PhD thesis or M2 internship or PLR

Correlation functions in the $1+1$ dimensional Sine-Gordon field theory

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Mathematical physics, quantum integrable models, universality and asymptotics, special functions beyond the Painlevé class.

Abstract of the thesis:

The construction of a genuine, non-perturbative, rigorously defined and truly interacting quantum field theory is a very hard problem on which only partial progress could have been made. Bringing such a program to an end means to be able to characterise, in an explicit and manageable way, all the correlation functions of the theory. In other words, to be able to provide closed formulae for the model's correlation functions -*viz.* the physically observable quantities- and to be able to extract from these all the physically interesting information. Achieving the full program, even for some specific instance of a non-trivial, *viz.* interacting, quantum field theory, would help to forge a deeper understanding, on rigorous ground, of quantum field theories beyond the constructive renormalisation scheme. In particular, it could shed light on new tools of analysis, alternative to the renormalisation be it the constructive or formal one, allowing one to study such theories.

Following the pioneering work of Karowski, Weisz [2] and Kirillov, Smirnov [3], there emerged an efficient technique, called the bootstrap program, allowing one to compute explicitly the correlation functions in numerous completely integrable massive 1+1 dimensional quantum field theories [4]. More precisely, by relying on the factorisability of the multipatricle scattering into two-particle processes which is present in such models, the bootstrap program allows one to provide explicit multiple integral representations for the matrix elements of local operators $\mathcal{O}(0,0)$ taken between the asymptotic states of the theory

$$_{\rm out}\langle(\beta_1,\mu_1),\dots,(\beta_n,\mu_n)\,|\,\mathcal{O}(0,0)\,|\,(\alpha_1,\nu_1),\dots,(\alpha_m,\nu_m)\,\rangle_{\rm in}\ .$$

Here, β_k, resp. α_k, label the rapidities of the asymptotic particles and μ_k, respectively ν_k, the associated internal degrees of freedom (spin, charge, flavour,...). By using the completeness of the asymptotic states one may then compute any multi-point function by means of the form factor expansion. For instance, within the method, a vacuum-to-vacuum two-point function is expressed as

$$\langle\mathcal{O}^\dagger(x,t)\mathcal{O}(0,0)\rangle\ =\ \sum_{n\geq 0}\frac{1}{n!}\int_{\mathbb{R}^n}\mathrm{d}^n\beta\sum_{\mu_1,\dots,\mu_n}\Big|\langle 0\,|\,\mathcal{O}(0,0)\,|\,(\beta_1,\mu_1),\dots,(\beta_n,\mu_n)\,\rangle_{\rm in}\Big|^2\cdot\prod_{a=1}^n\Big\{\mathrm{e}^{\mathrm{i}xp_{\mu_k}(\beta_k)-\mathrm{i}t\varepsilon_{\mu_k}(\beta_k)}\Big\}\ .$$

Above, $p_{\mu_k}(\beta)$ and $\varepsilon_{\mu_k}(\beta)$ are the momentum and energy of the asymptotic particles. While constituting a certain form of an answer, the results provided by the bootstrap program cannot be considered as the end of the story. Indeed, to fully solve the theory, one should develop techniques so as to characterise the above two-point functions explicitly. In this respect, there are several open challenging questions. First of all, the well-definiteness of such representation, and more importantly the proof of the locality of the so-constructed operators, relies on the assumption of convergence of such series [4]. Second, on physical grounds, one is mostly interested in the short-time and/or short-distance behaviour of such correlation function where one expects the emergence of universality through the convergence of an appropriately normalised scaling limit of the correlators to those in a conformal field theory. It is unclear how to access to such regimes directly from the above expansion.

The aim of the thesis would be to develop techniques allowing one to settle some of these issues in the case of the so-called 1+1 dimensional Sine-Gordon model [**?**, 1, 4]. More precisely, the first goal will be to prove the convergence of series of multiple integrals which arise from the form factor expansions of the two-point correlation functions in this model. The second goal would consist in extracting from that expression the short distance behaviour of the two-point function and understand the mecanism at the root of the emergence of a conformal field theory in this regime.

Since the integrands arising in these series have some structural similarities with certain random multi-matrix models issued integrals, the first part will demand to extend and generalise various technique appearing in the random matrix theory and allowing one to estimate the large n behaviour of various classes of n-dimensional integrals. The second part will build on the Riemann–Hilbert problem based techniques that appeared fruitful in the asymptpotic analysis of certain Fredholm determinants and various densification techniques. It is expect that, on top of the physical spin-offs, the advance on this problem will also make important contribution to the modern theory of special function, especially relatively to building new classes of functions lying above the Painlevé transcendents.

The main technical tools that will be handled during the internship will belong to the field of complex analysis, special functions and probability theory. Also, the research carried out will demand a great deal of calculations.

References

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