Dynamical correlations of the XXZ chain at finite temperature

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Abstract of the thesis:

The emergence of new experimental techniques in the recent years allows for an active study of materials described by one dimensional models. The experiments take place at finite, but relatively low, temperature and often involve materials that are described by gapless models, *viz.* those having no gap, in the thermodynamic limit, between their ground state and the tower of excited states. Typically, the experiments measure the two-point dynamical correlation functions at finite temperature or, more precisely, their space and time Fourier transforms, the so-called dynamic response functions.

There is no hope of characterising these observables for a generic model. Certain heuristic approaches [2], generalising the non-linear Luttinger liquid theories, have been developed to grasp some of the expected universal features of such correlators. However, solely exact results may allow one to develop a firm and realistic grasp of the effects and mechanisms induced by the interactions. Some explicit results have been obtained, in the past, for free theories. Those results are however too specific and, by definition, are unable to capture the non-trivial effects induced by the interactions. Recently, Kozlowski *et al.* [1] made an important step forward on the problem by obtaining exact representations for the two-point dynamical correlation functions of the XXZ spin-1/2 chain at finite temperature. This spin chain model is the archetype of a truly interacting one-dimensional model.

The aim of the thesis will be to develop a satisfactory understanding of this representation and obtain on its basis a thorough and exact characterisation of the dynamical correlation functions of the XXZ spin-1/2 chain at finite temperature. This means that the research carried out during the thesis would aim at working out techniques that would allow one to extract from the results of [1]:

i) the form and value of the critical exponents and/or correlation lengths driving the decay of the correlators in the long-time and/or large-distance regime; this while being able to achieve a sufficiently precise control of the dependence on the temperature of the building blocks of the correlator's asymptotic expansion.

ii) The singularity structure of the dynamical response function in the vicinity of the dispersion curves of the elementary excitation modes of the model. The aim would be to obtain a fine control on the effects induced by temperature, be it the smearing out of certain singularities present at zero temperature, or other possible effects induced by the presence of a finite temperature

iii) The structure of the dynamical correlation at infinite temperature. The aim would be to grasp the effects of the interactions on the spin-spin autocorrelation function at infinite temperature, what constitutes a long-standing open question.

The success in accomplishing these tasks will provide, for the first time, an exact and satisfactory characterisation of the dynamical correlators at finite temperature in a non-trivial, *viz.* truly interacting, model at finite temperature. It would allow for a strong test of the validity, limitations and range of applicability of the predictions obtained in [2] for the various asymptotics in the low-temperature regime.

In a subsequent stage of the thesis, the physical intuition gained from studying the exact answer will be used so as to develop a much deeper understanding of the microscopic structure of the model's

Hilbert space that is at the root of generating a universal structure of the long-time and/or large distance asymptotic behaviour. Such a deepening of the understanding of the microscopic structure would also help to get a better grasp on what is happening in more general models that are not necessarily integrable hence potentially leading to the construction of a phenomenological approach that would embrace all universal effects occurring at finite temperature and that would, quite probably, go way beyond the aspects argued in [2].

The main tools that will be handled during the PhD thesis will belong to the field of complex analysis, special functions and Riemann-Hilbert problems. I insist that the research would also demand a great deal of calculations.

References

[1] F. Göhmann, M. Karbach, A. Klmper, K.K. Kozlowski, and J. Suzuki, *"Thermal form-factor approach to dynamical correlation functions of integrable lattice models."*, J. Stat. Mech. (2017), 113106, cond–mat.stat–mech: 1708.04062.

[2] C. Karrasch, R.G. Pereira, and J. Sirker, *"Low temperature dynamics of nonlinear Luttinger liquids."*, New J. Phys. **17** (2015), 103003.