New universality classes in combinatorial models of statistical mechanics

Proposition de thèse / proposed PhD thesis

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Suggested starting date: September 2019

There are deep connections between statistical mechanics and combinatorics, particularly within the realm of "exactly solvable models". Such models played historically an important role in the study of phase transitions and critical phenomena, they are still useful today to analyze for instance systems out of equilibrium, and to prove rigorous mathematical results. There are many examples of the fascinating interplay between physics and combinatorics: the 2D Ising model and its connections with dimer models, spanning trees and free fermions [1]; polymers and self-avoiding walks [2]; the ice/six-vertex model and alternating sign matrices [3], etc.

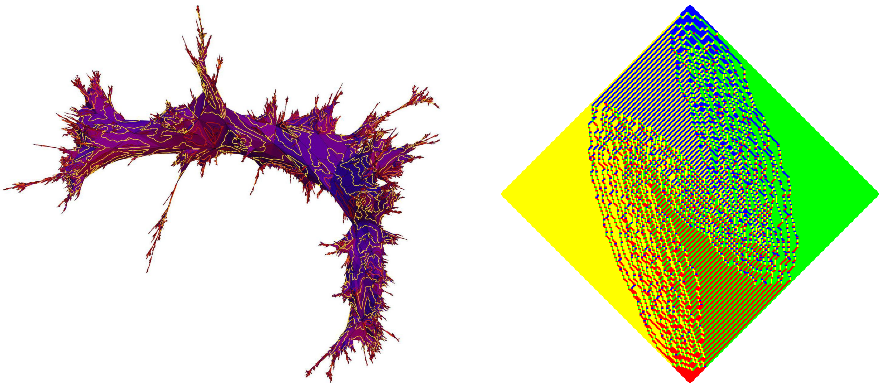

Figure 1: Left: a large loop-decorated random map (courtesy of J. Bettinelli). Right: a biased domino tiling of the Aztec diamond, displaying a nontrivial limit shape.

In this PhD thesis we propose two directions to investigate:

1. decorated random maps and 2D quantum gravity,
2. 2D dimer models, domino/lozenge tilings, random partitions and Schur processes.

Common to these two directions is the appearance of new universality classes which are still not fully understood.

Random maps are discretizations of random surfaces, used to model 2D quantum gravity. Much attention has been devoted to the study of the "uniform" distribution on random maps: it is for instance known that large planar maps have a fractal structure of dimension 4, whose essence is described by the so-called Brownian map [4]. But it is believed that the geometry changes drastically when the maps are decorated by a critical model of statistical physics, such as Ising/Potts spins or nonintersecting loops [5]. We propose to investigate some of the properties of such decorated random maps, through the use of suitable exploration processes [6].

Dimer models are among the simplest combinatorial models of statistical mechanics. Noninteracting fully-packed dimers in 2D are known to belong to the class of determinantal (or free fermionic) processes. Due to the nonoverlapping constraint, the boundary conditions may have long-range effects, and cause spatial phase separations. This is the "limit shape" or "arctic curve" phenomenon, which is now understood rigorously. Of particular interest is the interface between phases, where we observe universal scaling limits closely related with random matrices. We propose to investigate a certain family of models [7] and special boundary conditions [8] giving rise to new scaling limits.

References

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