

LARGE DEVIATION THEORY AND NON-EQUILIBRIUM STATISTICAL MECHANICS OF ATMOSPHERE JET DYNAMICS

# 1 Non equilibrium statistical mechanics of the large-scale organization of geophysical flows and climate

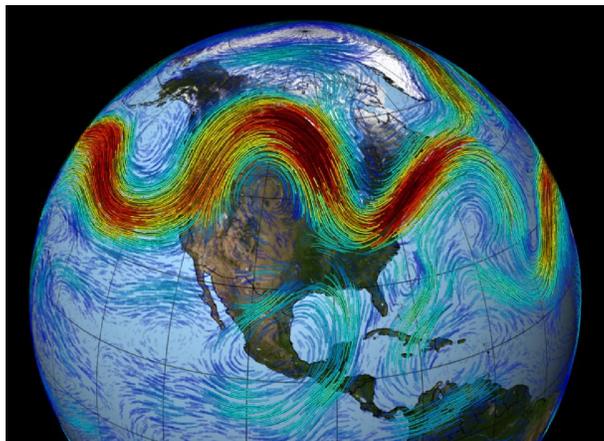


Figure 1.1: (a) Snapshot of the wind speed velocity at the top of the Earth troposphere, showing the jet stream over North America (from NASA). This jet stream is the main physical mechanism that explains midlatitude weather (cyclonic and anticyclonic weather situations). Is it possible to study the statistics of this jet using the same statistical mechanics tools as the one used in condensed matter theory? This would be a great step forward in the understanding of Atmosphere dynamics.

One of the most active domain in modern theoretical physics is the statistical mechanics of non equilibrium systems. Recently a large number of new fundamental results have been obtained: fluctuation theorems [5], stochastic thermodynamics [7], effective description of large scale dynamics through non equilibrium actions (macroscopic fluctuation theory [1]). Most of these new advances share common ideas for instance the classical statistical equilibrium ensembles should be replaced by statistics of trajectories, and the same theoretical tools mainly based on large deviation theory. A key question is to understand how deeply these fundamental theoretical results will have impact on genuine physics applications.

A fascinating application field for these new theoretical tools is the effective dynamics of the large scales of turbulent flows. Understanding the turbulent flows statistics is basically a statistical mechanics problem. However theoretical tools were lacking, mainly tools to deal with non equilibrium dynamical problems. The recent advances in non equilibrium statistical mechanics and large deviation theory open the way for a new field of research. One foresees applications for understanding the large scale dynamics of atmosphere dynamics, deeply related to climate. The possibility to make progress in understanding climate dynamics using statistical mechanics is fascinating subject.

Under the influence of rotation and stratification, geophysical turbulent flows tend to self-organize into coherent structures which can take the form of vortices and jets [2]. Such features are ubiquitous in planetary atmospheres: on Jupiter for instance, the planetary circulation is made of zonal bands with jets of alternating direction. Vortices are embedded in those jets. On Earth, zonal jets are also present, both in the atmosphere and the ocean, and play a major part in the global climate by transporting momentum, heat, and other constituents. Such features are stable enough to be observed, but also undergo variability on a large spectrum of time scales, and abrupt transitions. This kind of behavior is typical of non-equilibrium physical systems.

The statistical mechanics of geophysical flows has led to important theoretical and applied successes: a model for the Great Red Spot of Jupiter and of ocean vortices (see [2] for a review), a non equilibrium

statistical model of atmosphere jets [4], and explicit analytic prediction for Jupiter’s jet profiles [8]. The present proposal aims at developing a theoretical framework, rooted in statistical physics, to provide a deeper understanding of the emergence of self-organized regimes for geophysical flows, their intrinsic variability, and the transitions between different regimes. Following many recent advances in non-equilibrium statistical mechanics, the main theoretical tool will be large deviation theory.

## 2 Statistical mechanics of zonal jets

The project will be a theoretical one, working on deriving the equations, solving them in simple cases, and understanding them. We expect the student to be well trained with the classical concepts of Licence and Master statistical mechanics. Nothing else is expected, the relevant concepts of large deviation theory will be learnt during the project.

The aim of this project will be to develop the statistical mechanics of zonal (east-west) jets for the quasi-geostrophic barotropic model. This model is relevant as a first approximation for Jupiter’s troposphere jets. The aim is to compute the large deviation rate function for a jet profile  $U$ . The large deviation rate function plays the role of a non-equilibrium thermodynamical potential, and thus encodes all the statistics of the dynamics. The theory for computing such a large deviation rate function has already been developed recently within a quasilinear approximation [3]. The aim is to develop the theory beyond this approximation.

The first part of the work is a very interesting exercise in statistical mechanics, which is well delineated and should be performed without much difficulty within a few weeks. The student will have to study the analogy between the recent computation of large deviation rate functions for heat transport in harmonic chains [6] related to the classical phonon transmission function (Landauer, Casher and Lebowitz), on one hand, and the large deviation rate function of Reynolds stresses in the quasilinear theory [3], on the other hand. This analogy will give explicit formula for the large deviation rate function for Reynolds stresses.

The second part of the work will be more challenging. The aim will be to develop the complete theory for the statistical mechanics of zonal jets, using the quasilinear theory as a leading order approximation and the transmission function as a key tool. Once derived, we will look for symmetrized version of the equations that can be solved explicitly, and in the more general case we will develop a strategy for solving the equations.

This project and the possible PhD thesis are in theoretical physics. The student should be interested by theoretical approaches, developing new theoretical concepts and by understanding their relevance for important applications.

## Practical informations

**Scope:** This is intended as a few month research internship at the Master’s level, and could potentially be continued as a PhD project. Start date is flexible.

**Profile:** We are looking for candidates with a background in physics, in particular in nonlinear dynamics, statistical physics and fluid dynamics.

**Location:** Laboratoire de Physique de l’ENS de Lyon, Lyon, France.

**Supervisors:** Freddy Bouchet (CNRS, ENS de Lyon) and Corentin Herbert (CNRS, ENS de Lyon).

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## References

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