

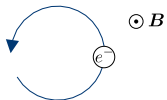
Synthetic topological matter with quantum gases and light

Nathan Goldman

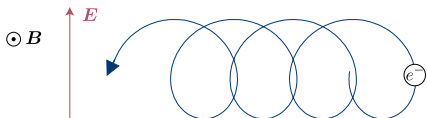
Lyon, May 17th 2021

Materials immersed in a uniform magnetic field

- An electron in a magnetic field : classical trajectories



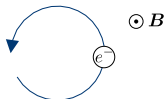
cyclotron orbit



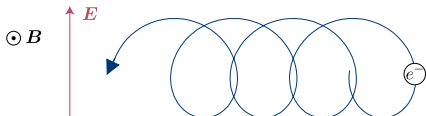
net drift (transverse to applied E field)

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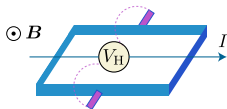


cyclotron orbit



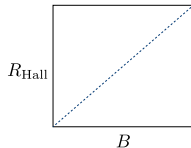
net drift (transverse to applied E field)

- The Hall effect (1879) : The conductivity of materials in an external magnetic field



Hall resistance

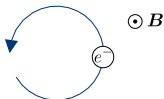
$$R_{\text{Hall}} = V_H / I$$



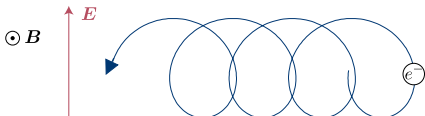
Application : Hall effect sensors

Materials immersed in a uniform magnetic field

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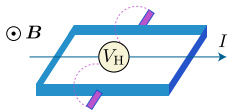


cyclotron orbit



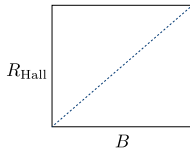
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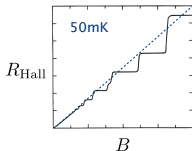
- The **quantized** Hall effect (1980) : 2D systems, low temperature and strong magnetic field



1985



1998



Quantized Hall plateaus

$$R_{\text{Hall}} = (h/e^2) \times (1/\nu)$$

$$\nu \in \mathbb{Z}$$

- $R_K = h/e^2$: "the Klitzing" (universal resistance : SI)

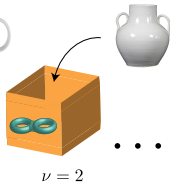
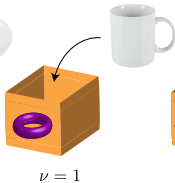
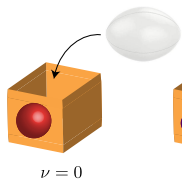
- The integers $\nu \in \mathbb{Z}$ are **topological** !



2016

Topological matter

- Topology ?



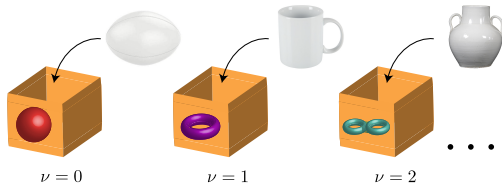
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A **topological invariant** : a **label** for each class

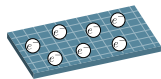
Topological matter

● Topology ?



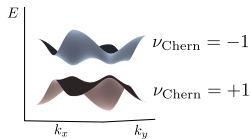
A **topological invariant** : a label for each class

● Topology in solids ?



Schrödinger's equation

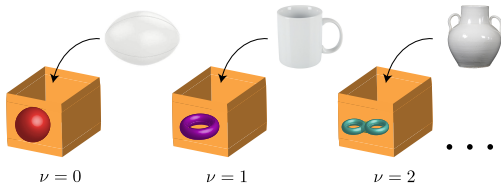
$$\hat{H}|\Psi_\lambda\rangle = E_\lambda|\Psi_\lambda\rangle$$



The **Chern number** $\nu_{\text{Chern}} \in \mathbb{Z}$
is a **topological invariant**
for each energy band

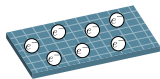
Topological matter

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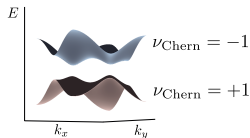
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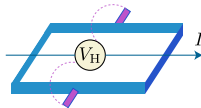
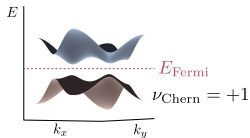
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for each energy band

● The quantized Hall effect and the notion of “Chern insulators”



Quantized Hall resistance

$$R_H = (h/e^2) \times (1/\nu_{\text{Chern}})$$

$$\nu_{\text{Chern}} \in \mathbb{Z}$$

Thouless et al. '82



How to generate a Chern insulator ?

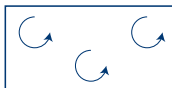
- Theorem: one has to break time reversal symmetry (Haldane, 1988)

→ "impose a privileged orientation (chirality) in the system "



time evolution

\neq

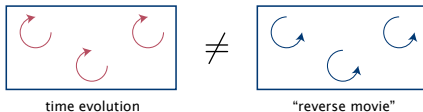


"reverse movie"

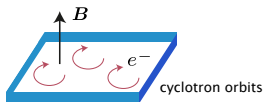
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- Immersion in a magnetic field

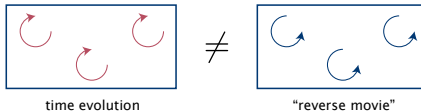


→ discovery of the quantized Hall effect (1980)

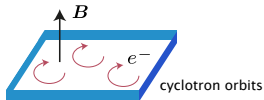
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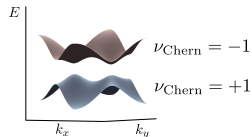
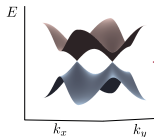
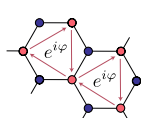


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- **Haldane's model (1988) : Chern insulator "without external magnetic field"**



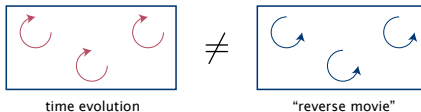
2016



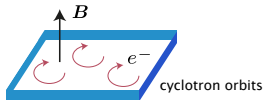
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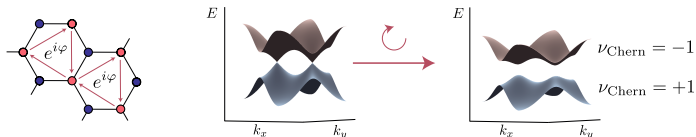


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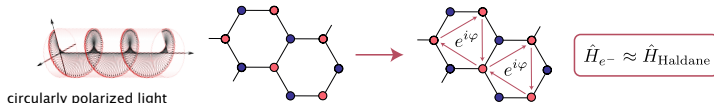
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2016



- **Light-induced methods :**



th: Oka & Aoki PRB '08

exp: Cavalleri et al. '19

A zoo of topological insulators and superconductors

- Are there other topological states of matter ? What are the relevant topological invariants ?

A zoo of topological insulators and superconductors

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Mathematics (Altland & Zirnbauer '97, Kitaev '09, Ryu-Schnyder-Furusaki-Ludwig '08) : presence of **symmetries** !!

classes	symmetries			spatial dimensions							
	Θ	Ξ	Π	1	2	3	4	5	6	7	8
A	0	0	0	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}
AIII	0	0	1	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0
AI	1	0	0	0	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}
BDI	1	1	1	\mathbb{Z}	0	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2
D	0	1	0	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}	0	\mathbb{Z}_2
DIII	-1	1	1	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}	0
AII	-1	0	0	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}
CII	-1	-1	1	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0
C	0	-1	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0
CI	1	-1	1	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0

time reversal symmetry

- Some concrete examples :

 : 2D Chern insulators $\longrightarrow \nu_{\text{Chern}} \in \mathbb{Z}$ classe A = "Chern insulators" (even dims.)

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D	0	1	0	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}	0	\mathbb{Z}_2
DIII	-1	1	1	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}	0
AII	-1	0	0	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}
CII	-1	-1	1	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0
C	0	-1	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0
CI	1	-1	1	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0

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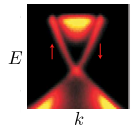
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CII	-1	-1	1	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0
C	0	-1	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0
CI	1	-1	1	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0

time reversal symmetry

- Some concrete examples :

 : 2D Chern insulators $\longrightarrow \nu_{\text{Chern}} \in \mathbb{Z}$ classe A = "Chern insulators" (even dims.)

surface states

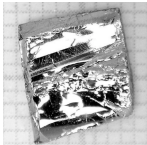


 : topological superconductors (th: '00 et exp: '14)

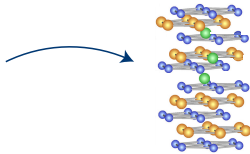
 : topological insulators (th: '05 et exp: '07)

Creating synthetic topological matter

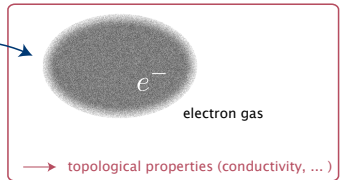
- Topological matter :



a material (ex. topological insulator)

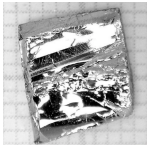


atomic structure

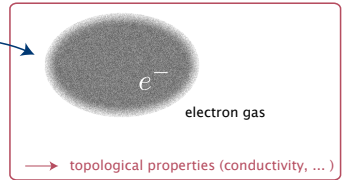
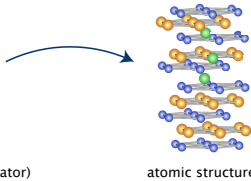


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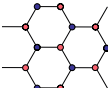




● Theoretical description : design a simple model

Schrödinger's equation : $\hat{H}_{e^-}^{\text{topo}} |\psi\rangle_{e^-} = E |\psi\rangle_{e^-}$

E : energy spectrum (band structure)

Hamiltonian operator : $\hat{H}_{e^-}^{\text{topo}} =$


+

+


lattice

motion (kinetics)

interactions



additional effects (couplings)

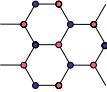
● coupling to an external field : ex. magnetic field (Hall effect)

● intrinsic coupling : ex. spin-orbit coupling (topological insulators)

Synthetic realizations


- Can one create synthetic topological matter in the lab ?

$$\hat{H}_{e^-}^{\text{topo}} =$$




lattice

+




motion (kinetics)

+



interactions

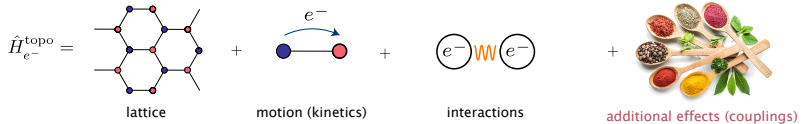
+



additional effects (couplings)

Synthetic realizations

- Can one create synthetic topological matter in the lab ?

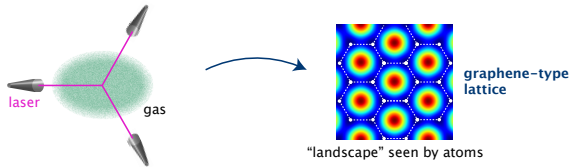
$$\hat{H}_{e^-}^{\text{topo}} = \text{lattice} + \text{motion (kinetics)} + \text{interactions} + \text{additional effects (couplings)}$$


The diagram illustrates the components of the topological Hamiltonian $\hat{H}_{e^-}^{\text{topo}}$. It is composed of four terms: a lattice (represented by a honeycomb lattice structure), motion (kinetics) (represented by an electron moving between two sites), interactions (represented by two electrons interacting via a wavy line), and additional effects (couplings) (represented by a collection of spices in spoons).

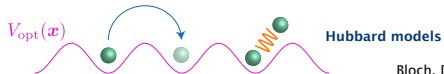
- Quantum simulation with ultracold atomic gases:

$$e^- \longrightarrow \text{(neutral atoms)}$$

- lattice: **optical lattice**



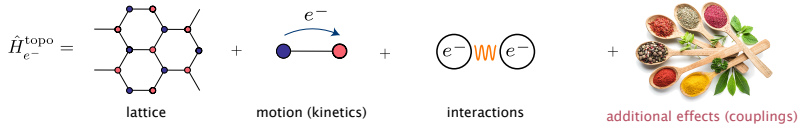
- hopping and interactions



Bloch, Dalibard, Zwerger, RMP '08

Synthetic realizations

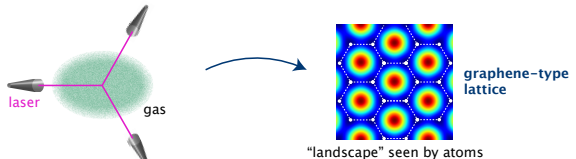
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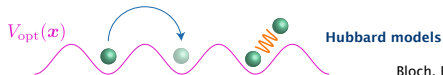
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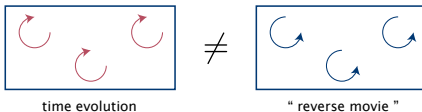


synthetic topological matter ? ? ?

A synthetic Chern insulator ?

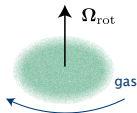
- Haldane (1988) : one has to break time reversal symmetry

→ "impose a privileged orientation (chirality) in the system"



- In ultracold atomic gases

- Rotation



$$\hat{H}_{\text{atom}} \approx \hat{H}_{\text{Hall}}$$

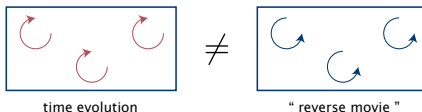
$$\Omega_{\text{rot}} \approx B : \text{artificial magnetic field}$$

exp: Dalibard '00, Ketterle '01, Cornell '04

A synthetic Chern insulator ?

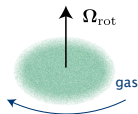
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- Rotation

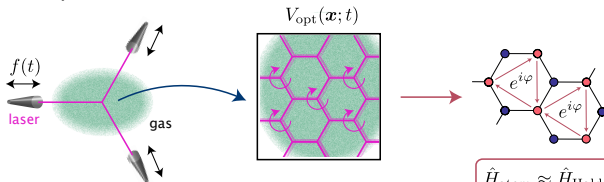


$$\hat{H}_{\text{atom}} \approx \hat{H}_{\text{Hall}}$$

$$\Omega_{\text{rot}} \approx B : \text{artificial magnetic field}$$

exp: Dalibard '00, Ketterle '01, Cornell '04

- Shaken optical lattices



$$\hat{H}_{\text{atom}} \approx \hat{H}_{\text{Haldane}}$$

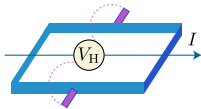
→ topological bands with $\nu_{\text{Chern}} \neq 0$

exp: Arimondo '07, Sengstock '11, Bloch '12, Ketterle '13, Esslinger '14, ...

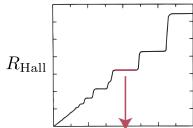
Probing synthetic topological matter

- Standard probes in solid states :

- Transport (conductivity)

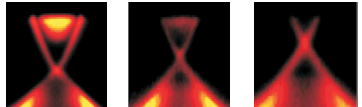


$$R_{\text{Hall}} = V_H / I : \text{Hall resistance}$$



$$(R_{\text{Hall}})^{-1} \sim \nu_{\text{Chern}}$$

- Spectroscopy (ARPES)

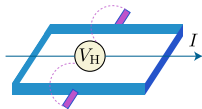


exp: Hasan et al. '09

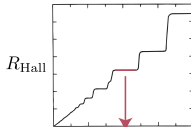
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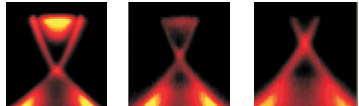


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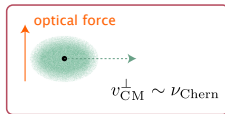
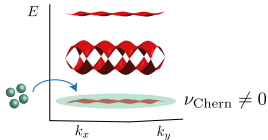
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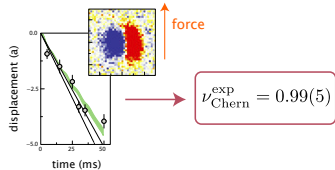
exp: Hasan et al. '09

● Examples of probes in ultracold topological matter :

● Transport (centre of mass) : Munich experiment (Aidelsburger et al. '15)



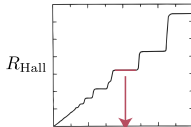
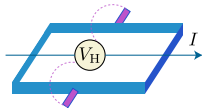
th: Dauphin & Goldman '13



Probing synthetic topological matter

Standard probes in solid states :

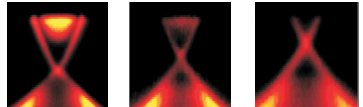
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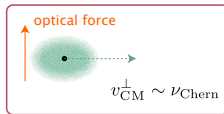
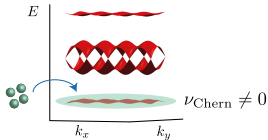
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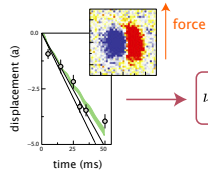
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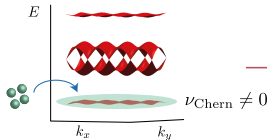
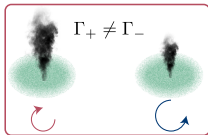


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$$\nu_{\text{Chern}}^{\text{exp}} = 0.99(5)$$

Circular dichroism : Hambourg experiment (Asteria et al. '19)

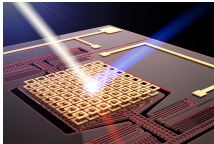


$$\int \Gamma_+ - \Gamma_- \sim \nu_{\text{Chern}} \text{ quantized}$$

th: Tran et al. '17

Replacing matter by light : topological photonics

- Band structures for light : photonic crystals

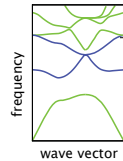


Zhang lab (Boston)

Maxwell' equations as an eigenproblem

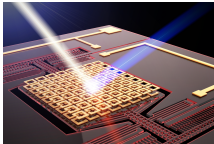
$$\hat{\Theta}|\mathbf{E}\rangle = \omega^2|\mathbf{E}\rangle$$

ω : modes frequency



Replacing matter by light : topological photonics

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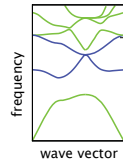


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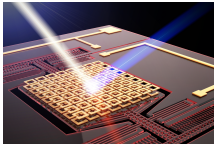
- Create Chern bands ? The idea of Haldane and Raghu ('05-'08) : **break time reversal symmetry**

- Basic ingredient : magneto-optic materials [**nonreciprocal** medium (Faraday effect)]



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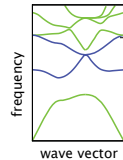


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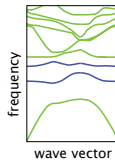
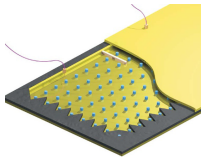


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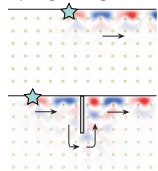
- Periodic setting : photonic crystal (experiment at MIT, Wang et al. '09)



$\nu_{\text{Chern}} \neq 0$



topological edge states



Explore the zoo of topological states with synthetic systems

- The periodic table is rich and broad:

examples discussed today

classes	symmetries			spatial dimensions							
	Θ	Ξ	Π	1	2	3	4	5	6	7	8
A	0	0	0	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}
AIII	0	0	1	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0
AI	1	0	0	0	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}
BDI	1	1	1	\mathbb{Z}	0	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2
D	0	1	0	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}	0	\mathbb{Z}_2
DIII	-1	1	1	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}	0
AII	-1	0	0	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}
CII	-1	-1	1	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0
C	0	-1	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0
CI	1	-1	1	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0

Reviews : ● Cold atoms : Cooper–Dalibard–Spielman RMP '19

● Photonics : Ozawa et al. RMP '19

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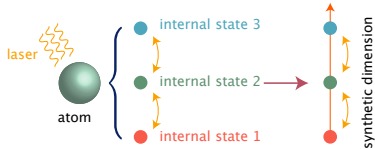
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D	0	1	0	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}	0	\mathbb{Z}_2
DIII	-1	1	1	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}	0
AII	-1	0	0	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}
CII	-1	-1	1	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0
C	0	-1	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0
CI	1	-1	1	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0

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Explore the zoo of topological states with synthetic systems

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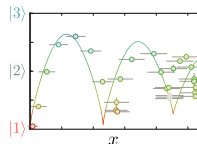
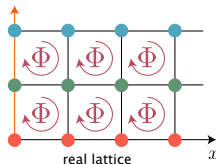
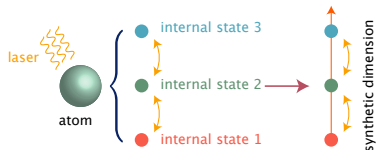
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D	0	1	0	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}	0	\mathbb{Z}_2
DIII	-1	1	1	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}	0
AII	-1	0	0	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}
CII	-1	-1	1	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0
C	0	-1	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0
CI	1	-1	1	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0

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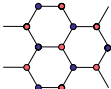
exp: Florence (Fallani) '15, NIST (Spielman) '15, ... Collège de France (Dalibard) '20: **17 internal states (dysprosium)**

th: Price et al. '15 (**4D QHE with cold atoms**), Ozawa et al. '16 (**4D photonics**), ...

Challenge: strongly-correlated synthetic topological matter


- Our model Hamiltonian :

$$\hat{H}_{e^-}^{\text{topo}} =$$




lattice

+




motion

+



interactions

+

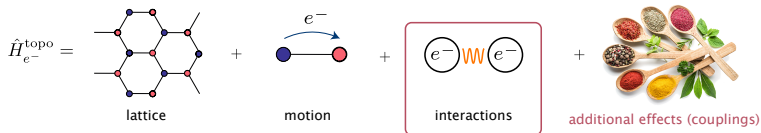


additional effects (couplings)

→ strongly-correlated states (ex. fractional quantum Hall liquids)

Challenge: strongly-correlated synthetic topological matter

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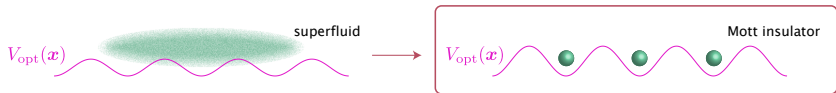
$$\hat{H}_{e^-}^{\text{topo}} = \text{lattice} + \text{motion} + \text{interactions} + \text{additional effects (couplings)}$$


The diagram illustrates the components of the model Hamiltonian $\hat{H}_{e^-}^{\text{topo}}$. It is a sum of four terms: 1. 'lattice', represented by a honeycomb lattice structure; 2. 'motion', represented by a blue electron moving from a blue site to a red site; 3. 'interactions', represented by a box containing two electrons (e^-) with a wavy line between them; 4. 'additional effects (couplings)', represented by an image of various spices and herbs.

→ strongly-correlated states (ex. fractional quantum Hall liquids)

- Cold atoms :

- Strongly-correlated states (non topological) :



exp: Greiner et al. '02

Challenge: strongly-correlated synthetic topological matter

● Our model Hamiltonian :

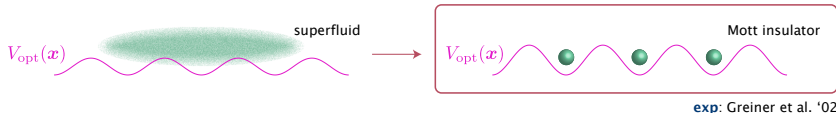
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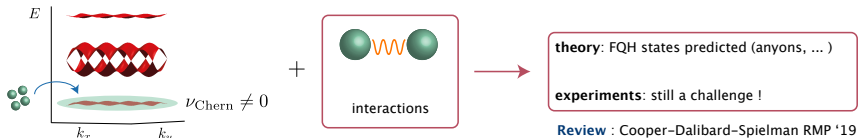
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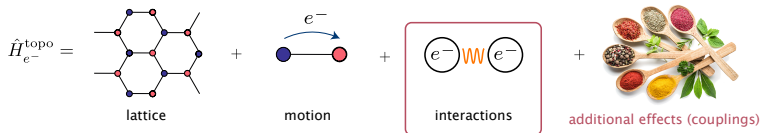


● Strongly-correlated topological states :



Challenge: strongly-correlated synthetic topological matter

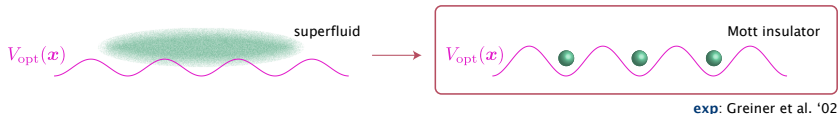
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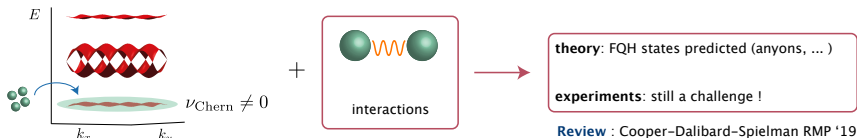
strongly-correlated states (ex. fractional quantum Hall liquids)

● Cold atoms :

● Strongly-correlated states (non topological) :



● Strongly-correlated topological states :



● Photonics :

- Create strong photon-photon interactions **Reviews** : Carusotto & Cui RMP '13, Ozawa et al. RMP '19
- Stabilize many-photon states despite losses (out-of-equilibrium systems)

Some reading ...

- **Ultracold atoms and topological matter :**

- Topological quantum matter with ultracold gases in optical lattices, N. Goldman, J. C. Budich and P. Zoller, [Nature Physics 12, 639 \(2016\)](#)
- Topological bands for ultracold atoms, N. R. Cooper, J. Dalibard, and I. B. Spielman, [Rev. Mod. Phys. 91, 015005 \(2019\)](#)

- **Topological photonics :**

- Topological photonics, T. Ozawa, H. M. Price, A. Amo, N. Goldman, M. Hafezi, L. Lu, M. C. Rechtsman, D. Schuster, J. Simon, O. Zilberberg, and I. Carusotto, [Rev. Mod. Phys. 91, 015006 \(2019\)](#)

- **Artificial gauge fields:**

- Artificial gauge fields in materials and engineered systems, M. Aidelsburger, S. Nascimbene and N. Goldman, [Comptes Rendus Physique, 19, 394 \(2018\)](#)