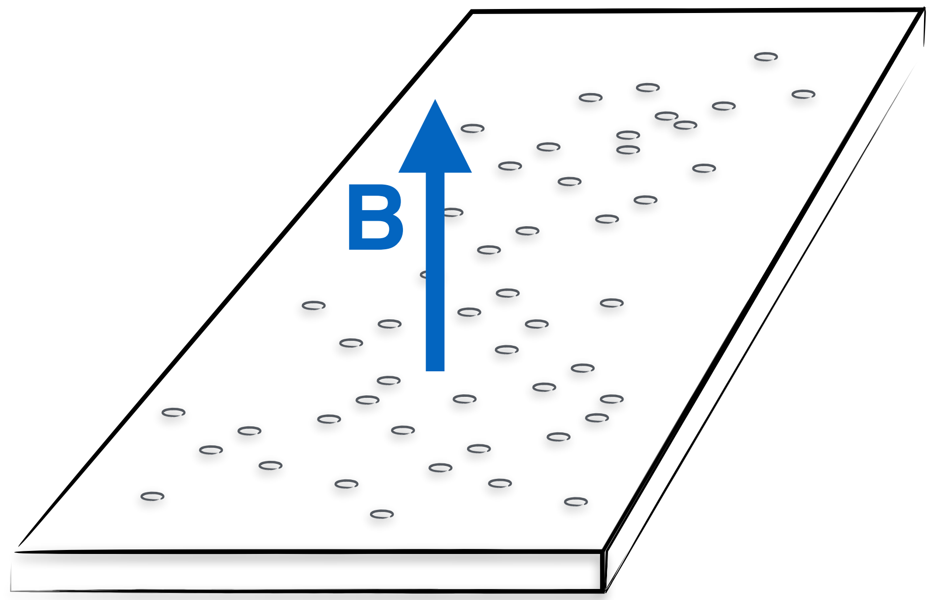
PHS : **x3** (0,+1,-1)

SLS = TRS . PHS (**+1**)

Symmetries and topology : classification

Chern insulators : ex. Quantum Hall Effect

- d=2
- breaks all symmetries (TRS)
- Topological index : Chern number



2DEG (Heterojunction GaAs/AlGaAs)

Thouless, Kohmoto, Nightingale and den Nijs (1982)
Niu, Thouless, and Wu (1985)
Haldane (1985)

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Symmetries and topology : classification

Topological insulators :

- d=2 and d=3
- TRS with spin 1/2 : spin-orbit
- Topological index : Kane-Mele

Kane and Mele (2005)
Bernevig, Hughes, and Zhang (2006)
Fu, Kane et Mele (2007)
Moore and Balents (2007)
Roy (2009)
Fu and Kane (2007)

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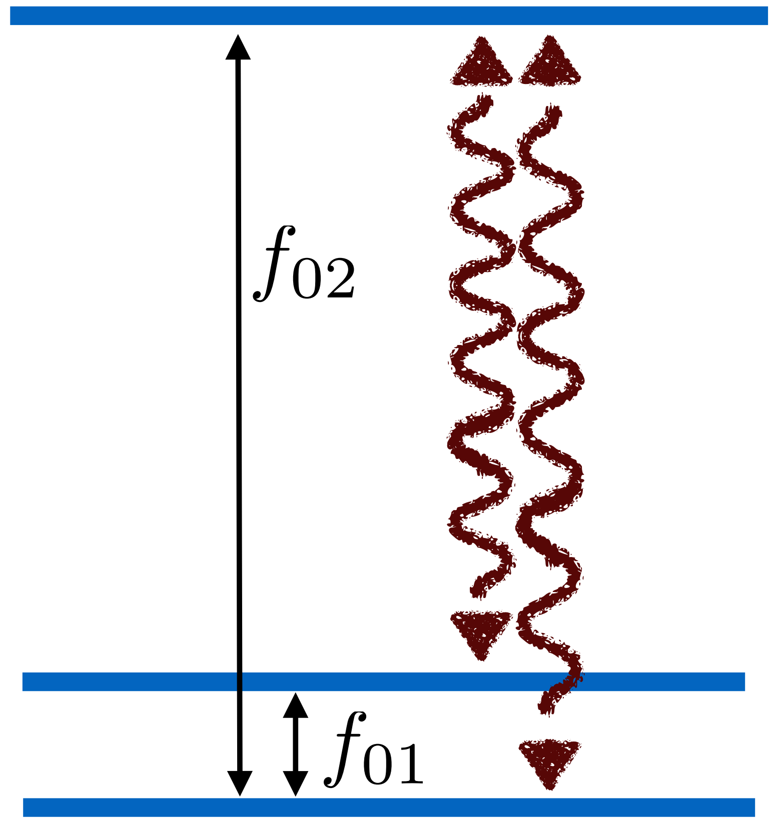
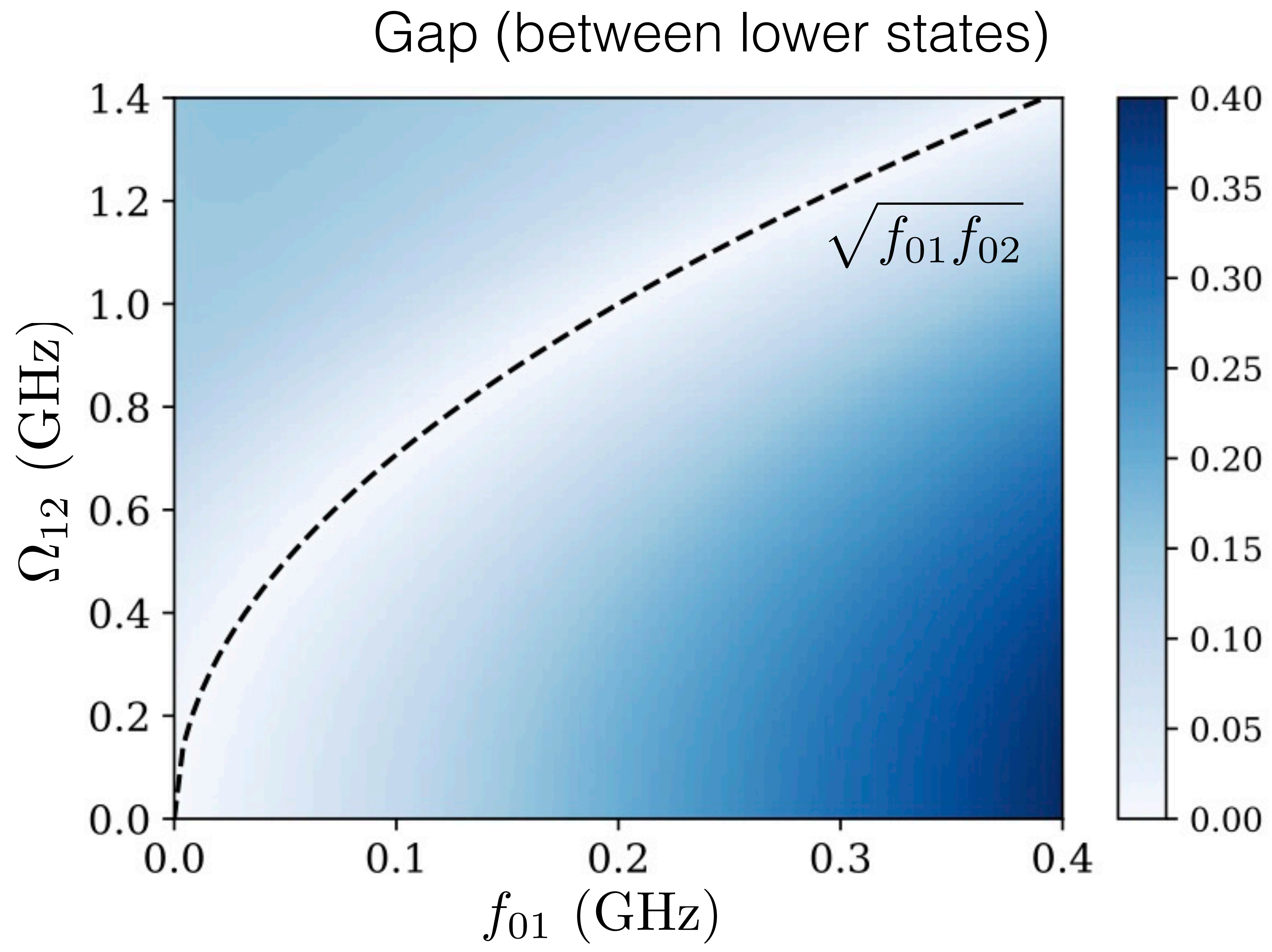
Topological Pump : effective 2 levels (qubit)

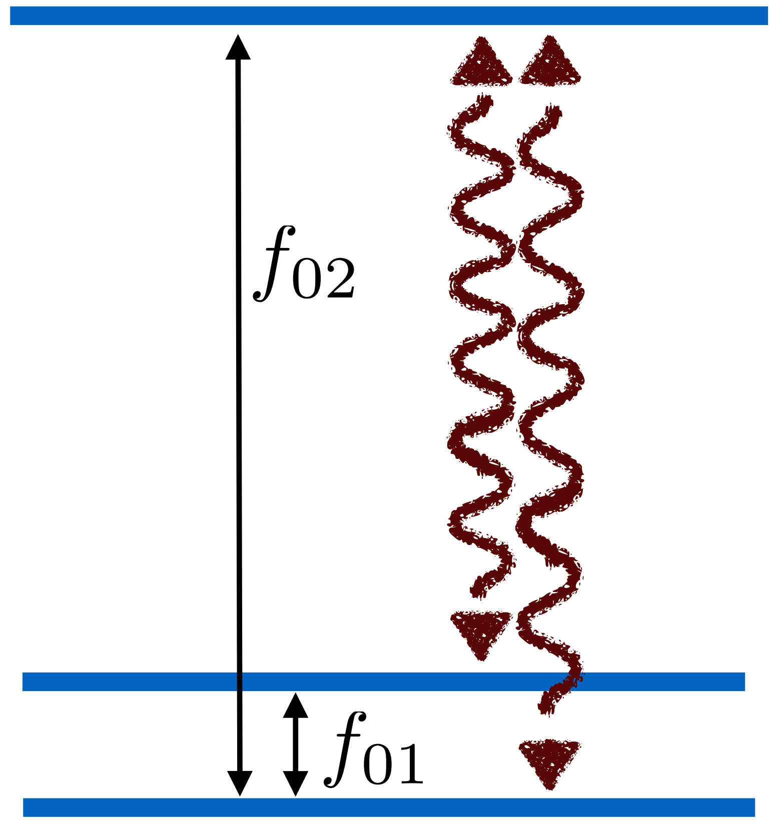


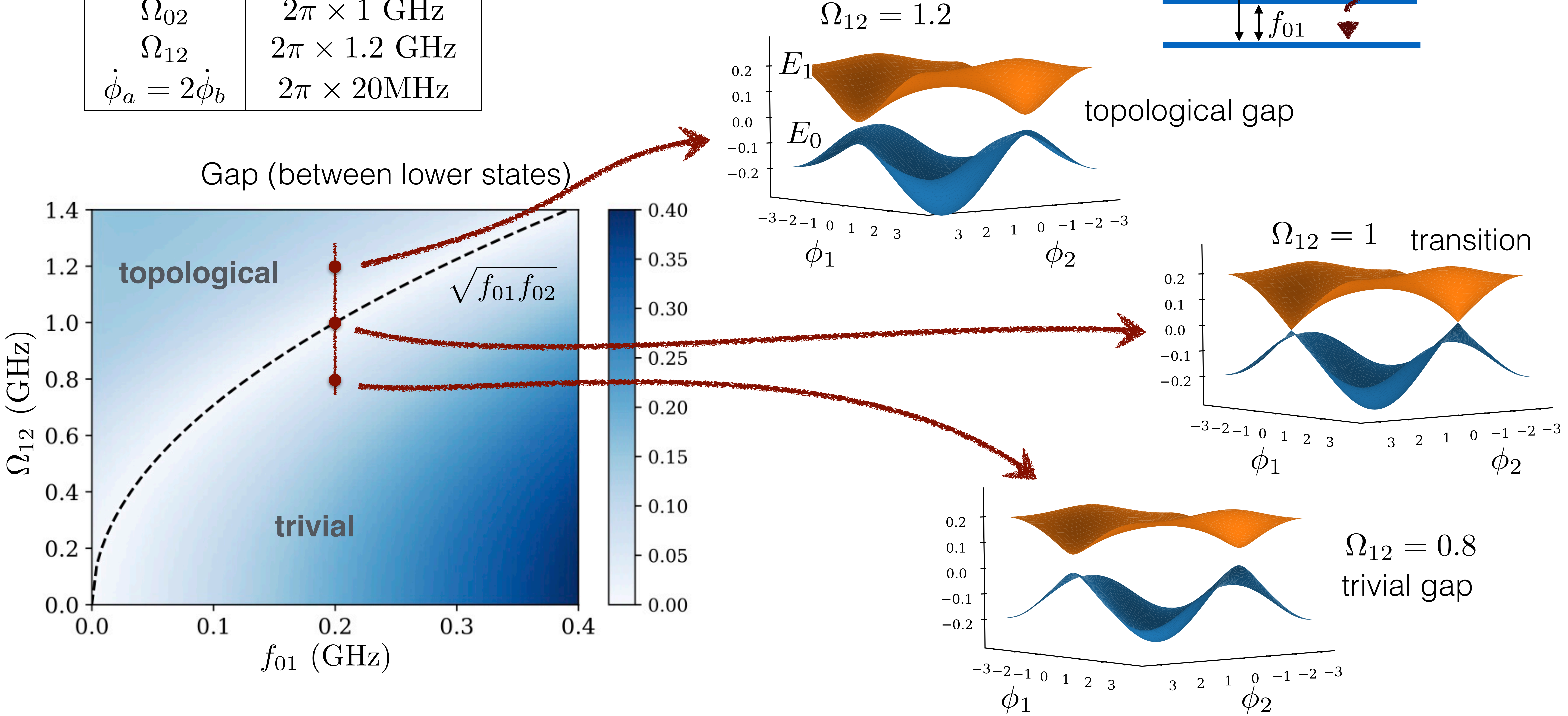
ω_{01}	$2\pi \times 200$ MHz
ω_{02}	$2\pi \times 5$ GHz
Ω_{01}	$2\pi \times 100$ MHz
Ω_{02}	$2\pi \times 1$ GHz
Ω_{12}	$2\pi \times 1.2$ GHz
$\dot{\phi}_a = 2\dot{\phi}_b$	$2\pi \times 20$MHz



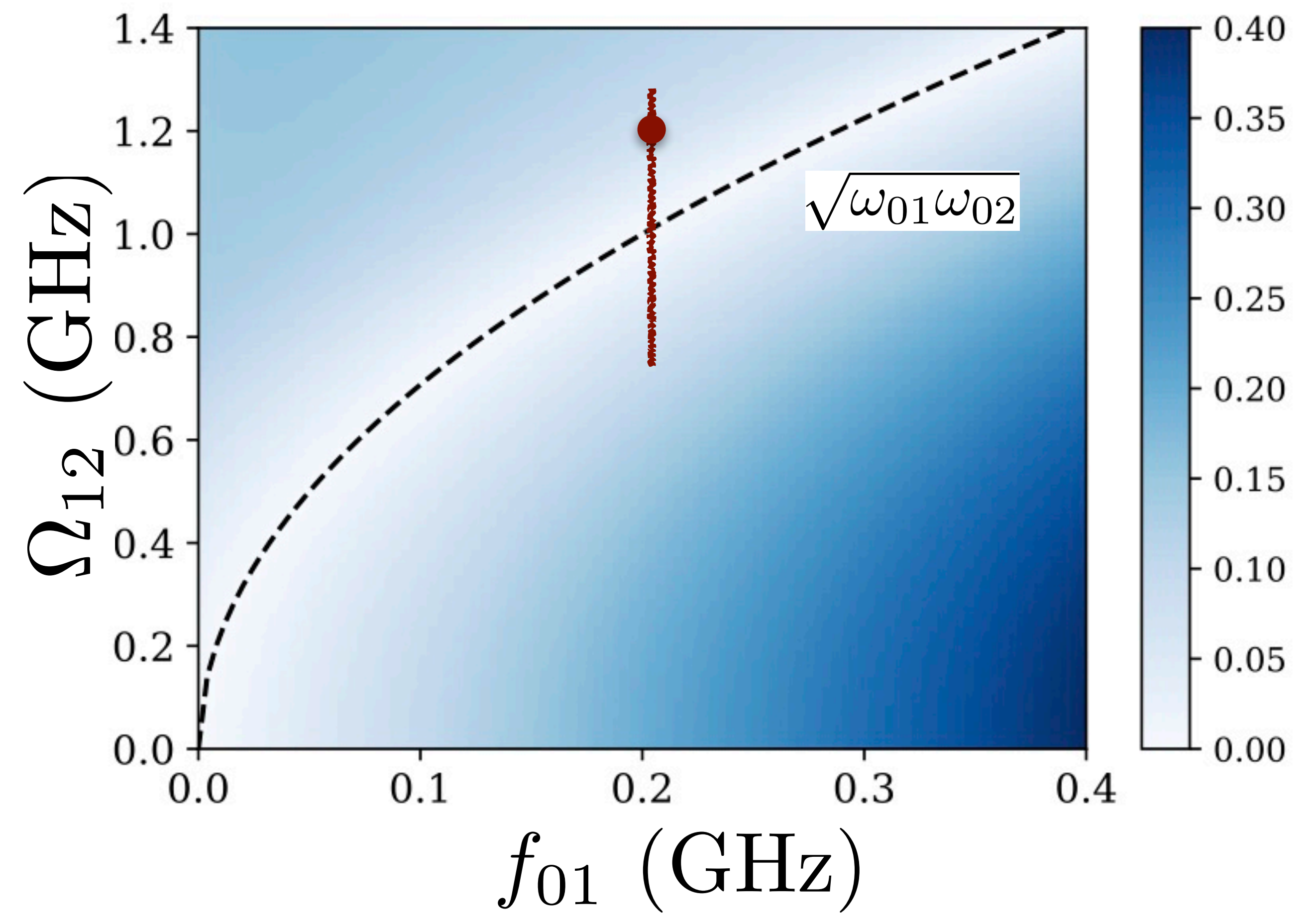
Topological Pump : effective 2 levels (qubit)

ω_{01}	$2\pi \times 200$ MHz
ω_{02}	$2\pi \times 5$ GHz
Ω_{01}	$2\pi \times 100$ MHz
Ω_{02}	$2\pi \times 1$ GHz
Ω_{12}	$2\pi \times 1.2$ GHz
$\dot{\phi}_a = 2\dot{\phi}_b$	$2\pi \times 20$MHz





Topological Pump : effective 2 levels (qubit)

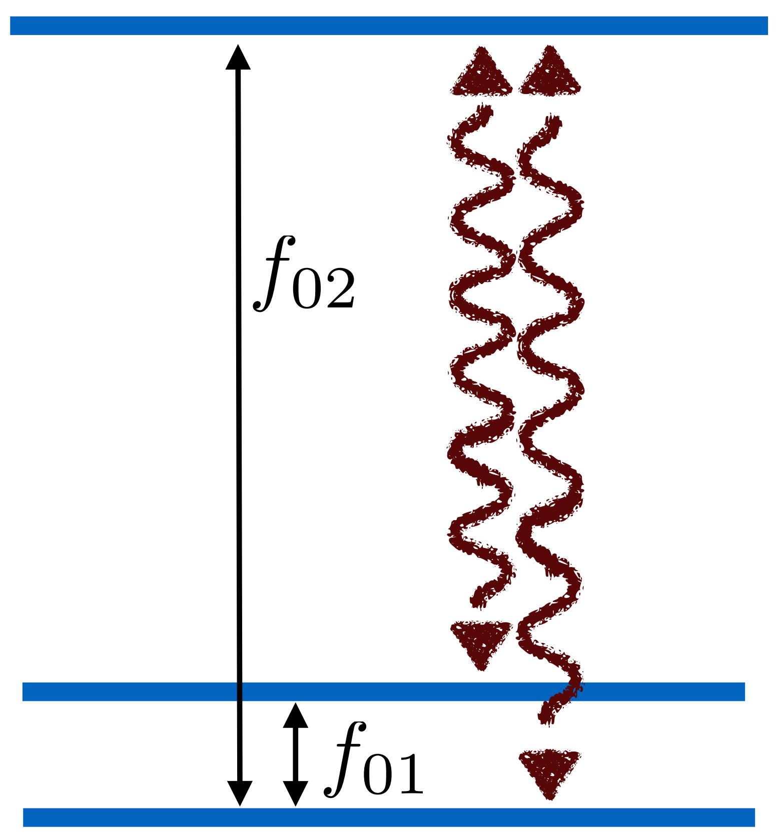


$\omega_1 = 10$ MHz

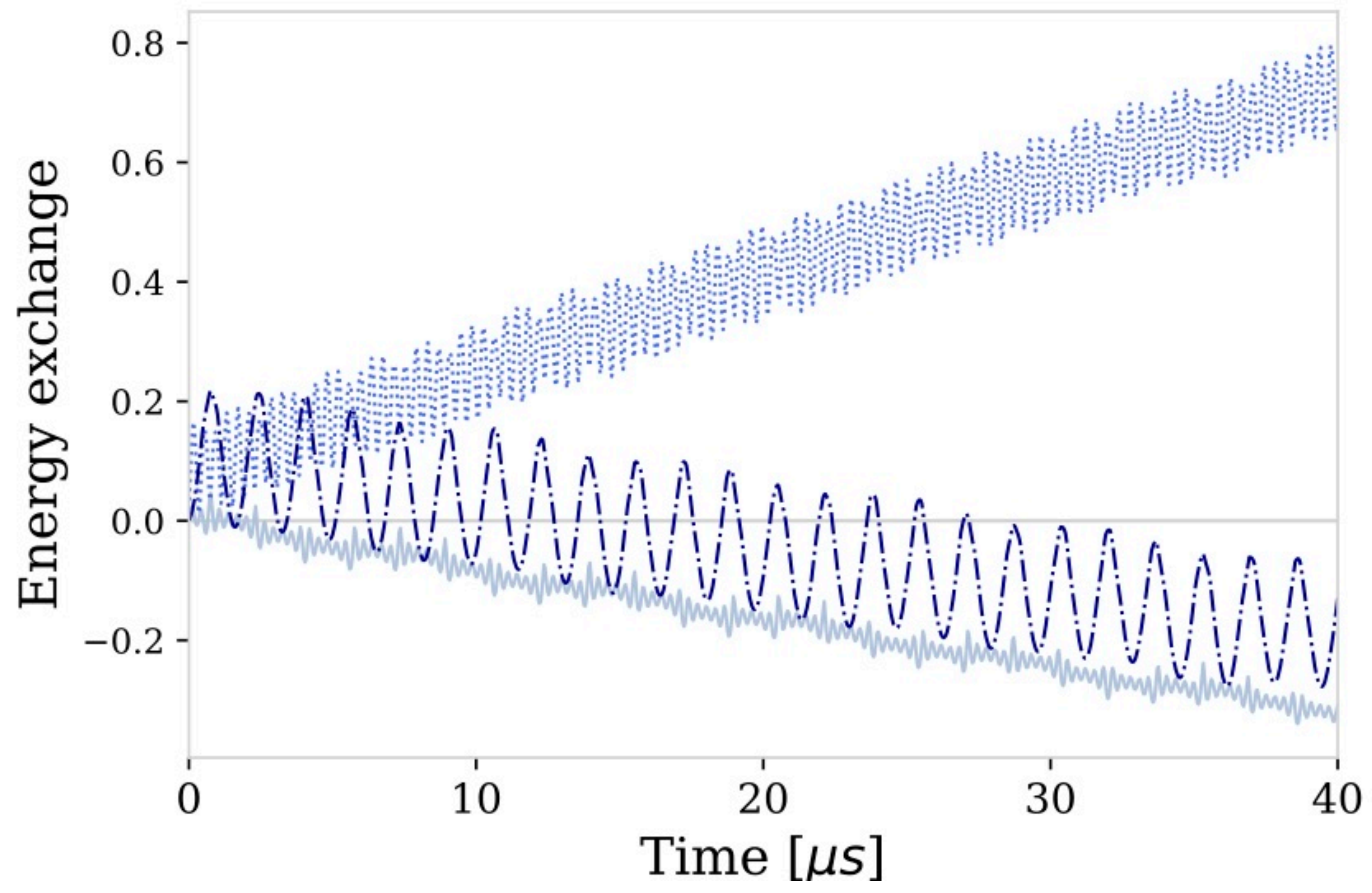
$\omega_2 \simeq 8$ MHz

$\Omega_{12} = 1.2$

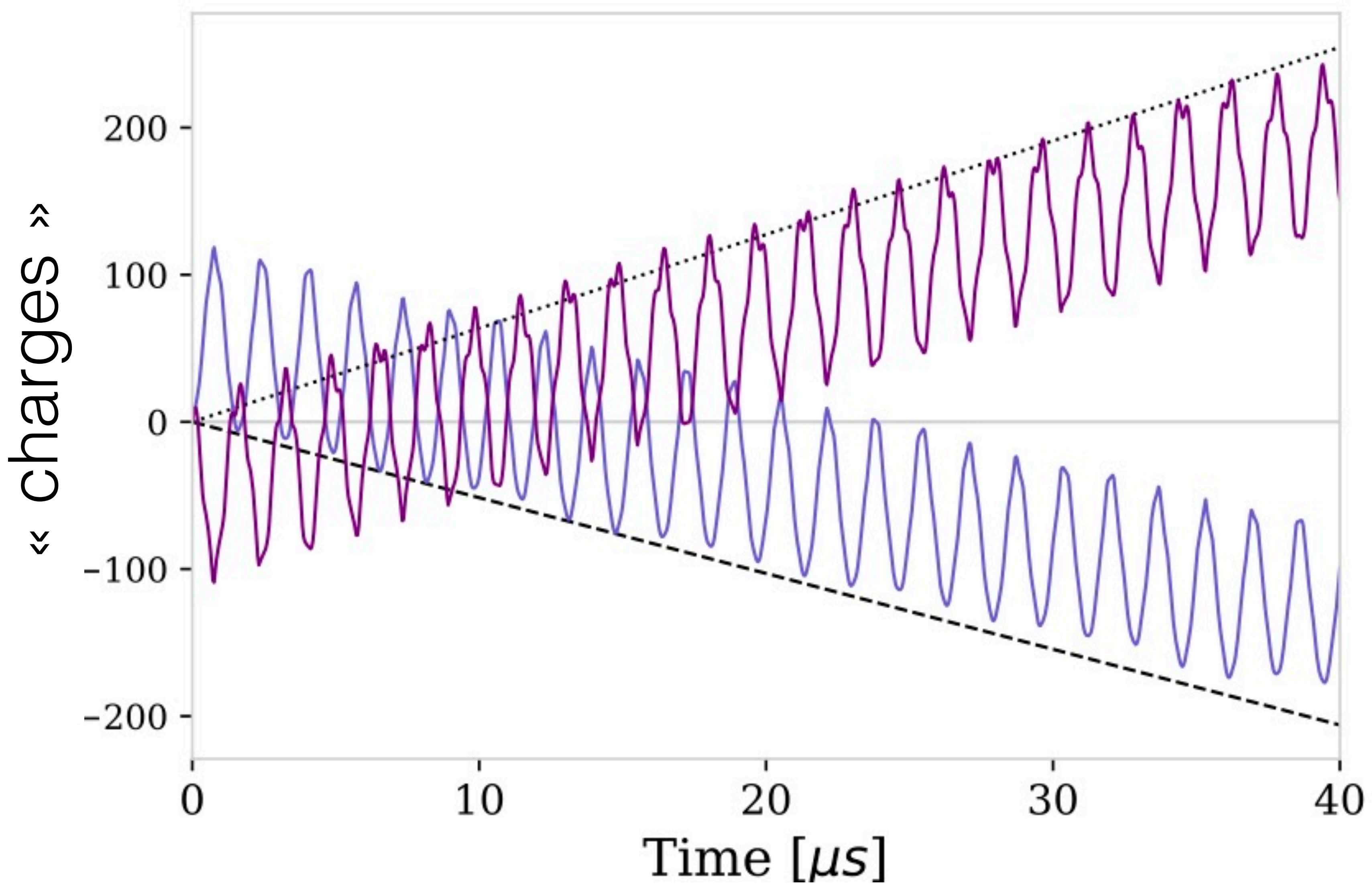
Prepare qutrit in ground state $|\psi_0\rangle$



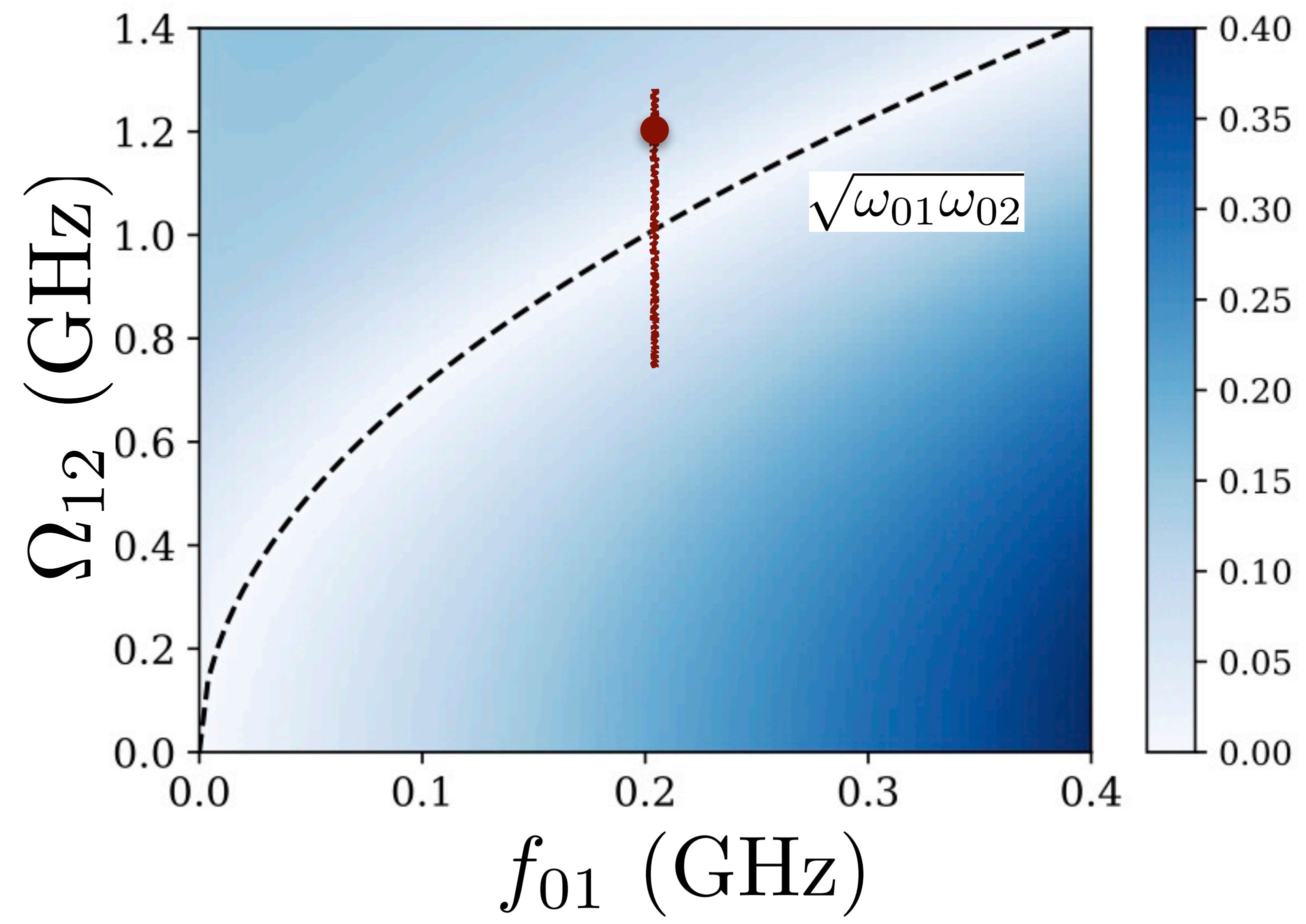
Energy exchange $\Delta\mathcal{E}_i$



« charges » $n_1 + n_3, n_2 - n_3$



Topological Pump : effective 2 levels (qubit)

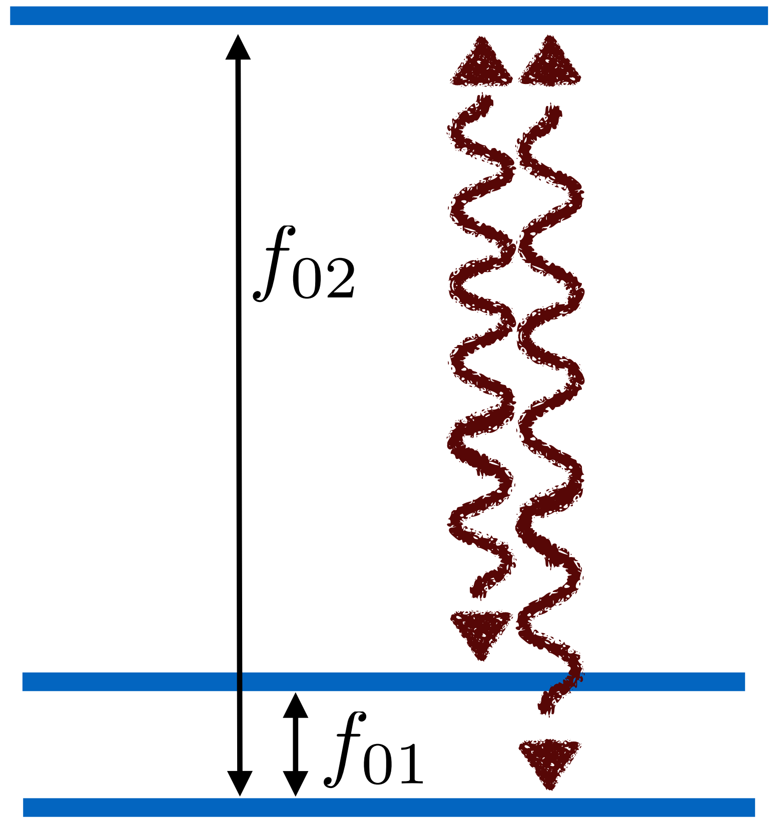


$\omega_1 = 10$ MHz

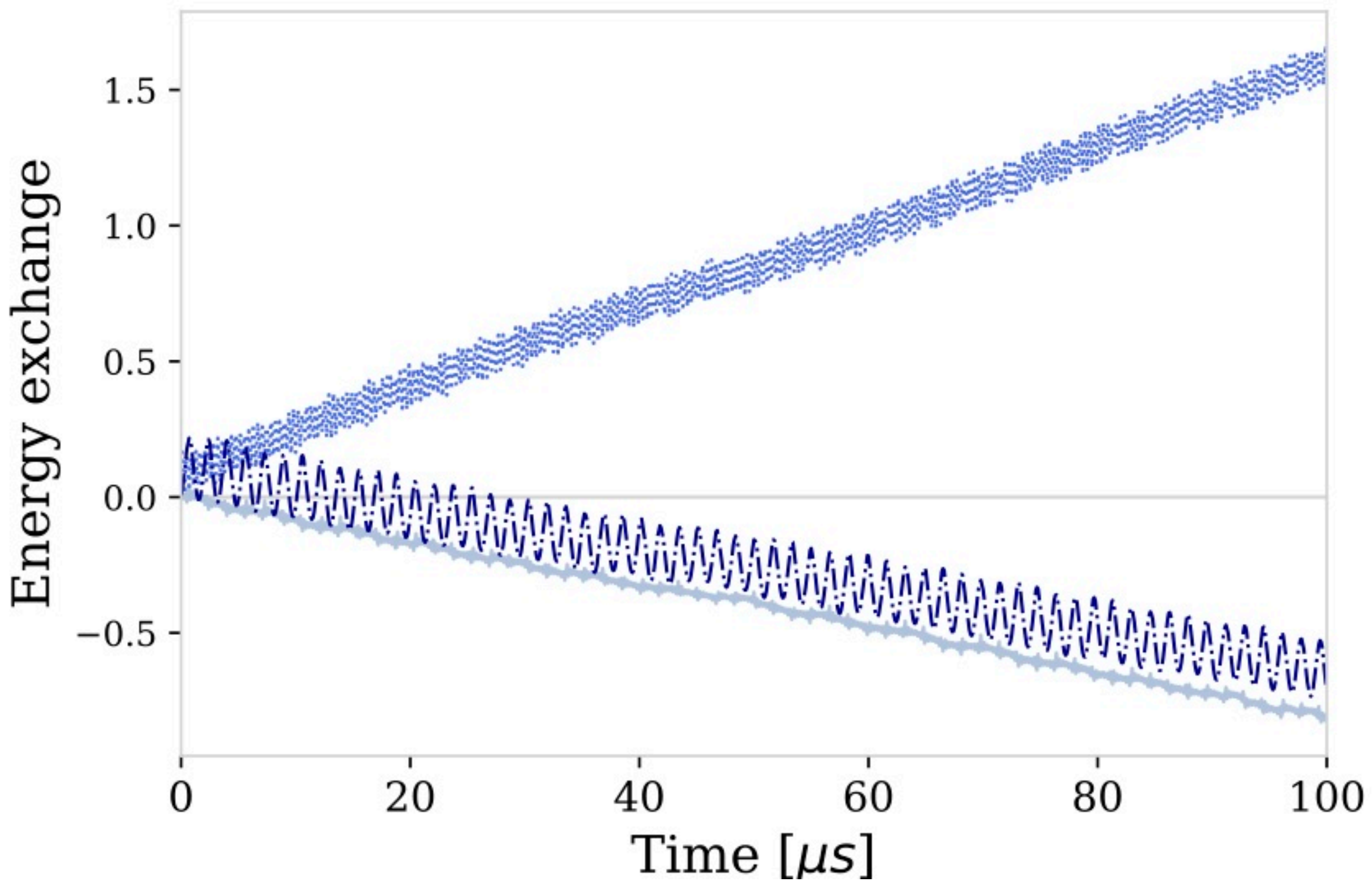
$\omega_2 \simeq 8$ MHz

$\Omega_{12} = 1.2$

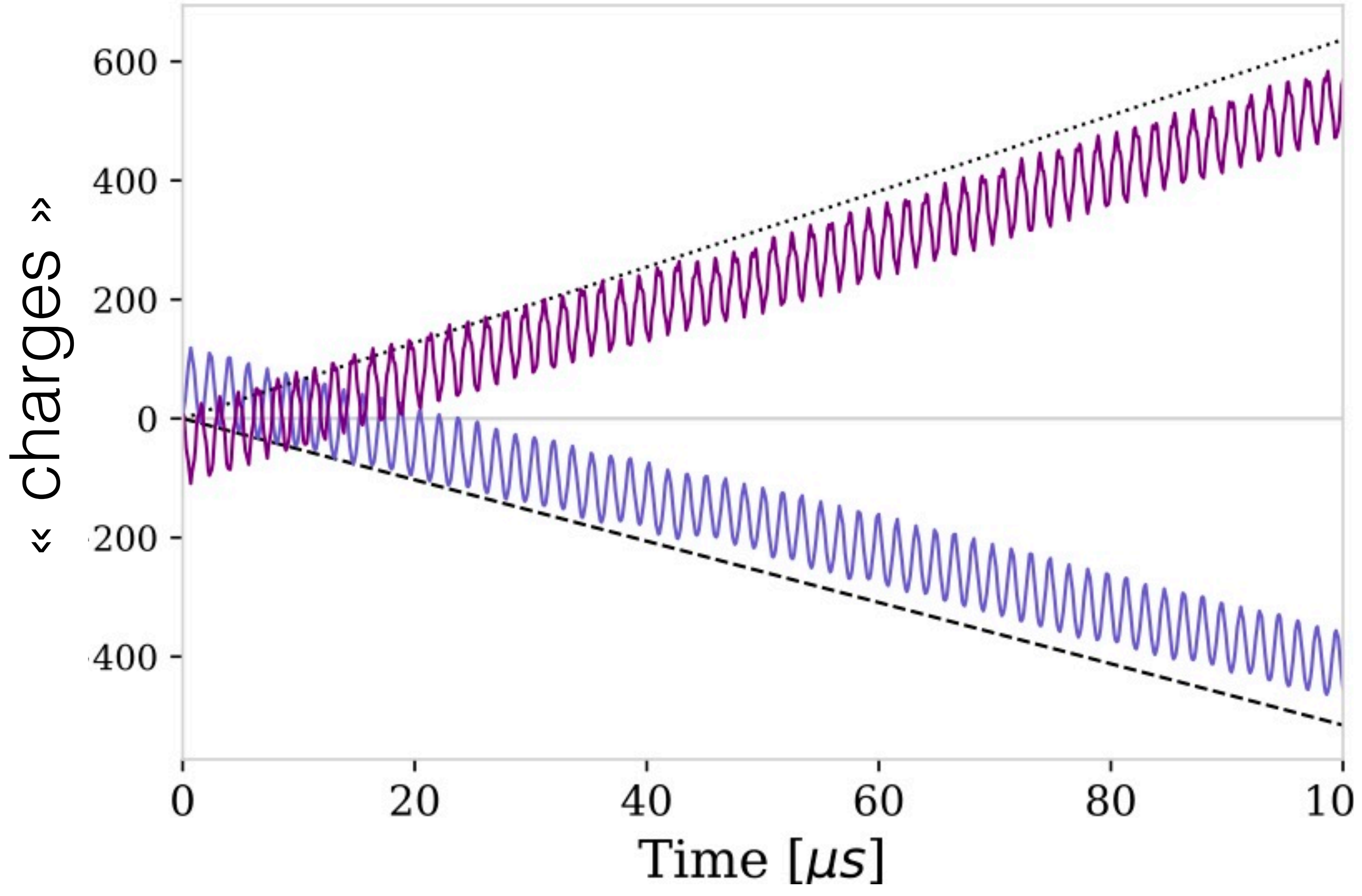
Prepare qutrit in ground state $|\psi_0\rangle$

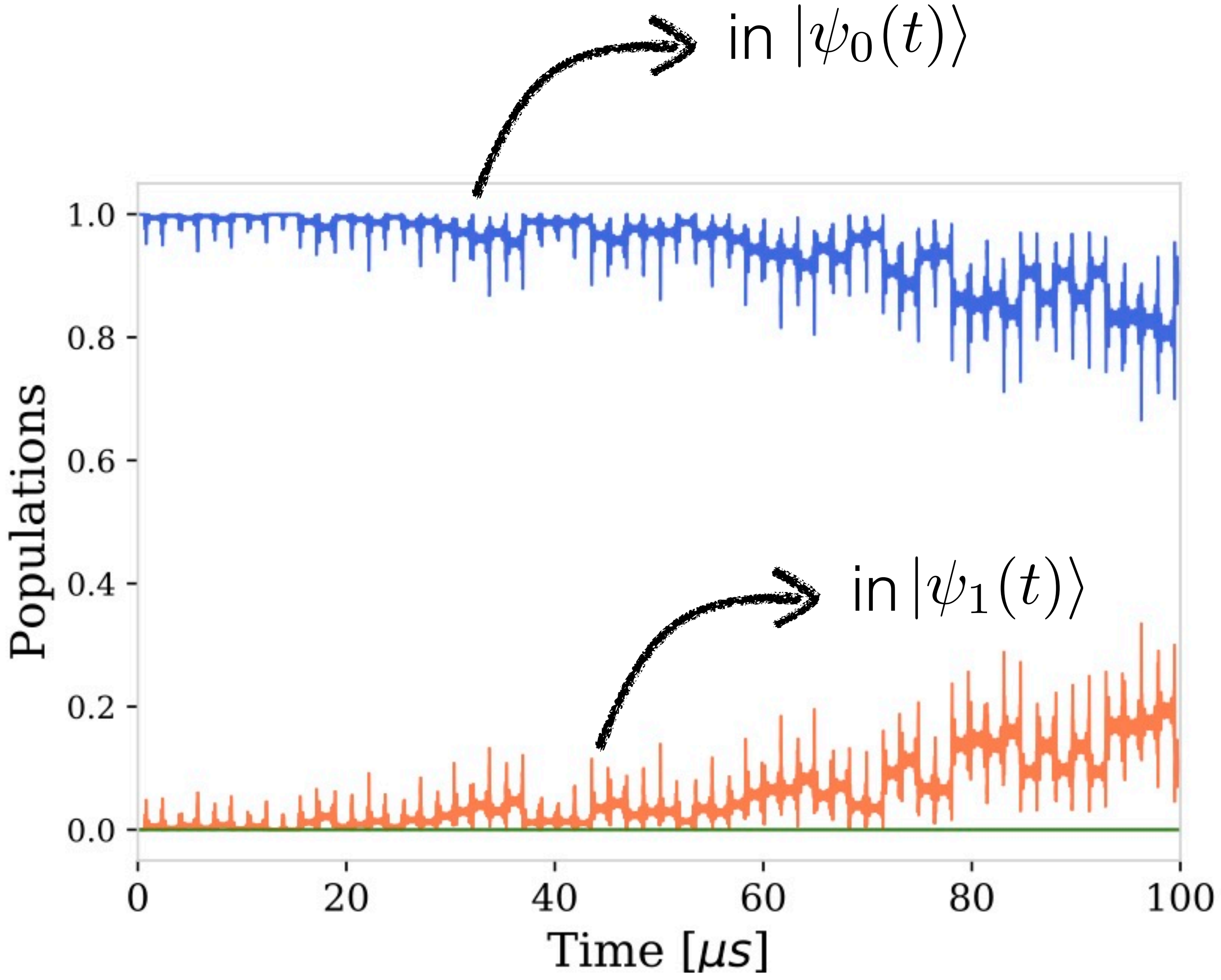


Energy exchange $\Delta\mathcal{E}_i$

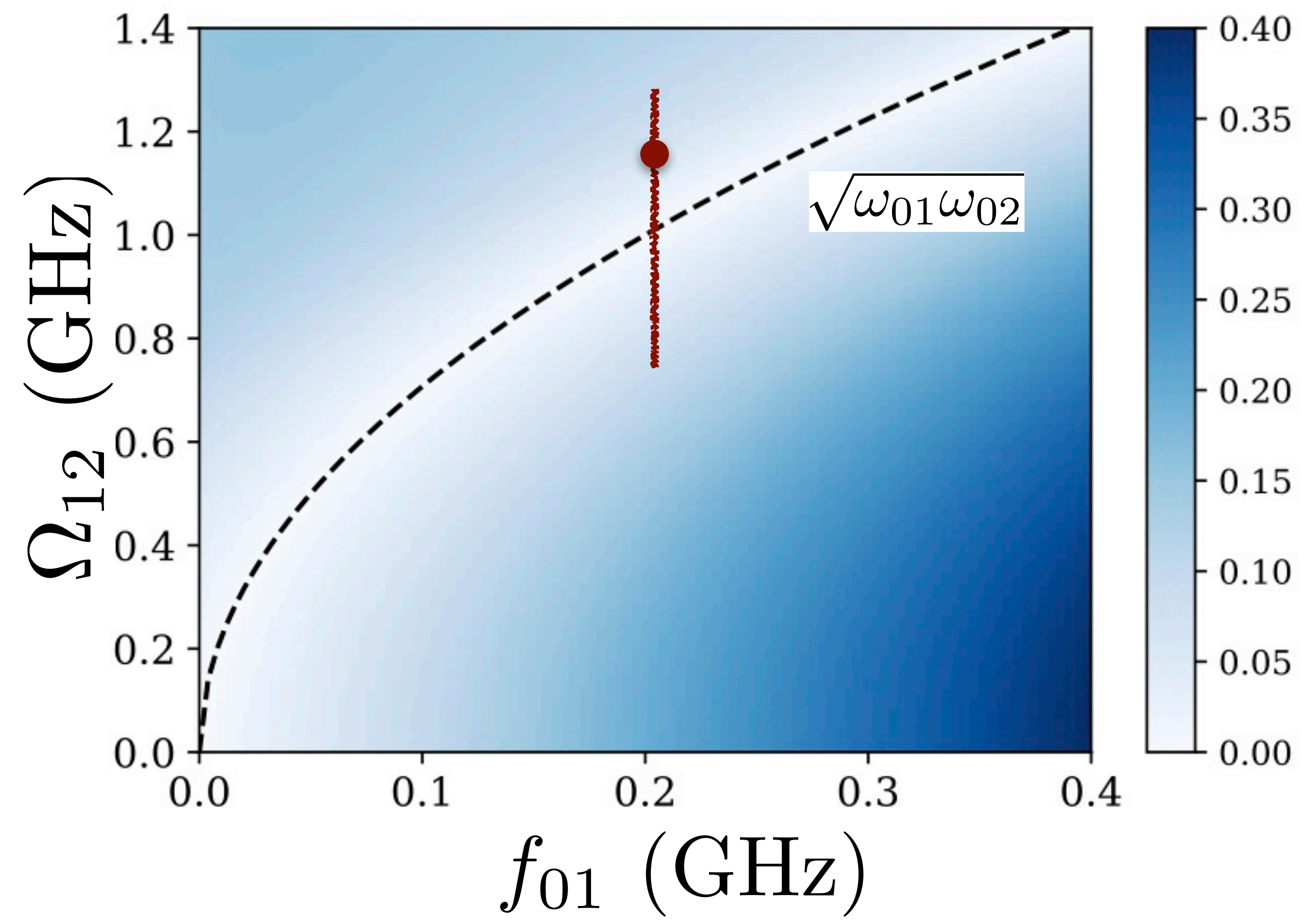


« charges » $n_1 + n_3, n_2 - n_3$



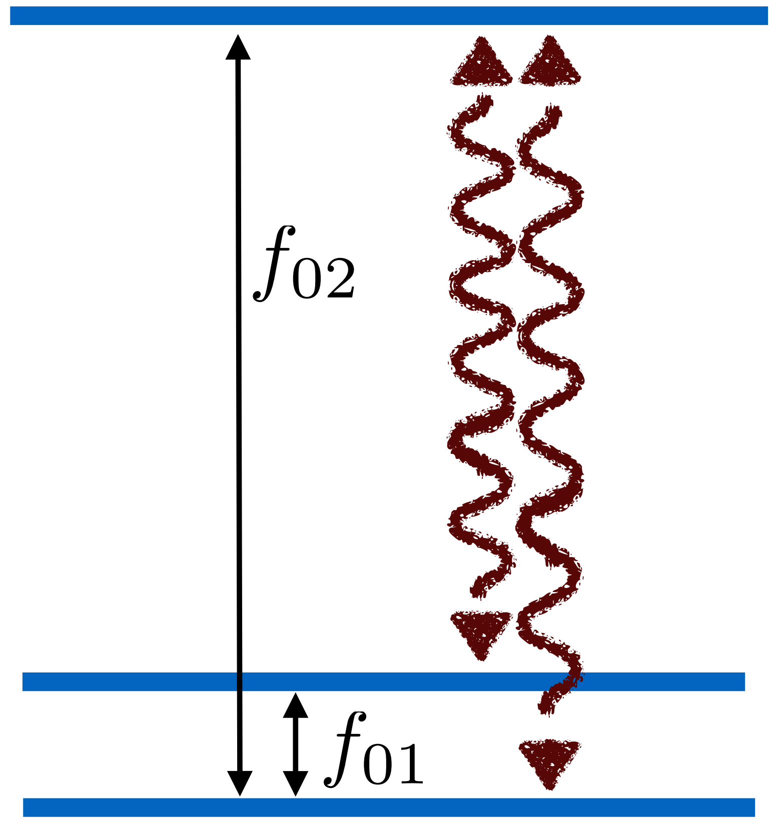


Topological Pump : effective 2 levels (qubit)

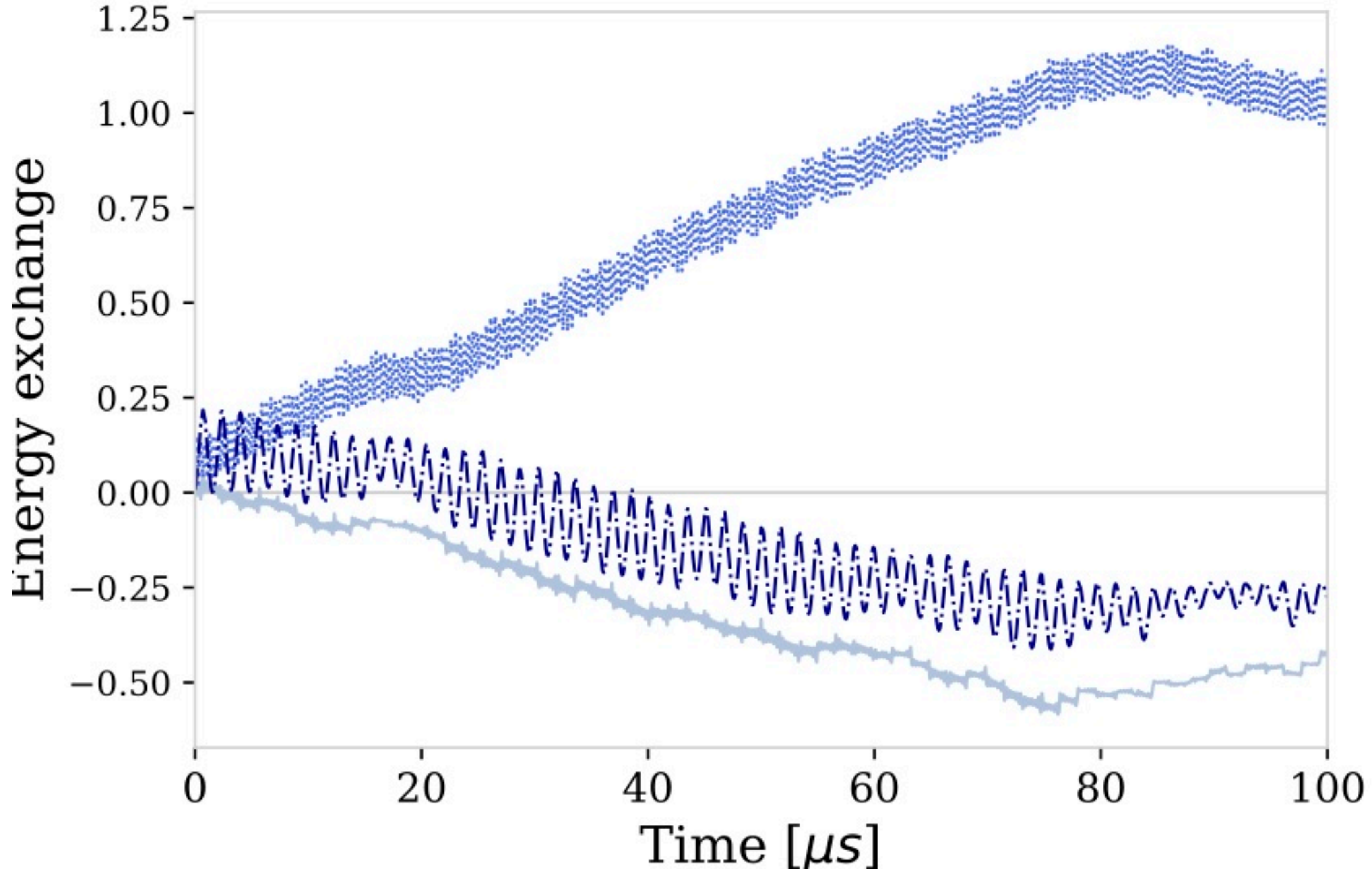


$$\omega_1 = 10 \text{ MHz}$$
$$\omega_2 \simeq 8 \text{ MHz}$$
$$\Omega_{12} = 1.15$$

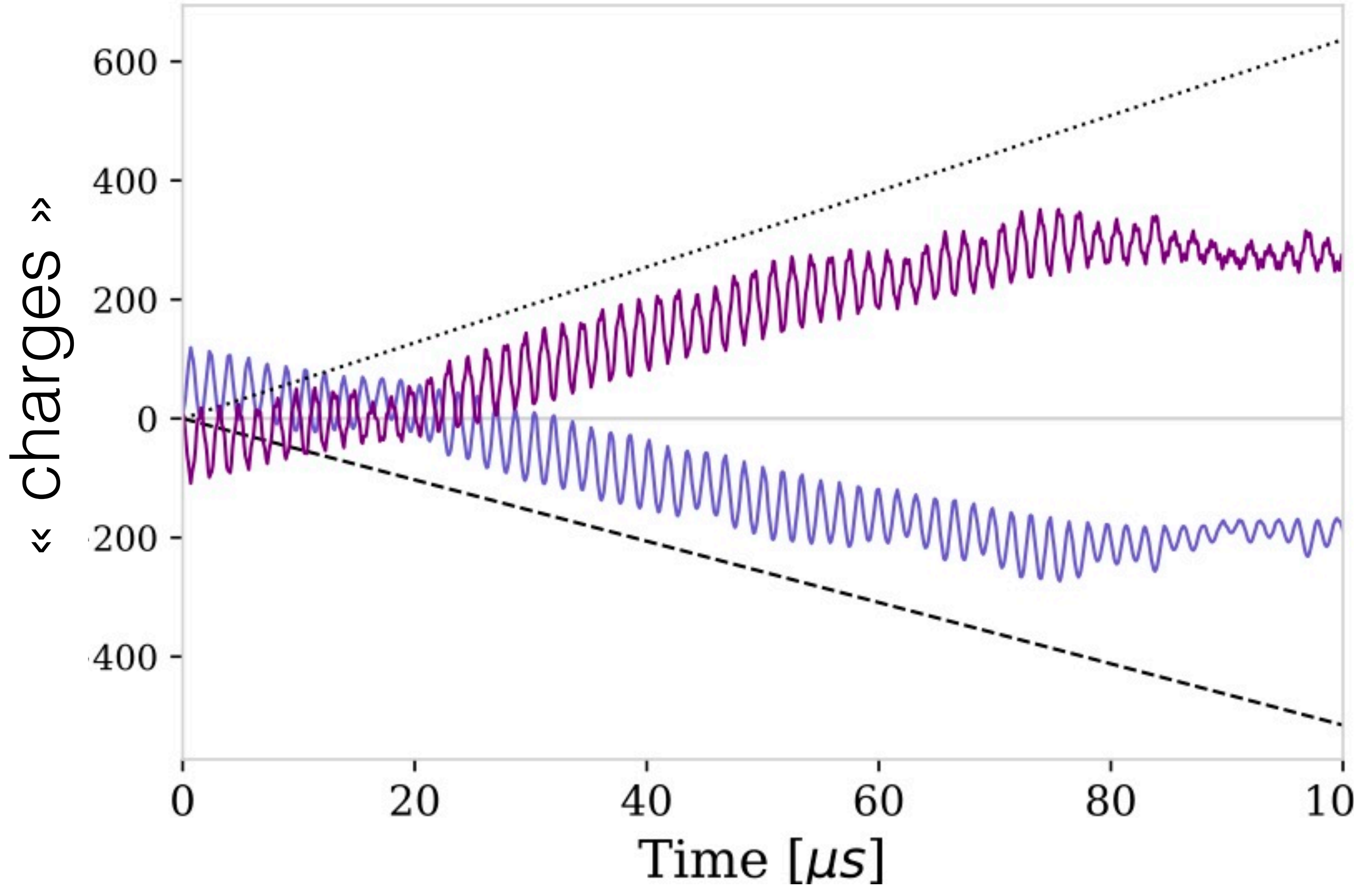
Prepare qutrit in ground state $|\psi_0\rangle$

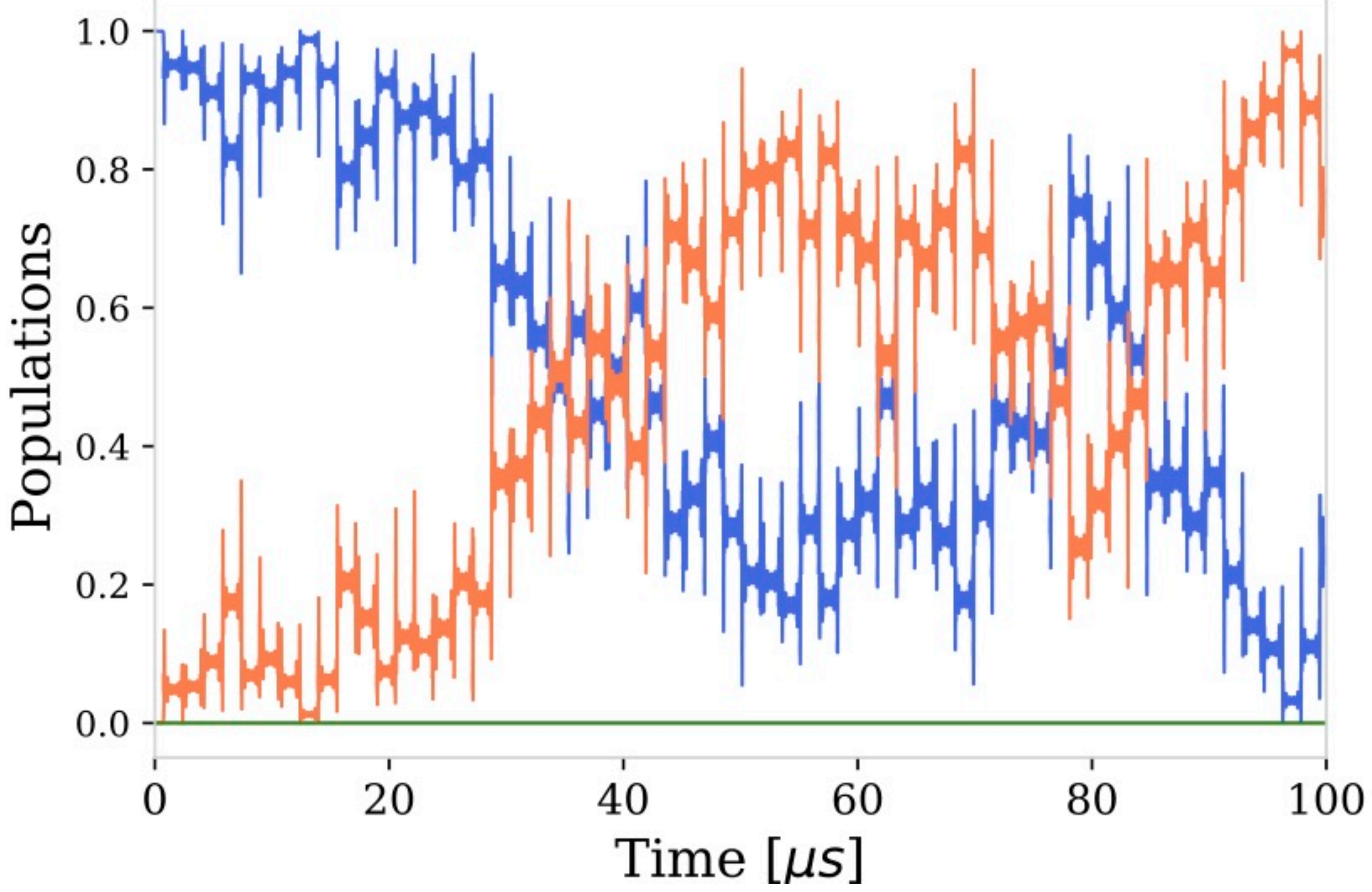


Energy exchange $\Delta\mathcal{E}_i$

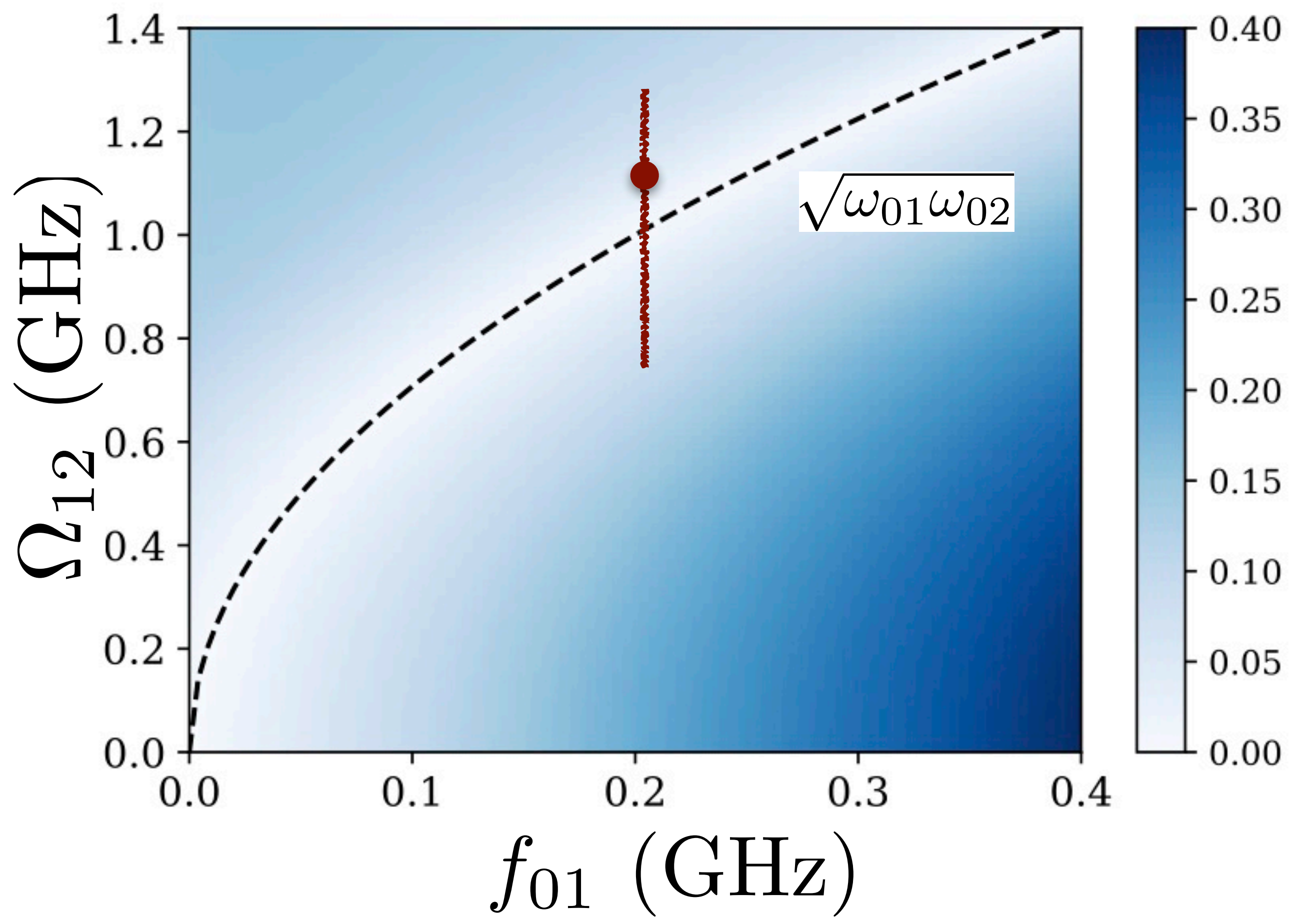


« charges » $n_1 + n_3, n_2 - n_3$





Topological Pump : effective 2 levels (qubit)

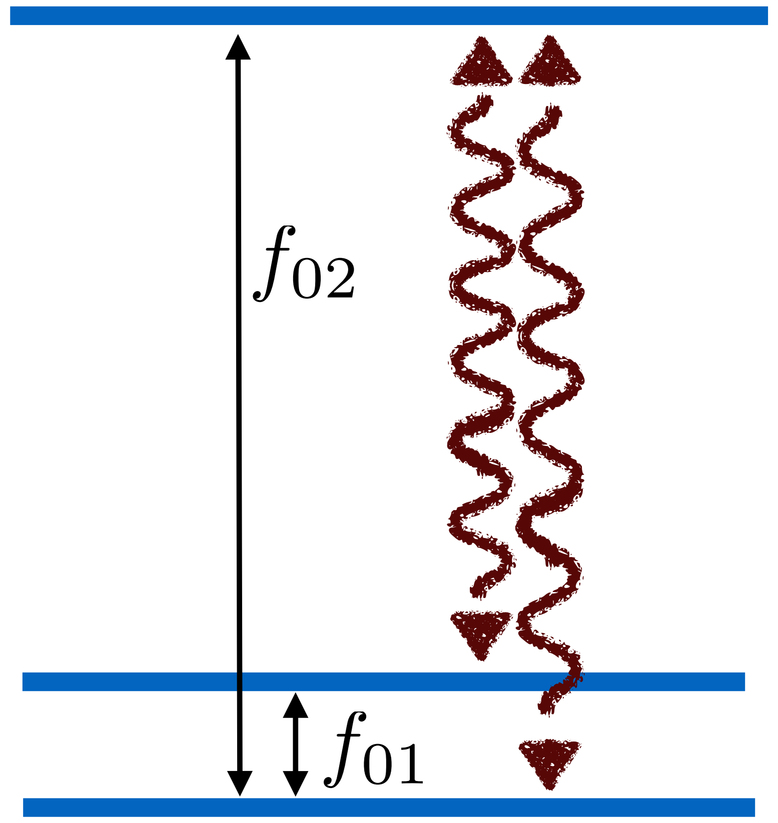


$\omega_1 = 10$ MHz

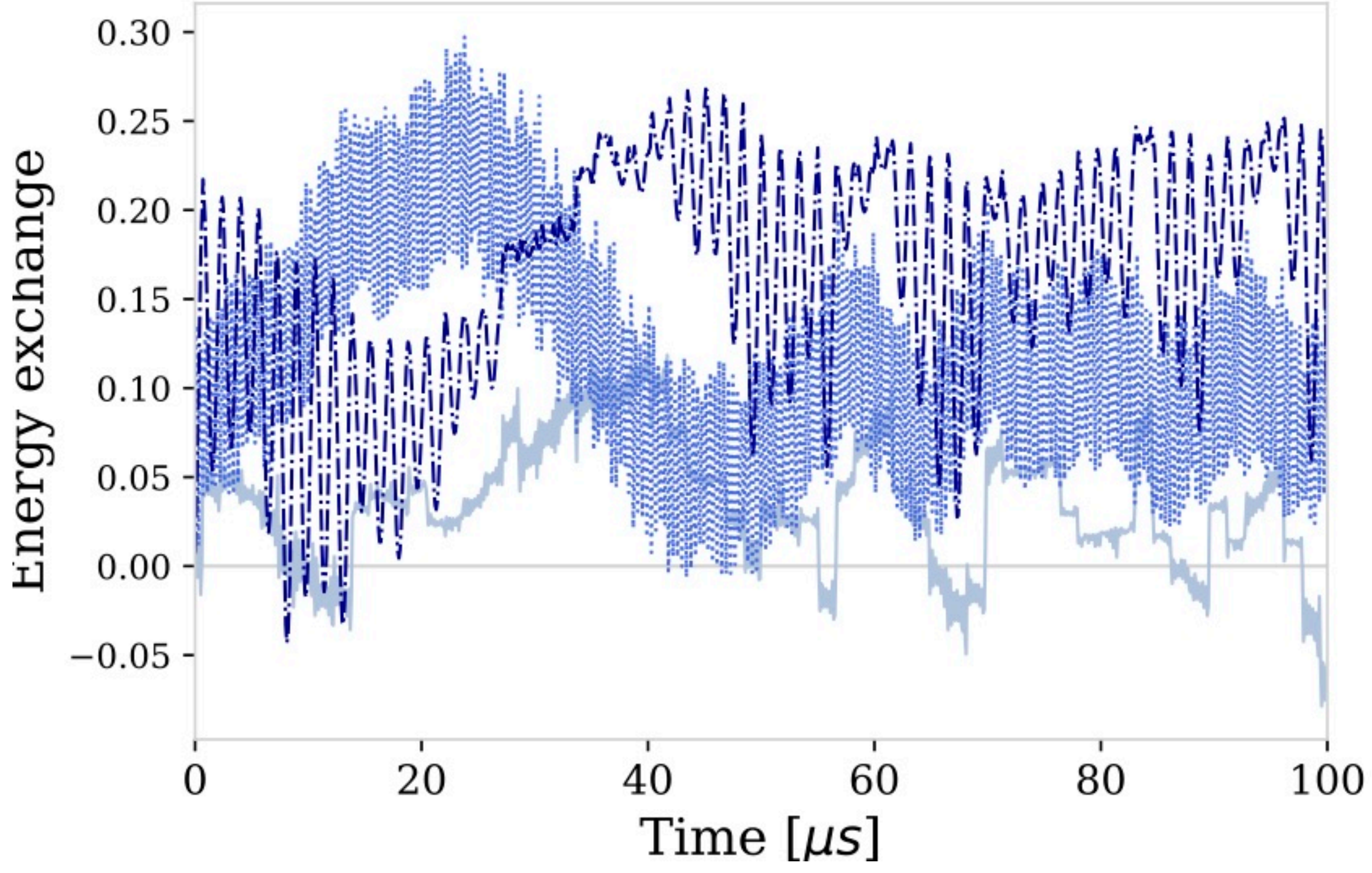
$\omega_2 \simeq 8$ MHz

$\Omega_{12} = 1.10$

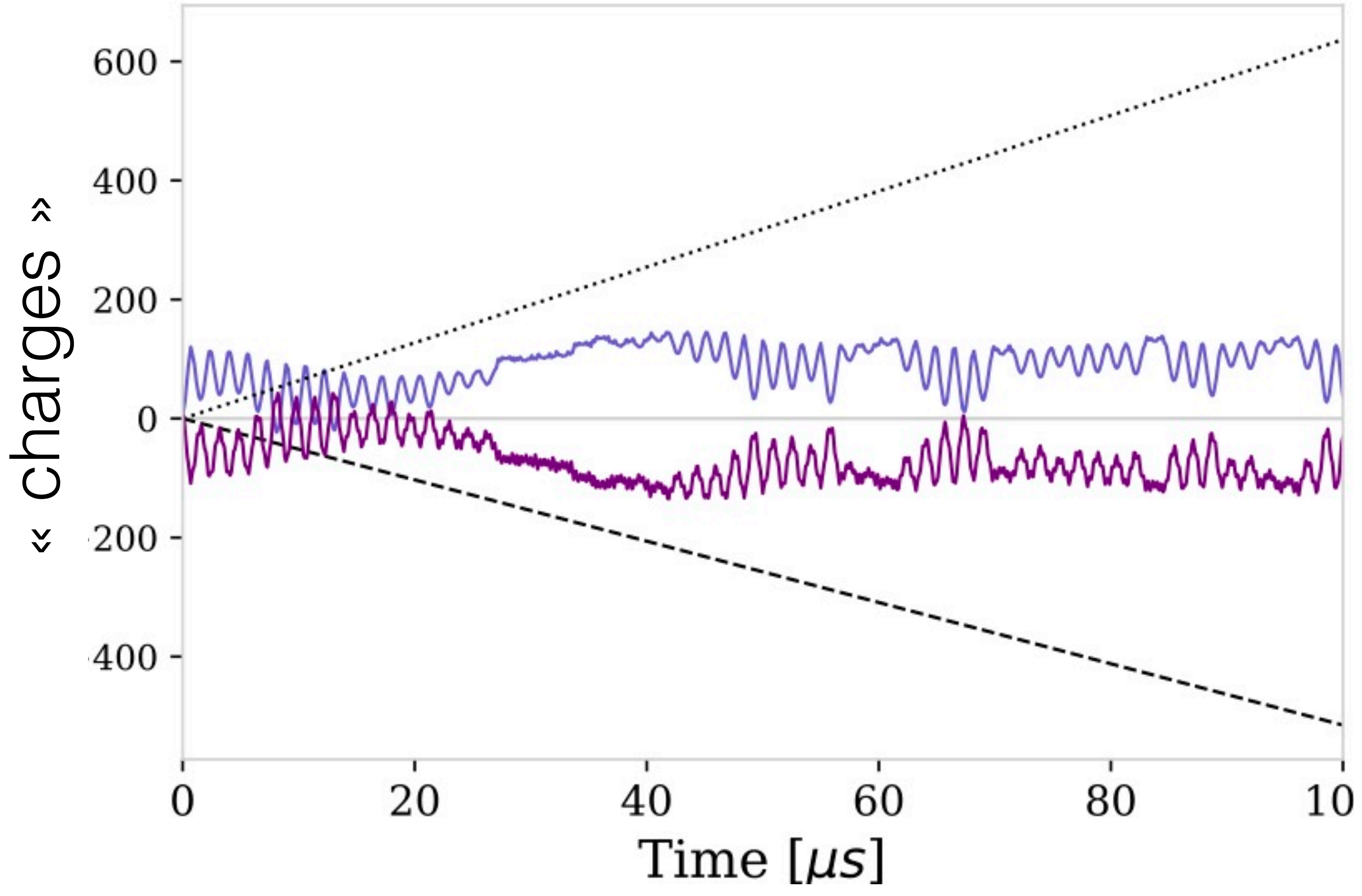
Prepare qutrit in ground state $|\psi_0\rangle$

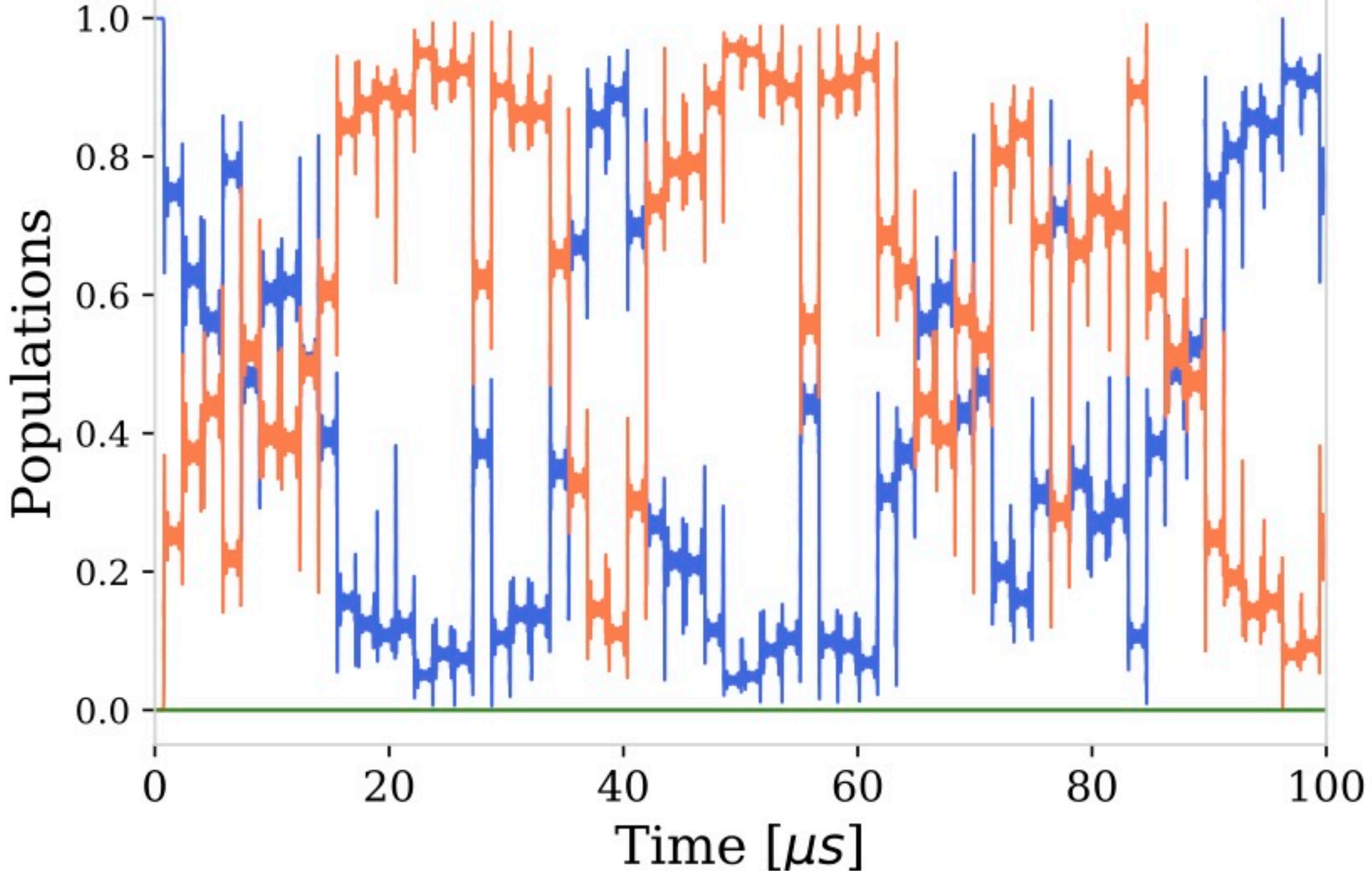


Energy exchange $\Delta\mathcal{E}_i$

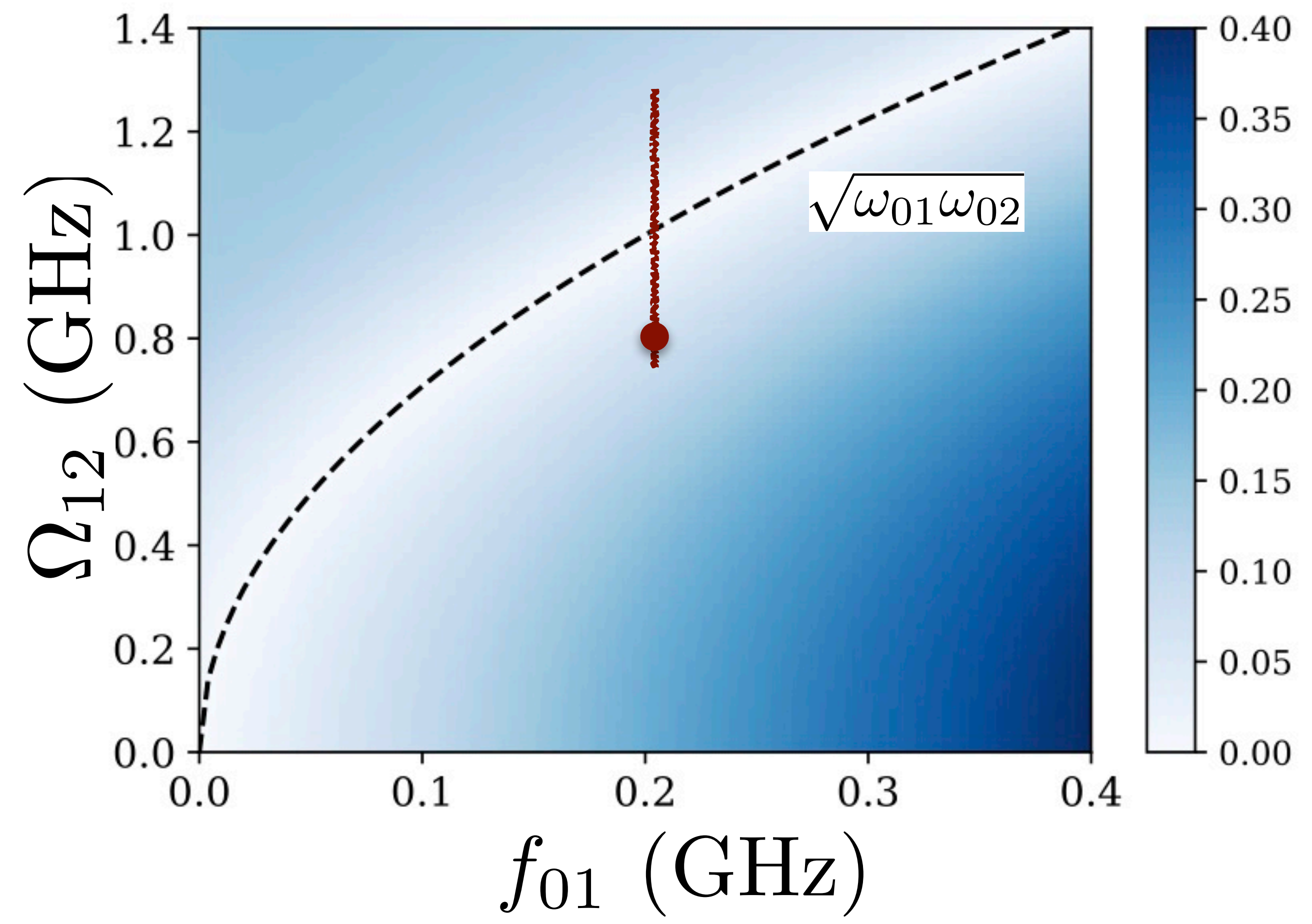


« charges » $n_1 + n_3, n_2 - n_3$



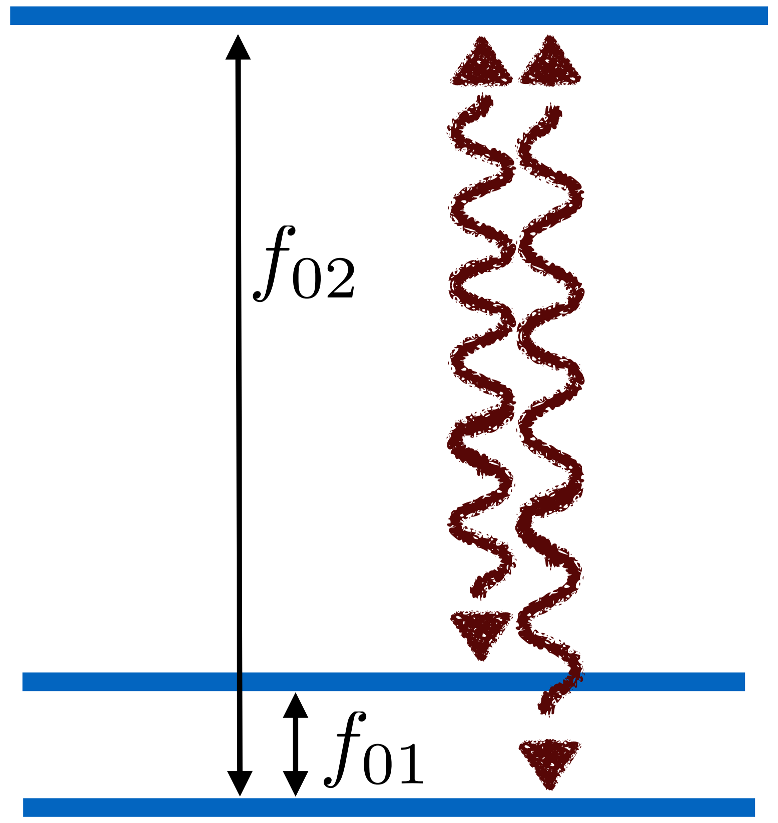


Topological Pump : effective 2 levels (qubit)

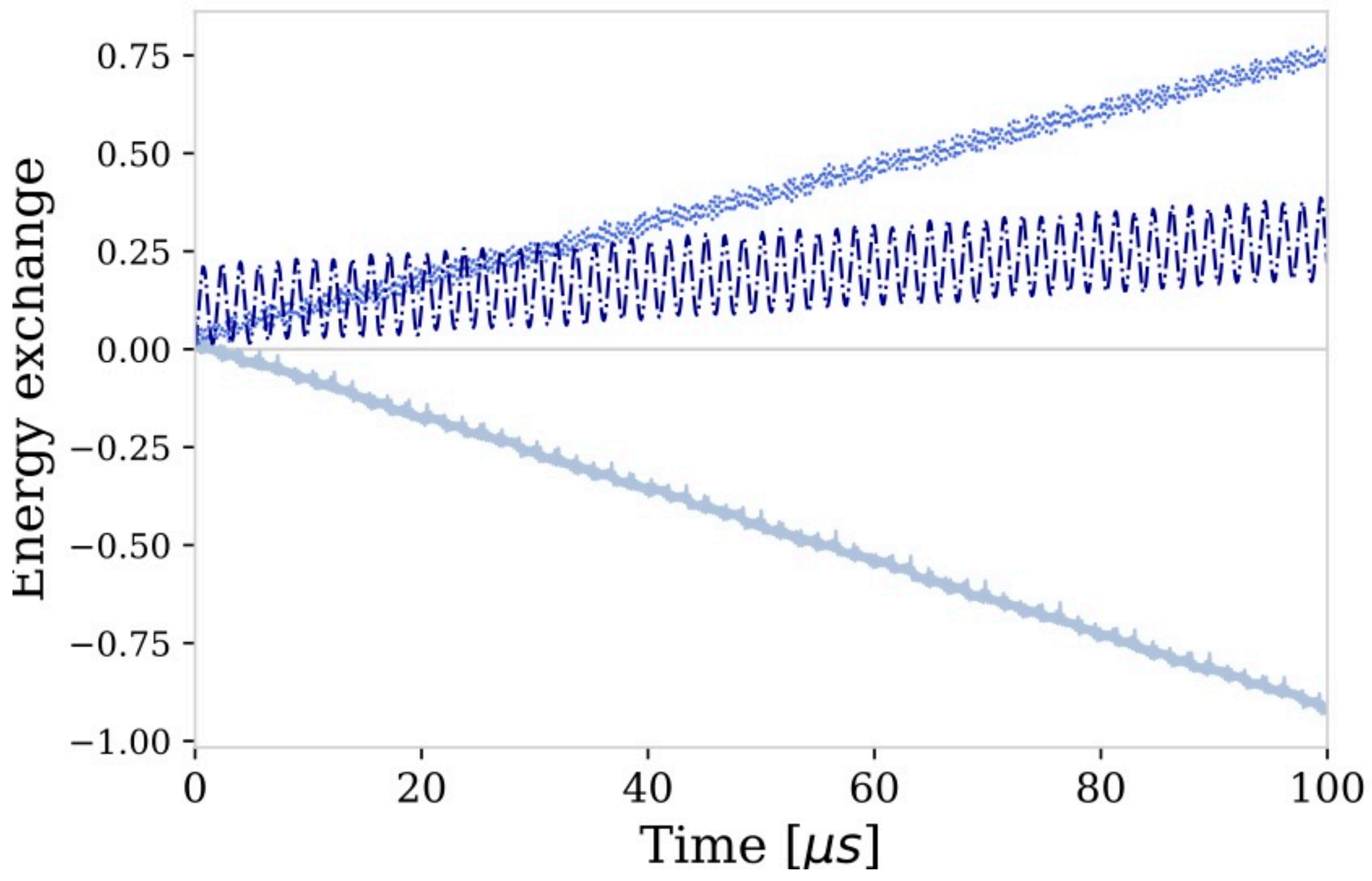


$$\omega_1 = 10 \text{ MHz}$$
$$\omega_2 \simeq 8 \text{ MHz}$$
$$\Omega_{12} = 0.80$$

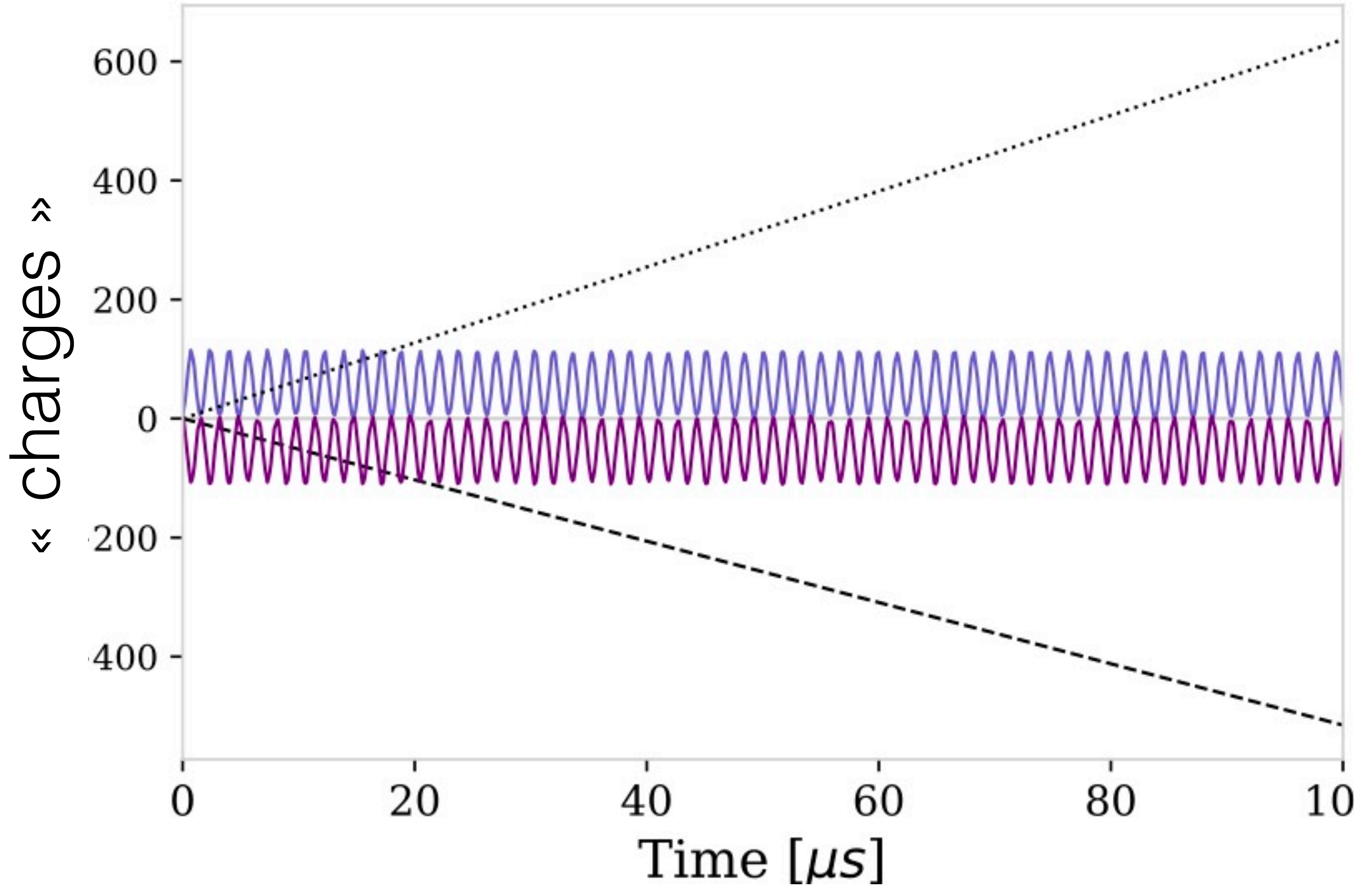
Prepare qutrit in ground state $|\psi_0\rangle$



Energy exchange $\Delta\mathcal{E}_i$



« charges » $n_1 + n_3, n_2 - n_3$



Populations